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A STUDY OF SWEEP EFFICIENCY IN ENHANCED OIL RECOVERY AND ITS APPLICATION FOR MAXIMIZING OIL PRODUCTION

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abstract

Waterflooding of oil reservoirs for the purpose of sweeping out the oil is a very well-established method in the industry today. Success of many of these operations is, however, far from assured. This is mainly due to an almost infinite variety of reservoirs and reservoir fluids in addition to the complex nature of the science and technology of water flooding. A thorough and up-to-date study of the elements involved in maximizing oil recovery in waterflooding operations is notably missing in the literature.

A study of reservoir and reservoir fluid properties, as well as computational and operational criteria, for the purpose of maximizing oil recovery in linear and two-dimensional immiscible water flooding has been undertaken. Numerical models were set up and the most efficient methods of simulation employed using microcomputers as well as a main frame computer. The outcomes of the computation were weighed against experimental results. Satisfactory agreements were achieved. A unique, dimensionless, saturation-distribution equation involving a "Scaling Factor" was developed. Behavior of major variables against Sweep Efficiency was plotted. A complete and thorough quantitative, as well as qualitative analysis, is presented.

Introduction

Early in 1941 a piece of purely mathematical work (1) in the area of immiscible fluid displacement in petroleum reservoirs was published, which did not receive the attention it deserved. However, today, Buckley-Leverett is a household name in petroleum reservoir engineering. Many theoretical, as well as experimental works, followed this original work (2 through 11) and with availability of computers and development of reservoir simulations, the science and technology of immiscible and incompressible fluid displacement (mainly water flooding of oil reservoirs), for the sole purpose of increasing oil recovery, has indeed reached an advanced level.

Complexity of the matter and vast variety of the reservoirs and reservoir fluid types have made it so far impossible to achieve the science and the technology needed to assure success in every flooding operation. In this sense, the investigative work on the subject is far from over. Published works on the nature of the operations are many. (References are at the end of the text.) But a complete and comprehensive study of the elements involved in maximizing oil recovery is noticeably missing. The intention of this work, therefore is to offer an up-to-date study of single and two-dimensional immiscible, incompressible, fluid displacement (mainly oil and water) for the sole purpose of maximizing recovery in petroleum reservoirs, using the latest methods in calculations and reservoir simulation computations.

Background

Using the reservoir's self energy to produce its fluid (primary production), even at very favorable conditions, in many cases will leave more than one-half o.f the original fluid in place, where it requires other artificial (external) forces to be brought to the surface.

Pushing and eventually sweeping oil out of the reservoir by a cheaper and available medium, like other methods of artificial oil recovery, initially happened by accident, and it is not new by any account. What is new is its use to such a wide scale that today it is indeed responsible for many millions of barrels of recovered oil in the United States and Canada.

Water emerged as the most efficient and suitable medium for two basic and specific purposes—reservoir pressure maintenance and flooding of the reservoir for sweeping out its oil. Numerous theoretical and experimental projects on the subject have been carried out and published results are available. Discussing the method of calculation, computation and/or operations, therefore, is not the purpose of this work. However, the main objective of this work is the study and understanding the effects of various elements—from reservoir and reservoir fluid properties to the method of computation on the sweep efficiency of waterflooding operations, with the intention of maximizing oil recovery.

Methodology

The work performed was basically in two sections—one-dimensional and two-dimensional systems. In the one-dimensional system a Buckley-Levertt type model was set up and numerous operations under different conditions were carried out and the results were plotted for further investigation. In this fashion it is assumed the gravity forces and capillary pressures are not determining factors and have no significant effects in a qualitative study of sweep efficiency. Setting up a reliable single dimensional mathematical-numerical model, therefore, was thought to be satisfactory for our purposes. In the two-dimensional system, the basic assumptions are the same, although due to the introduction of another dimension, the computation takes a much more complicated course. In both cases water and oil was assumed to be incompressible and immiscible in each other under the operating reservoir conditions, and furthermore, no gas was assumed to be present throughout the sand and no chemical reaction took place between injected fluids and reservoir fluids.

The Case of Linear Waterflooding

Theory-

When water, as an incompressible fluid and immiscible with reservoir fluids under operating reservoir conditions, is introduced (injected) into a relatively long layer of sand Fig. 1, under continuous and practically steady flow, water contacts the oil, forms a front, and begins to push it towards the producing well located at the other end of the sand where, theoretically, oil is produced at the same water injection rate.

Major factors involved in this operation are as follows:

1 -Reservoir properties such as;
- -Homogeneity, mean porosity (its magnitude and range of change), faults, fractures, lenses, sealing, and non-sealing shale, etc.
- -Isotropicity, absolute and relative permeabilities. Their relative magnitude and range.
- -Connate water and residual oil saturation
- -Rock, water or oil preference
- -Physical size and shape of the sand

2 -Reservoir fluid properties such as;
- -chemical nature of the fluid
- -Gravity and viscosity of the fluid

3 -Operation conditions such as;
- -Injection rate or pressure
- -Chemical nature of the injecting water
- -Density and viscosity of injecting water

4 -Method of investigation such as;
- -the mathematics of calculations
- — Simulation
- -Numerical solution

Due to inherent limitations of the numerical simulation method, it is practically impossible to observe the quantitative effects of all the above factors on the sweep efficiency or reservoir waterflooding. Therefore, in some cases the investigative process is unfortunately limited to qualitative observation and analysis. With this in mind, it has been possible to quantitatively demonstrate the effects of the following factors against sweep efficiency:

- -Injection rate and/or pressure
- -Absolute permeability
- -Porosi ty
- -Oil viscosity (oil-water viscosity ratio, or mobility ratio)
- -Physical size of the sand
- -Numerical modeling (grid block size)

Mathematical Development—

The general equation of conservation of mass in a two-phase, immiscible, incompressible flow of fluids in a linear system (x direction only) under constant flow rate, where gravity and capillary forces are not appreciable, may be described as follows:

for water,	3Vw/3* = - Φ ðSw / 9t		(1)
initial condition;	Sw = Swc	at t = 0	for all x
boundary condition;	Sw = 1	at x = 0	for t > 0

assuming :

So + Sw = 1
Vt = Vo + Vw

Vo = - KK ro/u o * dp/dx

Vw = - KK rw/Uw* dp/dx

let, f = Vw/Vt
or , Vw = f * vt
therefore :

ЭVw/Эx = Vt 9f/ Э x

substituting in equation (1) results,

Vt 9f/Эx = - Φ Э S/3t

where, S = Sw

or , Vt 9f/ðx + Φ ðS/ðt = 0

or in general.

$$Vt\ Vf + \Phi\ 3S/3\ t = 0 \qquad (2)$$

where, $f = (KKrw/\ \mu\ w)/(KKrw/\mu\ w + KKro/\mu o)$

also ,

$Vt = Vo + Vw$

$Vt = (-KKro/\mu o - KKrwAw)\ V\ p$

or, $Vt = -\ (\lambda\ o + Aw)\ V\ P$

or $Vt=-At*VP$

since Kro and Krw are only functions of water saturation, f is only a function of water saturation, too.

Equation (1) along with initial and boundary conditions determine the saturation of water (or oil) at any time at any point in the sand. However, assuming unsteady state flow, flow of fluids is governed by the following:

for oil: $V\ P - Qo\nabla\lambda\ \Phi\ \theta\ (So/Bo)\ /\ni\ t$

for water: $V\ Aw\ V\ P + Qw = \Phi\ \ni\ (Sw/Bw)\ /t$

or: $v\ At\ v\ P + Qw - Qo = 0$ (3)

since: $g(So + Sw)/\ t = 0$

Equation (3) along with its appropriate boundary and initial conditions determines pressure distribution along the sand at any point and time.

Assumption of unsteady state flow, although theoretically valid, is not justified considering the nature of the operation where rapidly approaches almost steady state condition, after the stabilization of the operation, that is continuous injection of water at some fixed rate. This is mainly due to the assumption of incompressibility of oil and water. Therefore, analysis of saturation distribution in the one-dimensional model Equation (1) will adequately serve our purpose in this part of the work.

Analytical solution of Equation (1) is possible only in some trivial cases. Even then the solution is subject to questions of discontinuity, shock wave, or triple saturation points. These problems have been discussed in detail in a number of published materials with no apparent practical (industrial) applications. Discussions of these results are not within the scope of this work.

With increasing availability of fast computing capability along with the development of numerical solutions, it is refreshing to know that the time has come to solve these or even much more complicated types of partial differential equations (such as the one in a two-dimensional model), through numerical methods with increasingly satisfactory outcomes for industrial applications.

Numerical Solution —

It is required to secure a set of primary, workable numerical data to set up a numerical model; it is quite essential. To determine water fraction at any saturation, relative permeabilities as functions of saturation are needed. The following effective permeability functions are chosen to represent this relationship:

$$Kw = (Sw - Swc)^{2}$$

$$Ko = (1 - Sw - Swc)^{2}$$

A numerical model was set up and partial differential Equation (1) was turned into partial difference equation, using a backward difference approximation scheme.

In computation of saturation distribution, numerical dispersion becomes a serious problem. Consequently, a sharp front, which is one of the criteria of two-phase immiscible flow, seemed unattainable by employing conventional methods and a much more sophisticated method of computation was needed.

A search among many available methods of computation for reducing numerical dispersion (12 through 20) revealed that the "Second Order Godunov Method" (12) is quite capable of generating a sharp front see Fig. 2 by minimizing numerical dispersion. Although, it is not by any means the easiest or the simplest one. See Appendix C for detail.

Running the Model—

Numerous trial runs were carried out. In order to test the reliability of the model, experimental input data (10) were fed in and the results were weighed against the experimental results. Close and satisfactory agreement was obtained. See Fig. 3.

Sweep Efficiency—

As it is defined in this work, Sweep Efficiency (SE) is the recovered portion of oil (due to waterflooding) divided by the recoverable oil initially (at the start of the flood) in place. To observe the effects of various reservoir and reservoir fluid properties, as well as flooding and simulation criteria on SE, different values of the variables were fed into the model, respective SE computed, and the results were plotted for visual inspection and further qualitative investigation. The variables which were considered essential and for which runs were made are as follows :

- Injection rate, which is directly related to injection pressure (reservoir pressure) and reservoir injectivity.

- Viscosity ratio, VR (oil/water), which is directly related to mobility ratio in a defined system.
- Absolute permeability
- Grid block size, which is directly related to physical size of the sand or number of grid blocks

There are obviously other influential elements which directly or indirectly affect the recovery. The effects of these elements will be analyzed and discussed only qualitatively. This is mainly due to the inherent limited ability of the quantitative investigation of the present method.

Discussion of Results

Viscosity Ratio—

Theoretically speaking, in a linear two-phase immiscible flui'd flow system (piston type motion), the fluids form a front at contact where a drastic change in fluid saturation occurs. Generation and stability of this front is directly responsible for most of the oil recovery, resulting in water sweeping the sand effectively. Experimental works (3,4,11) have shown that favorable mobility ratios $L < 1$ (or viscosity ratio in a defined system), are the key in generation and stability of the front. Lack of this front will cause viscous fingering, channeling and tonguing, resulting in water bypassing oil and leaving it behind. This causes appreciable reduction in oil recovery. Fig. 2 represents the effects of viscosity ratio on generation of a sharp front. Subsequently, Fig. 4 shows the effects of various viscosity ration (VR) from unfavorable (VR = 15) to favorable (VR =.10), on Sweep Efficiency of the waterflooded sand.

Water Injection Rate—

As injection starts and water spreads in the reservoir, higher water rates certainly result in more oil recovery. This short and temporary increase in recovery continues until the rate reaches some limit, which is a function of the reservoir injectivity (including porosity and permeability), bottom hole pressure, and surface facilities. Beyond this rate, however, there will be no increase in oil recovery by increasing water injection rate. Therefore, in any water flooding operation, it is significant to determine this operating level and keep the operation continuously at or above this level as long as desired or practically possible. See Figs. 5 and 6.

Reservoir Homogeneity and Isotropicity—

Investigation of SE in heterogeneous reservoirs is beyond the scope of this work, mainly due to inherent limitations of the method of investigation. A homogenous reservoir has been, therefore, assumed throughout this work. But change in reservoir characteristics could become crucial in the success of any waterflooding project. The following remarks are noteworthy:

- absolute permeability;

Except in reservoir injectivity, the mere size of the absolute permeability has no effect on the generation and stability of the front and, therefore, sweep efficiency. However, it is clear that change in the permeability of the sand (unisotropicity) within one reservoir, such as occurrence of fractures, vugs, lenses, channels, sand grain size or configuration, and interbedding shale appearances will undoubtedly interfere with the water sweeping pattern; it may facilitate front instability and fingering.

-Porosity;

Likewise, porosity in itself has no apparent effect on relative oil recovery but its variation, due to change in the structure of the sand, will certainly have the same unpleasant effect of unisotropicity.

-Variation in other sand properties, such as connate water, residual oil saturation, traces or pockets of gas, loose or tight sand, artificial or natural formation damage, etc., will have the same effect as preceding ones. If connate water becomes significant, its viscosity might cause concern. If connate water viscosity differs significantly with that of injecting water, it may interfere with the balance of oil-water viscosity ratio and in some cases become detrimental to the stability of the front.

Pysical Size—

Linearity is a major assumption in this part of the experiment. In comparison with the sand cross section, sand length should be enough to assure flow of fluid in only one direction. Furthermore, generation and stability of the front requires a certain time and space. Sand, therefore, should be long enough to minimize inlet and end effects and to allow the flow to form its assumed front. Except for these rather significant considerations, physical size of the sand has no bearing on the recovery outcome.

Modeling Criteria—

The problems arising in mathematical modeling and numerical solution are mainly numerical dispersion and selection of appropriate space and time steps. In particular, numerical dispersion should not be confused with the true fluid front that is directly related to mobility ratio and flow regime. Methods are available to automatically determine time steps based on input data, including grid block size. Selection of appropriate grid block size is of particular interest. Grid block size (or number of grid blocks) is related to physical size of the sand as well as the computation cost. It is clear that the relative accuracy of computed numerical values increases with a decrease in grid block size (or increases with the number of grid blocks), resulting in an increase in apparent oil recovery (or SE). This is due to the nature of linear mathematical modeling, which treats each grid block volume as a single point. There is a limit, however, to this increase, beyond which the effects of the grid block size becomes quite minimal. See Fig. 7. Economical reasons and other considerations should be used as guidelines in setting up the most appropriate model for each particular case.

The Case of Two-Dimensional Waterflooding

A five-spot water flooding pattern was considered in this part of the study, assuming all other conditions to be the same as the single dimensional model. In this fashion if water is injected into the lower-left corner of the sand then production of oil and/or water takes place from its upper-right corner.

Mathematical Development—

Conservation of mass may be used to derive flow equations for a two-phase, immiscible and incompressible fluid in porous media, as follows :

for pressure,

in oil phase VÀo V p + vo = Φ 3(So/Bo)/ Э t

in water phase VÀw VP + Vw = Φ 3(Sw/Bw)/ Э t

assuming gravity and capillary forces are minimal.
Since the total saturation is fixed,

So + Sw = 1

therefore ;

| VÀt V P + v = 0 | (4)

where,

V = Vo + Vw

The convention of a positive sign for injection rate and a negative sign for production rate were commonly assigned.
For saturation,

Vt V f + φ 3S/3t = 0 (5)

where S = Sw,

and f = Vw/Vt, as water flow fraction

Similar equations may also be written for the oil phase.

Numerical Solution—

Analytical solution of Equations 4 an 5 to determine pressure and saturation distribution is practically impossible on the same grounds as mentioned above for the one-dimensional model. The only practical and reliable method known today is a numerical solution of the reservoir mathematical model.

An oil-depleted, M by M grid block, reservoir model was set up. Water was injected in the lower-left corner and simultaneously oil was produced from the upper-right corner of the model (a five-spot pattern). The reservoir was assumed to be homogeneous and isotropic; furthermore, the sand was assumed to have no preference for oil or water, although the selected relative permeability data is from a preferentially oil wet sand (10).

Pressure Distribution—

Equation (4) was discretized by a "Forward Difference Approximation Scheme," and the SOR (Successive Over Relaxed) method was used to solve for pressure implicity. No-flow boundary conditions were used by setting transmissibilities equal to zero at boundaries :

P(0,j) = P(1,j), j = 1,2. . .M
P (i,0) = P (i,l) , i = 1,2. . .M
P(M+1,j) = P(M,j), j = 1,2. . .M
P(i,M+l) = P(i,M), i = 1,2. . .M

where, i and j represent horizontal and vertical grid block numbers, respectively.
Also ,
P(i,l) = reservoir injection pressure, as a given datum directly related to injection rate.
See Appendix (D), for further details.

Saturation Distribution—

Using the method of "Splitting Alternative Direction," Equation (5) may be written as follows:

Vx Sf/Эx + фЭБ/ЭС = 0 (5a)

Vy 9f/3y + фЭв/ЭГ = 0 (5b)

where, Vx + Vy = Vt
also, . Vx =- At dP/dx
and, Vy =- At dP/dy

If pressure of each cell and total mobility is known, Vx and Vy could be determined and substituted in Equations (5a) and (5b). These equations, then discretized by "Backward Difference Approximation Scheme," and using the "Splitting Alternative Direction," and Godunov method, along with the following initial and boundary conditions,

initial conditions, S(l,l) = 1, for t > 0
boundary conditions S(O,j) = S(l,j), j = 1,2...M
S(i,0) = S (i,1) , i = 1,2. . .M
S(M+l,j) = S(M,j), j = 1,2...M
S(i,M+l) = S(i,M), i = 1,2...M

saturation distribution will be closely approximated. See Appendix (D) for further details.

Relative Permeability Data—

Water and oil mobilities, as well as water fraction values, f(S), require relative permeability data for the above computation. Relative permeability data, as a function of water saturation, has been selected from laboratory data (10) in a curve fitting fashion, for this part of the work. Also, special subroutines were set up for interpolation of relative permeabilities for given water saturations.

Reliability of the Model --

Numerous trial runs were carried out, and in order to test the reliability of the model, experimental input data (10) were fed in. Computed results were weighed against experimental data. A close and satisfactory agreement was obtained. See Fig. 9.

Discussion of Results-

To investigate the effects of reservoir and reservoir fluid properties as well as operation and modeling criteria, a process similar to the single dimensional model was carried out as follows:

Viscosity Ratio (VR)—

Under similar conditions the model was run for different values of viscosity ratio (mobility ratio in a defined system). Saturation distribution for each case was computed and plotted. See Figs. 10,11,12. Oil recovery for each case was computed and plotted. See Fig. 13. Sweep efficiency was also computed in each case and finally plotted against viscosity ratios. See Fig. 14.

Results of this experiment show the unique effect of viscosity ratio in generation and stability of a sharp front at the fluid's contact. At VR values of one or less (favorable conditions), saturation gradients are quite significant at the front, and on the contrary, as VR increases (unfavorable conditions) , the front almost dissipates. The effects of VR on SE, based on this experiment, is predictably evident in Fig. 14. As stated before, viscosity ratio is the most important factor in generation and stability of the fluid front which in turn is the key in sweeping of the reservoir by waterflooding. Any change or interference in the reservoir and reservoir fluid characteristics and/or operation, such as reservoir heterogeneity, isotropicity, operation break down, etc. will undoubtedly result in instability and dissipation of the front and emergence of fingering or tonguing, which will cause water to break into the oil front and for the most part bypass it. It should be stated that left-behind-oil , may never be recovered under the same reservoir and operational conditions. This is probably the single most misunderstood, ignored, or hopelessly tried source of failure in waterflooding operations for.oil recovery today.

Water Injection Rate—

Models were run for different water injection rates (directly related to reservoir operating pressure) keeping other input data (such as VR = 1 or 5.42 cp) unchanged. Results in this case are in complete support of the findings in the one-dimensional model. That is, at the start of the operation, and before the flow of fluid becomes stabilized in the reservoir, recovery increases with an increase of the rate. This trend continues until it reaches a certain limit which is set by a particular operation. Beyond this limit however, an increase in recovery becomes minimal and sweep becomes independent of injection rate (reservoir operating pressure). See Fig. 15. This behavior is also reflected in the rate-sweep efficiency plot of Fig. 16. It should be stated that at low injection rate capillary and gravity forces become more significant and certainly detrimental to the formation of a steep front. On the other hand, at high rates of injection (above the limiting rate), flow is more under the influence of the viscous forces, particularly in a homogeneous reservoir, where the condition is quite favorable for generation and stability of the fluids front.

It is clear that lower production rates (in this work assumed to be the same as injection rate) will prolong the stability of the front and it is encouraged, although this is again subject to the economical guidelines and prevailing reservoir pressure.

Reservoir Homogeneity and Isotropicity—

Although reservoir permeability has a direct effect on the injectivity and other operating criteria, generally speaking, oil recovery seems to be quite independent of permeability as long as the assumption of reservoir homogeneity and isotropicity have been met. See Fig. 17. Similar are the effects of porosity on sweep efficiency. See Fig. 18. For further details on this subject refer to the discussion under the same title in linear flooding above.

Physical Size—

The assumption of only two dimensions in the model is a major one and sand dimensions should be such that only areal fluid flow is assumed. Basically, no vertical flow will take place under reservoir operating conditions. Furthermore, it requires some space and time to minimize the inlet and end effects and also to allow the flow of the water phase to form its assumed contact with the oil phase. Except for these rather significant operating considerations, the sheer size of the sand has no bearing on oil recovery, keeping all other elements unchanged.

Modeling Criteria—

Although the process of mathematical modeling and numerical solution in this case required a much more complicated method than the one-dimensional method, experience from the one-dimensional model, did facilitate the matter by avoiding any numerical dispersion or saturation creep. The time step was determined automatically, based on other input data such as selection of the space steps (in x and y directions). Selection of these scales will be directly reflected in the size of the numerical error. The smaller these steps (more grid blocks), the less the size of the error and the closer the approximated results, which means higher apparent sweep efficiency values. This behavior has been depicted in Fig. 19. In real operations, however, economical as well as other particulars of the operations should be used as guidelines in selection of proper values for time and space steps.

Operational Considerations—

Sweep Efficiency of all the above experiments was considered at the time of water breakthrough (BT). That is when the head of the injecting water phase reaches the producing well. Theoretically, all of the recoverable oil will be eventually recovered provided water injection continues indefinitely. In real operations however, this could not possibly take place for the following reasons:

-A foremost fact is that left-behind-oii will not again receive enough sweeping push to get it out of the rock pores and to the producing well—no matter how long water injection continues—so long as the front either does not materialize or for some reason breaks down and dissipates. This is more so in oil-wet rocks, where additional favorable conditions facilitates further the flow of water through the sand.

