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The Mississippi Mineral Resources Institute
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FINAL REPORT

Investigation of Relative Permeability Correlations for Multiphase Fluid Flow in Porous Media

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ABSTRACT

The major objective of this study was to develop correlations for predicting two-phase relative permeability curves of consolidated porous media. There are many situations where relative permeability data is not available or not representative of the reservoir. In most of these situations the models developed here could be used. The models or equations that were developed apply to the most common types of systems that exist in hydrocarbon reservoirs. These models were compared to experimental data and other existing models.

A detailed literature review was performed to determine what variables influence relative permeability curves. This information was used to group and select the data used in developing the models. Only data obtained from the Penn State and Hassler methods were used. The selected data was grouped according to rock lithology, wettability and types of flowing phases (that is, water/oil or gas/oil).

With information from the literature review and an evaluation of the available data, the types of programs needed to improve the models have been recommended.
Some of these programs are currently under investigation.

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NOMENCLATURE

Cov(x,y) = Covariance between random variables x and y.

d.F. = Degrees of freedom associated with a random variable.

E = Error in a prediction (predicted - measured).

κ = Permeability (md).

κ_{j} = Effective permeability of the j'th phase (md).

κ_{rj} = Relative permeability of the j'th phase (fraction).

Log = Base ten logarithm.

Ln = Base e logarithm.

P = Pressure (psi or atm).

P/L = Pressure gradient (atm/cm or psi/ft).

q/A = Macroscopic fluid velocity (cm/sec or ft/day).

r_c = Capillary or constriction radius (cm).

S = Saturation (fraction).

S_{wl} = Non-flowable initial water saturation (fraction).

S_{orw} = Residual oil saturation to a waterflood initiated at an initial non-flowable water saturation (fraction).

S_{org} = Residual oil saturation to a gasflood initiated at an initial non-flowable water saturation (fraction).

$$\text{SOFG} = \frac{S_0 - S_{org}}{1 - S_{wl} - S_{org}}$$

$$\text{SOFW} = \frac{1 - s - s_{w} \quad {}_{wl}}{1 - s_{wl} - s_{orw}}$$

$S^* = \dfrac{S_w - S_{wI}}{1 - S_w - S_{orw}}$

SSE = Sum of the squares of the errors or residuals .

S_e^2 = Mean square of pure error.

v = Velocity (cm/sec or ft/day).

V(x) = Variance of random variable x.

w = Weighting function.

$\overline{x}$ = Expected value of random variable x.

Greek Symbols

Θ = Angle (degrees).

σ = Interfacial tension (dyn/cm).

μ = Viscosity (cp).

Δ = Change in.

Subscripts

d = Displacing phase.

gc = Critical gas saturation.

nw = Non-wetting phase.

o = Oil phase.

wt = Wetting phase.

CHAPTER I

INTRODUCTION

Depleting hydrocarbon reservoirs in an optimum manner is both an economical and conservational necessity. Reservoir flow characteristics are an important aspect of reservoir performance and an influential factor in regard to reservoir developement plans. Reservoir flow characteristics dictate production rates of gas, oil and water throughout the depletion of the reservoir. Therefore, in order to more effectively evaluate reservoir potential and plan reservoir depletion schemes it is necessary to predict reservoir flow characteristics .

The components of the reservoir that are of primary interest are the porous media (reservoir rock), hydrocarbon liquid (crude oil), hydrocarbon gas and liquid water (brine). The reservoir fluids, which exist as individual phases, reside in the pores of the reservoir rock. The water phase and hydrocarbon phases (liquid and/or gas) may flow simultaneously in the porous media. This phenomenon is called multiphase flow in porous media. Thus, in order to predict the flow characteristics of an entire reservoir it is necessary to study multiphase flow characteristics of consolidated porous media.

Fluid flow in porous materials that are 100 per cent saturated with one phase is called single phase flow. A phase is any part of a system that is physically distinct, separated from the other parts of the system by definite boundaries, and homogenous (it has similar chemical and physical properties throughout).[1] The equation that is usually used to describe single phase flow in porous materials is Darcy’s law which defines the absolute permeability, k, of a reservoir rock. For horizontal rectilinear flow through an isotropic porous medium, Darcy's law may be expressed as follows:

$$\frac{q}{A} = \left(\frac{-\kappa}{\mu}\right)\left(\frac{\Delta P}{L}\right) \tag{1.1}$$

In physical terms this equation declares that the macroscopic fluid velocity, q/A, is inversely proportional to the fluid viscosity, and directly proportional to the pressure gradient, $\Delta P/L$, where permeability is the constant of proportionality.

Larson[2] was able to arrive at this equation from the Navier-Stokes equation for conditions in which the following premises are adhered:

(1) The fluid is Newtonian with constant viscosity.

