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Statistical Tool for the Cost Accountant

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The classification of costs as fixed or variable is far from being an exact science. It is, in fact, a kind of model building. To help measure the adequacy of this model, the author proposes —

A STATISTICAL TOOL FOR THE COST ACCOUNTANT

by Donald S. Brightly
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THE SEPARATION of fixed and variable costs is a chronic problem for cost accountants. It is a prerequisite to many of the reports made to management. Direct costing, budgets, breakeven analysis, cost-volume-price relationships, and pricing decisions all depend on an adequate and accurate separation of fixed and variable costs.

The natural questions are "What is adequate?" and "What is accurate?" Textbooks warn students about such possibilities as non-linear, semi-variable, and lump sum costs. How can the man on the job decide when costs have been properly segregated?

This article suggests that accountants should go through the following three-step evaluative process:

1. Analyze and segregate costs.
2. Measure the accuracy of the assumptions about what is fixed and what is variable.
3. Decide to accept or reject the accuracy obtained. (If rejection occurs, return to Step 1 and attempt to improve the analysis.)

The article lays out a simplified example of the approach and suggests some guidelines for application to more complex problems.

The standard definitions of fixed and variable costs assume not only that fixed costs are fixed but also that variable costs bear a linear relationship to volume. These assumed relationships are, of course, an oversimplification of what happens in the real world. For example, a 10 per cent increase in volume does not always result in a 10 per cent increase in direct labor; sometimes direct labor will increase 10.2 per cent; other times, 9.8 per cent. Nor do fixed costs always remain the same. Thus, classifying all costs as either fixed or variable is, in fact, building a model.

This model is similar to the scale models used in wind tunnels by aircraft designers. Their function is to help predict how real planes will react in real flights. When real planes are actually built, no two flights will be exactly the same in all details. However, the model will have predicted the interrelationships of such variables as speed,

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P & L STATEMENT USED AS SAMPLE

	January	February	March	April	Total
SALES	20,000	30,000	25,000	25,000	100,000
VARIABLE COSTS					
Direct Labor	3,800	5,800	5,100	4,900	19,600
Raw Material	10,600	15,000	12,000	12,600	50,200
FIXED COSTS					
Repairs	2,100	2,050	1,850	2,000	8,000
Supplies	3,200	2,700	2,800	2,900	11,600
TOTAL COSTS	19,700	25,550	21,750	22,400	89,400
PROFIT	300	4,450	3,250	2,600	10,600

EXHIBIT I

VARIANCE COMPUTATION

MONTH	r	$r-\bar{R}$	$(r-\bar{R})^2$	s	$s-\bar{S}$	$(s-S)^2$
Jan.	2,100	+100	10,000	3,200	+300	90,000
Feb.	2,050	+ 50	2,500	2,700	—200	40,000
Mar.	1,850	—150	22,500	2,800	—100	10,000
April	2,000	—0—	—0—	2,900	—0—	—0—
		TOTAL	35,000		TOTAL	140,000

$$V \text{ of } r = \frac{35,000}{3} = 11,667 \qquad V \text{ of } s = \frac{140,000}{3} = 43,333$$

