1-11-2021

Evidence of the four-quark nature of f0 (980) and f0 (500)

N. N. Achasov
Sobolev Institute of Mathematics of the Siberian Branch of the Russian Academy of Sciences

J. V. Bennett
University of Mississippi

A. V. Kiselev
Sobolev Institute of Mathematics of the Siberian Branch of the Russian Academy of Sciences

E. A. Kozyrev
Novosibirsk State University

G. N. Shestakov
Sobolev Institute of Mathematics of the Siberian Branch of the Russian Academy of Sciences

Recommended Citation

Achasov, N. N., Bennett, J. V., Kiselev, A. V., Kozyrev, E. A., & Shestakov, G. N. (2021). Evidence of the four-quark nature of f 0 (980) and f 0 (500). Physical Review D, 103(1), 014010. https://doi.org/10.1103/PhysRevD.103.014010

PHYSICAL REVIEW D **103,** 014010 (2021)

Evidence of the four-quark nature of $f_0(980)$ and $f_0(500)$

N. N. Achasov,[1,*] J. V. Bennett,[2,†] A. V. Kiselev,[1,3,‡] E. A. Kozyrev,[4,3,§] and G. N. Shestakov[1,∥]

[1]*Laboratory of Theoretical Physics, S. L. Sobolev Institute for Mathematics, 630090 Novosibirsk, Russia*
[2]*University of Mississippi, Oxford, Mississippi 38677, USA*
[3]*Novosibirsk State University, Novosibirsk 630090, Russia*
[4]*Budker Institute of Nuclear Physics SB RAS, Novosibirsk 630090, Russia*

(Received 9 September 2020; accepted 11 December 2020; published 11 January 2021)

There exists a great deal of concrete evidence in favor of the exotic four-quark nature of light scalars. At the same time, the further expansion of the area of the $q^2\bar{q}^2$ model validity for light scalars on ever new processes seems extremely interesting and important. We analyze the BESIII data on the decay $J/\psi \to \gamma\pi^0\pi^0$ and show that the results of this high-statistics experiment can be interpreted in favor of the four-quark nature of light scalar mesons $f_0(980)$ and $f_0(500)$.

DOI: 10.1103/PhysRevD.103.014010

I. INTRODUCTION

To date, an impressive array of data on light scalar mesons has been accumulated [1]. However, the nontrivial properties of light scalar mesons remain incompletely understood and continue to cause controversy. In particular, there is much evidence in favor of their four-quark $q^2\bar{q}^2$ structure, and it is a matter for lively discussions. The number of publications devoted to light scalar mesons is immense. Some understanding of how theoretical and experimental explorations related to light scalar mesons have been developing can be gained, for example, from the following works and reviews [1–11].

The light scalar mesons has been studied, for example, in πN and KN scattering, $p\bar{p}$ annihilation, central hadronic production, J/ψ, ψ'-, B-, D-, and K-meson decays, $\gamma\gamma$ formation, and ϕ radiative decays [1]. The aim of our work is to obtain information on the quark structure of the $f_0(980)$ and $f_0(500)$ resonances from the modern data presented by the BESIII Collaboration on the radiative decay $J/\psi \to \gamma\pi^0\pi^0$ [12,13]. We note that very interesting mechanisms of the $f_0(500)$, $f_0(980)$, and $a_0(980)$ production in $J/\psi \to \gamma\pi\pi$ and $J/\psi \to \gamma\eta\pi$ were considered recently in Refs. [14,15]. They discussed the unusual properties of the above states. The authors of Ref. [14] take into account contributions mediated by a K^+K^- triangle loop with a photon line attached. In Ref. [15] it is assumed that the $J/\psi \to \gamma\pi^+\pi^-$ and $J/\psi \to \gamma\pi^0\eta$ decays come from the $J/\psi \to \phi(\omega)\pi^+\pi^-, \rho^0\pi^0\eta$ reactions, where the ρ^0, ω, and ϕ get converted into a photon via vector meson dominance. It was found that the probabilities of the $J/\psi \to \gamma f_0(500)$ and $J/\psi \to \gamma f_0(980)$ decays corresponding to the considered production mechanisms are quite small. No attempts were made to describe the BESIII data [13] in these works.