-Economy of the waterflood ing operation will not allow indefinite water injection at injecting well(s) and sufficient water-oil separation and water disposal or recycling facilities at the producing wellhead (s). Maximization of sweep efficiency without economical and practical considerations has no virtue.

It is therefore, the sweep efficiency at breakthrough which mostly reflects the success or failure of the operation. In many cases, however, the final recovery, which is subject to the operator’s economy as well as operational limitations will be determined long after water breakthrough. In successful operation (except in tilted reservoirs where gravity forces are in effect), most of the recoverable oil has been produced by BT time. In such operations, occurrence of BT is considered a beginning of the end.

Scaling Factor

Experiments on viscosity ratios, injection rates, and sand physical sizes indicate the existence of a limit below which recovery is sensitive to these factors and their increase will result in higher recovery. Beyond this limit, however, flow patterns and oil recovery will practically remain independent of these factors.

To show the relative importance of these essential factors, Equation (1) is turned into a dimensionless one as follows:

we had, Φ 3s/3t + Vt 9f / Эx = o

or , Φ 3s/ 3t + q/A 3f / Эx = 0

or, 3s/3t + q/ (A*Φ) 3 f/ 3χ = o

or , 3 s/ 3t + q/(A*LM)L3f/3x = 0

Assuming Q as injection rate in terms of pore volume,

then, 3s/3t + QL 3f/3x = 0 (6)

multiply both sides by μθ/ţjw (a none zero value) results,

μθ/yW 3s/31 + Q L μo/uw 9f/ 9 x = O

let, , Sd =uo /uw S

also let, SF = Q LHo/Uw

substitute in (6), then, Sd/t + SF 3f/3x = O

or in general, | 3Sd/3t + SF V f = O | (7)

This is a unique dimensionless equation for all two-phase incompressible immiscible fluid flow in porous media.

SF is defined as "Scaling Factor" and has the following distinguished properties:

- From a theoretical standpoint (Eq. 7), All floods of similar SF values, behave similarly.
- From the above experiments, There exists a critical value for SF, in any water flooding operation, where at lower values the assumed flow pattern (the front) has not been established yet (not stabilized) and oil recovery is highly sensitive to SF and continuously increases with an increase in SF. At values higher than critical SF, however, the flow pattern (the front) has been established (stabilized) and the recovery becomes almost independent of SF.

In any real operation it is therefore essential to estimate the critical SF and keep the level of operation above this value. Critical SF could be determined by trial and error and/or· pilot plant studies.

Rapoport and Leas (3), in their excellent and fundamental work, have reached a similar conclusion and from theoretical work they defined "Scaling Coefficient" as c = UWVL, and experimentally showed a critical value for any flood where, beyond that, recovery remained constant with an increase of c. In their work, although they specifically expressed their findings in terms of viscosity ratio, formulation of c. The interchangeability of involved factors for SF to reach a critical value is also reported in this work. However, this matter although theoretically sound seems doubtful and requires further experimental study. Douglas and Wagner (11) also selected a similar group of paramenters as Q L.pν/(K dPc/dx), called "Rate Paremeter," which is closely defined as the Scaling Coefficient.

Conclusion

Injected water as an incompressible fluid should flow through sand pores and contact oil (also an incompressible fluid) and theoretically should form a sharp front (immiscible fluids) where saturation gradients are greatest. If sand homogeneity, favorable mobility ratio, and optimum injection and production rates are provided water will sweep out 100 pc of the recoverable oil, and sweep efficiency, by definition, is maximized. The key to this highly-theoretical, maximum efficiency is threefold:

1 .Favorable mobility ratio (viscosity ratio in a defined system),
2 .Homogenous sand, and
3 .Minimum required flow rate with respect to the physical size of the sand.

Any deviation from these basic principles is bound to have substantial impact on recovery and directly result in reduced sweep efficiency. In actual reservoirs and in field operations, control over these key factors is very difficult, if not in many cases impossible.

Experiments carried out in this work, along with many other studies, clearly show that favorable mobility ratio (VR $\leq$ 1) is directly responsible in generation -and stability of the front, which in turn is the foremost factor in having a highly efficient sweep. Lack of favorable viscosities will make the operation highly susceptible to the stability of the front and results in fingering, branching, channel ing,tonguing and eventual dissipation of the high-saturation gradient front. Branched out water, will bypass the oil, and recovery will be relatively dismal.

Reservoir homogeneity is indeed an uncontrollable yet crucial factor in the efficiency of the sweep. Understanding of the reservoir characteristics—from depositional condition, to grain size and configuration and existence of lenses, pockets, cracks, channels, vugs and shale discontinuity and finally reservoir boundaries—are essential in design, operation and maintenance of a continuous and successful water flooding.

Effects of injection rate, viscosity ratio and relative physical size of the sand on oil recovery have a certain limit. A "Scaling Factor," as a combined element representing these essential factors, could be used to determine their combined influential limit. This limiting value is called "Critical Scaling Factor".

Keeping the level of the flooding operation above this critical value in the design, operation and maintenance of a continuous waterflood ing operation will assure maximized oil recovery possible under prevailing operating conditions.

Recommendation

Waterflood ing of petroleum reservoirs is one of the established and foremost methods of enhancing oil recovery today. Although the science and technology of waterflooding have advanced notably, due to the inherent complicated nature of the subject and almost infinite variety of reservoir and reservoir fluid types, success of a vast majority of operations can not be guaranteed.

On the other hand, waterflooding as a primary means of enhancing oil recovery can not be employed in many petroleum reservoirs containing gas or very light hydrocarbons. In some cases water can not be used due to high cost of available water, environmental considerations, or availability of other types of displacing media such as carbon dioxide or nitrogen. There are also other reasons to suggest waterflooding has a limit and the industry is rapidly approaching it.

Consider the following:

1. Discovery of carbon dioxide reservoirs (a major one in Mississippi) .
2. Availability of nitrogen everywhere.
3. Gradual decrease in manufacturing, installation, and operation cost of surface facilities.
4. Almost day by day shortage, and in some cases scarcity of water, (such as the Middle East and populated areas, especially in the western hemisphere).
5. Environmental considerations and environmental protection organization's imposition of strict rules and regulations on industrial water consumptions.
6. Emergence of reliable miscible flooding technical know how, etc.

Therefore, the use of other media besides water, mainly carbon dioxide and nitrogen, as a logical and practical alternative in flooding hydrocarbon reservoirs for the sole purpose of increasing recovery will be progressively more in demand.

Any investigative work in the area of sweep efficiency in the reservoir flooding, such as this work, for the purpose of enhancing oil recovery, can not be complete and may not receive the attention and credit it really deserves, without the further challenging study of miscible flooding.

Therefore, the second part of this project, namely, "The Study of Sweep Efficiency in Miscible Flooding of the Hydrocarbon Reservoirs for Maximizing the Recoverable Oil and Gas," as has been proposed to MMRI, as a vital part of this project, needs to be undertäken .

Research on all aspects of water flooding, and much more so on miscible flooding, should therefore continue, and I believe it will, so long as the complexity and unknowns in the technology remain, so long as progress continues in sweeping more and more oil from exhausted oil fields, so long as it . stays viable to the oil industry, and more so, to the economy of the country.

REFERENCES

1. Buckley, S. E. and Leverett, M. C.; "Mechanism of Fluid Displacement in Sand." TRANS. AIME, Pet. Tech. 1941, T.P. 1337.

2. Douglas, P. M. and Blair, R. J. and Wagner, R. J.; "Calculation of Linear Waterflood Behavior Including The Effects of Capillary Pressure." Trans. AIME, Vol. 213, 1958, T.P. 8013.

3. Rapoport, L.A. and Leas, W. J.; "Properties of Linear Waterflooding." Trans. AIME, Vol. 198, 1953.

4. Kyte, J. R. and Rapoport, L. A.; "Linear Waterflood Behavior and End Effects in Water-Wet Porous Media." Trans. AIME, Vol. 213, 1958.

5. Richardson, J. G. and Perkins, F. M., Jr.; "A Laboratory Investigation of the Effects of Rate on Recovery of Oil by Waterflood ing." Trans. AIME, Vol. 210, 1957 .

6. Allen, Maddison and Puckett; "Theoretical and Experimental Studies on Rate Dependent Two-Phase Immiscible . Flow." SPE, T.P. 10972, 1982.

7. Sheldon, Zondek and Cardwell; "One-Dimensional Imcompressible None Capillary, Two-Phase Fluid Flow in a Porous Media." Trans AIME, Vol. 216, T.P. 8089, 1959.

8. Rapoport, L. A.; "Scaling Laws for Use in Design and Operation of Water-Oil Flow Models." Trans. AIME, Vol. 204, 1955.

9. Craig, F. F., Jr.; "The Reservoir Aspects of Waterflooding." SPE Monograph.

10. Rapoport, Carpenter and Leas; "Laboratory Studies of Five-Spot Waterflood Performance." Trans. AIME, Vol. 213, T.P. 8015, 1958.

11. Douglas, J., Jr. and Wagner, R. J.; "Calculation of Linear Waterflood Behavior Including the Effects of Capillary Pressure." Trans. AIME, V 218, T.P 8013,1958.

12. Bell, J. B. and Shubin, G. R.; "Higher-Order Godunov Methods for Reducing Numerical Dispersion in Reservoir Simulation." SPE 13514, 1985.

13. Leventhal, S. H.; "The Operator Compact Implicit Method for Reservoir Simulation." SPE 7678, 1979.

14. Carr, A. H. and Christie, M. A.; "Controlling Numerical Diffusion in Reservoir Simulation." SPE 12235, 1983.

15. Douglas, Darlow, and Kendall; "Self-Adaptive Galerkin Methods for One-Dimensional, Two-Phase Flow." SPE 7679, 1979.

16. Sethian, Chorin, and Conçus; "Numerical Solution of the Buckley-Leverett Equations."

17. Douglas, Peaceman, and Rachford; "A Method for Calculating Multi-Dimensional Immiscible Displacement." Trans. AIME V 216, T. P. 8090, 1959.

18. Li, K. M. G.; "Random Choice Method for Treating the Convection-Diffusion Equation." SPE, 12237, 1983.

19. Banks, Cheshire, and Pollard; "A Technique For Controlling Numerical Dispersion in Finite-Difference Oil Reservoir Simulation." Proceeding of the Bail I Conference Held at Trinity College, Dublin, Ireland, June 3-6, 1980.

20. McBryan, O. A.; "Elliptic and Hyperbolic Interface in Two-Phase Flow." Proceedings of the Bail I Conference held at Trinity College, Dublin, Ireland, June 3-6, 1980.

21. Breston, J. N. and Hughes, R. V.; "Relation Between Pressure and Recovery in Long Core Water Floods." AIME, T.P. 2591 AP 1949.

NOMENCLATURE

A .... Cross Section
Bo, Bw. . . . Reservoir Volume Factor (Oil, Water)
Cte.......... Constant
f............ Water Flow Rate Fraction
ϕ .... Porosity
i,j.......... Index (x,y direction)
K .... Absolute Permeability
Ko,Kw .... Effective Permeability (Oil, Water)
Kro,Krw . . . Relative Permeability (Oil, Water)
L .... Length
λ .... Mobility Ratio(Oil, Water)
At........... Total Mobility
M............ Number of Grid Blocks on Each Direction
Po Uw .... Viscosity (Oil, Water)
P .... Pressure (Oil Phase, Water Phase)
q............ Total FlowRate
qo , qw ... Flow Rate (Oil, Water)
Q............ Total Flow Rate in Pore Volume (Dimensionless)
S............ Sw
So,Sw .... Saturation (Oil, Water)
Sd. ..... цо /$ μ_w
SE........... Sweep Efficiency = Total Oil Recovered/Tota1 Recoverable Oil * 100
Swc .... Connate Water Saturation
t............ Time
V............ Total Flow Rate Per Unit Cross Section
Vo,Vw .... Flow Rate (Oil, Water)
VR........... Viscosity Ratio
x,y .... Space Dimensions

APPENDIX A

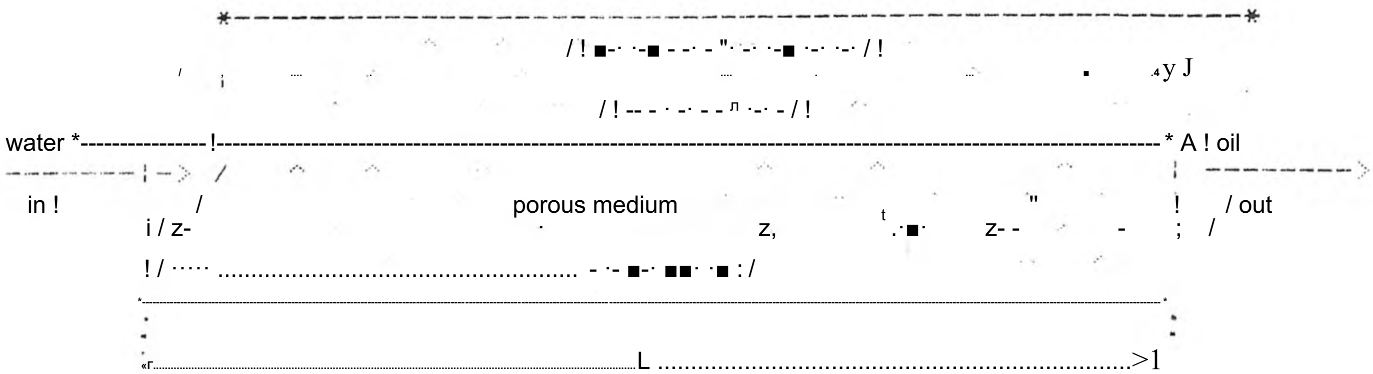


Fig.l. Linear moclel.

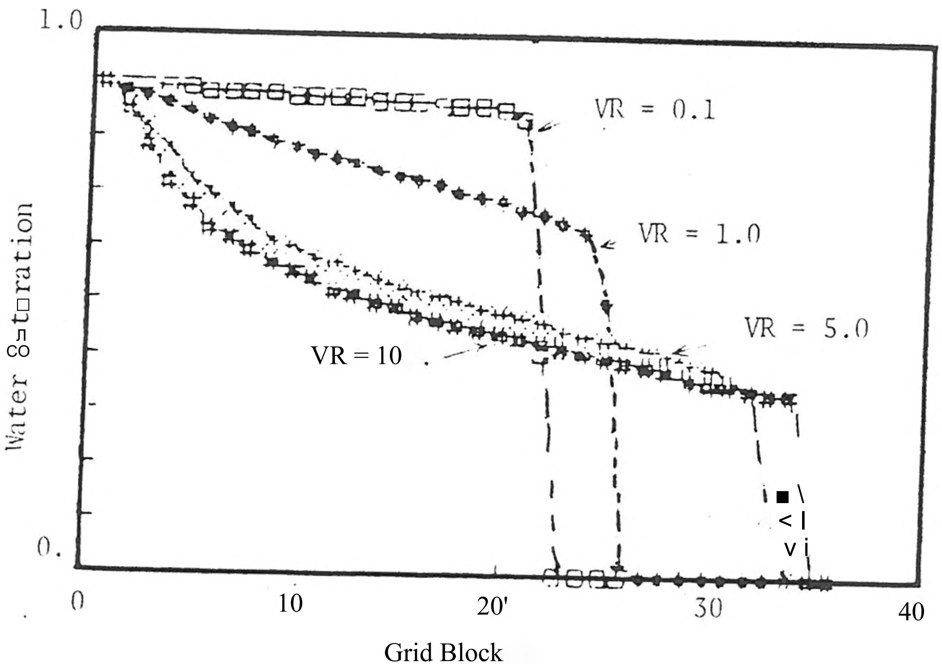


Fig. 2 Effect of VR on Water Saturation Distribution

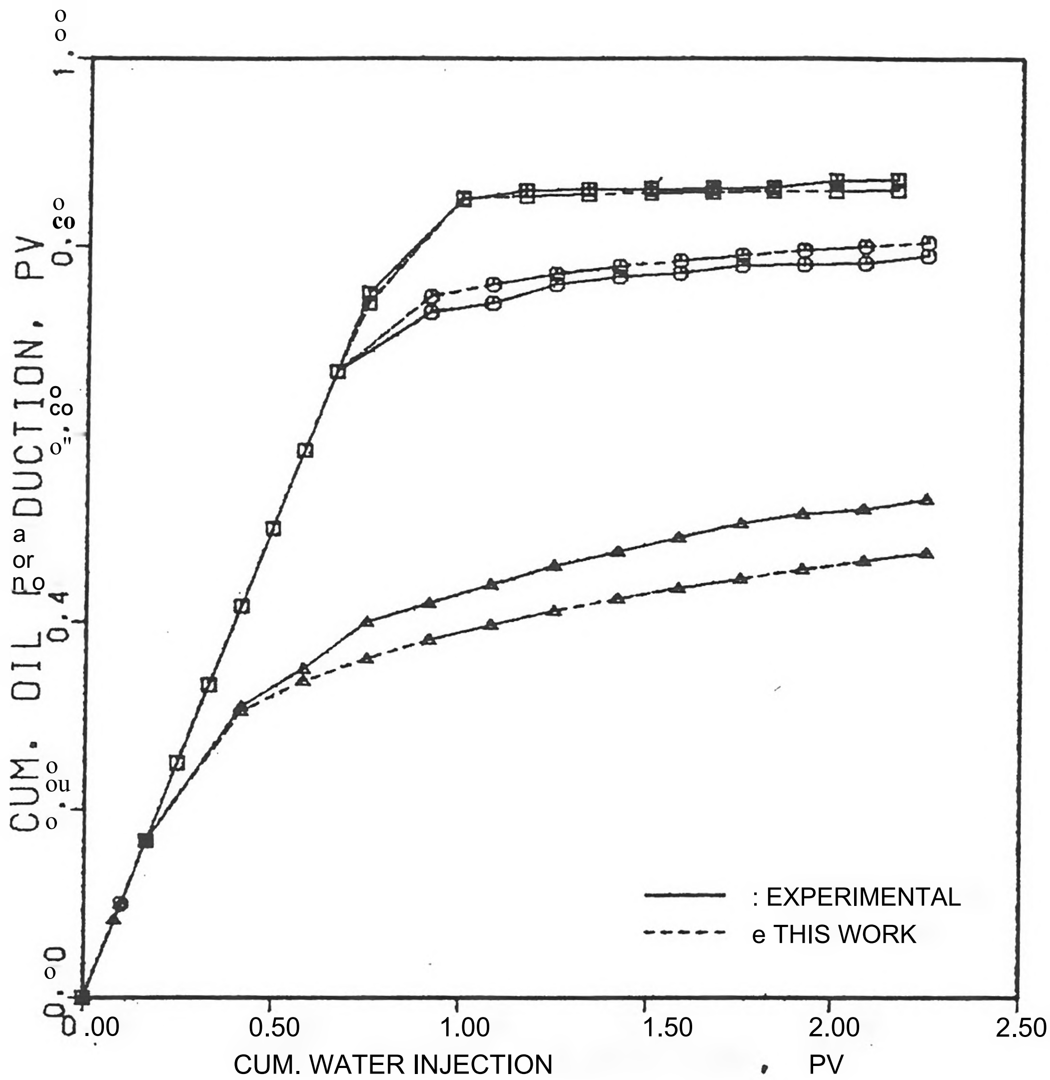


FIG.3 -COMPARISON BETWEEN LABORATORY AND COMPUTER MODELS

ID * VR = o. i

O · VR = 1.0

A · VR = 4D.