(2) The fluid is incompressible; the density is constant.

(3) The flow is steady state.

(4) The flow is creeping (very small Reynolds number) .

(5) The solid matrix or pore geometry is non-changing.

(6) The fluid does not slip along the pore walls.

Darcy's law has also been extended to describe multiphase flow. Symbolically

$$\frac{q_i}{A} = \left(\frac{-k_i}{\mu_i}\right)\left(\frac{\Delta P_i}{L}\right) \qquad (1.2)$$

where the subscript represents the i'th phase in the media, and k_i is the effective permeability of the i'th phase. In physical terms this equation is similar to equation 1.1. Equation 1.2 states that the macroscopic fluid velocity of the i ' th phase, Q_i/A, is inversely proportional to the i'th phase fluid viscosity, directly proportional to the pressure gradient of the i'th phase, $\Delta P/L$, where effective permeability is the constant of proportionality. Equation 1.2 defines effective permeability. In the petroleum industry, effective permeability is commonly expressed in terms of relative permeability as follows:

$$k_{ri} = \kappa_{i}/\kappa_{nw}(S_{wI}) \qquad (1.3)$$

where κ_{ri} is the relative permeability of the medium, and $k_{nw}(S_{wI})$ (sometimes called the base value) is the effective permeability to oil (or the non-wetting phase) at initial non-flowable water saturation (or initial non-flowable wetting phase saturation). One advantage of this type of normalization is that it allows comparisons of the relative flow characteristics between rocks having different base permeabilities. Equation 1.3 defines relative permeability.

Larson[2] was also able to arrive at equation 1.2 from the Navier-Stokes equation with use of the above and the following premises:

(7) The fluid distribution is fixed in the media.
(8) The overall pressure drop for each phase is the same .
(9) Both velocity and tangential stress must be continuous across the fluid interfaces.
(10) The phases must be immiscible.
(11) Buoyancy effects are negligible.

While the above premises are highly restrictive, experimentally, equation 1.2 has been found to adequately describe multiphase flow properties accurately enough for engineering purposes at conditions found in hydrocarbon reservoirs -- even in systems that deviate slightly from the conditions imposed by the above premises.[3,4] It has also been found through empirical methods that equation 1.2 will not make adequate predictions if there are major violations of the above premises as, for example, in cases in which the solid matrix changes because of clay swelling. Therefore[3], judgment must be used when applying equation 1.2 to systems in which the premises are not strictly adhered.

Relative permeability values are needed to predict reservoir flow characteristics. The most common way to obtain relative permeability values is to determine them in the laboratory.[3] A wide variety of factors exist that may influence relative permeability values. If the estimates of relative permeability from the laboratory are to represent the values that exist in another environment (reservoir conditions), then at each saturation point the influencing variables must be at the same "value" in the laboratory as they are in the environment of interest. The relative permeability to the ilth phase and the corresponding phase saturation are the quantities that are reported. The entire set of relative permeability values and their corresponding phase saturations is called a relative permeability curve .

Quantitative measurements of some of the variables that effect relative permeability curves, such as wettability, are not yet available. This fact causes serious difficulties when trying to match the "value" of a variable in the laboratory to the "value" of this variable in the reservoir. How do you match the "value" of a variable that you cannot quantitatively measure? Furthermore, it is experimentally arduous to exactly match the "value" of some of the variables that can be measured. These difficulties cause laboratory measurements to be extremely expensive, time consuming and often of questionable accuracy (Note: In this work the accuracy of relative permeability values refers to the difference between relative permeability values measured in the laboratory and the value that would be measured if all influencing variables could be matched exactly to those of the reservoir.)

Due to the problems of obtaining accurate laboratory data, it is often advantageous to be able to make estimates of relative permeability curves from data that is experimentally easier and/or cheaper to obtain. The main objective of this project was to develope a suite of equations with which to make these estimates. A suite of predicative equations that allow estimates of relative permeability values from readily determinable system properties have been developed with statistical techniques.[5] This suite of equations is believed to be better than other published correlations because of the following reasons: the suite is more complete; only data from the Penn State and Hassler[6] methods was used (These techniques have come to be industry standards.); the suite is relatively easy to apply; the suite makes interval estimates (Interval estimates are usually preferred because they not only give the expected value but also give information relating to the accuracy of the predicted value.); and most importantly it is believed that the suite generally makes better predictions. Also, this work makes recommendations as to the types of experimental programs that are needed to improve the suite of equations. The recommendations were based on the literature review of the theoretical and experimental aspects of multiphase flow combined with a statistical evaluation of the available data.