This paper is organized as follows. In Sec. II we consider the BESIII data on the S- and D-wave $\pi^0\pi^0$ mass spectra in the $J/\psi \to \gamma\pi^0\pi^0$ decay and demonstrate the qualitative difference between the contributions of the $f_0(500)$ and $f_0(980)$ and higher-lying resonance states. We conclude that the production mechanism of the light scalars $f_0(500)$ and $f_0(980)$ is very different in comparison with that of the tensor meson $f_2(1270)$ and heavy scalar states. We also compare the $f_0(500)$ and $f_0(980)$ production with the case of the light pseudoscalar mesons η and $\eta'(958)$ in a model of the Nambu-Jona-Lasinio type. In Sec. III we describe the shape of the S-wave $\pi^0\pi^0$ mass spectrum in the $f_0(500)$ and $f_0(980)$ resonance region. The four-quark mechanism of the formation of these states considered by us is in good agreement with the BESIII data. Thus, from the $J/\psi \to \gamma\pi^0\pi^0$ decay we have once more evidence in favor of the four-quark nature of the $f_0(980)$ and $f_0(500)$ resonances.

II. BESIII RESULTS ON $J/\psi \to \gamma\pi^0\pi^0$

Clear indications of the unusual nature of $f_0(980)$ and $a_0(980)$ mesons were given by experiments on hadronic and radiative J/ψ decays, the results of which are collected in Table I. It was found that decays with the participation of light scalars $f_0(980)$ and $a_0(980)$ are strongly suppressed

---

[*] achasov@math.nsc.ru
[†] jvbennet@olemiss.edu
[‡] kiselev@math.nsc.ru
[§] E.A.Kozyrev@inp.nsk.su
[∥] shestako@math.nsc.ru

TABLE I. Branching fractions of J/ψ decays taken mainly from the Particle Data Group [1] and also from Refs. [16–18].

$J/\psi \to a_2(1320)\rho$	$f_2(1270)\omega$	$f_2'(1525)\phi$	$\gamma f_2(1270)$, $\gamma f_2'(1525)$
$(1.09 \pm 0.22)\%$	$(4.3 \pm 0.6) \times 10^{-3}$	$(8 \pm 4) \times 10^{-4}$	$(1.64 \pm 0.12) \times 10^{-3}$, $(5.7\pm^{0.8}_{0.5}) \times 10^{-4}$
$J/\psi \to a_0(980)\rho$	$f_0(980)\omega$	$f_0(980)\phi$	$\gamma f_0(980)$
$<4.4 \times 10^{-4}$ [16,18]	$(1.4 \pm 0.5) \times 10^{-4}$	$(3.2 \pm 0.9) \times 10^{-4}$	$<1.4 \times 10^{-5}$ [17]

in comparison with rather intensive decays involving the classical tensor $q\bar{q}$ states $a_2(1320)$, $f_2(1270)$, and $f_2'(1525)$ [1,16–18]. These facts are difficult to understand from the point of view of the $q\bar{q}$ model, according to which the tensor $a_2(1320)$, $f_2(1270)$, and $f_2'(1525)$ and scalar $a_0(980)$ and $f_0(980)$ mesons are P-wave states in the usual $q\bar{q}$ system. However, they can be easily qualitatively explained, which was done in Ref. [19], in terms of the four-quark structure of $a_0(980)$ and $f_0(980)$ mesons [3,7], by the presence of an additional $s\bar{s}$ pair in their wave functions.

In the last decade, the BESIII Collaboration has made considerable efforts to improve the knowledge of the scalar and tensor meson sector with a series of the partial wave analyses of radiative J/ψ decays to $\eta\eta$ [20], $\pi^0\pi^0$ [12,13], $\eta\pi^0$ [21], and $K_S K_S$ [22]. The BESIII results on the $J/\psi \to \gamma\pi^0\pi^0$ decay that are important for our following discussion of the dynamics of the $f_0(980)$ and $f_0(500)$ production are shown in Fig. 1. The novelty of the presented data consists in their incredible statistical precision and purity provided by $(1.311 \pm 0.011) \times 10^9$ J/ψ decays collected by the BESIII detector. According to the mass-independent amplitude analysis [12,13], the full $\pi^0\pi^0$ mass spectrum presented in Fig. 1(a) is saturated with the contributions of the $D(2^{++})$ and $S(0^{++})$ waves shown in Figs. 1(b) and 1(c), respectively. We denote the invariant mass of the $\pi^0\pi^0$ system as $\sqrt{s}$. For our purpose, it is quite sufficient to use the data presented in Refs. [12,13] for one of the two ambiguous solutions, especially since there is only one solution in the $\sqrt{s}$ region up to 1 GeV.