VR: OIL TO WATER VISCOSITY RATIO

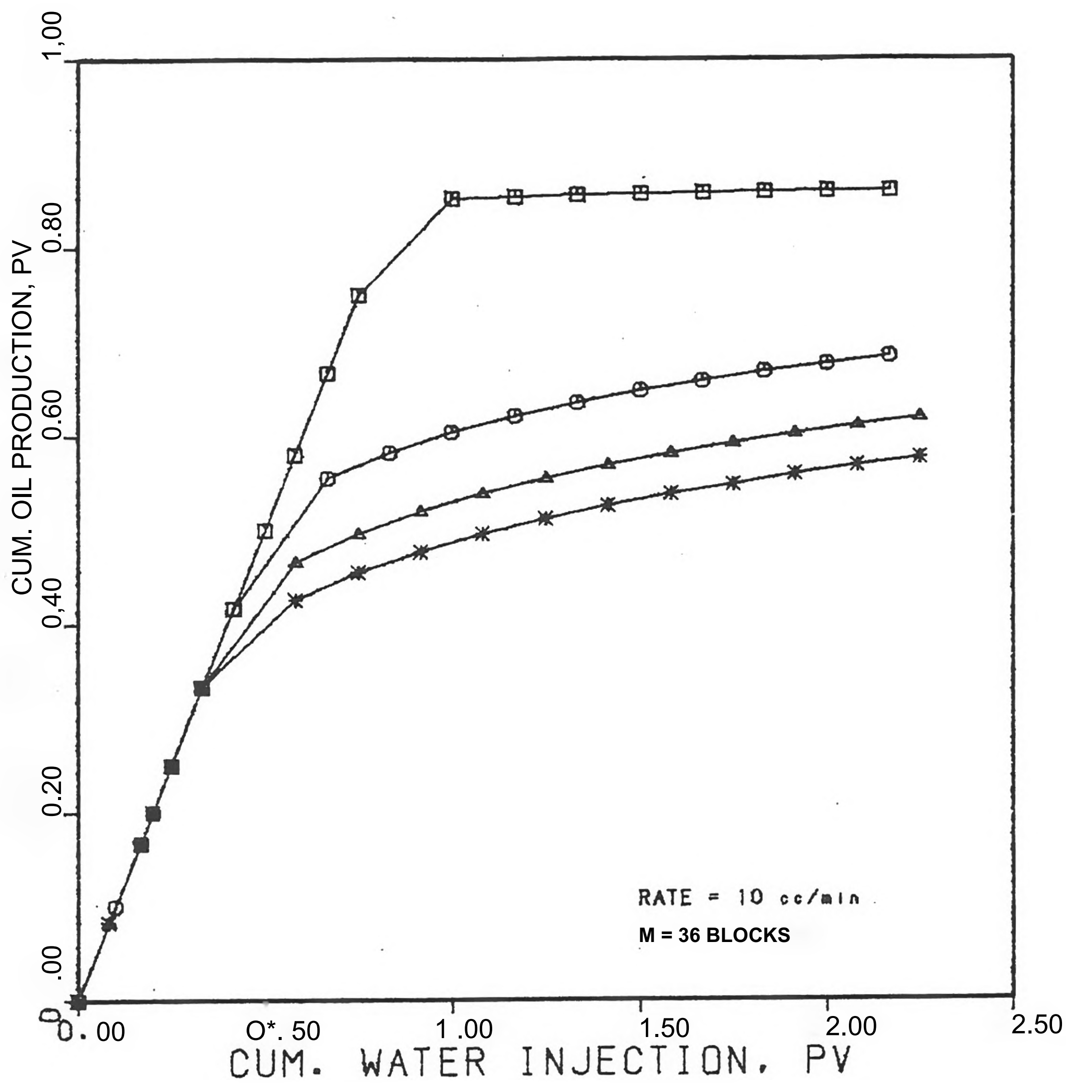


Fig. 4 -EFFECT OF VR ON OIL RECOVERY

□ **VR - .10** | **0 VR - 5.0**

д **VR s 10.** | **VR 3 15.**

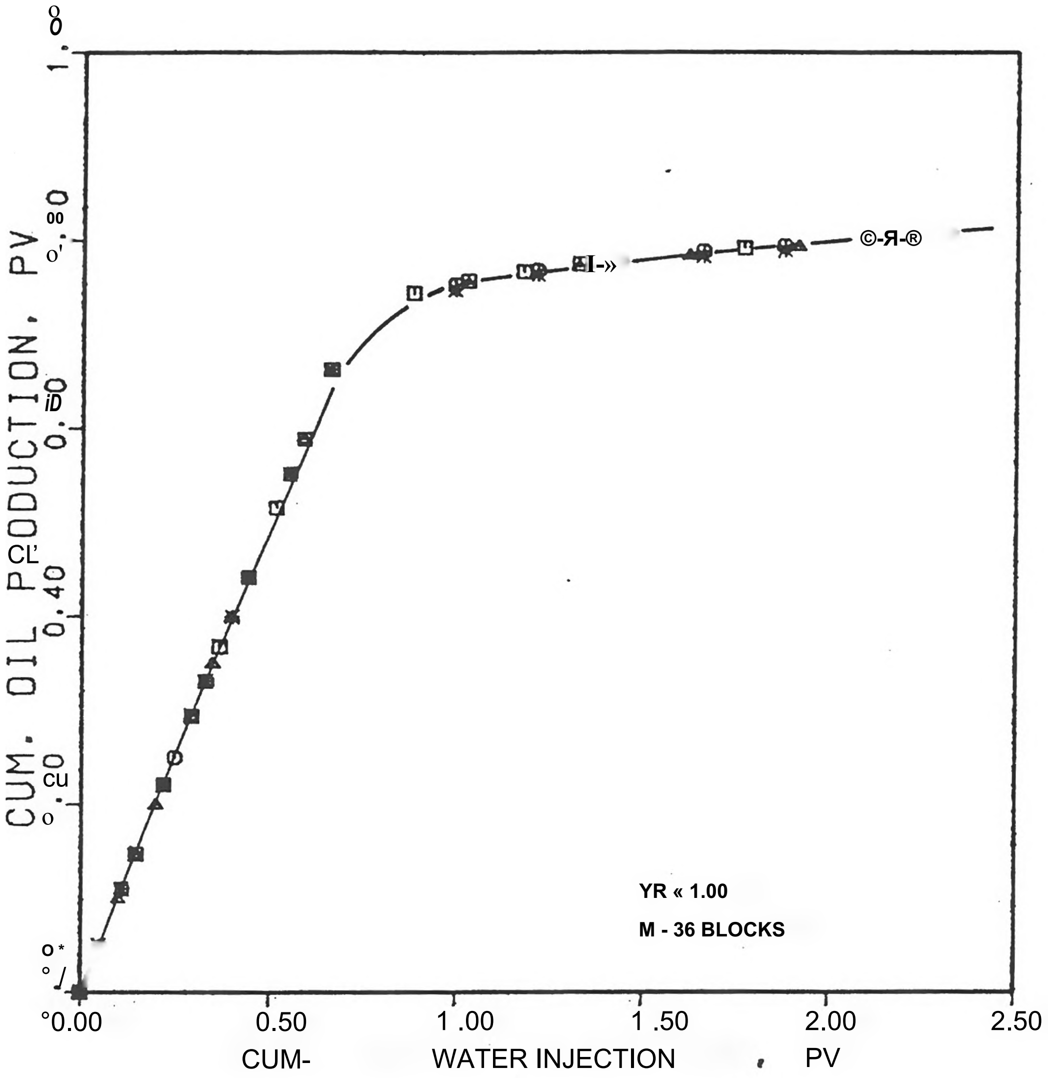


Fig. 5 -EFFECT OF RATE ON OIL RECOVERY

Q RATE - 5 CC/SEC Q RATE » 10 CC/SEC
RATE - 20 CC/SEC * RATE « 30 CC/SEC

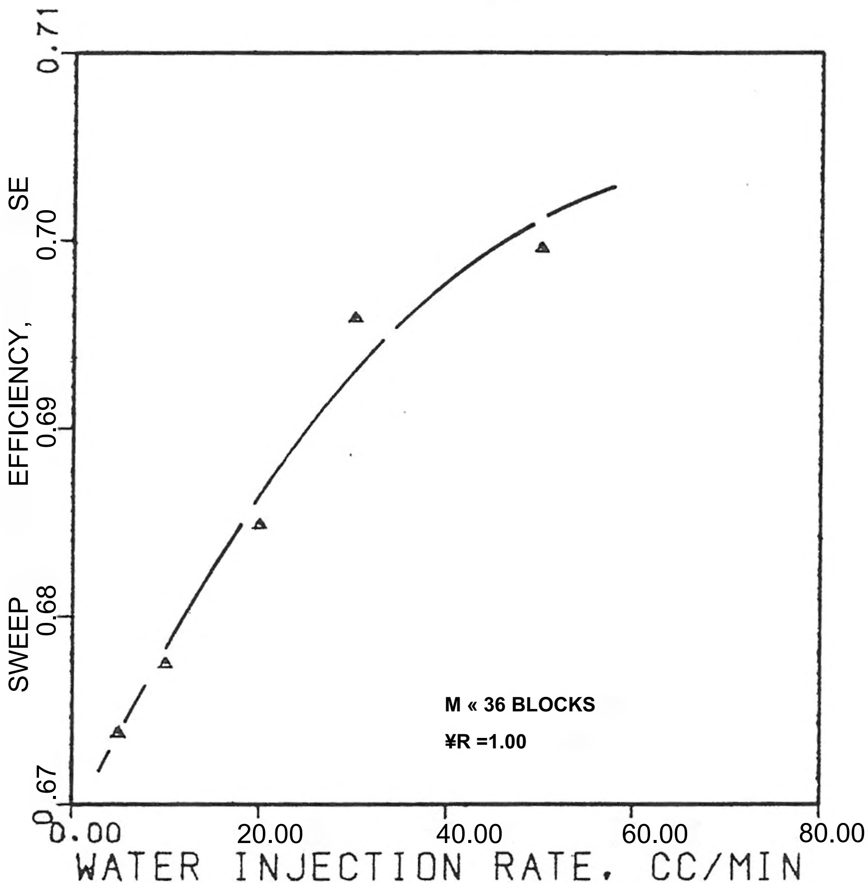


Fig. 6 -EFFECT OF RATES ON SE

д At Breakthrough

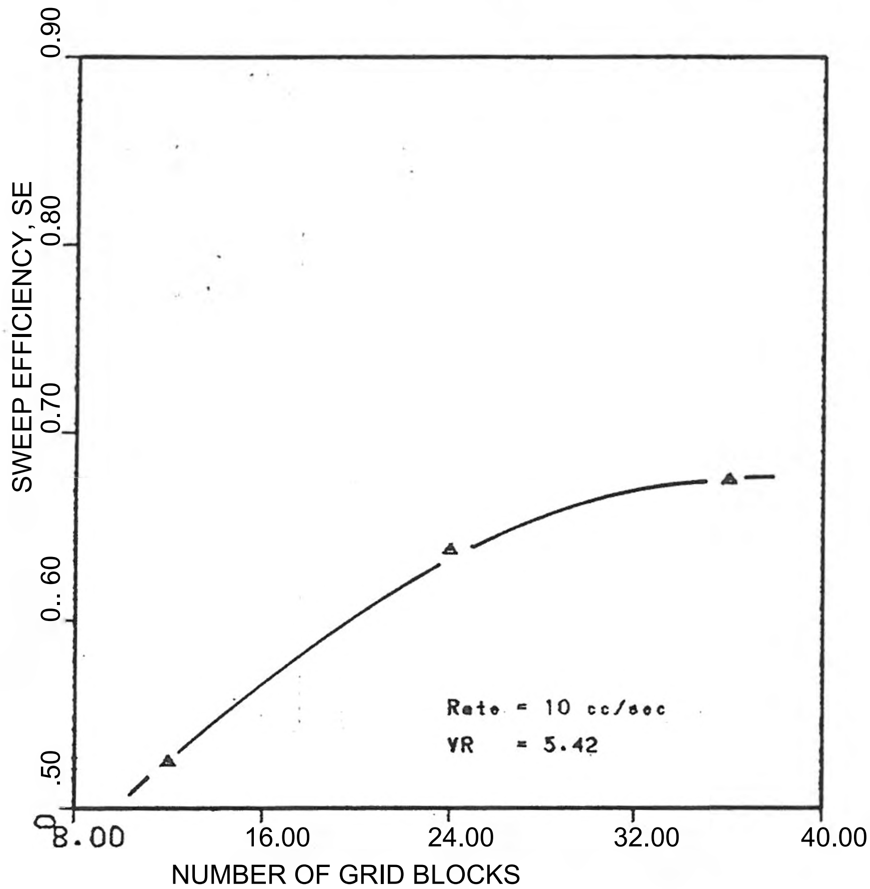


Fig. 7 -EFFECT OF GRID SIZE ON SE

Δ Ai Breakthrough

APPENDIX B

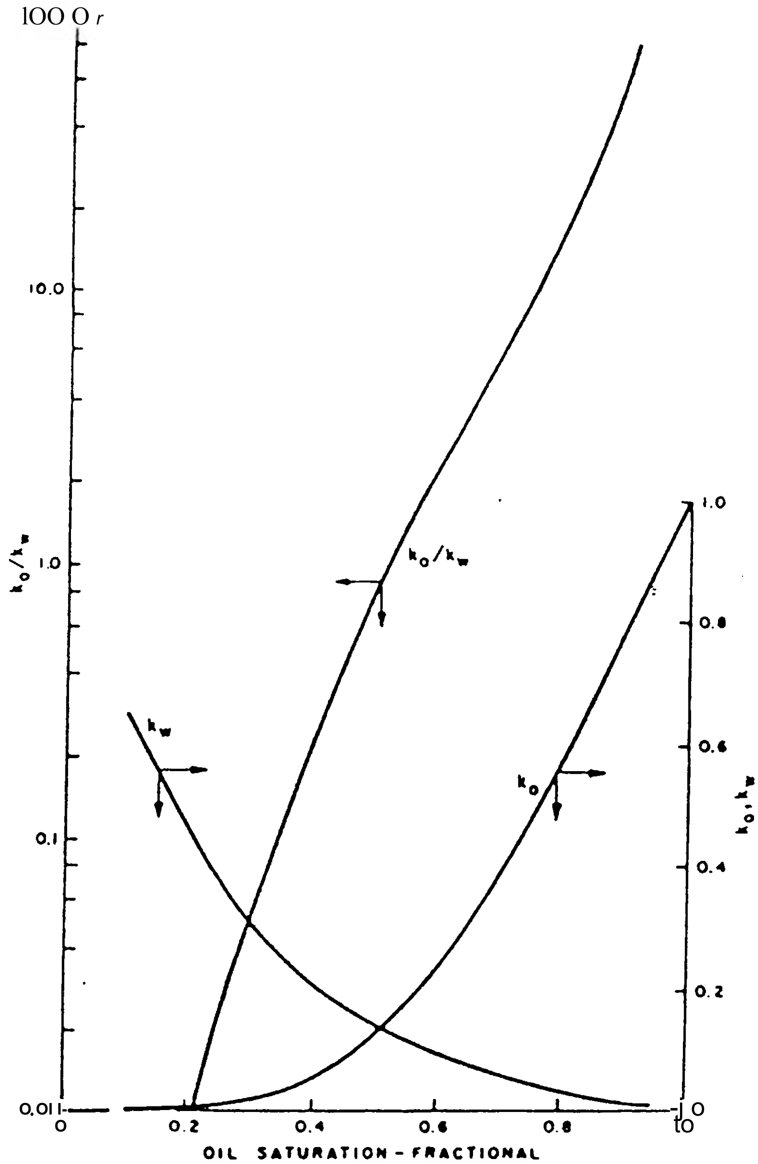


FIG. 8—RELATIVE PERMEABILITIES. DHI-FILMED GLASS BEAD PACKING. (10)

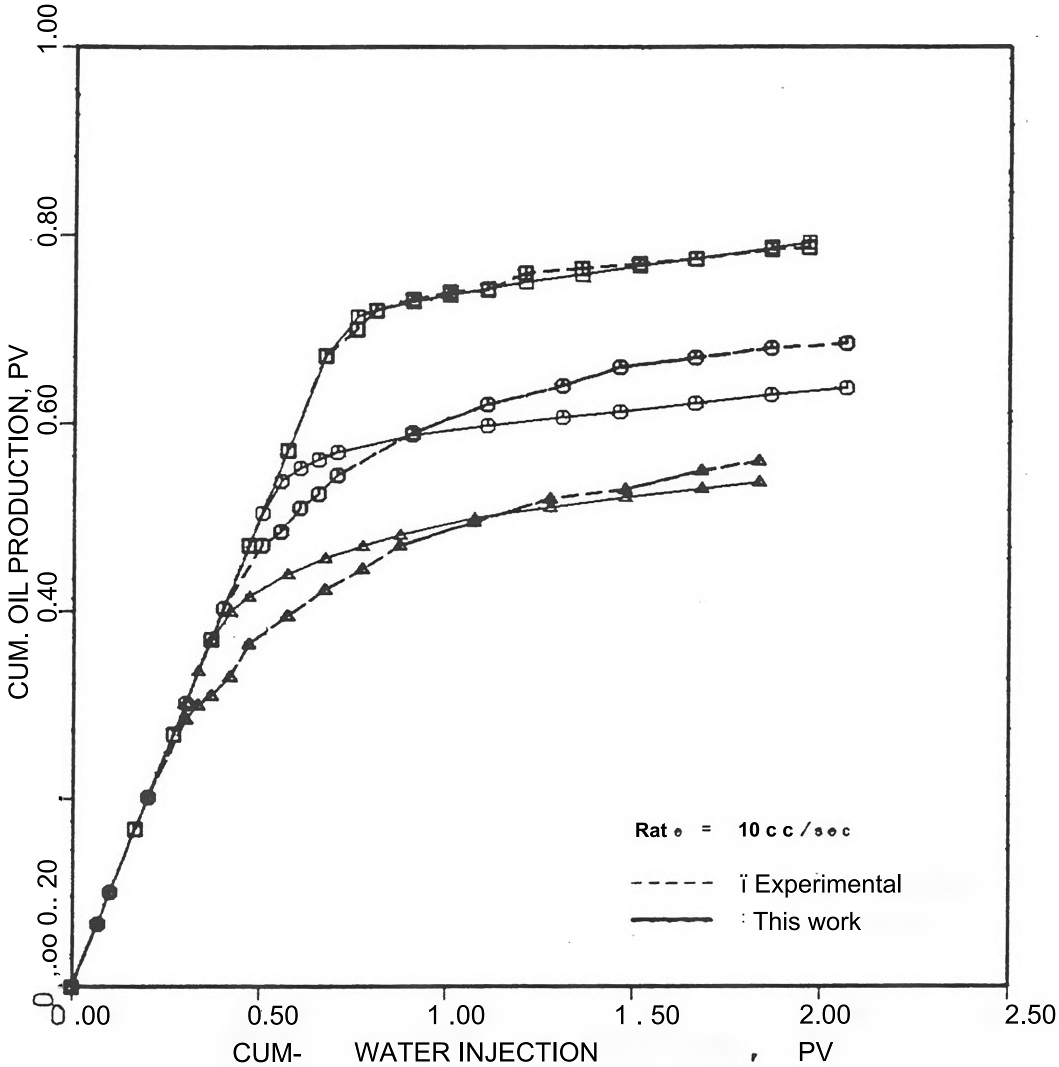
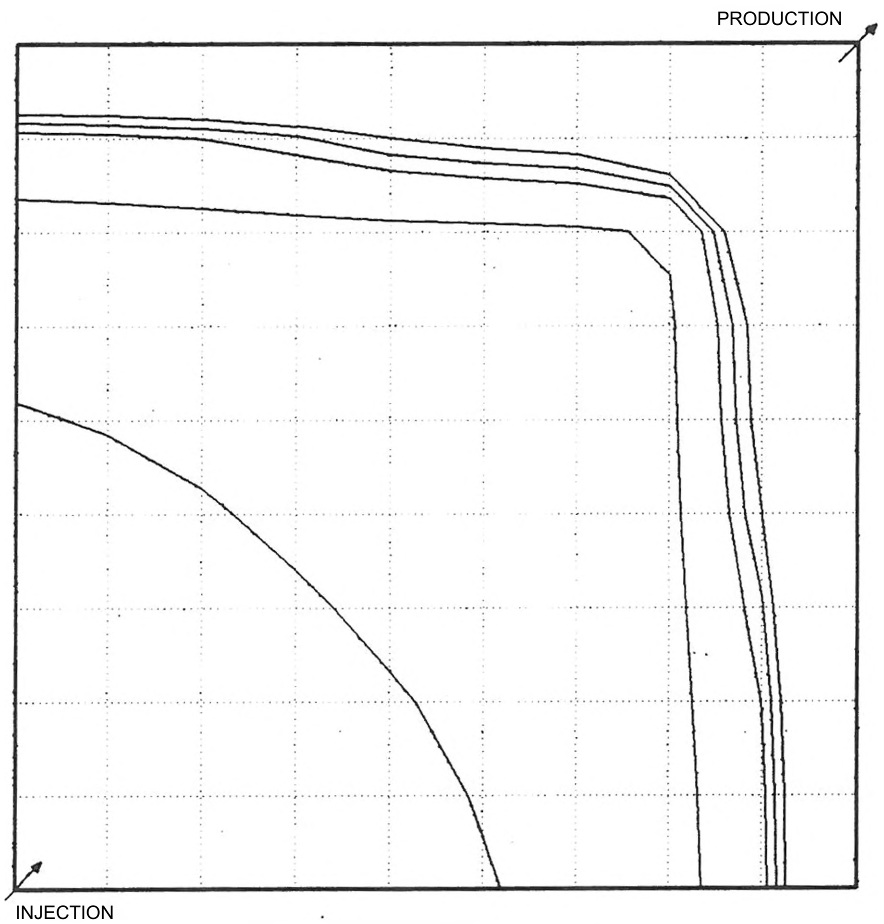


FIG. 9 -COMPARISON BETWEEN LABORATORY AND COMPUTER MODELS

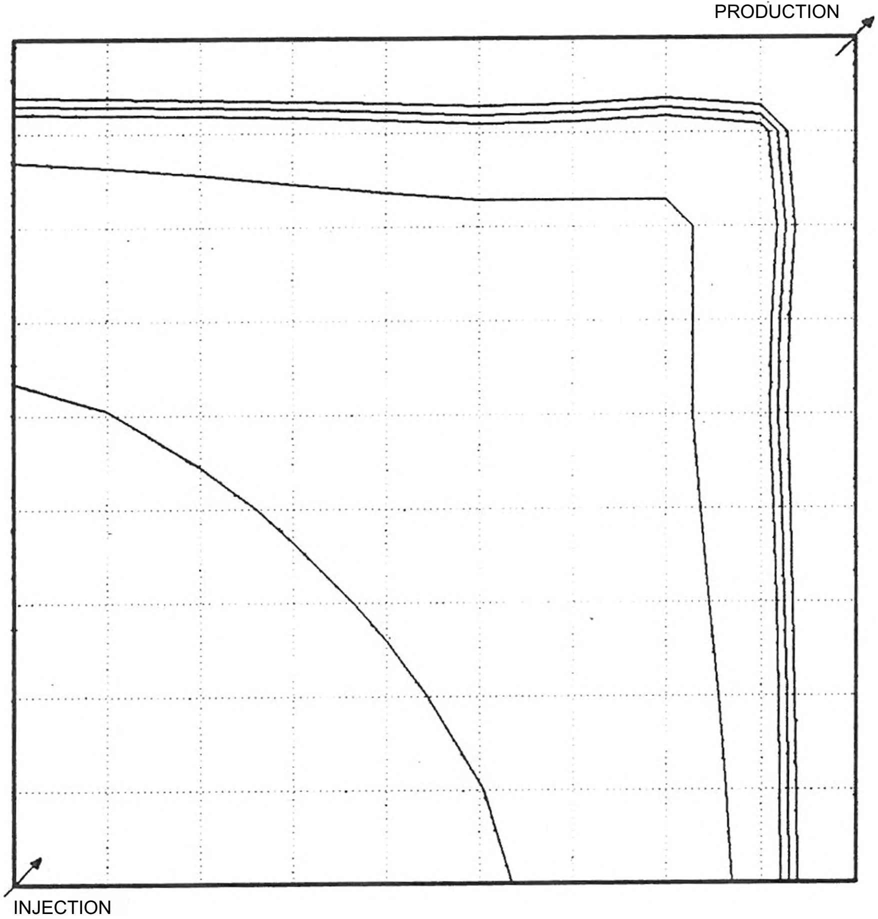
щ « VR ³ 1.02

Q » VR = 5.42

д « VR = 15-7



FI G-i OBSATURAT i ON CONTOUR PLOT FOR VR = 1.02
AT TIME = 700. SEC., (RATE = **10 cc/sec).**



F I G -1 ObSATURAT]ON CONTOUR PLOT FOR VR = 1.02
AT TIME = 749. SEC.. (RATE = 10 cc/3ec).