There is an immediate need for equations that make estimates of relative permeability values from easily obtainable data. Many situations exist in which the suite of equations that has been developed in this work would be useful. Some of these situations are listed below:

(1) Often relative permeability data is simply not available. This problem commonly occurs in leasing and field purchase situations. Also, there are many reservoirs for which relative permeability data was never obtained. In many smaller reservoirs it may not be economically justified to obtain relative permeability data. Moreover, many smaller companies do not obtain relative permeability data for any reservoirs.

(2) There are usually large time lags between coring the well (obtaining samples of the reservoir) and

receiving results from the laboratory. In many cases the estimates of relative permeability curves are needed before the laboratory results arrive.

(3) Some of the laboratory techniques commonly used to obtain relative permeability data may yield suspicious results. For example, in many instances data obtained with the dynamic displacement method has been found to be erroneous. Also, as discussed above, the experimental problems may cause the flow characteristics of the sample to be different from those of the bulk reservoir. For these cases the models could be used to check or in some cases even replace the laboratory data.

(Д) Some measurement techniques do not give all the data that is needed. For example, the centrifuge method, which is becoming more popular because the technique can be automated, does not give the relative permeability of the displacing phase. The models developed here could be used to estimate the relative permeability values of this phase. Often the dynamic displacement gives only one or two relative permeability - saturation points, and these points may be in positions of least interest. In these situations a common practice is to extrapolate from the available data points to the region of interest; however, often the direction of the extrapolation has been determined with little theoretical or experimental basis. The models developed here could be used to help make these types of extrapolations.

(5) Institutions with limited expertise in the area of multiphase flow in porous media may find these models to be of enormous practical value. For example, governmental agencies can use the models to check the consistency of oil company reported data. The agencies can also use the models to estimate oil and gas production rates from state leases and therefore make estimates of tax revenue.

CHAPTER II

LITERATURE REVIEW

PREFACE

Several concepts and definitions will be used throughout this chapter. These fundamental ideas form the conceptual foundation needed to accurately describe most aspects of two phase flow in porous media.

MICROSTRUCTURE

With the use of the scanning electron microscope, clear pictures of actual pore space can be obtained[7]. It is evident from these pictures that pore-space possesses non-uniform curvature, roughness of pore walls and an irregular and chaotic nature. It is also apparent from these pictures that it would be practically impossible to consider all the geometric aspects of pore-space on a microscopic level. Therefore, the term "microstructure" will be used in an abstract fashion to denote all the spacial considerations of pore-space (This terminology is consistent with the more recent and significant publications). In this context microstructure refers to both measurable and non-measurable geometric properties of pore-space. Two measurable descriptions of microstructure that significantly influence transport properties in porous media are pore-space geometry and pore connectivity.

Pore connectivity is defined as a measure of the number of independent paths in a structure. Specifically, pore connectivity is the maximum number of closed curves that can be drawn on the surface of a structure without separating it.[8] For example, the structure in Figure 2.1(a) may be cut only once without becoming separated; on the other hand, figure 2.1(b) may be cut twice before it is separated. Thus, Figure 2.1(b) is said to have higher connectivity than Figure 2.1(a).

Figure 2.1 -Connectivity illustration.

Pore-space geometry refers to pore size and pore shape distributions. The specific measure of the size of pores or the shape of pore boundaries is arbitrary. These measures of size or shape are distributed in the media according to a probability density function. Thus , the term "pore size distribution" or "pore shape distribution" refers to the corresponding probability density function.[8, 9]

WETTABILITY AND CAPILLARITY

Figure 2.2 illustrates a system consisting of two fluid phases (which are immiscible) and one solid phase. The wettability of the system is qualitatively defined as the tendency of one fluid phase to spread on to the solid surface in the presence of the other fluid phase.[3]