The branching fraction of $J/\psi \to \gamma\pi^0\pi^0$ measured by BESIII [13] is determined to be $(1.15 \pm 0.05) \times 10^{-3}$, where the uncertainty is systematic only and the statistical uncertainty is negligible. Using this result at [13] containing the numerical values of the $\pi^0\pi^0$ mass distributions, we estimate the branching fractions for various resonance type enhancements observed in $\pi^0\pi^0$ mass spectra in the $J/\psi \to \gamma\pi^0\pi^0$ decay. The branching fractions are

$$\mathcal{B}(J/\psi \to \gamma f_0(500) \to \gamma\pi^0\pi^0) = \mathcal{B}(J/\psi \to \gamma\pi^0\pi^0) N_S(2m_\pi < \sqrt{s} < 0.9\ \text{GeV})/N_{\text{tot}} = 0.324 \times 10^{-4}, \tag{1}$$

$$\mathcal{B}(J/\psi \to \gamma f_0(980) \to \gamma\pi^0\pi^0) = \mathcal{B}(J/\psi \to \gamma\pi^0\pi^0) N_S(0.9\ \text{GeV} < \sqrt{s} < 1\ \text{GeV})/N_{\text{tot}} = 0.425 \times 10^{-5}, \tag{2}$$

$$\mathcal{B}(J/\psi \to \gamma f_0(1440) \to \gamma\pi^0\pi^0) = \mathcal{B}(J/\psi \to \gamma\pi^0\pi^0) N_S(1\ \text{GeV} < \sqrt{s} < 1.5\ \text{GeV})/N_{\text{tot}} = 0.131 \times 10^{-3}, \tag{3}$$

$$\mathcal{B}(J/\psi \to \gamma f_0(1710) \to \gamma\pi^0\pi^0) = \mathcal{B}(J/\psi \to \gamma\pi^0\pi^0) N_S(1.5\ \text{GeV} < \sqrt{s} < 1.85\ \text{GeV})/N_{\text{tot}} = 0.152 \times 10^{-3}, \tag{4}$$

$$\mathcal{B}(J/\psi \to \gamma f_0(2020) \to \gamma\pi^0\pi^0) = \mathcal{B}(J/\psi \to \gamma\pi^0\pi^0) N_S(1.85\ \text{GeV} < \sqrt{s} < 2.25\ \text{GeV})/N_{\text{tot}} = 0.145 \times 10^{-3}, \tag{5}$$

$$\mathcal{B}(J/\psi \to \gamma f_2(1270) \to \gamma\pi^0\pi^0) = \mathcal{B}(J/\psi \to \gamma\pi^0\pi^0) N_D(2m_\pi < \sqrt{s} < 1.5\ \text{GeV})/N_{\text{tot}} = 0.482 \times 10^{-3}, \tag{6}$$

where N_S and N_D (for corresponding intervals of $\sqrt{s}$), and N_{tot} are the S and D wave, and total numbers of $J/\psi \to \gamma\pi^0\pi^0$ events, respectively; three S-wave enhancements above 1 GeV are conventionally labeled as $f_0(1440)$, $f_0(1710)$, and $f_0(2020)$ in accordance with the visible peak positions in Fig. 1(c).

Thus it is seen that the production of light scalar mesons is strongly suppressed in comparison with the production of the classical tensor meson $f_2(1270)$:

$$\frac{\mathcal{B}(J/\psi \to \gamma f_0(500) \to \gamma\pi^0\pi^0)}{\mathcal{B}(J/\psi \to \gamma f_2(1270) \to \gamma\pi^0\pi^0)} \approx 0.067,$$
$$\frac{\mathcal{B}(J/\psi \to \gamma f_0(980) \to \gamma\pi^0\pi^0)}{\mathcal{B}(J/\psi \to \gamma f_2(1270) \to \gamma\pi^0\pi^0)} \approx 0.0088. \tag{7}$$

Naturally, this may indicate a fundamental difference in the corresponding production mechanisms.