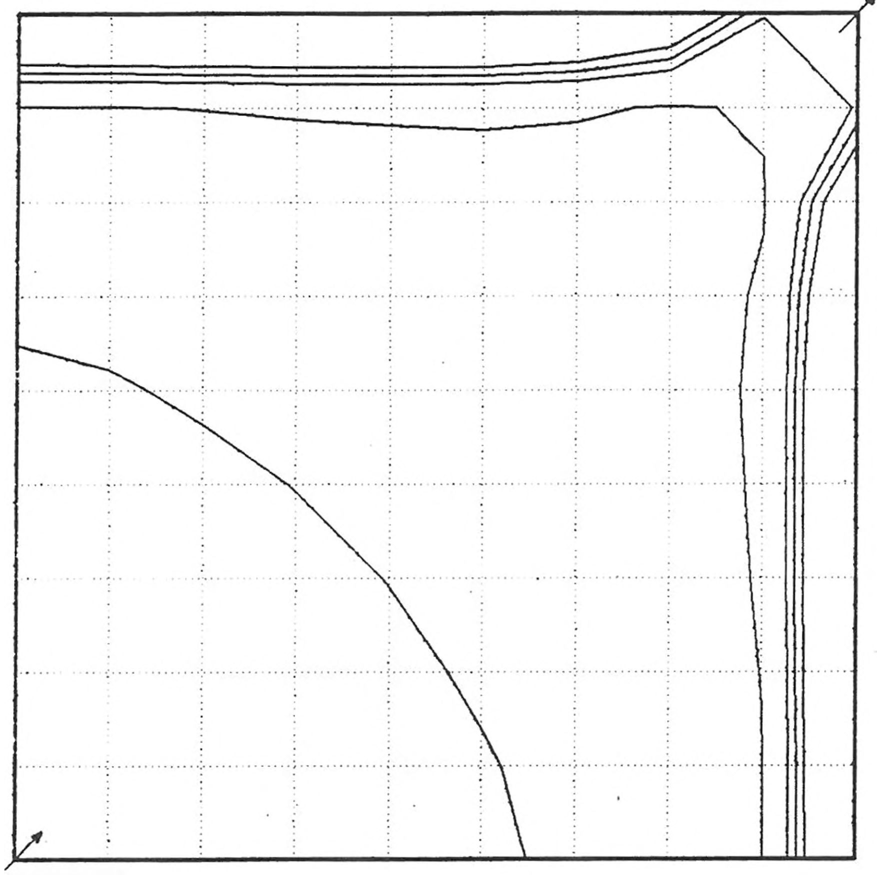
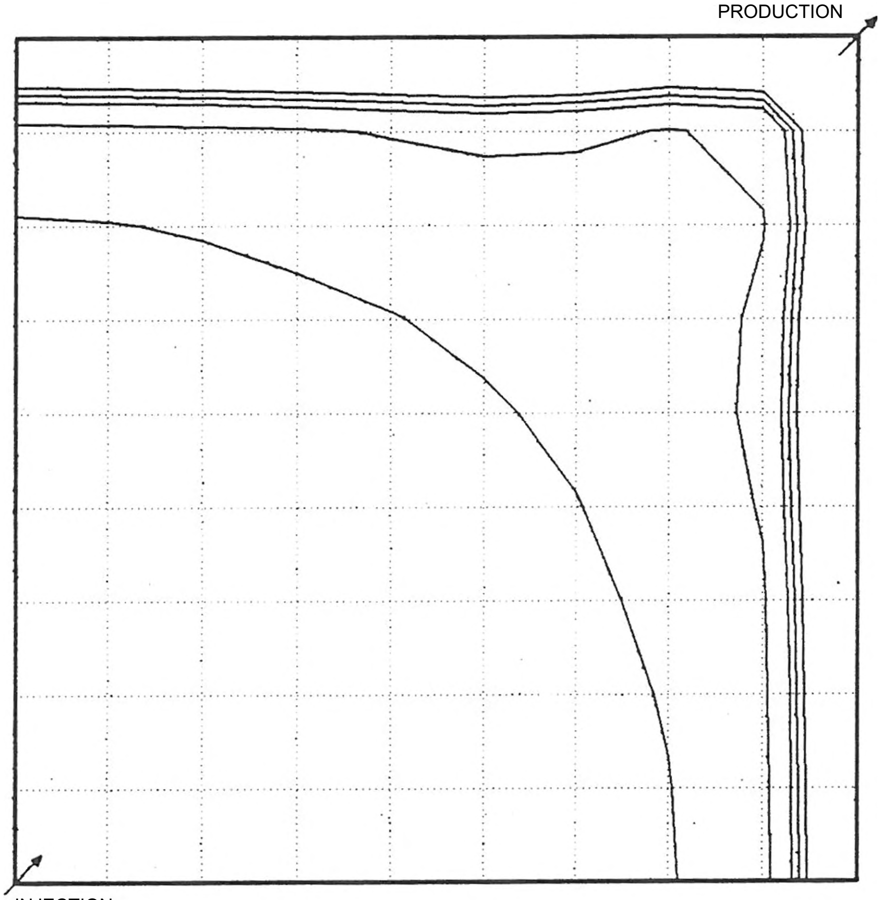


FIG-1 Oc SATURATION CONTOUR PLOT FOR VR = 1.02
AT TIME = 800. SEC., (RATE = 10 cc/sec).

FIG.I 1 a SATURATION CONTOUR PLOT FOR VR = 0.50
AT TIME = 700. **SEC., (RATE = 10** CC/MC).



FI G-1 1 b SATURAT I ON CONTOUR PLOT FOR VR = 0.50
AT TIME = 826. SEC.. (RATE = 10 cc/s·c).

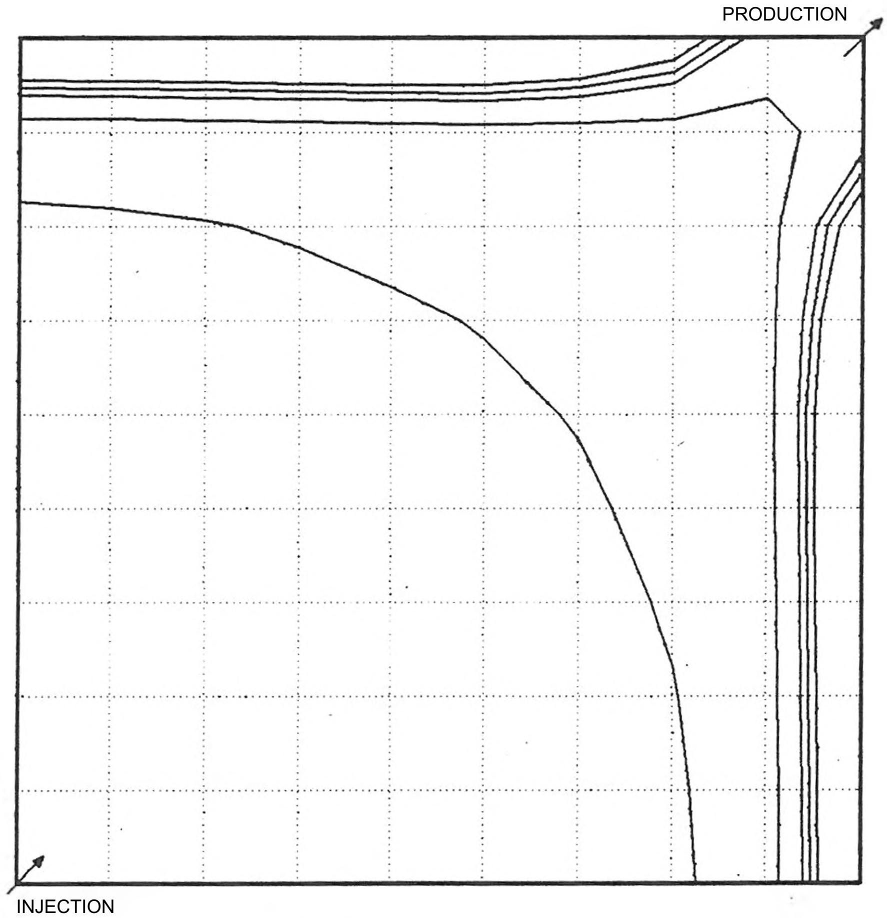
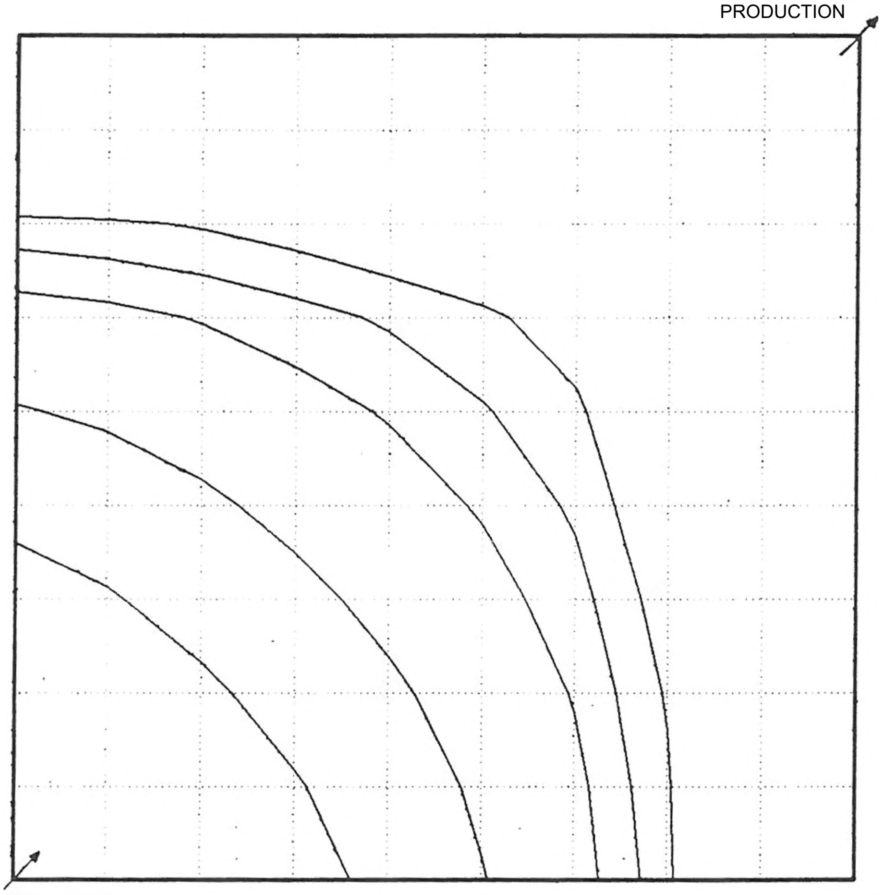


FIG.I 1cSATURATION CONTOUR PLOT FOR VR = 0.50
AT TIME = 880. SEC.. (RATE = 10 ec/soc).



FI G. 12aSAT URAT I ON CONTOUR PLOT FOR VR = 15.70
AT TIME = 340. SEC.. (RATE = 10 ccMc).

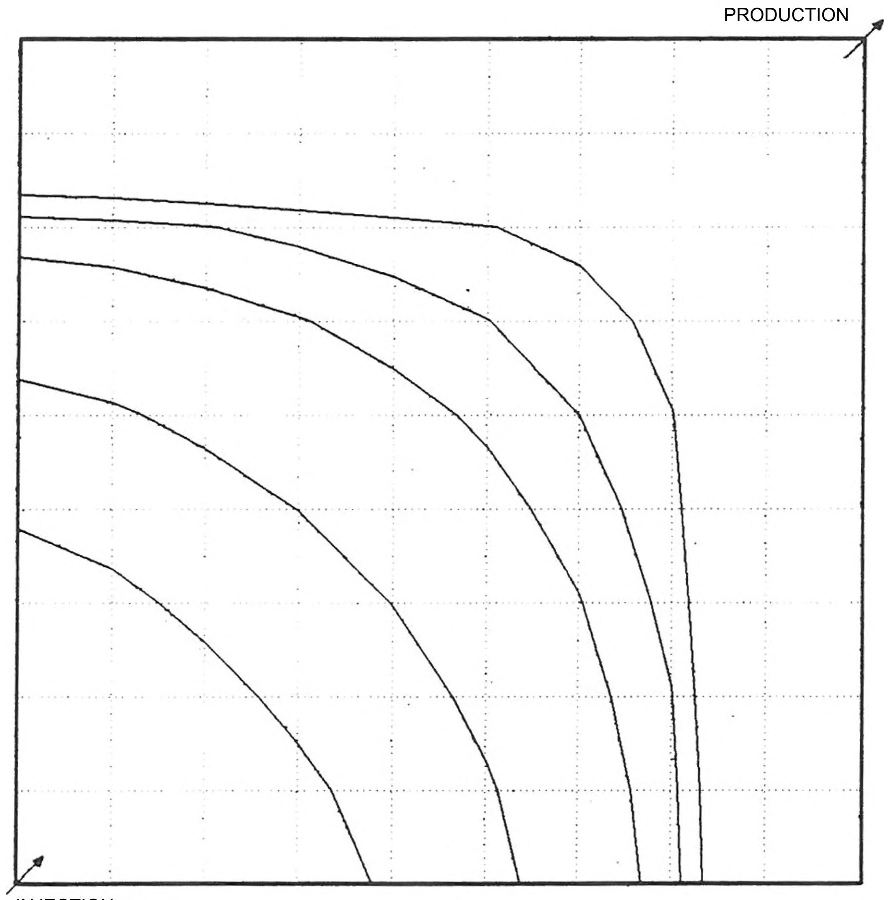


FIG.I 2b SATURATION CONTOUR PLOT FOR VR = 15.70
AT TIME = 391. SEC·. (RATE = 10 cc/eec).

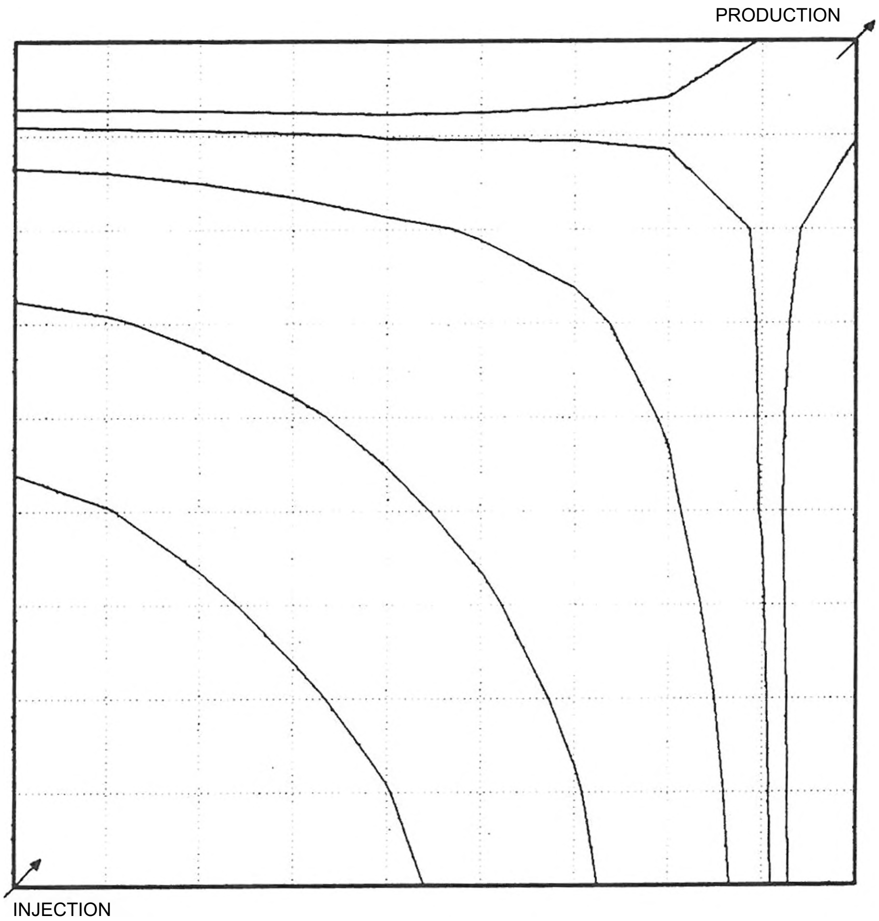


FIG-1 2c SATURATION CONTOUR PLOT FOR VR = 15.70
AT TIME = 560. SEC-. (RATE = 10 cc/sec).