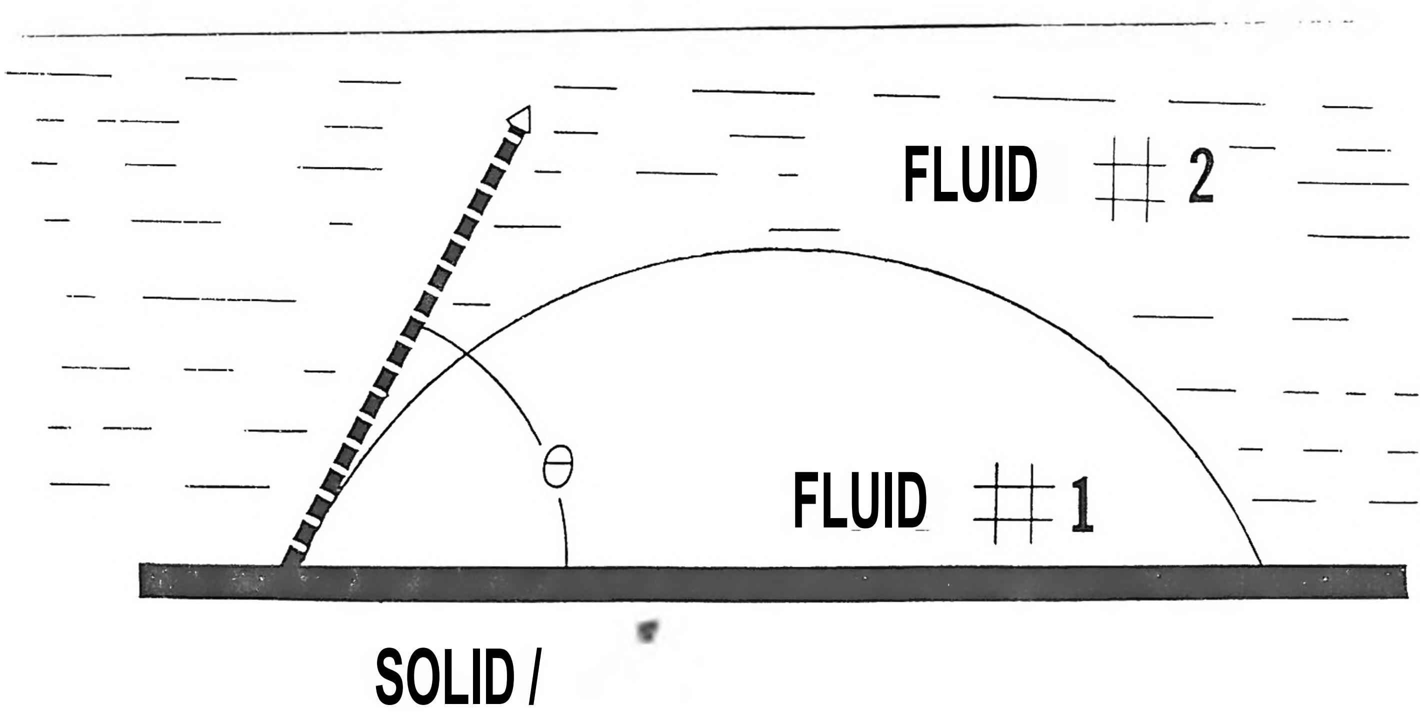


Figure 2.2 -Three phase immiscible system.

When this system is in equilibrium equation 2.1 must be satisfied. This equation (labled Young's equation) was developed from an energy balance.[1]

$$\sigma_{so} - \sigma_{sw} - \sigma_{wo} - \quad x \; (\cos \Theta) \tag{2.1}$$

The contact angle,Q , is defined as the angle of the line tangent to the more dense fluid phase at the three phase intersection. The location of the contact angle within the three phase system is illustrated in Figure 2.2. By convention, the contact angle is taken through the more dense fluid phase. Therefore, as the denser phase becomes more wetting, the contact angle becomes smaller. The contact angle is often used as a measure of the wettability of a system.

Consider a system consisting of immiscible fluids that are located in small constrictions, such as capillaries or pores-space, with one of the fluids wetting. At equilibrium a pressure difference will exist across the interface of these fluids. This pressure difference is called capillary pressure and is defined as follows:[1]

$$P_C = P_{nw} - P_w \tag{2.2}$$

Capillary pressure is related to wettability by the following equation:

$$P_c = \frac{\sigma_{wo} \times (\cos \Theta)}{r_C} \tag{2.3}$$

This equation gives a relationship between a measure of pore size, ηθ; a measure of wettability; the interfacial tension between the fluids; and the pressure difference between the immiscible phases which are in the media. When equilibrium has established between the immiscible phases that are in the pore-space, the fluids must be distributed in arrangements that satisfy equation 2.3 at each interface. Because of the irregular geometry of pore space many different fluid distributions satisfy this requirement. Therefore, capillary pressure is not a unique function of saturation, and many additional factors influence the distributions of immiscible fluids within porous materials. Some of the factors that influence fluid distributions will be discussed in detail in the following sections.

ADDITIONAL TERMINOLOGY

If the wetting phase is increasing in saturation, the flow system is called an imbibition system. If the wetting phase is decreasing in saturation, the flow system is called a drainage system.