EXHIBIT 2

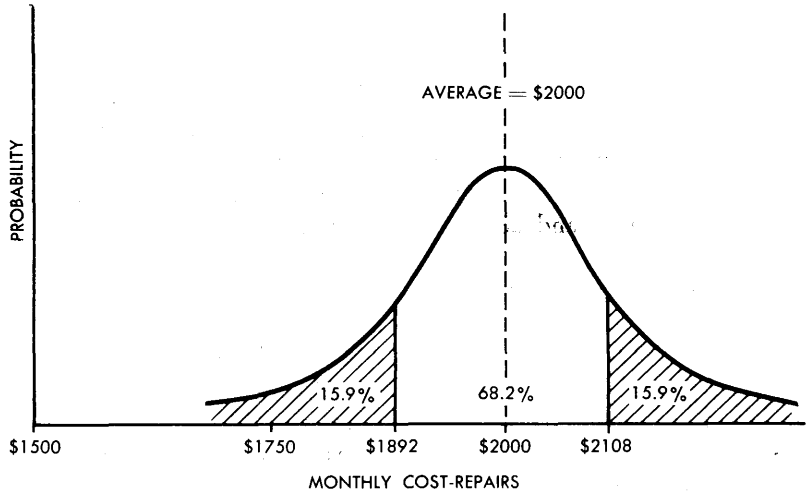

FIGURE I

thrust, lift, and drag. The fixed-variable-cost model is asked to predict the interrelationships of two variables—cost and volume. The substance of the fixed-variable model is categories and formulas used to produce a profit and loss statement each month. Once the system of categories and formulas is set up, the paramount question is "How well does the model reflect what really happens?"

The answer to this question is found with the aid of the following five-step process:

1. A statistical measurement is made of the variation of each type of fixed cost.

2. The same measurement is made of the variation of each type of variable cost.

3. The cost types that show the greatest fluctuations from model values are found.

4. Further analysis can then be done on the cost types showing the most unpredictability to see whether further segregation of fixed and variable components is practical.

5. When further segregation becomes impractical, the measurements are used as an index of overall accuracy.

To measure statistically the month-to-month variation in fixed costs, the statistician has two tools—variance and standard deviation. Variance is defined as the sum of the squares of the differences between individual numbers and the average value of the numbers—this sum being divided by the quantity of numbers in the group minus one. It may be expressed as a formula as follows:

$$\text{Variance } (V_x) = \frac{\sum_{i=1}^{i=n} (x_i - \bar{X})^2}{(n-1)}$$

DEFINITIONS

x_i = individual quantities

$\bar{X}$ = average of all individual quantities

n = number of individual numbers

$\sum_{i=1}^{i=n}$ = the summation for all x's from the 1st to the nth

Take as examples the fixed cost categories of repairs and supplies shown in the simplified profit and loss statement presented as Exhibit 1 above. (The symbols r and s in the following calculations refer to repairs and supplies, respectively.)

The first computation is that of average value.[1]

$$\bar{r} = 8,000 \div 4 = 2,000$$
$$\bar{s} = 11,600 \div 4 = 2,900$$

The computation of variances (V of r and V of s) is shown in Exhibit

[1] For ease of illustration all figures are rounded to two significant figures.

2, appearing on the preceding page.

The second measure (standard deviation) is simply the square root of the variance. For the previous example:

$$\text{SD of } r = \sqrt{11{,}667} = 108$$

$$\text{SD of } s = \sqrt{43{,}333} = 208$$

The standard deviation of any set of numbers has great significance under certain conditions.[2] For instance, if a set of numbers or a population has a natural distribution that approximates the well known bell-shaped normal curve, an important conclusion can be drawn. Such a conclusion applied to repairs in the example is as follows:

The monthly cost of repairs will fall between $1,892 and $2,108 in 68.2 per cent of the months.

The reasoning behind this statement is as follows:

1. The SD of repairs is $108.
2. The average cost of repairs is $2,000.
3. In a bell-shaped or normal distribution 68.2 per cent of all values fall within one standard deviation of the average.
4. Thus $2,000 + $108 is the upper limit and $2,000 − $108 is the lower limit of the zone containing 68.2 per cent of all monthly repair costs. Figure 1 on page 14 presents this fact in graphic form.[3]

Combined variance

If in the preceding example the variance of the combined cost category "Repairs and Supplies" was required, how would it be obtained? The answer is by simply adding the individual variances together. This is done below for the example that we have been using:

$$\begin{aligned} V \text{ of } r &= 11{,}667 \\ V \text{ of } s &= +43{,}333 \\ \hline V \text{ of } r+s &= 55{,}000 \end{aligned}$$

As before, the standard deviation is the square root of the variance.