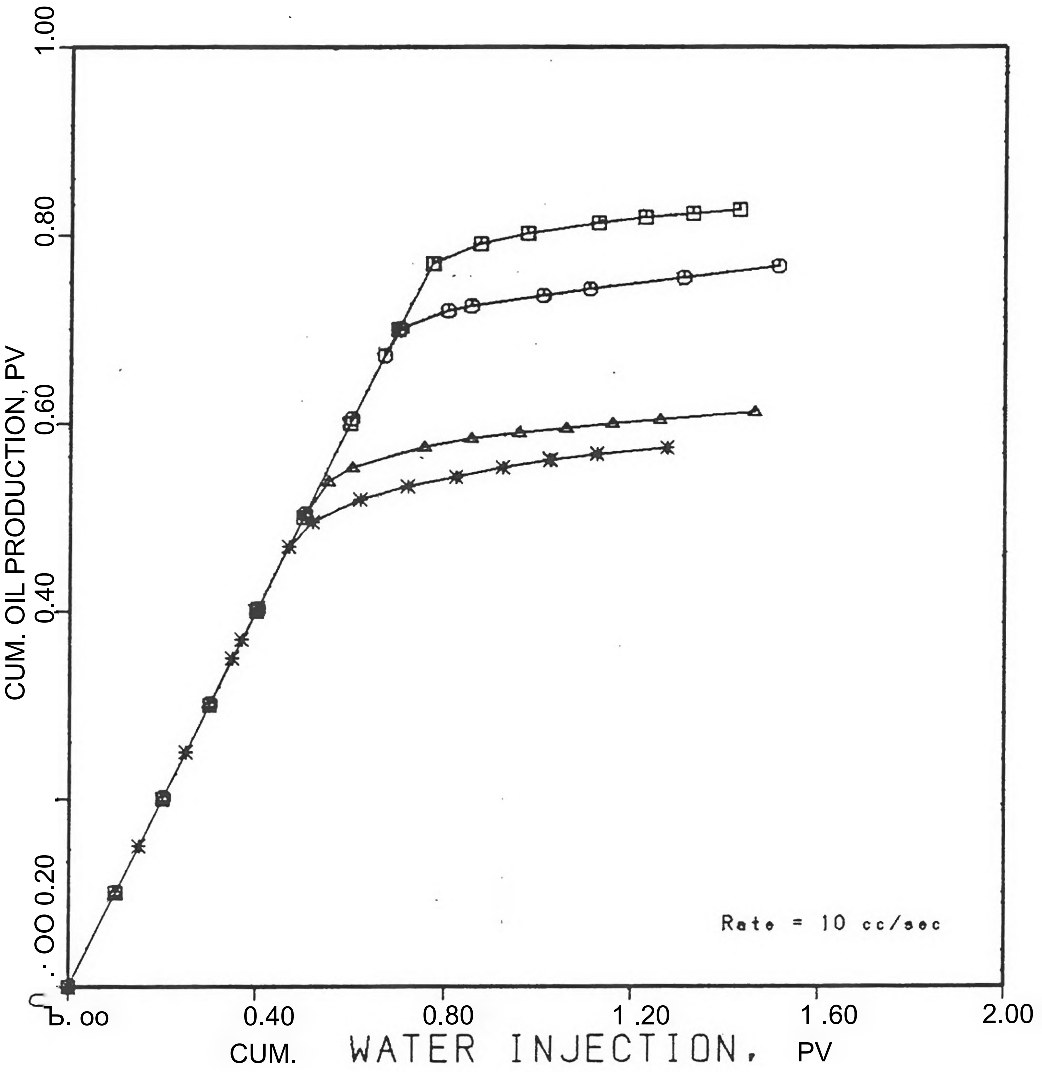


Fig. 13 - EFFECT OF VR ON OIL RECOVERY

Д VR = 0.50 Q VR = 1.02

д VR = 5.42 ✳ VR = B.00

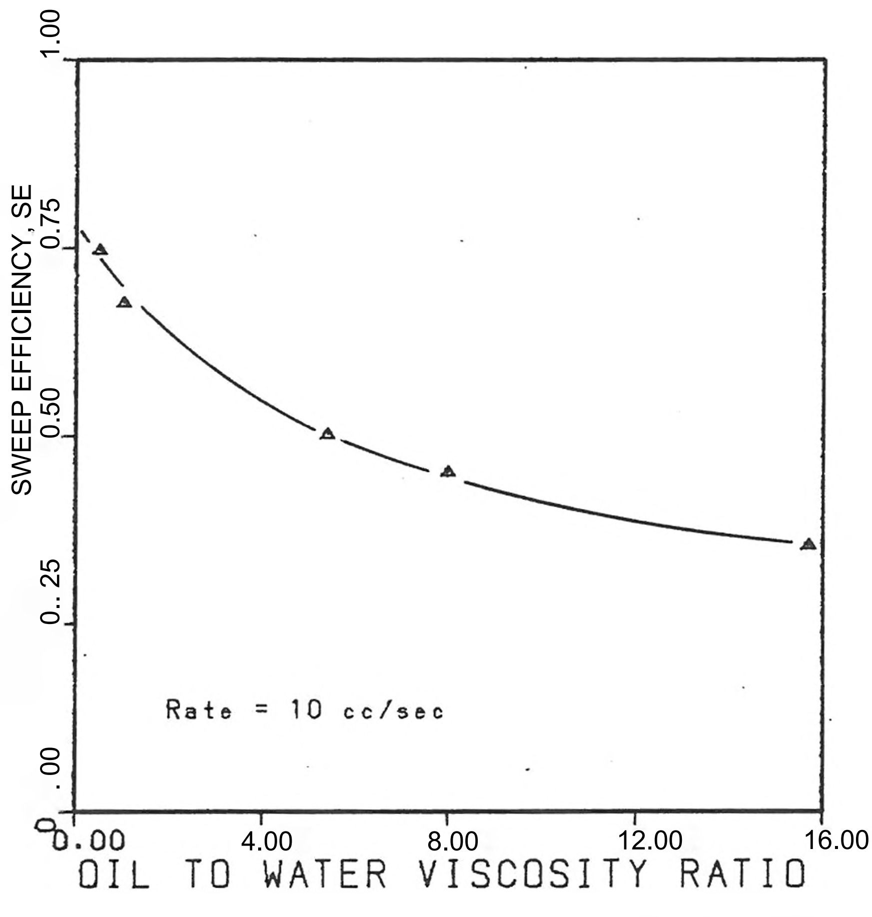


Fig. 14 -EFFECT OF VR ON SE AT BREAKTHROUGH

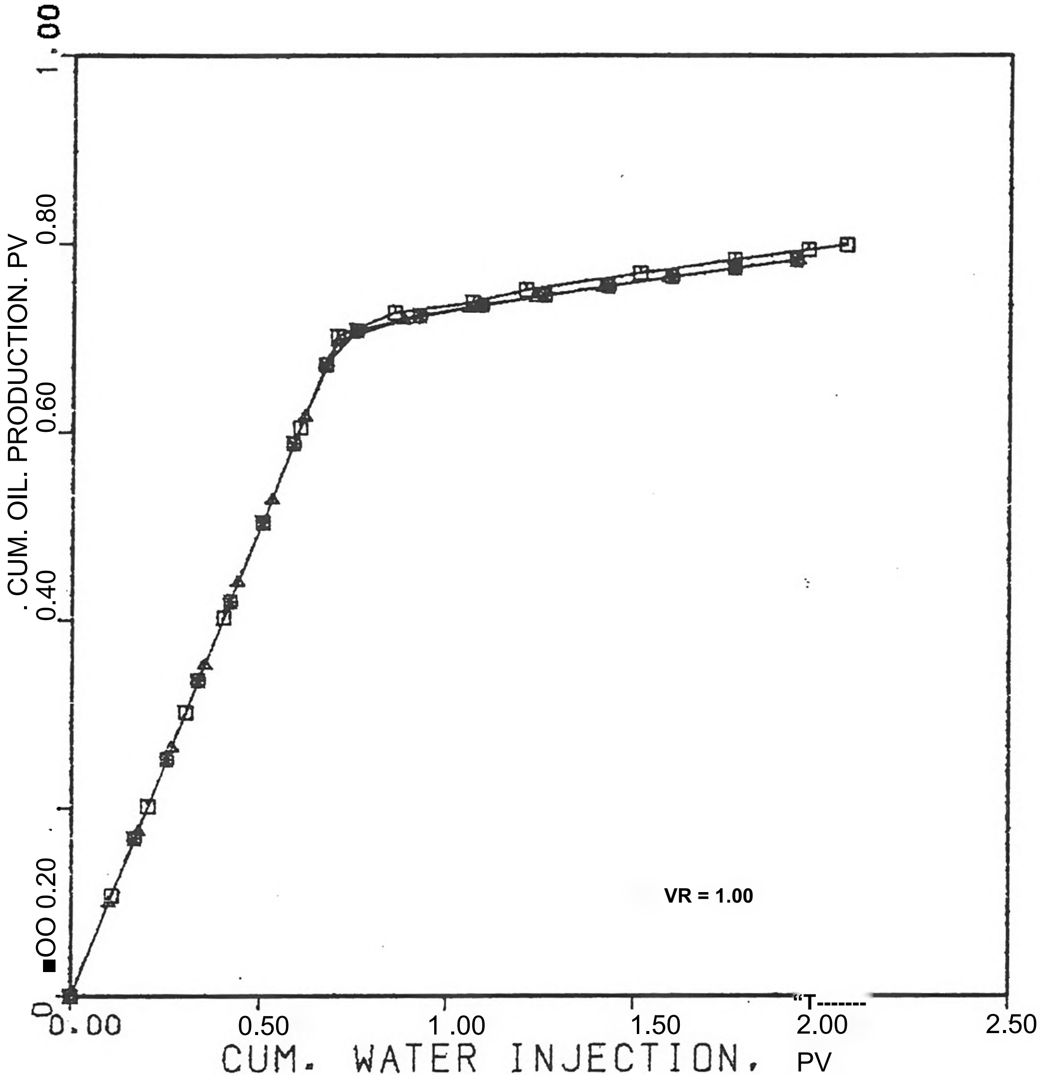


Fig. 15a -EFFECT OF RATE ON OIL RECOVERY

щ RATE = 10 CC/SEC Q RATE * 25 CC/SEC
Д RATE = 35 CÇ/SEC £ RATE * 50 CC/SEC

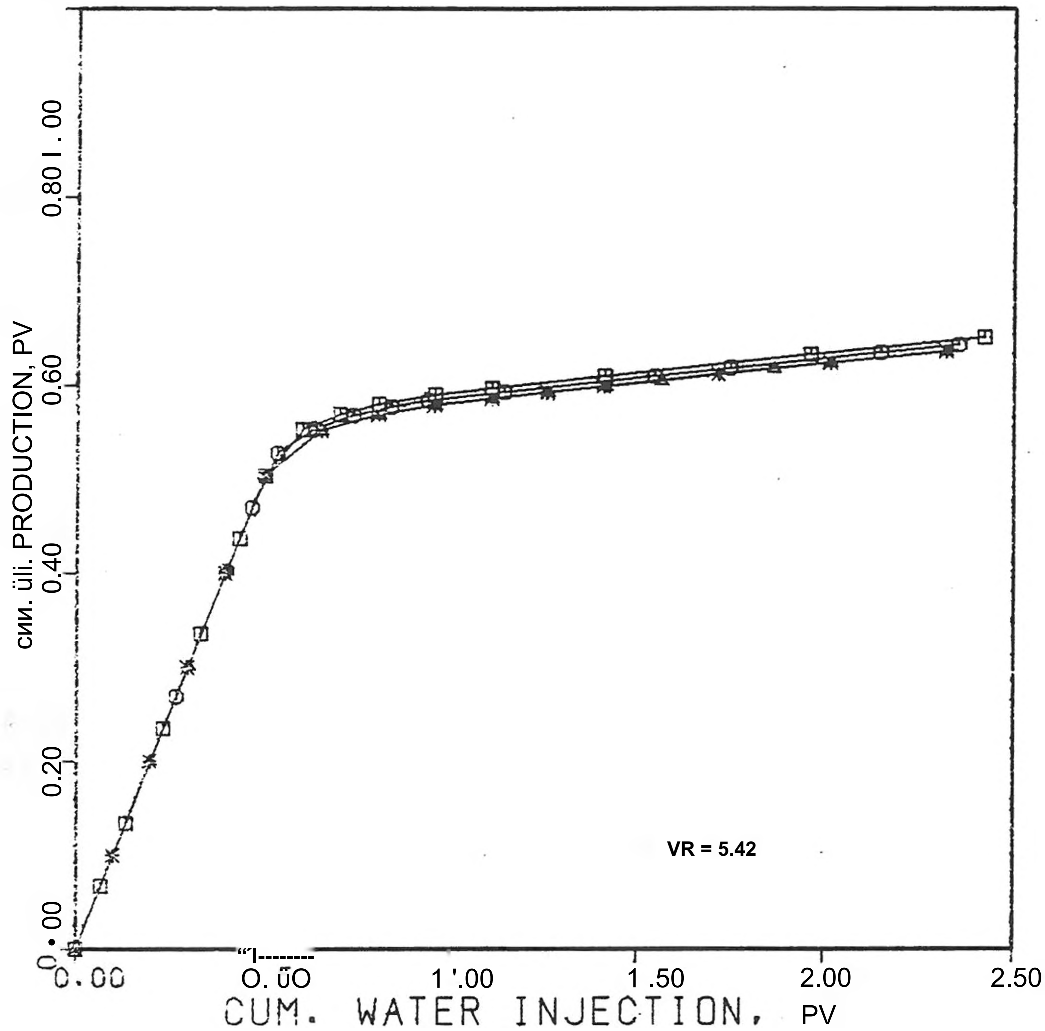


Fig. 15b-EFFECT OF RATE ON OIL RECOVERY

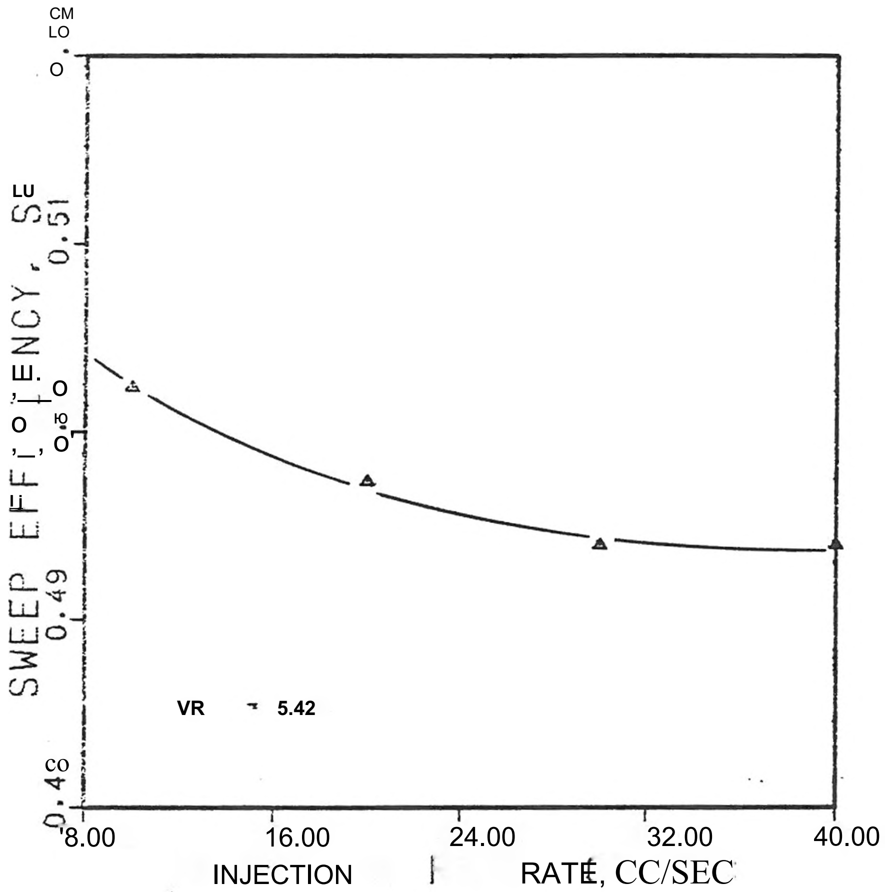


Fig. 16 -EFFECT OF RATE ON SE

Δ At SraakThrough

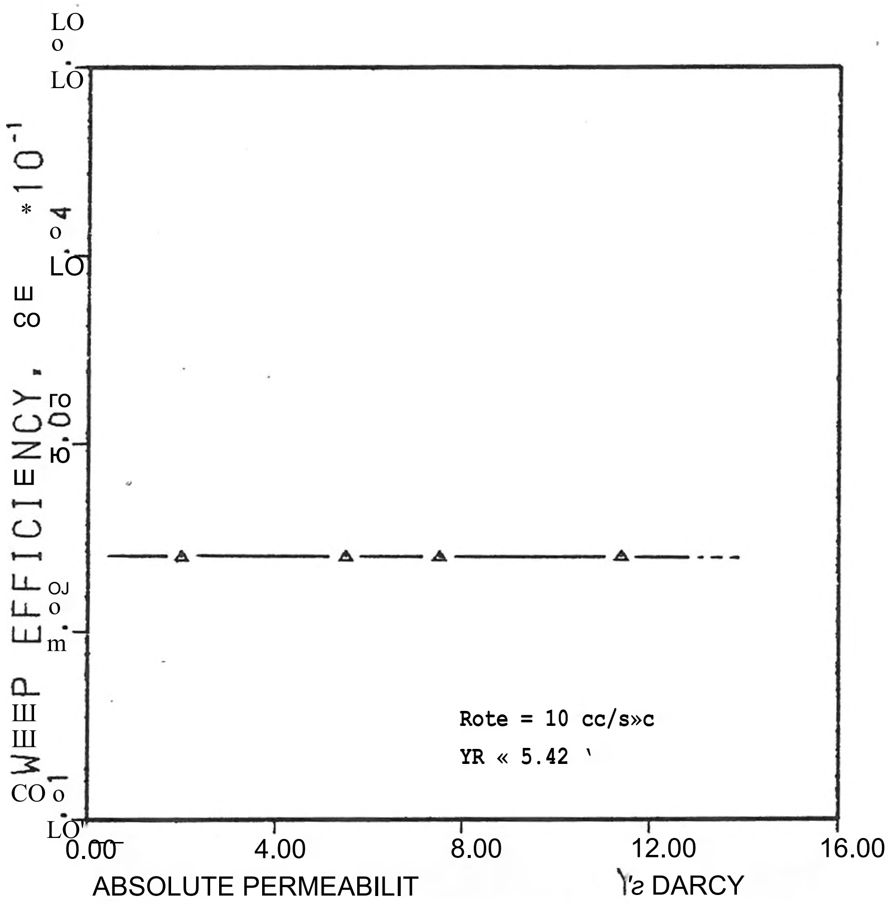


Fig. 17 -EFFECT OF PERMEABILITY ON SE

Δ At Breakthrough

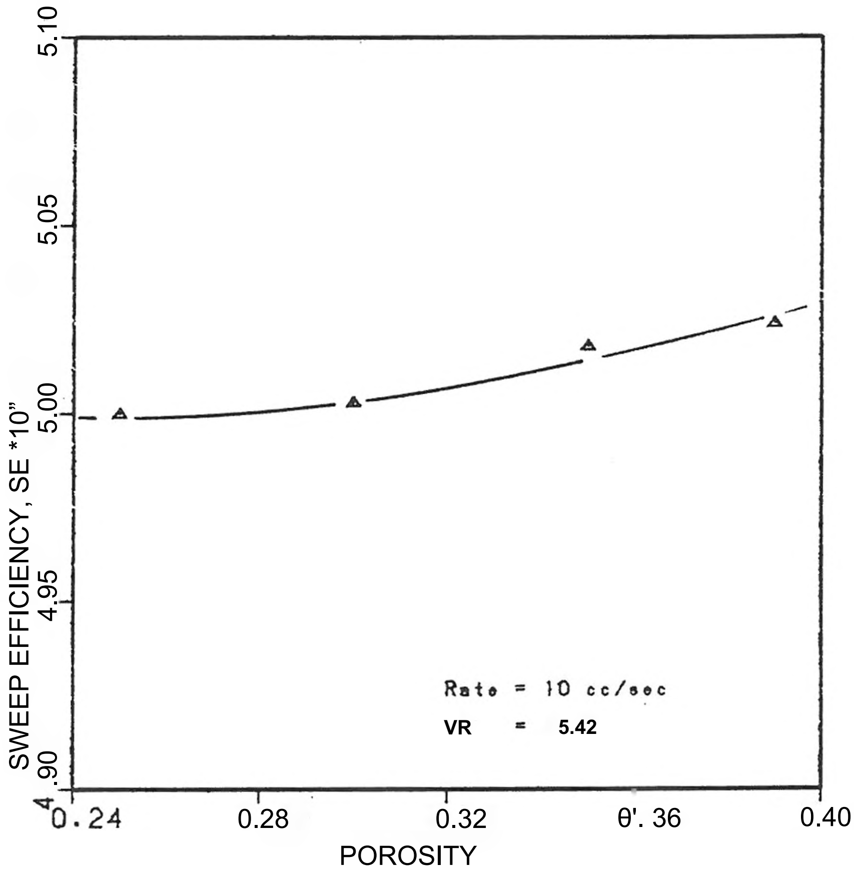


Fig. 18 -EFFECT OF POROSITY ON SE

A At 8reakthrough

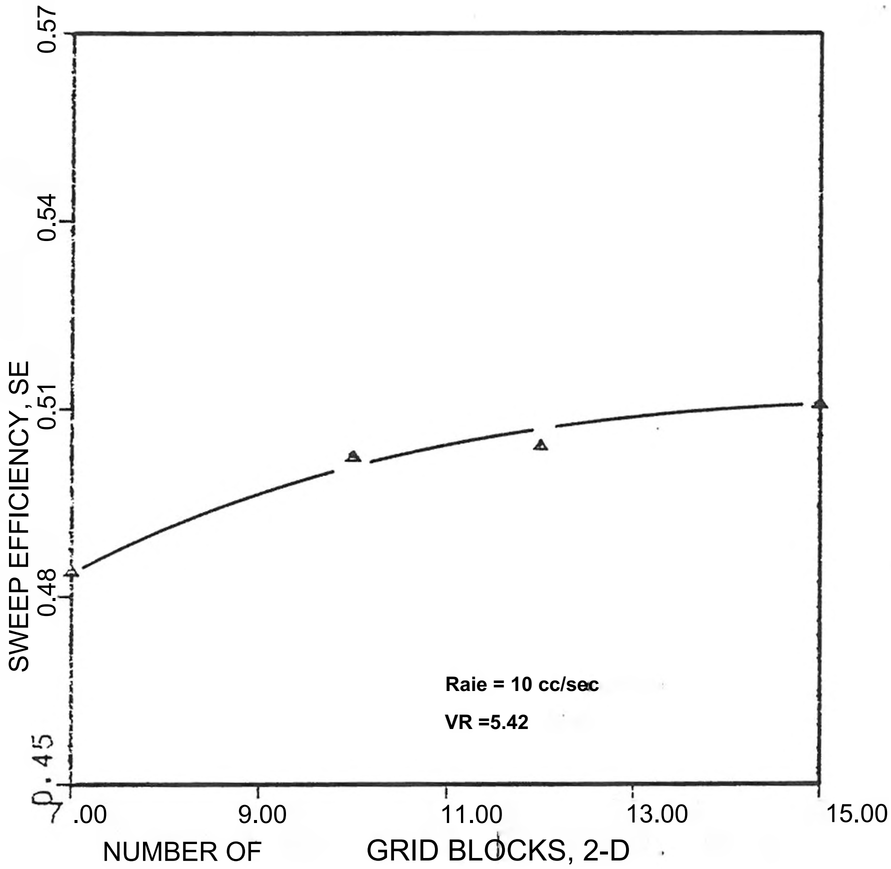


Fig. 19 -EFFECT OF GRID BLOCK ON SE

Δ At Breakthrough

APPENDIX C

OhJEI' DIMENSIONAL MODEL

I. MAIN PROGRAM

A. Array Declaration:

CHIJÍ200) .... : Cue. water injection, pv
C0PDÍ200) .... : Cua. oil production, pv
SN(20> .... : Hater saturation, fraction
KR0(20) .... : Oil relative pereeability
KRW(20) .... : Hater relative pereeability
FH(20) .... : Hater fractional flow values
S(50) .... : Hater saturation

B. Input Data Variables:

M .... : **Nusber of grid blocks**
XLSTH .... : Model length, ce
AREA................. : Cross sectional area, sq.ca
PHI .... : Porosity, fraction
PERU................. : Absolute pereeability, Darcy
SHI .... : Initial water saturation, fraction
SDR................... : Residual oil saturation, fraction
OVIS................ : Oil viscosity, cp
HVIS.................. : Hater viscosity, cp
SHIJ .... : Hater injection rate, cc/sec
DT .... : Tiae step, sec
WORMAX .... : Maxinun allowable producing HOR
MKR .... : **Nuaber of rock data points**
SH,KRH,KRO ···: As explained

C. Output Control Variables:

MAXT .... : Maxiaum nuaber of calculations
ISTEP............. : Interval current result printed tiae
STOL............... : Hater saturation at breakthrough

D. Initialization:

TQ1J .... Cua: water injection, pv
TSWP................ : Cus. water production, pv
OREC .... : Cua. oil recovery, pv
HOR .... : Hater to oil ratio
INDEX .... : Index of CHIJ and CQPD arrays
INC .... : Increaent of ISTEP
ĐTHROU .... : Logical variable to check on breakthrough
ITINE................. : Counter of calculations
TIME .... : Tiae, sec

D. Interiediate Variables:

DWVR .... : Oil to water viscosity ratio
PV .... : Pore volute, cc
ROIP .... Recoverable oil in-place, cc
VB...................... : Unit grid block volute, cc
FAC .... : QWIJxDT/(PHIxVB)
EFF .... : Sweep efficiency, 1

II. SUBROUTINE SODUNV

SS(15) Hater saturation at half tine step
SLESS..................: Value of SCO)
OD,DI,D2.............. : **Dunny** variables
DS .... : Equal 0 or naxiaun value of (DD,D1,D2)
DF .... : Slope of FW at Si
FR,FL ....: Hater fractional flow values
SI6N ....: Sign of DS:
+ if DD > 0
- if DD < 0

III. SUBROUTINE FNDF

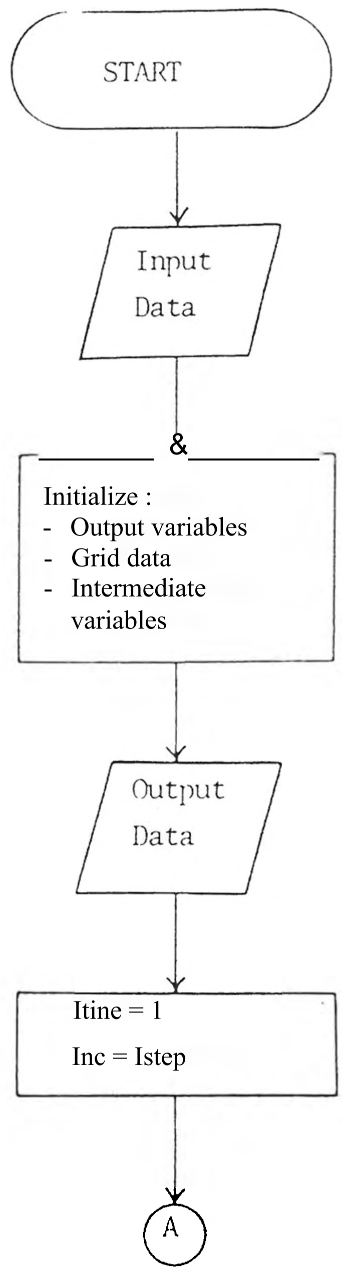
X .... : Value at which slope is calculated
XL,XR .... : Left and right values of X
FL,FR.................: FH values of XL and XR
DF .... : Slope of FW at X

IV. SUBROUTINE INTERP

XX(20),YY(20)..: Tetporary arrays
X .... : Interpolating value
FN .... : Result of interpolation