The term "initial water saturation" refers to the fraction of void space that is occupied by water placed into the media with the following sequence: the media is completely saturated with water, and oil is injected (displacing water) until the desired water saturation is obtained. If the oil is injected until no more water is produced then the resulting water saturation is termed "irreducible water saturation". It should be noted that the word "irreducible" does not strictly apply since the fraction of water left in the media may be changed by altering the viscosity of the oil or the rate at which the oil is injected.[10] When a sample is said to be at initial or irreducible water saturation the following equations give the saturation of each phase in the media:

$$S_o = 1 - S_{wI} \tag{2.4}$$

$$S_w = S_{wI} \tag{2.5}$$

$$S_g = 0 \tag{2.6}$$

If a sample is at initial water saturation and

If a sample is at initial water saturation and water saturation is increased until a maximum is obtained (no more oil will come out of the sample) the fraction of the void space occupied by the remaining oil is called the residual oil saturation to a waterflood. The following equations give the saturation of each phase in the media:

$$S_o = S_{orw} \qquad (2.7)$$

$$S_w = 1 - S_{orw} \qquad (2.8)$$

$$S_g = 0 \qquad (2.9)$$

Most data indicates that S_{orw} is affected by S_{wi}.[10]

If a sample is at initial water saturation and gas saturation is increased until a maximum is obtained (no more liquid will come out of the sample) the fraction of void space occupied by the remaining oil is called the residual oil saturation to a gasflood. When none of the initial water saturation is produced the following equations give the saturation of each phase in the media:

$$S_o = S_{org} \qquad (2.10)$$

$$S_w = S_{wi} \qquad (2.11)$$

$$S_g = 1 - S_{org} - S_{wi} \qquad (2.12)$$

Most data indicates that $S_{or\,g}$ is affected by S_{wi}.[10] The saturation at which the gas phase obtains a finite permeability is termed "critical gas saturation". The fluid saturations resulting from a completed gasflood or a waterflood initiated with the sample at an initial water saturation are called end point saturations.

The order in which fluid phases are introduced into the media affects fluid distributions and flow properties. Therefore, when multiphase flow properties (such as relative permeability) are being discussed it is necessary to specify the sequence in which the fluids are introduced into the media. The only flood sequences that will be considered in this work (unless otherwise stated) are those described above: gasfloods and waterfloods initiated with the sample at an initial water saturation. These are the injection sequences that most commonly occur in hydrocarbon reservoirs.

EXPERIMENTAL OBSERVATIONS

Most experimental investigations of the factors effecting flow properties were with systems having high interfacial tension between the fluids --commonly referred to as high interfacial tension systems. The systems of the investigations reviewed in the MICROSTRUCTURE, WETTABILITY and VISCOSITY sections were all high interfacial tension systems. Low interfacial tension systems are considered in the INTERFACIAL TENSION section.

MICROSTRUCTURE

Multiphase flow characteristics of porous materials are influenced by the microstructure of the pore-space. Morgan and Gordon[11] investigated, on a qualitative basis, the relationship between microstructure and relative permeability data. Thin-section analysis was used to make qualitative assessments about the microstructure, and the dynamic displacement method[4] was used to determine the water/oil relative permeability curves (the floods were initiated at irreducible water saturation). Their data indicates that rocks having a uniform pore size distribution of large pores, small specific surface area and well connected pores will generally have the following flood properties:

(1) The irreducible water saturation will be low.

(2) The effective permeability to water at residual oil saturation will be a large fraction of the absolute permeability.

(3) The effective permeability of oil at irreducible water saturation will be a large fraction of the absolute permeability.

Rocks having a uniform pore size distribution of small pores, large specific surface area, and well connected pores will have the following flood properties:

(1) The irreducible water saturation will be high.

(2) The effective permeability to water at residual oil saturation will be a small fraction of the absolute permeability.

(3) The effective permeability of oil at irreducible water saturation will be a small fraction of the absolute permeability.

It is not clear if the variations of the observed flood properties were caused only by changes in microstructure. This uncertainty arises because of the following: It is apparent that each of the systems were water wet; however, there was no measure of the magnitude or strength of the wettability, and it is not certain if the wettability remained constant from test to test. Relative permeability values from the dynamic displacement method in which the Johnson, Bossier and Neuman equations[4] are used are often in error.[10,12,13] Neither the viscosity of the fluids nor the interfacial tension between the fluids were reported; these variables may have fluctuated from test to test.

Several investigators have found a relationship between the irreducible wetting phase saturation and the specific surface area of the media.[14] The results agree with the findings of Morgan and Gordon[11], rocks with a large specific surface area will have a high irreducible wetting phase saturation. Irreducible wetting phase saturation has also been found to vary with pore size distribution, pore shape distribution and any heterogeneities that may exist in the media;[15] however, no definite relationships have emerged. (Note: it should be clear that any factor that changes end point saturations also changes the flow characteristics of the system --part iculary the relative permeability values in the neighborhood of the altered end point saturations.)