The $f_0(500)$ and $f_0(980)$ production is also significantly suppressed in comparison with the production of heavy

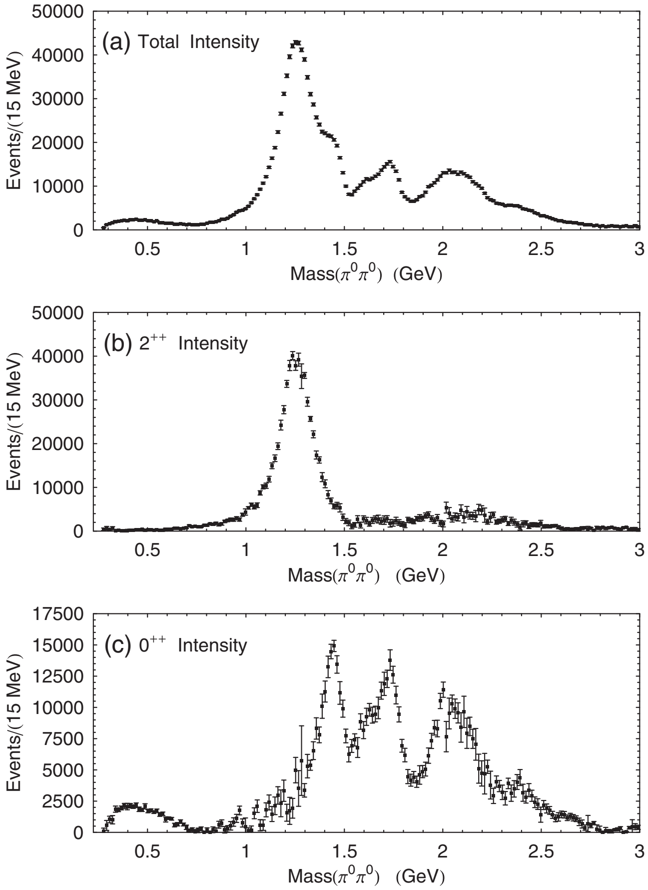

FIG. 1. The (a) total, (b) D-wave, and (c) S-wave $\pi^0\pi^0$ mass spectra in the $J/\psi \to \gamma\pi^0\pi^0$ decay presented by BESIII [13] for one of the two ambiguous solutions (the first of two nominal results). There is an unambiguous solution below 1 GeV.

scalar states. The $f_0(500) - f_0(980)$ resonance complex at $\sqrt{s} < 2m_{K^+}$ is coupled only with the $\pi\pi$ decay channel. The $f_2(1270)$ resonance is also practically elastic, $\mathcal{B}(f_2(1270) \to \pi\pi) \approx 85\%$ [1]. If the S-wave peak in the vicinity of 1440 MeV is related to the complex of conventional $f_0(1370)$ and $f_0(1500)$ resonances [1], then with taking into account their significant inelasticity [1] and Eqs. (3) and (6), it cannot be ruled out that the full branching fraction $\mathcal{B}(J/\psi \to \gamma f_0(1440)) \approx \mathcal{B}(J/\psi \to \gamma f_2(1270))$, i.e., much larger than $\mathcal{B}(J/\psi \to \gamma(f_0(500))$ and, certainly, $\mathcal{B}(J/\psi \to \gamma(f_0(980))$. The data on the $f_0(1710)$ resonance [1,22] and Eq. (4) also support a similar proportion. It is likely that the $f_0(2020)$ resonance, which needs confirmation, is also very inelastic [1].

Thus, one can conclude that the production mechanism of the light scalars $f_0(500)$ and $f_0(980)$ is very different in comparison with that of the tensor meson $f_2(1270)$ and heavy scalar states.