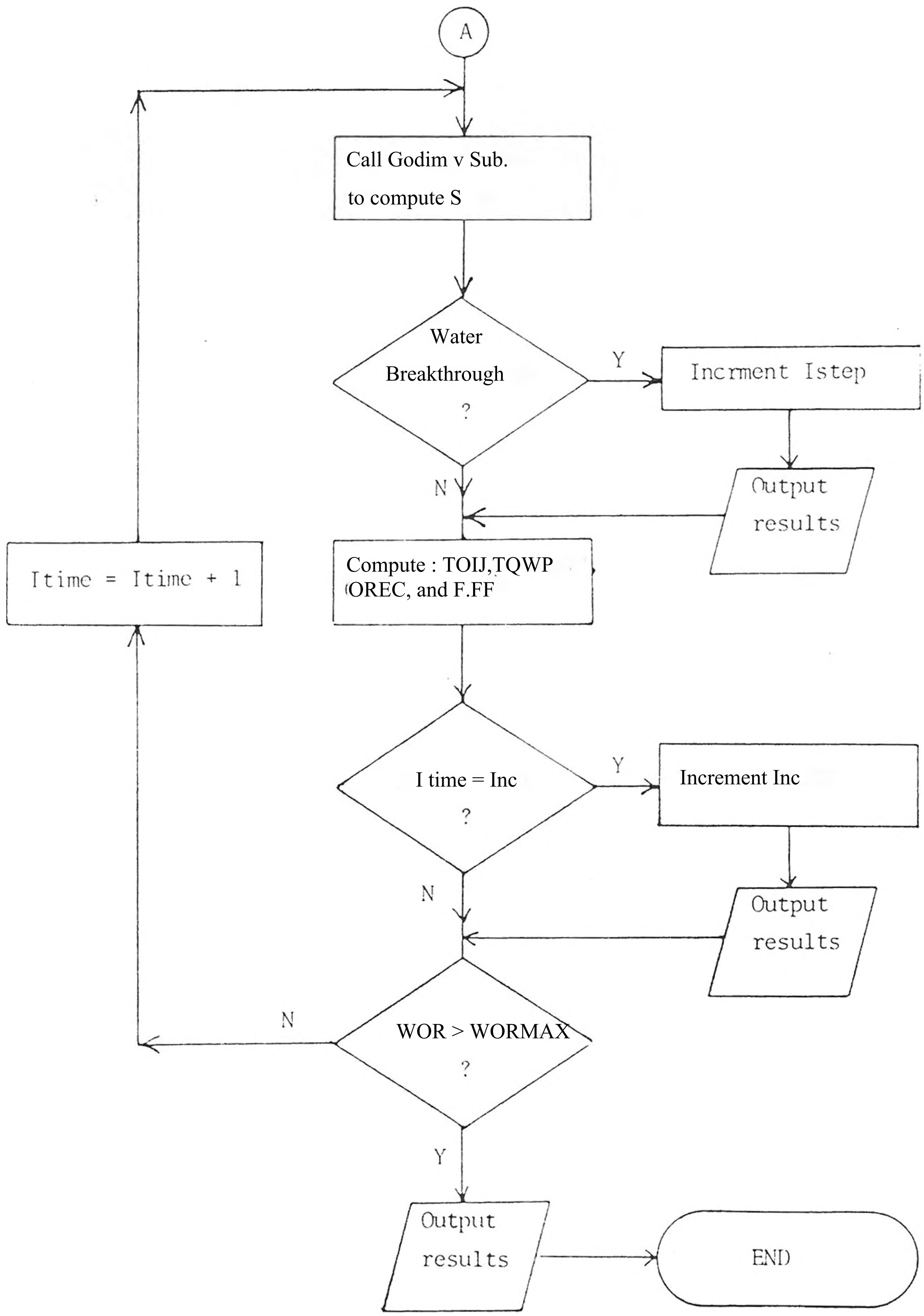
V. SUBROUTINE PRINT

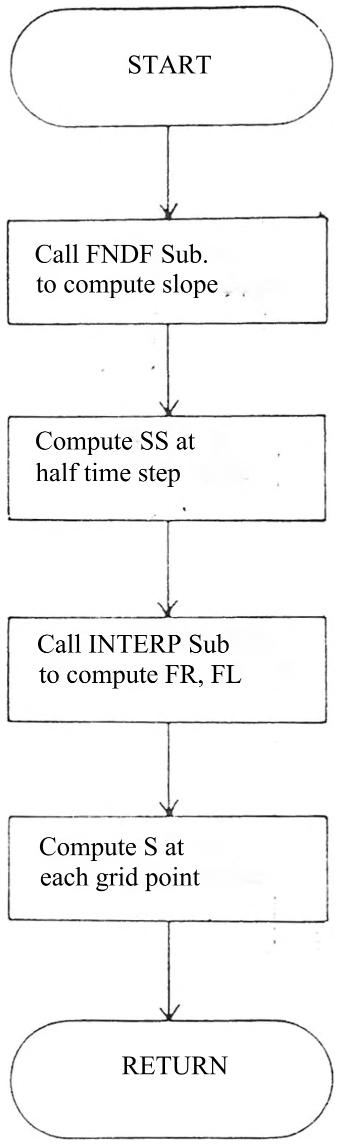
1



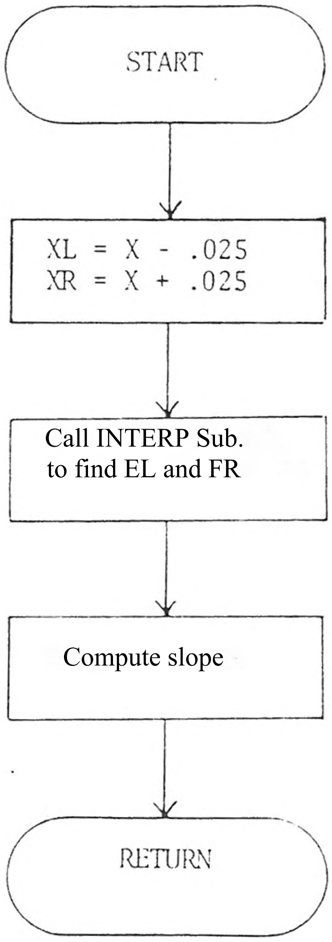
Main Program

2

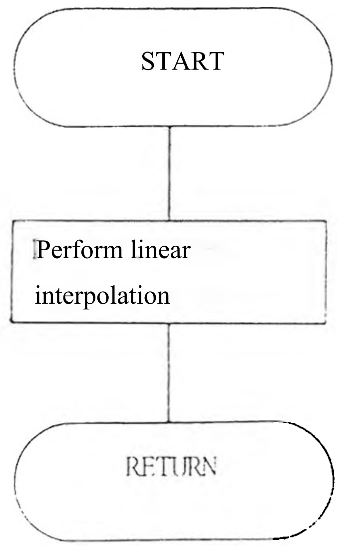

A
Call Godim v Sub.
to compute S
Water
Breakthrough
?
Y
Incrment Istep
Output
results
N
Compute : TOIJ,TQWP
OREC, and F.FF
Itime = Itime + 1
I time = Inc
?
Y
Increment Inc
Output
results
N
WOR > WORMAX
?
N
Y
Output
results
END



Subroutine Godunv



Subroutine Interp



Subroutine Interp

```
          Real CHIÙ(200),C0PD(200)
          Real 5wf20i .FW42O) ,KRÜ(2ö) ,№20)
          Ccsson MKR
          £oææor: /Sat/ S (50)
          Logical Bthrou
 *
 »        ================================================
 t        ===  ONE  DIMENSIONAL  RESERVOIR  SIMULATION  ===
 *        ===    A    STUDY    OF    SWEEP    EFFICIENCY    ===
 »        ===    Prepared    by:    Quyet    V.    Nguyen    ===
 t        === Date: April 10, 1985 -==
 4        ================================================
 i
 t        This program is designed to sinulate a linear
 ł        waterflooding laboratory model . The Bodunov's
 ♦ ·      sethod is used to solve for the saturation.
 *
 *        VARIABLES :
 *
 i           xlgth   ..........:  Length, c®
 *           area    ..........:  Cross sectional area, sq.cs
 *           psi     ..........:  Initial pressure, atm
 *           phi     ..........:  Porosity, fraction
 *           swi     ..........:  Initial water saturation, fraction
 *           sor     ..........:  Residuai oil saturation, fraction
 ♦'          qwi j   ..........:  Water injection rate. cc/sec
 ♦           Cris    ..........:  Oil viscosity, cp
 t           wvis    ..........:  Water viscosity, cp
 *           S       ..........:  Water saturation, fraction
 »           per®    ..........:  Absolute permeability, Darcy
 *           roip    ..........:  Recoverable oil in-place, cc
 *           M       ..........:  Number of grid blocks, M <= 36
 *           MKRW ...........: Number of SW.KRW, and KRO points
 »           dt      ..........:  'ime step, sec.
 4           wormax  .........:  Maximum wor tolerance
 4
 4        ---------------------------------------------------------------
 4
 4        ===    Input Data
 »
             Read(5,1Ô0) M
             Read (5, 1IO) xlgth,area,phi,perm
             Read(5,110) swi, sor, ovi s., wvis
             Read(5,115) qwij,dt,wor®ax
             Read(5,100) MKR
             Read(5,120) (SW(i),KR0(ii.KRWii)= i,MKR:
 4
 i        === Output Control variables
 4
             maxt = 2000
             istep = 20
             tol = .001
 4
 4        === Initialization
 »
             tqij = 0.
             tqwp = 0.
             oree = 0.
```

```
              i.'.Ptt - '
              inc = istap
              Bthrou - .false.
      t
              Do 10 i = 2,«
      10      S(D = 5W1
              3(11 = I.-sor
      +
              Do 15 i = 1,MKR
      15      FW(i? = KRWli)/iKRW(I>+wvis*KR0i i)/ovis)
      *
      t    === Intermediate Variables
      t
              cwvr = ovis/#yis
              pv = area»xlgth*phi
              roi p = pv»íl. -swi-sor
              vb = pv/Л
              fac = qwij*dt/vo
      i
      ♦    === Print out input cata
      j
              Write(6,11) M j x ! gth, ar es. oar o, DVí s. wvi s,phi,
           1 Horna:,5ki ,sor,qwi j,dt
              Kriteii,12)
              ЙГ11 a(6,13 ; ; 3И í i).KRH t s ?,KROí I>.Ри ú ). I=1,«KR)
              яг i ta (6. ÜI
      * .
      !
      ♦    === COMPUTATION BEGINS ===
      ♦
           Do 20 itise = I^axt
              tme = itis8*üt
              Cal 1 ßodunv (ň, f ac, 51í ƒ#!)
      »
      í    === Comte cil recovery. water production, ane HOR,
      *
           1+ (£(«).gt.stol) Ther
              Cali Interpol,Sli,KRM,г.<и)
              Cali In terp (S (H , SW ,I.RO. r ko?
              йor = owvr*rkw/rkc
           Endi f
              qc = quij/(1.+wor)
              qwp = qwij - qc
              cree = oree + qc*dt/roip
              tqwp = tqwp t qwp^dt/roip
              tqij = tqij t qwijłdt/rsip
              aí i = ЮOлcrec
      ♦
      i    === Check for water breaktńrcuoh
      ♦
              It ; (.Not.Btbroji.And.(SÍM:.gt.sto·'' 'hen
                  Bthrou = .true.
                  istap = Ifix(2*i=tepl
                  knta(6,21)
                  Call Print (tires.oree,tqi j,tqwp.wor,ет-ř)
         ' End i ŕ
      *
      ♦    === Check fer output condition
      i
```

```
                  :"z = inc » istep
                  incex = index ♦ 1
                  iCNIJ(index) = tqi j
                  COP'D (index) = oree
                  Cali Print(tiße,orec,tqij.tqwp,wor,eH)
               End if
*
»          == Check for ©sxiRus ЙP
*
               If (wer.gt.wcr.max} Goto 222
     20    Continue
*
     222   ¿ritéti,22)
           Cai; Pr i nt(11 де,or ec,t qi j,t qwp,wer.eff í
           Write(6,23)
           Writ e í µ 24) iCWiJíil ,CQPD(i/ ,i=l,index)
*
           STOP
     и     Formati ' 1 //5x,'===== INPUT DATA =====',/,
          V5x. Grid dimension ........... ........... . . . :    ',12,' Blocks',
          V5x, 'Length, es .................................  :    'ƒ8x
          V5x, '-rea. sc...cs ..........................  . . . . :    ' ,F8X
          V5x, Permeability, Darcy ........................  :     ƒ8.4,
          \/5x.'0il viscosity, co .........                  :    ■ ƒ8.4.
          \/5x,'Nater viscosity, cc ..........................  :    ,F8.4,
          Ł/5x, Porosity ..................                  :    ',F8.4,
          ./5x , 'Naxisus WDR ...............               :    ' ƒ8.4,
          W5x, 'Initial water saturation ..               :     ¿S.4,
          Vox.'Residual cii saturation ...                :     χ8.4, '
          V5x.'Water injec. rate, cc/sec                 :    ',F8.4,
          V5x, 'Tise stec, sec ........... .................  :     ƒ8.4)
     12    Forsat(//5x,Sw —',5x,-                          -- Krw ---' ,5x.
          V — Kro — ,5x,'--- Fw —')
     13    Fornat(/Bx,F6.4,9x,F6. 4.10x,Fi.4             , 10x,Fi.4)
     li    Formati T//5x,'—== CCHPUTATIÖN ===== ,/Л
     ?.*   Forcat(//5x,'===== BREAKTHROUGH =====',/)
     22    Fermatt//5x,SIMULATION RESULT ♦♦***,/)
     07    Fornat(//5x, Cus. Water Inj., pv                 Cua. Gil Fred., pv ,/)
     24    Forsat(lùx, FIO.i,15x,F10.i)
     lûo   Forsat(12)
     llð   Forsat(4F7.3)
     115   Forsat(3F7.3)
     120   Forsat(3F6.4)
           END
•
t          ====================:     ========================
               SUBROUTINE bodunv(N.fac,SW,FN)
           ====================     =========================
·
·
'          This subroutine uses           Godunov s methGö to solve
t          for water saturation           distribution.
»
           Real SW(20),FW(20)
           Rec-1 SS (15;
           Cosmon NKR
           Coscon /Sat/ S(5Ú;·
'
                 Do 610 i = 1,M
                     siess = S(i)
```

```
                    i· -u',i,£.c£:   "z'n"i
                    6: = 2.» Síi+lJ-slessi
                    dl = 2. »«Síi-Hl-Síi )
                    G2 = 2.*(S(i)-sless)
                    ds = 0.
                    Ií (dl*d2.gt.O.)· Then
                       sign = 1.
                       Ií (dd.it.O.) sign = -1.
                       ds = Absida)
                       If (Abs(dl).lt.d=) ds = Absidi
                       It ìAbs(d2).lt.ds) ds = Abs(d2>
                       ds = dsHign
                    End if
                    Cail FndfíSíiijSH.FW^f)
                    tesp = .5*il.-fac*df Hd=
                    SS ( i ) = Síi) * tenp
blu              Continue
*
                 Do 620 i = 1,M
                    siess = Síi;
                    If (i.gt.l) siess-= SS(i-l)
                    Cali Interpisless^WD
                    Call Intera(SS(i).S^Ei^fr)
                    temp = fac*(fl - fr)
                    Síi) - Síi: + temp
t-2?             Continue
             RETURN
             END
*
             ============================================
                  Subroutine Fndf(x,SW,FW,df)
             ============================================
*
*            "his subroutine calculates the slope of the
*            fractional flow curve at x. The slope, df.
*            is needed for the Godunov s method.
*
             Real SWi20),FM(20)
             Сожкс-л NKR
*
                  xl = x - .025
                  xr = x + .025
             Call InterpixrjSWjFHjfri
             Call Interpol,SW,FK,fl)
                  df = (fr-fl)/(xr-xl!
             RETURN
             END
*
             ==============================================
                  SUBROUTINE Interp(x,xx,yy,fn)
             ==============================================
*
*            This subroutine performs linear interpolation
*
             Real XX(20).yy(20)
             Common NKR
*
                  If ix.ge.xx(NKR)) Then
                       in = yy(MKR)
                  Else if Ix.le.xx(1)) Then
```

```
            fise
            Do 501 n = Í,M.R
            501 If (x.lt.xx(M) Goto 555
            555 hi = h - 1
            fn = yy(hl)+(x-xx(hl))*(yy(h)-yyihl))/
        i (xx(h)-xx(hD)
            End if
        RETURN
        END
            *
»       ================================================== ===
            SUBROUTINE Print(tise,orec,tqij,tqwp,wor,eff)
í       ================================================== ===
*
♦       This subroutine outputs the current condition,
♦
Comeon /Sat/ SÍ50;
*
        Write(6,701) time
' Writeí6,70ð)
        write(6,710)    (Síi),    i=l,12^
        Nrita(6,710)    (Síi),    1=13,24;
        hiteié,710)     (Síi),    i=25,3o)
        Write(6,7öó)    tqij,oree,tqwp,eff,wer
        *
        RETURN
        704 Porinatí/5x, »♦» Time: ',F!0.4, Sec. >
        700     Forsat(/5x, === SATURATION DISTRIBUTION ===')
        710     Forsat(/5x,12F10.4)
        706 Forsat'/5x,'Current Inj.^Prod. : ,/,
        i/5x, 'Cas.    water Inj.................:    '510.3.   pv',
        W5x,'Cuæ.    Oil Prod.................:     .F10.3,   pv ,
        L/5x/Cus.    Sater Prod.      ....:        '.F10.3,   pv',
        V5x, 'Displacement Ef...: '.F10.3, V,
      V5x,'Water to Oil Ratio            ..:    «F10.3,
        &/1x,'==============================================')
        END
```

APPENDIX D

TWO DIMENSIONAL FIODEL

MAIN PROGRAM

A. Array Declaration:

1. One Disensional Array:

CHIJ(200) ....: Cum. water injection, pv
C0PD(200> ....: Cue. oil production, pv
SN(20) ....: Hater saturation, fraction
KR0(20) .... : Oil relative perseability
KRW(20) .... Hater relative perneabi lity
FWÍ20) ....: Hater fractional flow values

2. Two Dimensional Array:

TL(15,15) ....: Total «ability
P(15,15) ....: Pressure, at»
5(15,15) ....: Hater saturation, fraction
VK15,15) .... : Velocity in x-direction, cn/sec
VY(15,15) ....: Velocity in y-direction, c»/sec
AE(15,15),AH(15,15).: Matrix coefficients
AN(15,15),AS(15,15).: Matrix coefficients

B. Input Data Variables:

I0P ....: Pressure output variable, (1 or -1)
M................ : MxM grid block dimension
XLGTH .... : Model length, c»
YLSTH ....: Model width, ca
DZ .... : Model thickness, cя
PHI ....: Porosity, fraction
PERM............. : Absolute peneability, Darcy
PHI ....: Initial pressure, at»
SHI ....: Initial water saturation, fraction
SOR .... : Residual oil saturation, fraction
OVIS.............. : Oil viscosity, cp
HVIS .... : Hater viscosity, cp
8HIJ ....: Hater injection rate, cc/sec
DT.......... : Tifie step, sec
WORMAX .... : Naxinun allowable producing NOR
NKR ....: Nueber of rock data points
SH,KRH,KRO.: As explained

C. Output Control Variables:

MAKT ....: Haxinua nueber of calculations
ISTEP........... : Interval current result printed ti«e
STOL .... : Hater saturation at breakthrough

D. Iteration Control Variables:

IMAX .... : Maxiaue nuaber of iterations
TOL .... : Naxiaun difference
BETA ....: SOR acceleration factor

E. Initialization:

TGIJ ....: Cua. water injection, pv
TSMP .... : Cua. oil production, pv
OREL............. : Cub. oil recovery, pv
KOR .... Hater to oil ratio, cc/cc
INDEX.................. Index of CMIJ and COPO arrays
INC ....: Increaent of ISTEP
BTHROU ....: Logical variable
ITINE ....: Counter of calculations
TINE ....: Tiae, sec

F. Interaediate Variables:

NLESS............ : N - 1
DI ....: Grid block length, ca
DY . ... : Grid block length, ca
PV...................: Pore volume, cc
ROIP................: Recoverable oil in-place, cc
IOWVR ....: Oil to water viscosity ratio
DX2,DY2 ....: Dunny variables
FACI ....: DT/(PHIxDX)
FACY..............: DT/(PHIxDY)
9Π .... : Velocity in x-directi'on, ca/sec
6IY................ : Velocity in y-direction, ca/sec
IQIAVE .... : Average velocity, ca/sec
80 ....: Oil production rate, cc/sec
9HP...................: Hater production rate, cc/sec
EFF ....: Sweep efficiency, ï

II. SUBROUTINE NOB

RKO .... : Oil relative peraeability
RKH .... : Hater relative peraeability

III. SUBROUTINE COEFS

IV. SUBROUTINE PSOR

ERROR ....: Maxinua difference of P
DELTA ....: Source/sink values
PXLESS.........: P(O,j), j = 1,2,...,H
PXPLX ....: P(NH,j), j = 1,2,...,N
PYLESS.........: P(i,0), i = 1,2,...,N
PYPLX ....: P(i,N+l), i = 1,2,...,N

TEMP .... : Temporary storage
PP .... : Retain part of SOR iteration

V. SUBROUTINE VELO

VI. SUBROUTINE GODUNV

SS(15) ....: Water saturation at half tiie step
SLESS ....: Value of S(0,i) or S(i,0), i = 1,2,..,«
DD,D1,D2 Duany variables
DS . ...: Equal 0 or naxinun value of (DD,D1,D2)
DE .... : Slope of FW at sote S
PR,FL ....: Water fractional flow values
SIGN ....: Sign of DS:
+ if DD > 0
- if DD < 0

VII. SUBROUTINE FNDF

X ....: Value at which slope is calculated
XL,XR .... : Left and right values of X
FL,FR........... : FN values of XL and XR
DF .... : Slope of FW at X

VIII. SUBROUTINE INTERP

XX,YY............ : Tesporary arrays
X ....: Interpolating value
FN.................: Result of interpolation

IX. SUBROUTINE PRINT

RESERVOIR SIMULATION

$$j \frac{dS}{d't} * V_t WS» - 0 \qquad (1)$$

$$V_t = Ć \qquad (2)$$

$$V_t = -\backslash(S)VP \qquad (3)$$

Eq. 1 is the aas5 conservation equation
Eq.2 is the continuity equation for iacospressible fluid
Eq.3 is the Darcy's law

SPLITTIN6 DIRECTION:

Eq.I can be rewritten in two disensional for· as follows:

$$X\frac{dS}{dt} + V_x\frac{dFŒI}{dx} = 0 \qquad (4.a)$$

$$Я\frac{dS}{dt} + V_y\frac{dHS»}{dy} = 0 \qquad «.bl$$

ALGORITHM TO OBTAIN S^{n+1} FROH S^n:

1. Select grid disensión, Fig.I
2. Initialize saturation and pressure at each grid point.
3. Coapute À/S) at each grid block
4. Solve for P implicitly at ti«e level nH fron Eq.2 and Eq.3
5. Cospute V_x and Vy at each grid block
6. Advance S fro· tine level n to n+1, using Splitting Alternate Direction and Godunov Methods
7. Check if water breakthrough occurs
8. Print out the results at selected ti»e step
9. Go to step 3 until aaxinus NOR is «et

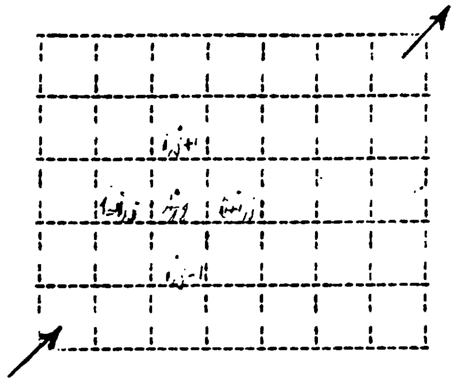


Fig.I - 6rid dimension

INITIAL AND BOUNDARY CONDITIONS:

Síi,-j) =Sni t >= O

S(I,I) =1. - Sor t >« O

P (i, j) sPwi t = 0

For i = 1,2,...,M and j = 1,2,...,M

$\frac{dS}{dn} = 0$

$\frac{dP}{dn} = 0$

where n is nonai vactor to ths boundaries.