$$\text{SD of } r+s = \sqrt{55{,}000} = 235$$

The statistical method used for fixed costs will now be applied to the variable cost component of the model in a slightly different way as illustrated in Figure 2 above. Figure 2A shows the sales and direct labor portion of Exhibit 1. Figure 2B graphically represents these figures with a separate line

FIGURE 2A

MONTH	SALES	DIRECT LABOR
JANUARY	$20,000	$3,800
FEBRUARY	30,000	5,800
MARCH	25,000	5,100
TOTAL	$75,000	
AVERAGE	$25,000	

FIGURE 2B

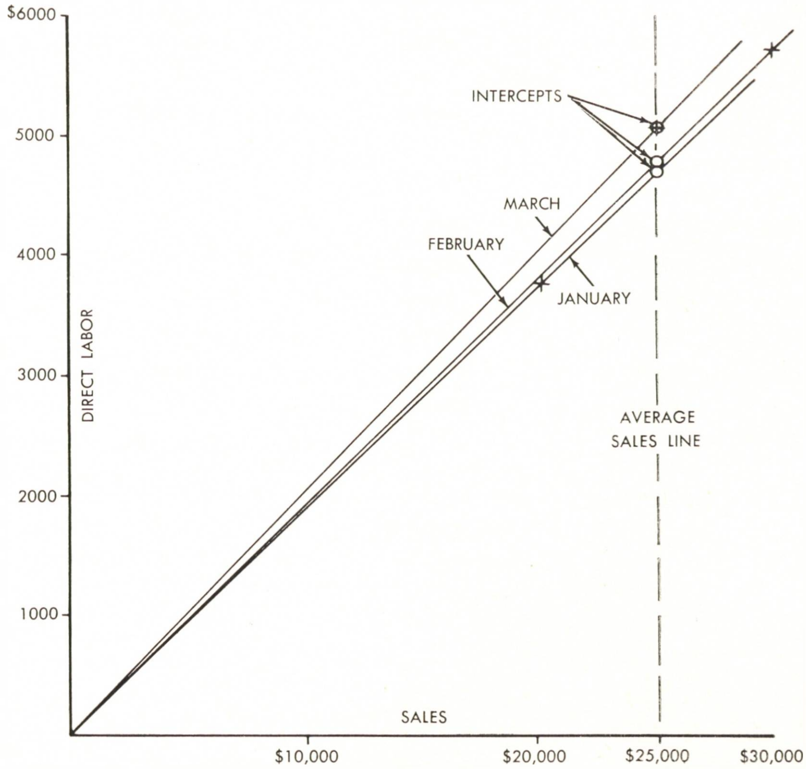

FIGURE 2A, 2B

COMPUTATION OF INTERCEPTS AT AVERAGE SALES
(Direct Labor & Raw Material)

MONTH	SALES	AVER. SALES + SALES	D. LABOR ACTUAL	D. LABOR INTERCEPT	RAW MAT. ACTUAL	RAW MAT. INTERCEPT
JAN.	$20,000	1.250	$3,800	$4,750	$10,600	$13,200
FEB.	30,000	.834	5,800	4,840	15,000	12,500
MAR.	25,000	1.000	5,100	5,100	12,000	12,000
APRIL	25,000	1.000	4,900	4,900	12,600	12,600

FORMULAS:

$$\text{D. Labor (Intercept)} = \frac{\text{(Aver. Sales)}}{\text{Sales}} \text{ (D. Labor Actual)}$$

$$\text{Raw Mat. (Intercept)} = \frac{\text{(Aver. Sales)}}{\text{Sales}} \text{ (Raw Mat. Actual)}$$

FIGURE 3

[2] Let it suffice for this article that the conditions required for interpretation of a standard deviation have been met. For statisticians, the assumption is that of a normal approximation of any population. If the population were somehow known to have a normal distribution, this would become a normal approximation of a student's T distribution.

[3] An alternate approach would be to consider the 2 sigma range within which 95% of the values fall. This would be $2,000 ± $216.