Note also that in a chiral-symmetric model of the Nambu-Jona-Lasinio type [23] it would be quite natural to expect the following approximate relations between the decay probabilities: $\mathcal{B}(J/\psi \to \gamma\eta) \approx \mathcal{B}(J/\psi \to \gamma f_0(500))$ and $\mathcal{B}(J/\psi \to \gamma\eta'(958)) \approx \mathcal{B}(J/\psi \to \gamma f_0(980))$. However, this is not the case. If $\mathcal{B}(J/\psi \to \gamma f_0(500)) \approx 0.97 \times 10^{-4}$ and $\mathcal{B}(J/\psi \to \gamma f_0(980)) \approx 1.27 \times 10^{-5}$ according to Eqs. (1) and (2), then according to the Particle Data Group [1], $\mathcal{B}(J/\psi \to \gamma\eta) \approx 1.11 \times 10^{-3}$ and $\mathcal{B}(J/\psi \to \gamma\eta'(958)) \approx 5.25 \times 10^{-3}$. This fact is also a clear confirmation of the unusual internal structure of the $f_0(500)$ and $f_0(980)$ resonances in comparison with the typical $q\bar{q}$ mesons η and $\eta'(958)$. Considerations related to the chiral-symmetric model of the Nambu-Jona-Lasinio type were used previously in Ref. [24].

III. DESCRIPTION OF THE $f_0(500)$ and $f_0(980)$ RESONANCE REGION

We now turn to description of the shape of the S-wave $\pi^0\pi^0$ mass spectrum in the $f_0(500)$ and $f_0(980)$ resonance region (see Fig. 3 below). This spectrum is defined by only one invariant amplitude $F_{J/\psi\to\gamma\pi^0\pi^0}(s)$ and can be written in the form

$$\frac{dN_S(s)}{d\sqrt{s}} = \frac{1}{12\pi}|F_{J/\psi\to\gamma\pi^0\pi^0}(s)|^2 k_\gamma^3(s)\frac{\rho_{\pi\pi}(s)}{32\pi}\frac{2\sqrt{s}}{\pi}, \quad (8)$$

where $k_\gamma(s) = (m^2_{J/\psi} - s)/(2m_{J/\psi})$ and $\rho_{\pi\pi}(s) = \sqrt{1 - 4m_\pi^2/s}$.

To determine $F_{J/\psi\to\gamma\pi^0\pi^0}(s)$, we use the following consideration about the dynamics of the $f_0(500)$ and $f_0(980)$ resonance production. In a series of papers [24–26], it has been shown that semileptonic decays of D and B mesons can be used as probes of constituent $q\bar{q}$ components in the wave functions of light scalar mesons decaying into pseudoscalar meson pairs. In particular, in Ref. [26] it was demonstrated that the model according to which the $f_0(500)$ and $f_0(980)$ production proceed via direct $q\bar{q} \to f_0(500), f_0(980)$ transitions ($D^+ \to d\bar{d}e^+\nu_e \to [f_0(500) + f_0(980)]e^+\nu_e \to \pi^+\pi^-e^+\nu_e$ and $D_s^+ \to s\bar{s}e^+\nu_e \to [f_0(500) + f_0(980)]e^+\nu_e \to \pi^+\pi^-e^+\nu_e$) cannot describe simultaneously the BESIII [27] and CLEO [28] data on the decays $D^+ \to \pi^+\pi^-e^+\nu_e$ and $D_s^+ \to \pi^+\pi^-e^+\nu_e$. Figuratively, one can say that the $q\bar{q}$ probe existing in semileptonic $(D^+, D_s^+) \to \pi^+\pi^-e^+\nu_e$ decays does not find, to a first approximation, the corresponding $q\bar{q}$ components in $f_0(500)$ and $f_0(980)$ mesons. However, the $f_0(500) - f_0(980)$ resonance complex can be produced via seed four-quark fluctuations $q\bar{q} \to \pi\pi$ and $q\bar{q} \to K\bar{K}$, which are then dressed by strong interactions in the final state. This four-quark production mechanism provide a good description of all details of the $\pi^+\pi^-$ mass spectra measured in above mentioned experiments [26].

In the case of the radiative $J/\psi \to \gamma\pi^0\pi^0$ decay, the role of the $q\bar{q}$ probe is played by the transition $J/\psi \to \gamma c\bar{c} \to \gamma gg \to \gamma q\bar{q}$, where g is a gluon. This source generates the classical $q\bar{q}$ resonance $f_2(1270)$ and its scalar P-wave partners in the $q\bar{q}$ multiplet. However, it does not work in the case of light scalar resonances. We assume that the

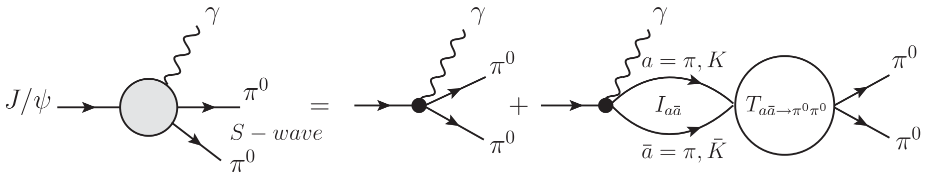

FIG. 2. Production of the $f_0(500) - f_0(980)$ resonance complex in $J/\psi \to \gamma\pi^0\pi^0$.