Boundary Condition lepi esentati on:

PWJł_:_PIhil = o / dx —> PÍO,j) = Pd,j), j =1,2,...,м

PIIÆC-PIlil' = 0 / ďy —> Pii,OI = Pii,1), i =1,2,...,N

P^ЫКс.РФхИ = 0 / dx —> PíWfP e ... J =1,2,...,N

Pií,NHI - P(i,N) = 0 / ďy —> Pii,MH) = Pii,N), i =1,2,...,N

$$\frac{S(0,j) - S(1,j)}{dx} = 0 \longrightarrow S(0,j) = S(1,j), \qquad j = 1,2,\ldots,N$$

$$\frac{S(i,0) - S(i,1)}{dy} = 0 \longrightarrow S(i,0) = S(i,1), \qquad i = 1,2,\ldots,N$$

$$\frac{S(M+1,j) - S(M,j)}{dx} = 0 \longrightarrow S(N+1,j) = S(N,j), \qquad j = 1,2,\ldots,N$$

$$\frac{S(i,N+1) - S(i,N)}{dy} = 0 \longrightarrow S(i,N+1) = S(i,N), \qquad i = 1,2,\ldots,N$$

PRESSURE EQUATION:

^\(S)VP = 0 (5)

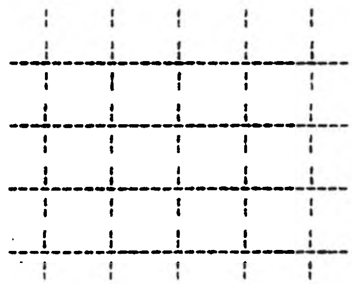


Fig.2 - Five-point difference schese

Five Point Difference:

let IL = ^(S), then:

^RVP = 0 (6)

The five-point difference schese of Eq.ó (see Fig,2 above) has the following fore:

-I-(TL(i-I/2,j)P(i-I,j)$^{n+i}$ - (TKi-I/2, j) + TKi+í/2. j))P(i,j)$^{n+I}$
DX^2 ♦ TL(i4/2,j)Hi+I,j)^{hf}}

+

_* (TL(i,j-I/2)P(i,-i (TL(i,j-1/2) ♦ TL(i,j4/2))P(i,j)$^{n+1}$
DY^2 ♦ TL(i,j+I/2)P(i,j*I)^{n}4

= 0 (7)

Let: AE(i,j) = TL(iH/2, j)/DX^2

AH(i,j) = TL(i-I/2,j)/DX^2

AN(i,j) = TL(i,j*I/2)/IY^2

AS(i,j) = TL(i,j-1/2)/DY^2

Then Eq. 7 becoses:

$$\begin{aligned} &AK(i,j)P(i-I,j)^{n*1} ♦ Ae(i,j)P(iH,j)^{n+1} - d(i,j)P(i,j)^{n+1} \\ ♦ \\ &AS(i,j)P(i,j-I)^{n+1} ♦ AX(i,j)P(i,j+1)^{"*1} = û \quad (8) \end{aligned}$$

Khere:

$$d(i,j) = AI(i,j) + AE(i,j) * AS(i,j) * AN(i,j)$$

THE SOR ITERATION METHOD:

The SOR aethod is used to solve for P ieplicitly, with the previous specified initial and boundary conditions. If the injection rate and the production rate are equal, then the SOR iteration schese is:

$$P(i,j)^{k+1} = P(i,j)^{k} ♦ beta((AH(i,j)P(i-I,j)^{kU} + AE(i,j)P(i4,j)^{nH}$$
$$* AS(i,j)P(i,j-D^{nH} ♦ AN(i,j)P(i,j*n^{nH} * deltal/d(i,j)$$
$$- P(i,j)^{n}I$$

(9)

Mherez delta ᶳ

⎧	Save	if i = I and j = 1
⎨	- Save	if i = Я and j = M
¿	0	Elsewhere

Dave = $\frac{Six + Qiy}{' 2."}$

Six = $\frac{QWIJ}{DYxDZ}$

8iy = $\frac{QWIJ}{DXxDZ}$

ÖWIJ : Nater injection rate, cc/sec
DX : Grid block length, ci
DY : Brid block length, ce
DZ : Nodel thickness, ce

t No flow boundary condition is isplenented by setting transnissibi 1 i ty at boundary equal to 0.

SATURATION EQUATION:

Fro· Eq.4a and Eq.4b:

-gas * Vx dE(S)
It ¿X (4.a)

-gas * Vy dE(S 1 * ○
■at ¿y (4.b)

60DUMOV'S METHOD

1. I direction:

S(i,j)n+l ' S(M)n ♦ dt VX(i,j)(F(S(i-I/2,j)n+^2) -
ddy F(S(i4/2,j)n+I/2»}
(10)

Where:

S(i*I/2,j)n+^/2 = S(i,j)n ♦ .5(1 - _dt_ VX(i,j)F'(S(i,j)n)}ds(i,j)
Йx

ds(i,j) 3 j Hin<|dd<i,j>J,|dt(i,j>I,fd2(i,j>|I<Si9n of dd(i,j> if dl(i,j)xd2(i,j) > 0
I o Otherwise

dd(i,j) = 2{S(i4,j)n - S(i-1,j)n>

dl(i,j) = 2(S(iH,j)n - S(i,j)n)

d2(i,j) = 2{S(i,j)n - S(i-i,j)n)

vx(i,j) = - TL(i-i/2,j) .ElixiL-PlizLiiL
dx

2. Y Direction:

$$S(i,j)^{n*\wedge} « S(i,j)^{n} + \frac{dt}{\phi dy} VY(i,jHF(S(i,j-I/2)^{*u/2}, F(S(i,jH/2)^{n+1/2})\} \quad (11)$$

Where:

$$S(i, j+I/2)^{n*\wedge 2} = S(i,j)^{n} * .5(1 - \frac{Jt}{\phi //} VY(i,j)F^{*}(S(i,j)^{n})>ds(i,j)$$

$$ds(i,j) = \begin{cases} Kin(|dd(i,j)i\ JdI(i,j)|,|d2(i,j)HxSign\ of\ dd(i,j) & \text{if } dI(i,j)xd2\ (i,j) > 0 \\ 0 & \text{Otherwise} \end{cases}$$

$$dd(i,j) « 2(S(i,jH)^{n} - S(i,j-I)^{n}\}$$

$$dI(i,j) = 2(S(i,j+i)^{n} - S(i,j)^{n})$$

$$d2(i,j) = 2(S(i,j)^{n} - S(i,j-i)^{n})$$

$$VY(i, j) = - TL\ (i, j-1/2)\ \frac{Pütil-Z-HLi:!!}{dy}$$

TOTAL NOBILITY :

TL(i,j) « K (Krnil,j) + Ä Kro(i,j)}
ƒ N /0 (12)

VELOCITY:

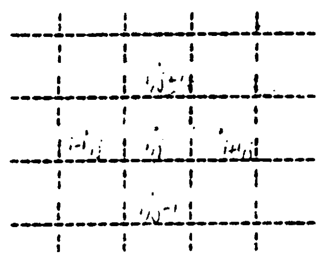


Fig.3 - Velocity at half grid block

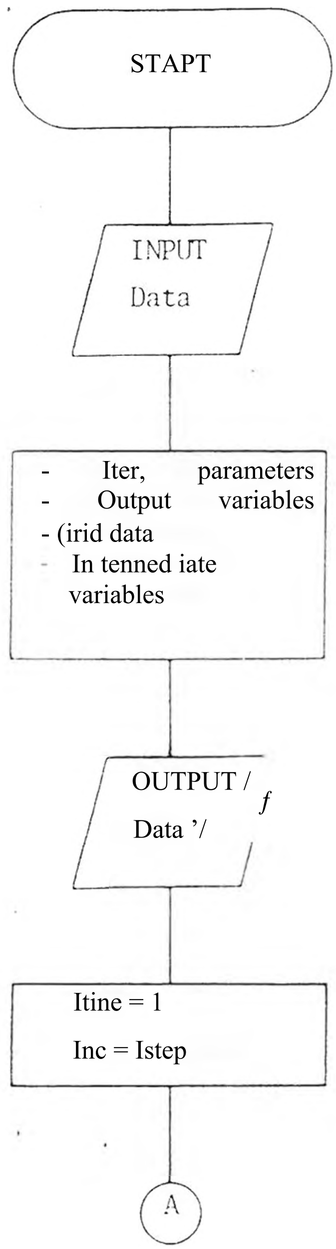
$$vx(i,j) = - TL(i-i/2,j) \frac{P(i,j) - P(i-1,j)}{dx}$$

VY(i,j) = - TL(i,j-i/2) -PlulLz-ükizll
dy

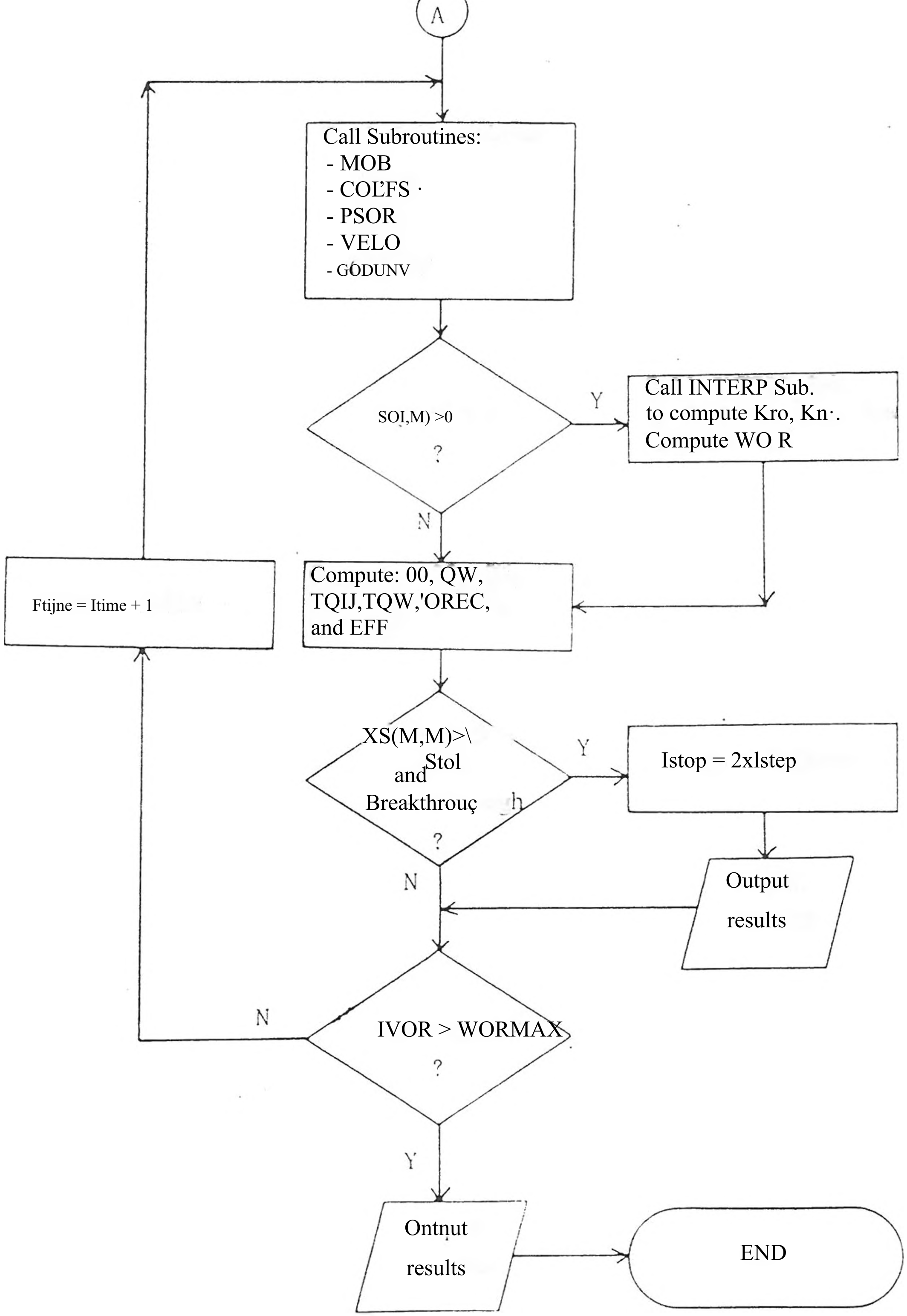
TL(i-i/2,j) = IL(iJLt.IUjzliiL
2

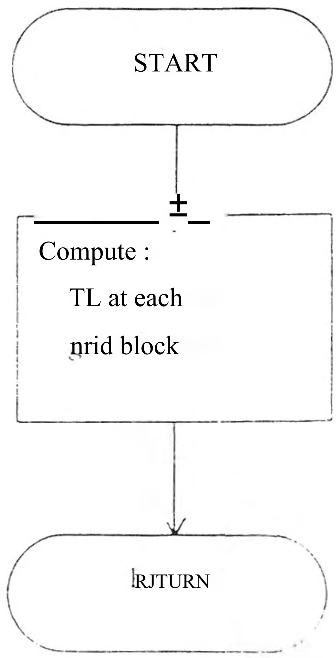
τι (i, j-1/2) = .TLiLiLt.IIIIďdI
2

1



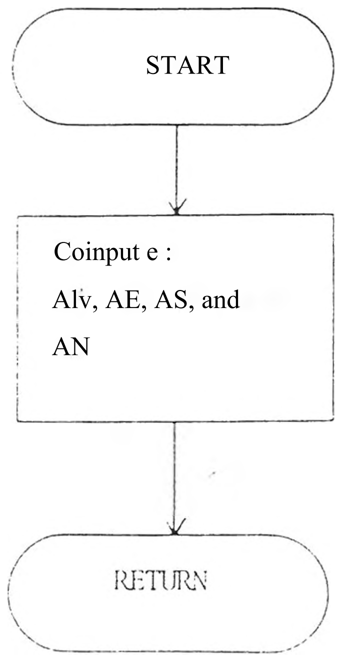
Main Program


A
Call Subroutines:
- MOB
- COLFS
- PSOR
- VELO
- GODUNV
SOI,M) >0
?
Y
Call INTERP Sub.
to compute Kro, Kn·.
Compute WO R
N
Compute: 00, QW,
TQIJ,TQW,'OREC,
and EFF
Ftijne = Itime + 1
XS(M,M)>\
Stol
and
Breakthrouç h
?
Y
Istop = 2xlstep
N
Output
results
N
IVOR > WORMAX
?
Y
Ontnut
results
END