In porous media with one fluid strongly wetting, the residual non-wetting phase will reside as isolated blobs in the larger pores (pore bodies).[17] The connectivity of the media is one of the factors that controls the residual non-wetting phase saturation of a consolidated porous material.[8][1,6] The influence of connectivity on the non-wetting phase saturation has been investigated in a quantitatively and experimentally rigorous manner by Pathak, et al.[8] They used synthetic porous media which was made from copper powder by a sintering process. The sintering process consists of heating a bed of unconsolidated particles (staying below the melting point) to form consolidated porous media. The major advantage of using synthetic porous media is that it allows the microstructure to be changed while holding the wetting properties constant. The microstructure of synthetic porous media may be controlled by varying the sizes and shapes of the powder grains. Pathak, et al. insured that changes in the residual oil saturation were caused only by changes in the microstructure of the media by controlling the wettability of the system (copper is strongly water wet), fluid viscosity and interfacial tension for each test. Their results show a strong correlation between connectivity and residual oil saturation in strongly water wet systems; that is, the higher the average connectivity of the pore space the lower the residual oil saturation.

WETTABILITY

Hydrocarbon bearing sedimentary rocks are composed of a variety of materials. Figure 2.3 illustrates the "variety of composition" typical for a sandstone.[3] Since the wetting properties of a system depend on the chemical nature of the phases that make up the system it would be expected that sedimentary rocks are of a heterogeneous wettability.[6,1,10] (Note: this is not to say that an entire sample cannot be strongly wetting to a phase, but that the "strength" will vary within the pore). A rock is said to be of a mixed wettability if it contains both water wet and oil wet pores.[19]

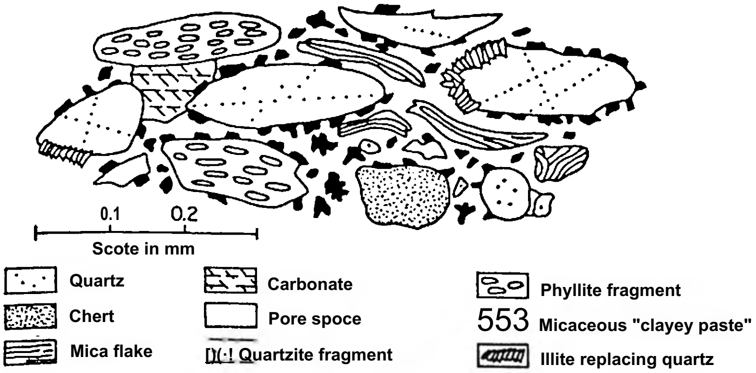


Figure 2.3 -Graywacke sandstone (Amex).[3]

Waterfloods into media that are strongly water wet (imbibition floods) will have the following production characteristics: at water breakthrough (the moment the first drop of water is produced) practically all oil production will cease; the residual oil saturation will be low; the flood will be efficient (low oil saturations are obtained at low water to oil production ratios). Gasfloods, waterfloods into strongly oil wet media, or in general drainage floods, will have the following production characteristics: significant amounts of oil will be produced after breakthrough of the injected fluid; the residual oil saturation will be high; the flood will not be efficient. Intermediate wetting systems will have production properties that fall between these two extremes. Figure 2.4 illustrates the relationships between wettability and oil production characteristics. Each test was started at the same initial water saturation; of course, each line represents a separate test. All of the tests were performed on Torpedo sandstone (constant microstructure); viscosity and interfacial tension were also held constant. Therefore, the variations of the production properties were caused only by changes in the wettability of the system. Owens, et al.[20] adjusted the wettability of the system by adding varying amounts of barium dinonyl napthalene sulfonate to the oil. The water oil contact angle was measured on a smooth quartz crystal (the contact angle was measured through the water phase; thus, 0° contact angle represents a strongly water wet system). The circle in the illustration represents the moment at which the producing water to oil ratio is 25.