$f_0(500)$ and $f_0(980)$ states are created in the radiative J/ψ decay due to all possible seed four-quark fluctuations $gg \to q\bar{q}q\bar{q} \to \pi\pi, K\bar{K}$. The described picture of the creation of light four-quark resonances in $J/\psi \to \gamma\pi^0\pi^0$ can be effectively realized in the language of hadronic states, as this is shown diagrammatically in Fig. 2.

According to this figure, we write the amplitude $F_{J/\psi\to\gamma\pi^0\pi^0}(s)$ from Eq. (8) in the form

$$F_{J/\psi\to\gamma\pi^0\pi^0}(s) = \lambda_{\pi\pi}[1 + I_{\pi^+\pi^-}(s)T_0^0(s)] + \lambda_{K\bar{K}}[I_{K^+K^-}(s) + I_{K^0\bar{K}^0}(s)]T_{K^+\bar{K}^+\to\pi^0\pi^0}(s), \tag{9}$$

where $T_0^0(s) = T_{\pi^+\pi^-\to\pi^0\pi^0}(s) + \frac{1}{2}T_{\pi^0\pi^0\to\pi^0\pi^0}(s)$ is the S-wave amplitude of the reaction $\pi\pi \to \pi\pi$ in the channel with isospin $I = 0$ composed of the amplitudes related to individual charge channels; $T_0^0(s) = [\eta_0^0(s)\exp(2i\delta_0^0(s)) - 1]/(2i\rho_{\pi\pi}(s))$, where $\eta_0^0(s)$ and $\delta_0^0(s)$ are the corresponding inelasticity and phase of $\pi\pi$ scattering [29]; $T_{K^+K^-\to\pi^0\pi^0}(s)$ is the amplitude of the S-wave transition $K^+K^- \to \pi^0\pi^0$; $T_{K^+K^-\to\pi^0\pi^0}(s) = T_{K^0\bar{K}^0\to\pi^0\pi^0}(s)$ [29]. Functions $I_{a\bar{a}}(s)$ (where $a\bar{a} = \pi^+\pi^-, \pi^0\pi^0, K^+K^-, K^0\bar{K}^0$) are the amplitudes of the loop diagrams describing $a\bar{a} \to a\bar{a} \to$ (the scalar state with a virtual mass equaling $\sqrt{s}$) transitions in which initial $a\bar{a}$ pairs are produced by the underlain gluon source, $gg \to q\bar{q}q\bar{q} \to a\bar{a}$, described by coupling constants $\lambda_{a\bar{a}}$. Above the $a\bar{a}$ threshold, $I_{a\bar{a}}(s)$ has the form [29]

$$I_{a\bar{a}}(s) = \tilde{C}_{a\bar{a}} + \rho_{a\bar{a}}(s)\left(i - \frac{1}{\pi}\ln\frac{1 + \rho_{a\bar{a}}(s)}{1 - \rho_{a\bar{a}}(s)}\right), \tag{10}$$

where $\rho_{a\bar{a}}(s) = \sqrt{1 - 4m_a^2/s}$ (we put $m_\pi \equiv m_{\pi^0} = m_{\pi^+}$ and take into account the mass difference of K^+ and K^0); if $\sqrt{s} < 2m_K$, then $\rho_{K\bar{K}}(s) \to i|\rho_{K\bar{K}}(s)|$; $\tilde{C}_{\pi^+\pi^-} = \tilde{C}_{\pi^0\pi^0}$ and $\tilde{C}_{K^+K^-} = \tilde{C}_{K^0\bar{K}^0}$ are subtraction constants in the loops.