VARIANCE AND STANDARD DEVIATION

DIRECT LABOR

Month	Direct Labor Intercept	$dl-\overline{DL}$	$(dl-\overline{DL})^2$
Jan.	4,750	−150	22,500
Feb.	4,840	− 60	3,600
Mar.	5,100	+200	40,000
April	4,900	—0—	—0—
	19,590		66,100

$\overline{DL} = \frac{19,590}{4} = 4,900$

$\sum(dl-\overline{DL})^2 = 66,100$

$V \text{ of } dl = \frac{66,100}{3} = 22,030$

$SD \text{ of } dl = \sqrt{22,030} = 148$

RAW MATERIAL

Month	Raw Material Intercept	$rm-\overline{RM}$	$(rm-\overline{RM})^2$
Jan.	13,200	+600	360,000
Feb.	12,500	−100	10,000
Mar.	12,000	−600	360,000
April	12,600	—0—	—0—
	50,300		730,000

$\overline{RM} = \frac{37,700}{3} = 12,600$

$(rm-\overline{RM})^2 = 730,000$

$V \text{ of } rm = \frac{730,000}{3} = 243,000$

$SD \text{ of } rm = 243,000 = 493$

EXHIBIT 3

plotted for January, February, and March. Note that the average monthly sales have been computed in 2A and a vertical line drawn in 2B at Sales = $25,000. Each month's line intercepts the average sales line at a different point shown by a circle. If the model were perfect, there would be but one line (the same for all months) and one intercept. This month-to-month fluctuation of a variable cost can be treated in the same way variations in fixed cost were treated.[4]

The points of intercept can be computed using the formula and method shown in Figure 3 on page 15. The variance and standard deviation computations are performed on the intercept values as shown in Exhibit 3 on this page.

Using the method described for combined standard deviation, the variances of direct labor and raw material are added together (22,030 + 243,000 = 265,030). The standard deviation of the combined cost "Raw Material plus Direct Labor" is the square root of 265,030 or 514.

Standard deviation comparison

The third step in the procedure, comparison of standard deviations, is shown in Figure 4 on this page. Figure 4A tabulates all the standard deviations and average values that have been computed. The variable costs show a standard error (another name for standard deviation) of 514 versus a fixed costs standard error of 235. These can be combined (Figure 4B) to give a total costs standard error of 546. Thus, the primary cause of deviation from model conditions is variable costs since that group has the largest standard error. Also, the total costs standard error of 546 will only be reduced if the variable costs standard error of 514 is reduced. Finally, within the group entitled Variable Costs raw material is the model's weakest link.

COMPARISON OF STANDARD DEVIATIONS

FIGURE 4A

Cost Group		Cost Type	Average Value*	Standard Deviation
Variable				
		Raw Material	12,900	493
		Direct Labor	4,900	148
	Combined		17,800	514
Fixed		Repairs	2,000	108
		Supplies	2,900	208
	Combined		4,900	235

FIGURE 4B

		Average	Standard Deviation
Sales		25,000	
Variable Costs	17,800		514
Fixed Costs	4,900		235
Total Costs		22,700	546
PROFIT		2,300	

*The average values shown for variable costs are the averages of intercept values—not actual monthly figures.

FIGURE 4

[4] A second method involving the error variance of a regression line could also be used.