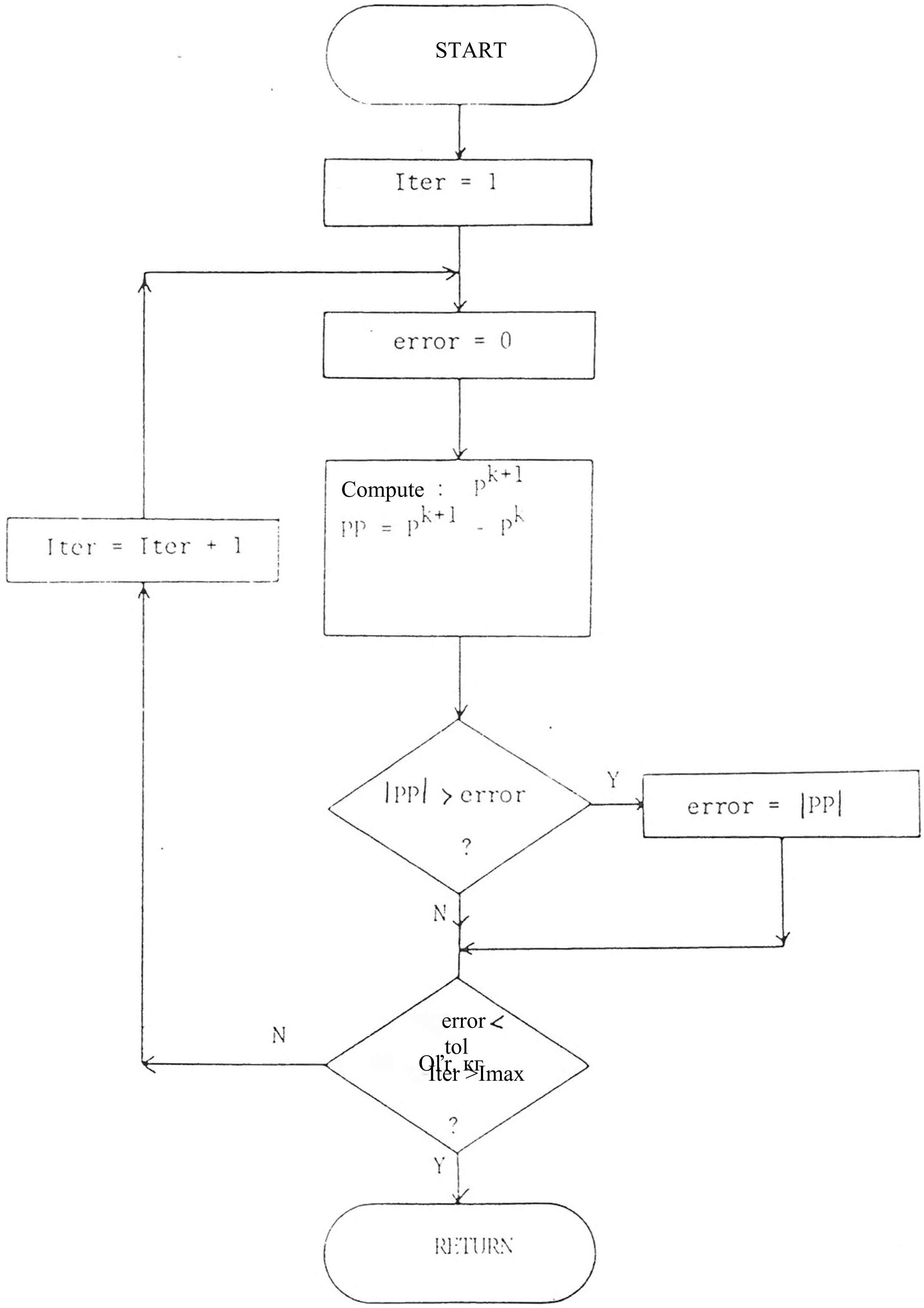


Subroutine Mob

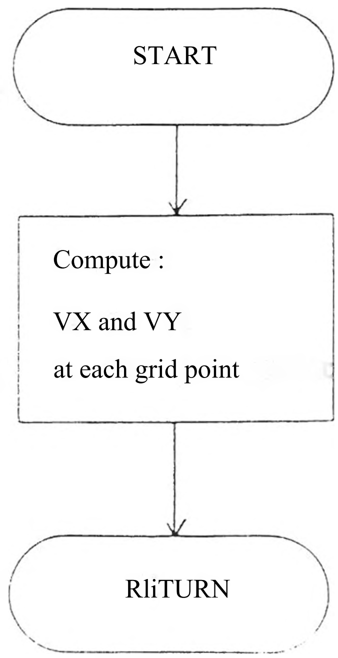
4



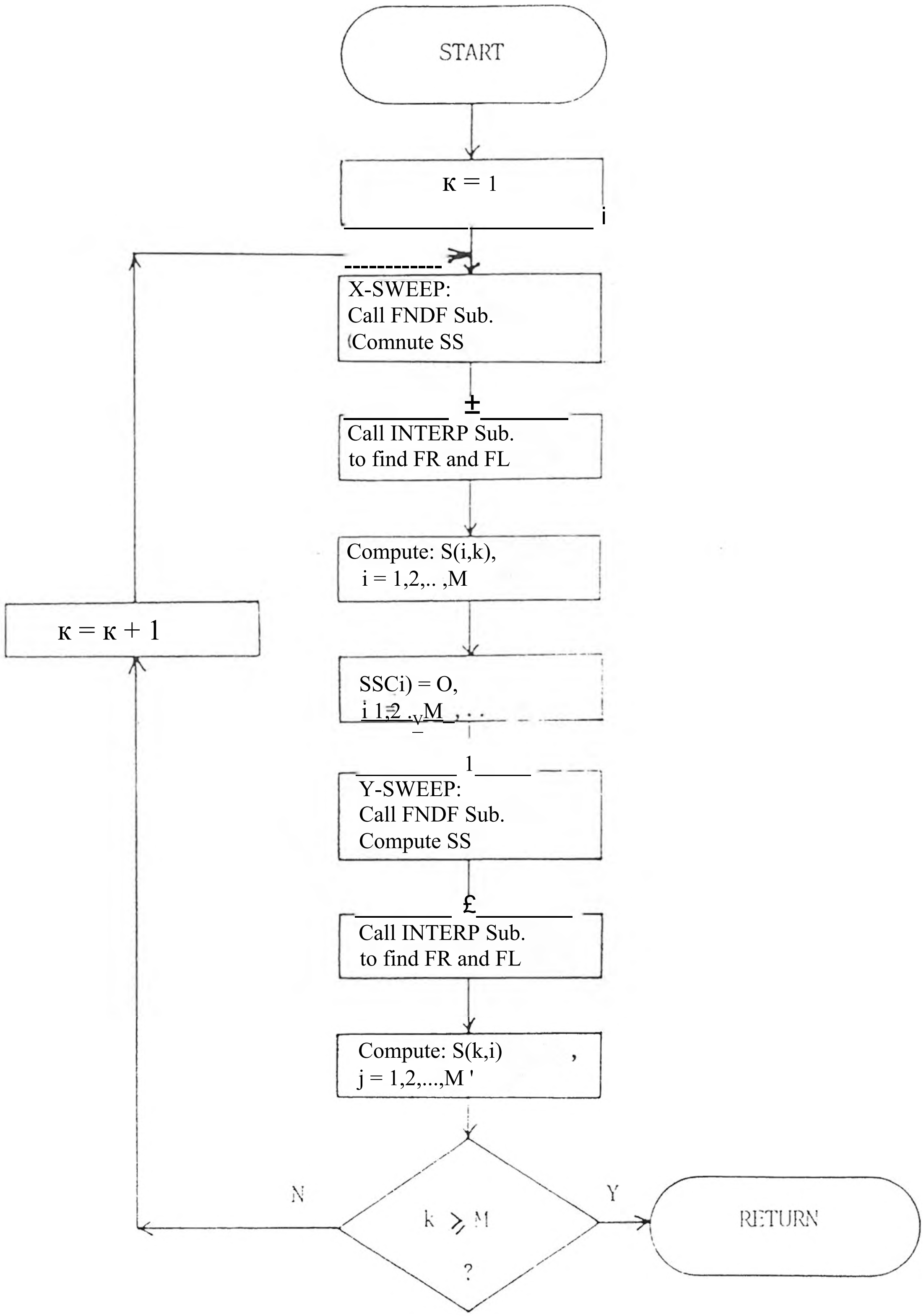
Subroutine Coefs



Subroutine Psor

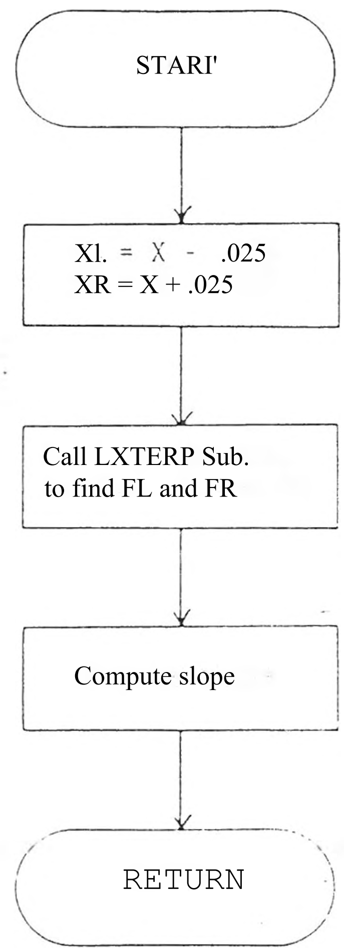


Subroutine Velo

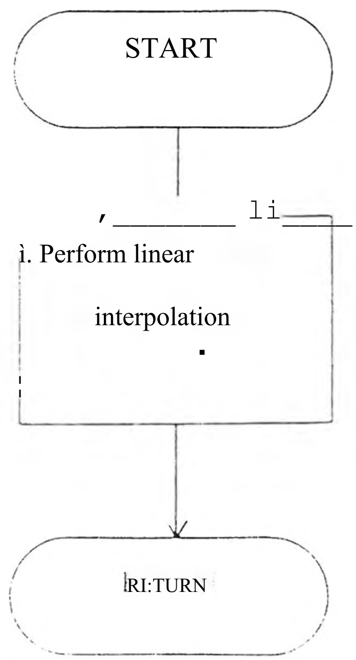


Subroutine Godunv

8



Subroutine Fndf



Subroutine Interp

```
*        Real CWIJ(20Ú).C0PĐÍ200)
            Real SW(20),FN(2Ö),KR0(20),KRW(20)
            Common ΙΌ
            Соавол      /Pout/ юр
            Common     /Tool/ TL(15,15)
         Cesser          ,^-i/ P(15,15),S(15,15)
            Common     /Rate/ Vi(15,15),V¥(i5,15)
            Сойиоп /Cosi/ AE\15,15),АИ(15,15),AN(15,151,ASi15,15)
            Common        /Itrs/ beta,error,imax,tol,iter
         Logicai Bthrou
         ♦
         |================================================
     a   ===     TWO DIMENSIONAL RESERVOIR SIMULATION          ===
         ♦            === A STUDY OF SHEEP EFFICIENCY ===
     *   ===     Prepared by: Suvet V. Nguyen                   ===
     *   ===     Date:                 April 10, 1985          ===
         J ==================r=z====x=====z====z=z=zr====
         »
     *   This program is designed to simulate a 4ve-spot
     *   watertlooding laboratory model. The SOP method
     *   is used to solve tor pressure implicitly and the
     ♦   Godunov and the Alternate Splitting Methods are
     *   used to solve for water saturation. Darcy s unit
     »   is used in the model.
         #
* * VARIABLES:
         ♦
     t       xlgtn ....................:   Length, cm
     ♦       ylgth ................... :   Thickness, cm
     *       de ...................... :   Thickness, cm
     *       pwi .......................:   Initial pressure. atm
     *       phi .... ................ ...:   Porosity, fraction
     *       sai ..................... :   Initial water saturation,              fraction
     *       sor ..................... :   Residual oil saturation,              fraction
'+ qwij .............................. .....:   Hater injection röte, cc/sec
     ♦       ovis .... ............... :   Oil viscosity, op
     »       wvis ................ .....:  Hater viscosity, cp
     *       S ....................... :  Hater saturation, fraction
     *       per® ..................…:  Absolute permeability, Darcy
        * P ...................... pressure, atm
     *       roip .................... :  Recoverable oil in-piace, cc
     *       I'! ......................  ;  Number of grid blocks
     ♦       ΙΌ ......................:  Number of Sk^Kh. ano KO points
     ♦       dt .......................:  Time step, sec.
     *       beta ................... :  SOR acceleration factor
     »       wormax ................:  Maximum wor tolerance
     *       SW .................... .:  Hater saturation, fraction
     ♦       KRN .................... :  Oil relative permeability, fraction
     *       KRO ...................:  Hater relative perneabilty , traction
     *       log ......................:  Pressure output variable, (1 or              -1)
        t
 f ----------------------------------------------------------------------------------
        »
        * === Input Data
        ♦
        Read(5,100)          iop
        » Read(5,100; M
            Read(5,110)      xlgth,ylgth.de,phi,per?
            Read(5,110)      pwi,swi,scr,ovis,wvis
```

```
      Read 15,115qwi j,ü t,worTan
      Read(5,100) MKR
      Read(5,120) (SW(i),KRÖ(i),KRW(i),i          = 1,MKR)
*
*    === Output Control Variables
*
      яax t = 2000
      istep = 20
      stol = .001
*
*    === Iteration Control Parameters
*
      iaax = 100
      tol = l.e-05
      beta =1.5
*
*    === Initialization
*
      tqij = 0.
      tqwp = 0.
      oree = 0.
      wor = 0.
      index = 0
      ine = istep
      Bthrou = .false.
*
*    === Pressure and Saturatior Distrid          ut:ons
*
      De 10 j = 1,H
        Do 10 i = l.M
          S(i,j) = ski
          P'i,j) = pwi
 10   Continue
          S(l,l) = 1. - sor
*
*    ===Calculate      Fw
*
      Do 15 i = 1,MKR
 15   FW(i) = KRW(i)/(KRWii) + wvis*KR0i          i .1 /ovis)
*
*    === Intermediate Variadles
*
      aless = M-1
      dx = xlgth/M
      dy = ylgth/M
      pv = x1gth*y1gth*dz*phi
      roip = pv*(l.-swi-sori
      GWVl' = ovis/Hvis
      dx2 = dx*dx
      dy2 = dy*dy
      faex = dt/(phi+dx)
      f acy = dt/(phi*dyi
*
*    === Flux flows in the x and y direc          tions
*
      qix = qwij/(dy*dz)
      qiy = qwij/(dx*dz)
      qiave = .5*(qix/dx + qiy/dy)
*
*    === Print out input data
```

```
*
        Wr i ta : 6,1 í ; M ,M, x 1 g t h, y 1 g t n, d :,pera,evi s. wvi s, p          h:,
     к woraax,pwi,swi,sor,qwij ,dt
        Write(6,12)
        Kr i t e ( 6,13 ) ( SW ( i ), KRM í i ), KRO í i ). FK i i ), i = 1, MKR )
        Mrite!6,14) imax , beta, tol
        Write(6,16)
ł
t
♦    === COMPUTATION BESINS ===
t
      Do 20 itise = I,aaxt
        time = itiaełdt
        Cal 1 Kob(К,ov i s,иv i =. parE.SK,KRK.KRO)
        Cal* Coaf s(M,к 1 ess.dx2.ay2'
        Call Psor(M,qiave)
        Call Velo(M,dx,dy'
        Ca!! войилу(M,1acx,Tacy,5K,rK!
♦
»    === Compute oil recovery, wãter production, and          WOR.
»
      Ir iSíM,M;.gt.stol! Then
        CaИ intarpiSIM.M) .SW,KRw ,гкн .
        Call Interp(S(M,M?.SW.KRū,rko.
        wer = owvr*rkw/rkc
      End if
   •    qo = qwij/il.+wof
        qwp = qwi j - qo
        oree = oree + co+dt/roip
        tqwp = tqwp + qwp*dt/roip
        tqi j = tqij t qwij*dt/roip
        eft = ЮОЛогес
*
»    === Check tor water oreakthrough
♦
        if ; í.Not«Bthrou) .And. iSiM.M) .gt. stol) i Then
          Pthrou = .true.
          istep = Ifix (2*istap)
          Nrite(6,21)
          Call Print (M,tifile,orec,tqij.tqwp,wcr,eff)
        End if
*
t    === Check for output condition
†
        if (itime.eq.inc) Then
          ine = inc + istep
          index = index i I
          CWIJ(index) = tqe;
          COPD iindex) = oree
          Cal 1 -ri nt i M, tise, oree, tqi i.towp,wor,eft)
        End if
*
*    === Check 4or '.axifiuf: WOR
♦
        If (wor.gt.woraax) Goto 222
20    Continue
ł
222   Writer,22)
      Call Print(M,tise,oree,tqij,tqwp,wor,effJ
      Krite(6,23)
```

```
        Write(6.24) (CWIJ (i ; ,CDPD(i ) ,i=l, index)
*
        STOP
        11 Formati 17/5x, ===== INPUT DATA —               ===== ' ,/ ,
        L/5x. Grid dimension ............................ :    ,12,        ,12,
        &/5x,'Length, cя ...................................:     ,F8.*.
        t/5x, Width, cя .....................................:     58.4.
        £/5x,'Thickness, cя .............................. :     ,F8J,
        Ь/5x, Permeability, Darcy ........................ :     ,F8.4,
        W5x,'0il viscosity, cp ............................ :     .F8.4.
        &/5x, 'Water viscosity, cp ........................ :     ,F8.4,
        &/5x,'Peresity ..................................... :     ,F8.4,
        V5x, 'Maximum MOR .............................:     .F8.4,
        V5x, ' initial pressure, ats ........................ :     .F8.4.
        W5x, Initial water saturation ..:                         ,F8.4.
        75x,'Residual oil saturation ,F8.4,            . . . :
        ?i/5xw Water injec. -ate. cc/sec :                 »FS.4.
        b'5x,'Tise step, sec ............................... :     ,F8.41
        12 Format7/5x. — Sw ™',5x,'™ ¡.rw — ,5x.
        Kro —',5x, — Fw —'l
13      Foreat(/3x,F6.4,9x,Fb.4,10x,F6«4,10x.Fc.4'
        14       Formati/5x, 'ITERATION CRITERIA: , / ,
      * à/8x, 'Maximum number or iterations ........................... ·. ,14,
        VSx, Relaxation Factor ............................................ ,F4.2.
        Ł/8x.'Error tolerance ........................................ : ,Fi0.9)
        là Formati 1'//5::, '===== COMPUTATION ===== ,7)
        21       Foreat(//5x,'=====   BREAKTHROUGH ===== ,/)
22      Format7/5x,'*****                SIMULATION RESULT ***** ./>
        23 Format íZ/5x,'Cue. Hater Inj., pv Cue. Oil Proc., pv 1
        24 Format(!0x,F10.6,15x,F10.6)
  100        Forsat(121
  110        Format (57.3)
  115        Format(3F7,3)
  120        Format(3F6.4)
        END
        ti : *
        *==============================================
        SUBROUTINE         Coefs(M,siess,dx2,dy2i
  ♦     ==============================================
  *
  *     'his subroutine couputes the matri:·-· coefficients.
  t
  Сожяоп /Taol/ TLÍ15.15)
  Cosaon /Co=F/ AE75.15; .Aiíí15,15) ,AN7J.15; .HS75.15
*
  *     === Jpstream weichting mobility
  *
  Do 210 j = 1,M
  Do 210 i = 1,siess
  AE(i,j) =                 TL(i,j)/dx2
  Atf(i+!,jj                = AE.i.j;
  AN(j,i) =                 TL!j,i)/dy2
  AS!j,i+l)                 = AN(j,i) .
  210 Continue
  *
  »     === No flow boundary condition
  *
  Do 220 i = 1,M
```

```
          AW(l,i) = 0.
          AE(N,ı) = 0.
          AS(i,î) = v.
          ANií,H; = 0.
 220  Continue
*
      RETURN
      END
t
*     ΣXΣΣΣΣΣΣ=ΣΣΣΣΣΣΣΣΣΣΣΣ==ΣΣΣΣΣΣΣΣ  =================
          SUBROUTINE Psor(M.qiave)
♦     ΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣ
*
♦     This subroutine uses SOR iteration method
*     to solve tor pressure distribution implicitly.
*
      Common /Tisol / TL(15,15)
      Common /Prša/ P'15,15),3(15,15)
      Common /Coef/ AEí 15,15: ,Atf(15,15i ,ANH5.15) ,AS(15,15/
      Common /'trs/ beta,error,irnax,tol,iter
»
      Do 305 iter = l,isax
          error = ð.
          Dû 31Ū i = Ì,M
              Dc 310 j = 1,M
                  celta = 0.
                  dd = AW ( i, j)+AE(i,j)+AS(i,j)+AN(i, j)
                  pxles = Píl.j)
                  pxplx = PiM.j)
                  pyles = P íi,i)
                  pypi x = Pii)
                  If ìi,gt.lì pxles = P(ı-1,u
                  IT ii.It.N! pxplx = Pliti,j!
                  If (j.gt.1) pyles = Pii, j-1)
                  If (j.lt.M) pyplx = P(i,j+1;
                  temp = AWii,j)*pxles + AE ( i,j)*pxplx +
i                 AS(i,jjŧpyles + ANÍ1,j)tpyplx
                  It ìii.eq.1;.And. (j.eq.D) delta = qiave
                  H ( (i.eq.M). And.j.eq.M) ) delta = -qiave
                  pp = öeta*( (tenp + delta)/dd-P(i,j))
                  If (Absipp).ot.error) error = Absipp)
                  P(i,j) = P(i,j) f pp
 310          Continue
              If (error.It.tol/ RETURN
 305  Continue
*
      RETURN
      END
*
♦     ΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣΣ
          SUBROUTINE Ve!o(M,dx,dy)
♦     ==========================================
*
«     This subroutine calculates the velocity
*     in the x and y directions.
»
      Common /Tisol/ TL<15,15)
      Cooon /Prsa/ Pi 15,15),3(15,15.
      Common /Rate/ VX(15,15).VY(15,151
*
```

```
          Do 500 j = 1. M
               Do 500 i = 2,11
                    txl = .5*(TL(i-l,j)+TL(i,j))
                    tyl = .5*(TL(j,i-l)+TL(j,i))
'                   VXÍiJ) = -txl*(P(i, j)-P(i-l, jH/dx
                    VY(j,i! = -tyl»(P(j,i)-P(j,i-l))/dy
500       Continue
     RETURN
     END
♦
*    ===============================================
          SUBROUTINE GodunvíM,facx , facy, SH, FW
*    ===============================================
»
í    This subroutine uses Oodunov s aethod to solve
*    for water saturation distribution.
*
     Real SW(20),FW(20)
     Real SS í 15)
     Cosaon NKR
     Cosison /Prsa/ PÍ15,15),5(15,15;
     Сомсп /Rate/ VX(15,15),VYi15,15¡
♦
          Do 600 к = 1 ,M
/
*    === X - Sweep
*
               Dc 610 i = I,M
                    siess = S(i,к)
                    If (i.gt.l; siess = Sii-l.k)
                    dd = 2.»(S(i+l,k;-sless)
                    dl = 2.*(S U + i,k)-S(i ,k) !
                    d2 = 2.*(S í i,k)-siess)
                    ds = 0.
                    If (di»d2.gt.Ů.) Then
                         sign = 1.
                         If idd.lt.0.) sign = -1.
                         ds = Abs(dd)
                         If (Abs(dl).lt.ds) ds = Absidi)
                         If (Abs(d2).lt.ds) ds = Abs(d2)
                         ds = ds+sign
                    End if
                    Call Fndf(S(i,k),SW,FW,df:
                    teap = .5*i!.-facx*vX(i,k)*df)*ds
             • SS(i) = 8(1 ,k) * teso
610            Continue
»
               Do 620. i = 1,N
                    siess = S(i,k)
                    If (i.gt.l) siess = SSli-1 '
                    Call InterpislessjSW.FW,f l)
                    Call Interp(SSli) ,SW,FW,fr)
                    teep = facxíVXü,k;*(fl - fr)
                    Sí,k) = Sii ,k) + teap
620            Continue
f
               Do 625 li = 1,N
625                 SS(ii) = 0.
♦
»    === Y - Sweep
```

```
*
                Do 640 j = !,N
                     siess = Sík, j)
                     If (j.gt.l) siess = S(k,j-li
                     dd = 2.*(S(k,j+l)-sless)
                     dl = 2.»(S(k,j+l)-5(k,jj)
                     d2 = 2.»(S(k,j)-sless)
                     ds = 0.
                     If (dl*d2.gt.0.1 Then
                          sien = 1.
                          It (dd.lt.0.J sign = -1.
                          ds = Absídd)
                          If (Absidi).it,ds/ ds = Absidi)
                          If (Abs(d2;.lt.ds: ds = Abs(d2;
                          ds = d5*sign
                     End if
                     Call Fndf (S(k,3),SW,FW,df)
                     temp=,5* ( 1. -f acy*VY (к, j) *df ? *d=
                     SS(j) = S(k.j) * teSip
  640           Continue
 i
                Do 650 i = J.N
                     siess = Sík,j)
                     If (j.ct. 1) siess = SS(j-1
                     Cail Interp(siess,SW,FW,fl ;
                     Call InterpíSS(j),SH,FN,fr)
                     teso = facy*VY(k,j)*(fl - fr)
                     S(k,j) = Sík,j) + temp
  650           Continue
  600      Continue
        RETURN
        END
 ł
 4      ==================================================
             SUBROUTINE     iiob (M, ovi s, wvi 5, per s, SW, KRk, KRO;
 *      ==================================================
 i
 *      This subroutine computes the total mobility of
 ł      the two fluids.
 ł
        Real SM(20),KRW(20),KR0(20)
        Coupon NKR
        Coм,on /Tuoi/ TL(15,15)
        ComiTiOn /Prša/ S(15,15) ,9(15,15/
 *
        Dc 410 i = i,N
             Dc 410 j = 1,N
                  Call Interp (Sí i, j. ,SW,kRW.rkw;
                  Cali Interp(Sti,/,5k, KRO, r кo )
                  TL(i,j) = perm*(r kw/wvis+rko/ovisř
  410   Continue
        RETURN
        END
 t
 *      ============================================
             Subroutine Fndtí x,SW,Fk,df)
 f      ============================================
 ł
 ł      This subroutine calculates the slope of the
 ł      fractional Hoи curve at x. The slope, df,
```

```
t is needed for the Sodunov's method.
*
 Real                SW(20),FW(20)
 Common MKR
*
          xl = x · .025
          xr = x + .025
       Call          Interpixr,SW,FW,fr)
       Call          Interp(xl,SW,FW,f1i
          df = (fr-fl)/(xr-xi)
       RETURN
       END
          ♦
*      =============================================
          SUBROUTINE Intermix,xx,yy,fn)
*      =============================================
          *
*      This subroutine perforas linear interpolation.
          *
       Real xx(20),yy(20)
       Сожаоп NKR
*
          .   If (x.ge.xx (MKR) ! Then
          fn = yy(MKR)
‘         Else if (x.le.xx(D) Then
          fn      =    yy(l)
          Else
          Do     501 h =    i,NKR
          501 If (x.lt.xx(h)) octo 555
          555  hl  =    h - 1
          fn      =yy ( h 1 ) + ix-xx      (n 1 ) ) ♦ ( yy (n )-yy (r l ) > /
      .Ł (xx(hi-xx (hl))
          End if
       RETURN
       END
          *
       ♦        ===================================================
          SUBROUTINE     FT i лt(M,ti®e, orsc,tqij,tqwp.wor,eft)
       ♦        ===================================================
          »
*      This subroutine outputs the current condition
*
       Cosmon /Pout/ iop
       Common /Prša/ P(15.15),5(15,15)
       Common /Itrs/ beta,error,imax,tol,it=r
*
       Write(6,701) tise
       If (iop.eq.l) Then
·          Write (6,702)
       Do 700 j = 1,0
 700       Write(6,703) (Pii,j),i=l,M)
       End if
       Write(6,704)
       Do 710 j = 1,M
       710 Hrite(6,705)    (S(i,j),i=l,N)
       Write(6,706)        tqij,orec,tqwp,eff,wor
       Write(6,707)        iter,error
       *
       RETURN
       701 Foraat (/5x, 'H* Tifie: ',F10.4, Sec.')
```

```
702 Forsat(/5x,'=== PRESSURE DISTRIBUTION ===')
703 Forsatí/5x,15F8.3)
704 Foraat (/5x,'"= SATURATION DISTRIBUTION ===')
705 Format i/5x,15F8.4)
706 Forsat</5x,'Current Inj.SfProd. : ,/,
№, Cua.              water Inj.          ,F10.3,    pv-,
V5x,'Cus.            Oil Proc.           ,F10,3,    p*   ,
W5x,'CuiR.           »ater Prod.         .F10.3.    pv   ,
    У5x.'Displacement EH.................. .■ ,Fí0.3. Г.
    ^/6x. »ater to Oil Ratio             .F10.3)
    707 Forsat i/5x, 'Nesber ot Iteration: ,14,
    W5x,    Naxisuffi DiHerence: ,F12.7.
    Vlx,    ==================^==================        ,/;
    END
```