We take the amplitudes $T_0^0(s)$ and $T_{K\bar{K}\to\pi\pi}(s) = T_{\pi\pi\to K\bar{K}}(s)$ from Ref. [29] (corresponding to fitting variant 1 for parameters from Table I therein). Note that our principal conclusions are independent of a concrete fitting variants presented in Refs. [29–31], containing the excellent simultaneous descriptions of the phase shifts, inelasticity, and mass distributions in the reactions $\pi\pi \to \pi\pi$, $\pi\pi \to K\bar{K}$, and $\phi \to \pi^0\pi^0\gamma$. The amplitudes $T_0^0(s)$ and $T_{\pi\pi\to K\bar{K}}(s)$ were described in Refs. [29–31] by the complex of the mixed $f_0(500)$ and $f_0(980)$ resonances and smooth background contributions. The constructed $\pi\pi$ scattering amplitude T_0^0 (s) [29–31] has regular analytical properties in the s complex plane and describes both experimental data and the results based on chiral expansion and Roy equations [5].

Note that the phase of the amplitude $F_{J/\psi\to\gamma\pi^0\pi^0}(s)$ in Eq. (9) [taking into account Eq. (10)] coincide with the $\pi\pi$ scattering phase $\delta_0^0(s)$ below the K^+K^- threshold where $\eta_0^0(s) = 1$ [as is the phase of the amplitude $T_{K^+K^-\to\pi^0\pi^0}(s)$ [29]].

For reasons of $SU(3)$ symmetry, we will assume that the seed coupling constants in Eq. (9) are the same: $\lambda_{\pi\pi} = \lambda_{K\bar{K}} \equiv \lambda$. Thus, λ defines the overall normalization. Since the amplitudes $T_0^0(s)$ and $T_{K^+K^-\to\pi^0\pi^0}(s)$ are known [29] from the analysis of the data on the reactions $\pi\pi \to \pi\pi$, $\pi\pi \to K\bar{K}$, and $\phi \to \pi^0\pi^0\gamma$, then we have only two parameters $\tilde{C}_{\pi^+\pi^-}$ and $\tilde{C}_{K^+K^-}$ to describe the shape of the $\pi^0\pi^0$ mass spectrum.

The choice of $\tilde{C}_{\pi^+\pi^-} = 1.052$ and $\tilde{C}_{K^+K^-} = 0.2396$ provides a good description of the BESIII data [13] on the S-wave $\pi^0\pi^0$ mass spectrum in the $J/\psi \to \gamma\pi^0\pi^0$ decay at $\sqrt{s} < 1$ GeV, see Fig. 3. The curve in Fig. 3 corresponds to $\chi^2/(n.d.f) \approx 57.4/(49 - 3) \approx 1.25$. In so doing the relative errors of the fitting parameters λ, $\tilde{C}_{\pi^+\pi^-}$, and $\tilde{C}_{K^+K^-}$ are approximately ±1.9%, ±2.4%, and ±9.3%, respectively. Thus, we obtained once more evidence in favor of the four-quark nature of the $f_0(980)$ and $f_0(500)$ resonances. Their production occurs due to four-quark transitions. It is interesting to compare the resulting picture on the $\pi^0\pi^0$ mass spectrum with the S-wave $\pi\pi$ elastic cross section. This cross section reaches the unitary limit in the

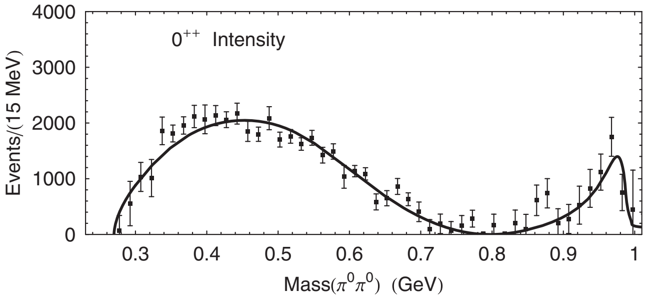

FIG. 3. The S-wave $\pi^0\pi^0$ mass spectrum in the $f_0(500)$ and $f_0(980)$ resonance region in the $J/\psi \to \gamma\pi^0\pi^0$ decay. The points with the error bars are the BESIII data [13]. The solid curve represents our fit.

energy range where the S-wave $\pi^0\pi^0$ mass spectrum in $J/\psi \to \gamma\pi^0\pi^0$ has practically zero minimum. It has also a narrow dip at the site of the $f_0(980)$ resonance (compare with Fig. 3). The S-wave $\pi\pi$ elastic cross section falls naturally in the inelastic region ($\sqrt{s} > 1$ GeV) with increasing energy, in contrast to the S-wave $\pi^0\pi^0$ mass spectrum in the $J/\psi \to \gamma\pi^0\pi^0$ decay [see Fig. 1(c)].