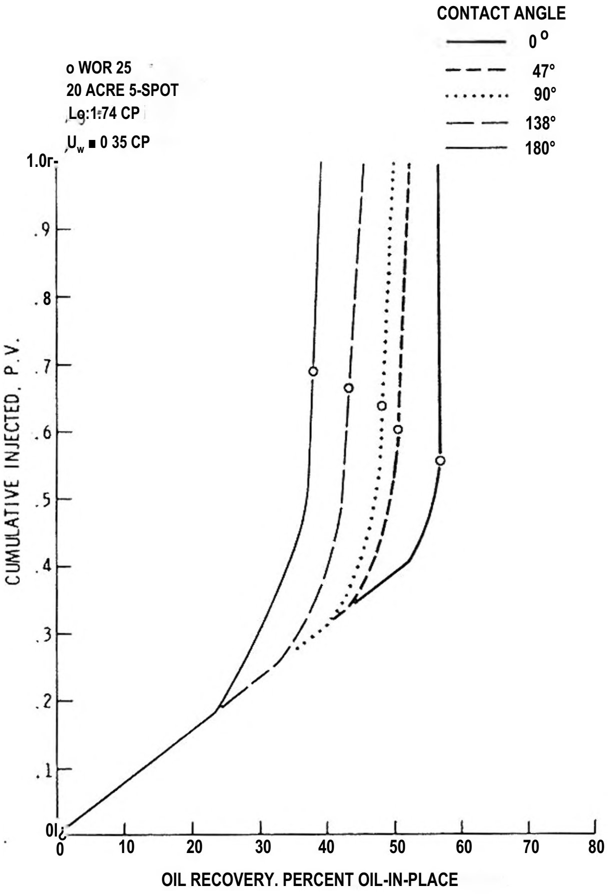


Figure 2.4 -Effects of wettability $_{2}$gn water flood performance (Owens, et al.).

In the same experiment described above, Owens et al.[20] also investigated the effects of wettability on relative permeability curves. These results are illustrated in Figure 2.5. As pointed out in the above paragraph, variations of flow properties were caused entirely by changes in system wettability. Some of the more important features of the effects of wettability on water/oil relative permeability that are illustrated in Figure 2.5 follow: When the media is at a high oil saturation the relative permeability to oil with the system in an oil wetting mode (180 ° contact angle) is much lower than the oil relative permeability with the system in a water wetting mode (0 ° contact angle). When the media is at residual oil saturation the relative permeability to water with the system in an oil wetting mode is much higher than the water relative permeability with the same system in a water wetting mode.

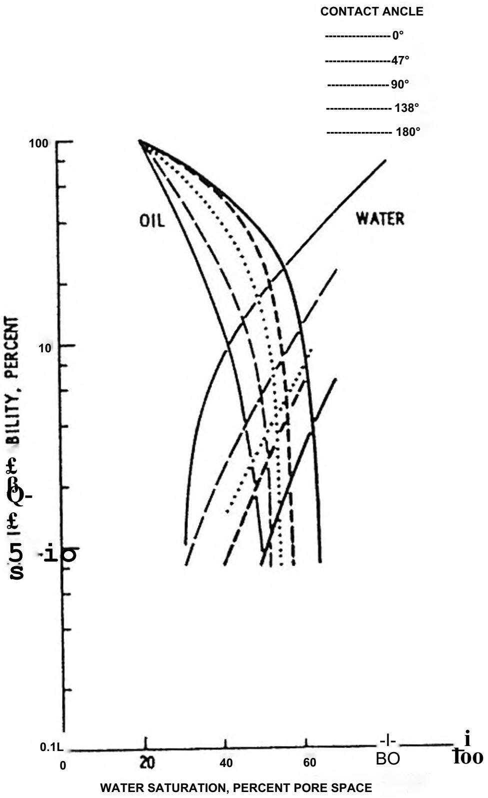


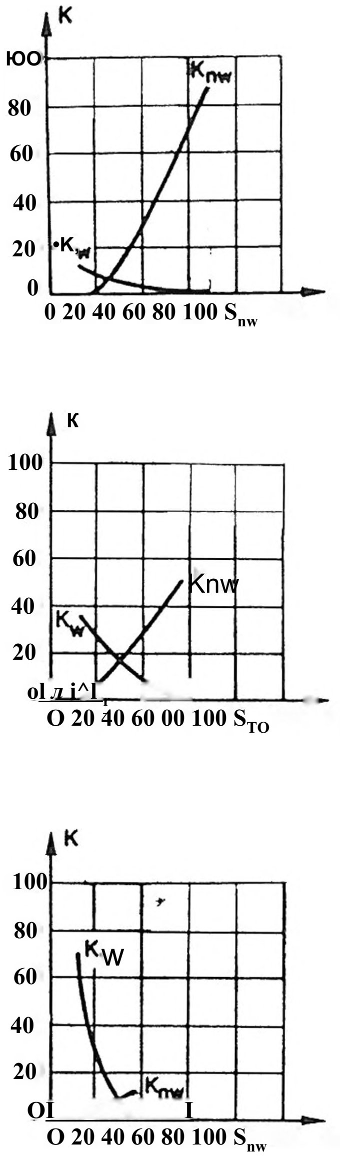
Figure 2.5 -Effects of wettability on water/oil relative permeability (Owens, et al. 26)

The tests on other rocks (specifically Berea sandstone, which changes the microstructure) showed less variations in the relative permeability curves with changes in wettability. Therefore, as would be expected from equation 2.3 the magniture of the effects of wettability are influenced by the microstructure of the media.