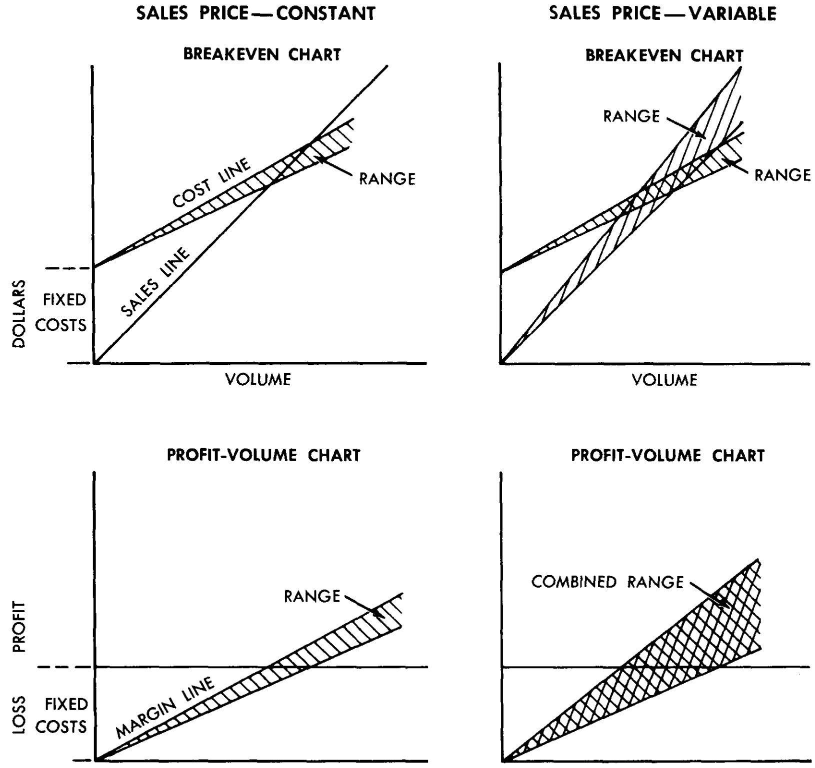

FIGURE 5

With the computations complete, a close look at their meaning is required. Originally certain costs were assumed to be fixed and others to be variable. A statistical measurement of the accuracy of these assumptions has been made. It has pinpointed the particular costs that cause the greatest error in the fixed-variable model.

Further analysis

The methods used to compute raw material cost should be checked by means of the cross-checking and re-auditing that are standard in cost accounting. It may be found, however, that the standard error reflects normal variation of product mix or plant efficiency. If this conclusion is reached, the fluctuation should no longer be considered an inaccuracy. The cost system has in fact done a good job of reflecting real costs. (The fact remains that the assumptions of the model fall short of mirroring what happens in the real world.)

Total system accuracy

If all cost types showing large standard errors have been analyzed and proved to reflect true cost accurately in spite of the large standard error, a decision to accept the results is in order. The reasoning behind the decision for the example shown is as follows:

1. Fixed costs are much closer to model conditions than variable costs.
2. But fluctuations in variable costs are legitimate and tend to increase system accuracy.
3. Therefore, the fixed-variable model—although not perfect—does give accurate answers and predictable total costs within $\pm$ 546 in 68.2 per cent of the months.
4. Furthermore, the unpredictability ($\pm$546) is caused mostly by variable costs.

In some practical applications the conclusions may be different. However, the approach will always be the same:

1. Find the expense types showing the largest deviations from model conditions.
2. Determine whether fixed and variable have been properly segregated for these accounts.
3. Use the standard deviation as a gauge of variation of actual costs from model conditions.

Discussion and alternate uses

Since this article has utilized an oversimplified example, some topics require amplification of the general approach required in actual use.

Sample Size—Assuming that monthly figures are used, the past twelve months provide a large enough sample. The reader will note that this means $n=12$, which is larger than the example where $n=4$.[5]

[5] From a statistical point of view an even larger sample would be helpful. However, this increases the quantity of data requiring analysis and possible re-analysis. It also introduces older data which may not typify current conditions.

Application to Departmental Figures—This method can be used on companywide or departmental figures.

Alternative to Variable Cost Analysis—Figure 5 on this page shows two breakeven charts. The chart under the heading Sales Price—Constant shows the range of costs due to fluctuation in the variable costs. In many applications it will be found that the sales price does not bear a fixed relation to volume (Sales Price—Variable). In such cases profit is the result of the interplay of two ranges (cost and sales price). The analysis is much simpler if a profit-volume chart is drawn as shown in Figure 5. The analysis of variable costs (Step 2) is replaced by an analysis of margin.

If the accountant will study the fixed-variable model as explained and its relation to actual results, he will gain a valuable tool for use in profitability accounting and forward planning.