As we have already noted in the Introduction, the nontrivial properties of light scalar mesons remain not fully understood and continue to cause controversy. Different modern points of view on the quark structure of the $f_0(980)$ and $f_0(500)$ mesons are remarkably presented in the minireviews "Scalar mesons below 2 GeV" [10] and "Non-$q\bar{q}$ states" [11]. We hope that our evidence will serve the general desire to better understand the physics of light scalars.

ACKNOWLEDGMENTS

The work of N. N. A., A. V. K., and G. N. S. was carried out within the framework of the state contract of the Sobolev Institute of Mathematics, Project No. 0314-2019-0021.

[1] P. A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. (**2020**), 083C01.
[2] M. Gell-Mann and M. Levy, Nuovo Cimento **16**, 705 (1960).
[3] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977); **15**, 281 (1977).
[4] C. Amsler and N. A. Tornqvist, Phys. Rep. **389**, 61 (2004).
[5] I. Caprini, G. Colangelo, and H. Leutwyler, Phys. Rev. Lett. **96**, 132001 (2006).
[6] E. Klempt and A. Zaitsev, Phys. Rep. **454**, 1 (2007).
[7] S. Weinberg, Phys. Rev. Lett. **110**, 261601 (2013).
[8] J. R. Pelaez, Phys. Rep. **658**, 1 (2016).
[9] N. N. Achasov and G. N. Shestakov, Usp. Fiz. Nauk, **189**, 3 (2019) [Phys. Usp. **62**, 3 (2019)].
[10] C. Amsler, S. Eidelman, T. Gutsche, C. Hanhart, and S. Spanier, Scalar mesons below 2 GeV, Mini-review in Ref. [1].
[11] C. Amsler and C Hanhart, Non-$q\bar{q}$ states, Mini-review in Ref. [1].
[12] J. V. Bennett, Ph.D. thesis, Indiana University, 2014.
[13] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **92**, 052003 (2015).
[14] C. W. Xiao, U.-G. Meissner, and J. A. Oller, Eur. Phys. J. A **56**, 23 (2020).
[15] S. Sakai, W. H. Liang, G. Toledo, and E. Oset, Phys. Rev. D **101**, 014005 (2020).
[16] U. Mallik, SLAC Report No. SLAC-PUB-4238, Stanford, 1987.
[17] G. Eigen, in *Proceedings of the XXIV International Conference on High Energy Physics*, edited by R. Kotthaus and J. H. Kühn (Springer-Verlag, Munich, 1988), Session 4, p. 590.
[18] L. Köpke and N. Wermes, Phys. Rep. **174**, 67 (1989).
[19] N. N. Achasov, Usp. Fiz. Nauk **168**, 1257 (1998) [Phys. Usp. **41**, 1149 (1998)].
[20] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **87**, 092009 (2013).
[21] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **94**, 072005 (2016).
[22] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **98**, 072003 (2018).
[23] M. K. Volkov and A. E. Radzhabov, Usp. Fiz. Nauk, **176**, 569 (2006) [Phys. Usp. **49**, 551 (2006)].
[24] N. N. Achasov and A. V. Kiselev, Phys. Rev. D **86**, 114010 (2012).
[25] N. N. Achasov, Phys. Part. Nucl. **51**, 632 (2020).
[26] N. N. Achasov, A. V. Kiselev, and G. N. Shestakov, Phys. Rev. D **102**, 016022 (2020).
[27] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **122**, 062001 (2019).
[28] K. M. Ecklund *et al.* (CLEO Collaboration), Phys. Rev. D **80**, 052009 (2009).
[29] N. N. Achasov and A. V. Kiselev, Phys. Rev. D **73**, 054029 (2006).
[30] N. N. Achasov and A. V. Kiselev, Phys. Rev. D **83**, 054008 (2011).
[31] N. N. Achasov and A. V. Kiselev, Phys. Rev. D **85**, 094016 (2012).