VISCOSITY

Leverett[21] found that for unconsolidated sand packs the relative permeability curves are independent of the viscosity ratio of the fluids in the media. Other researchers[22-25] have found that for consolidated media the relative permeability curves are significantly affected by the viscosity ratio of the fluids in the media. (In this work the viscosity ratio will be defined as the non-wetting phase viscosity divided by the wetting phase viscosity, $\mu \; 7_{o} \; \mu_{w} \; . \; ._{u}$).

Lefebvre[22] worked with sintered teflon media. Figure 2.6 illustrates the effects of viscosity ratio on the relative permeability curves of these samples. Each curve illustrated represents data from an imbibition flood which was initiated at an initial wetting phase saturation. Some fluctuation occurred in the wetting properties as indicated by the variation in the contact angle . Note: contact angle measurements are usually only reproducible within about ± five degrees.[20]



$\mu_{nw} + \mu_{w}$ = 26 . 5
μ_{w} = 0.9 cp
% = 23.8 cp
σ = 38 dyne/cm
0^{wo} = 18

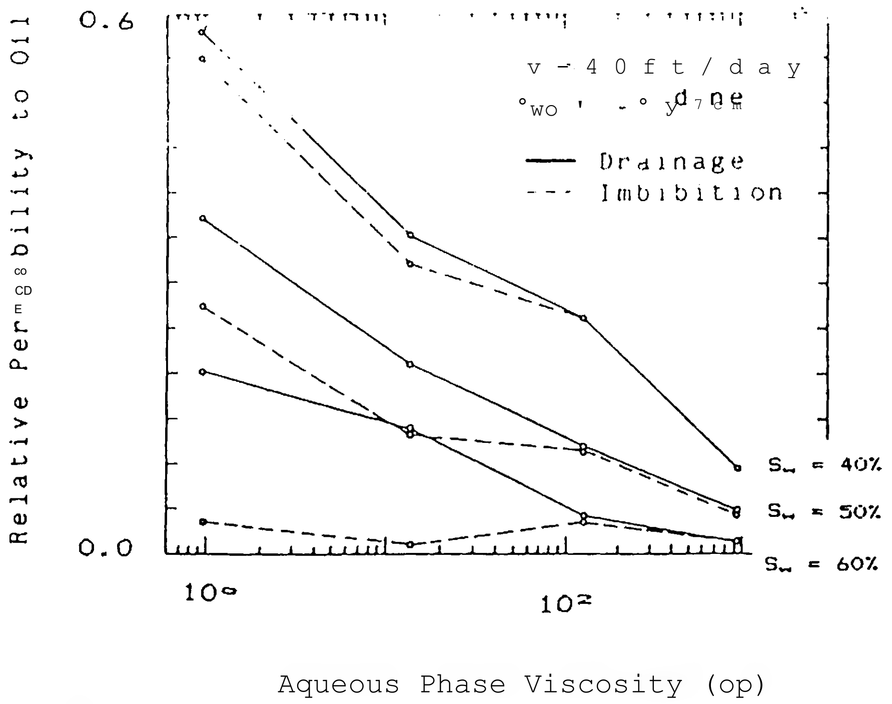
$\mu_{nw} + \mu_{w}$ = 1
% = 20.0 cp
% = 20.0 cp
σ = 19 dyne/cm
Θ = 29

^nw^1^ = 0.0026
% = 20.5 cp
μ_{o} = 0.5 cp
σ wo = 15 dyne/cm
Θ = 1 0

Figure 2.6 -The effects of viscosity rasio on relative permeability data (Lefebvre).

Odeh[23] found that for imbibition floods started with the sample at an initial wetting phase saturation (no measure of wettability was reported) the non-wetting phase relative permeability increases as the viscosity ratio increases. His results indicate that the wetting phase was not significantly affected by changes in the viscosity ratio. He also made theoretical arguments which suggest that the effects of the viscosity ratio would decrease as absolute permeability increases. These results are consistent with those of Leverett.

Fulcher, et al.[24] investigated the effects of viscosity for both imbibition and drainage floods over a wide range of viscosity ratios. These results are illustrated in Figure 2.7. The drainage floods were started with the rock (Berea sand) completely saturated with an aqueous phase, and the imbibition floods were initiated at an initial aqueous-phase saturation. (Note: The results from drainage floods initiated at 100 percent wetting phase saturation may not necessarly be applied to the drainage systems initially saturated as described in the preface; however, similar results would be expected.) The aqueous-phase was strongly wetting; however, no measure of wettability was reported. The oil viscosity was constant for each test (about 2 cp.); thus, increasing aqueous-phase viscosity (as plotted in Figure 2.7) corresponds to a decreasing viscosity ratio.



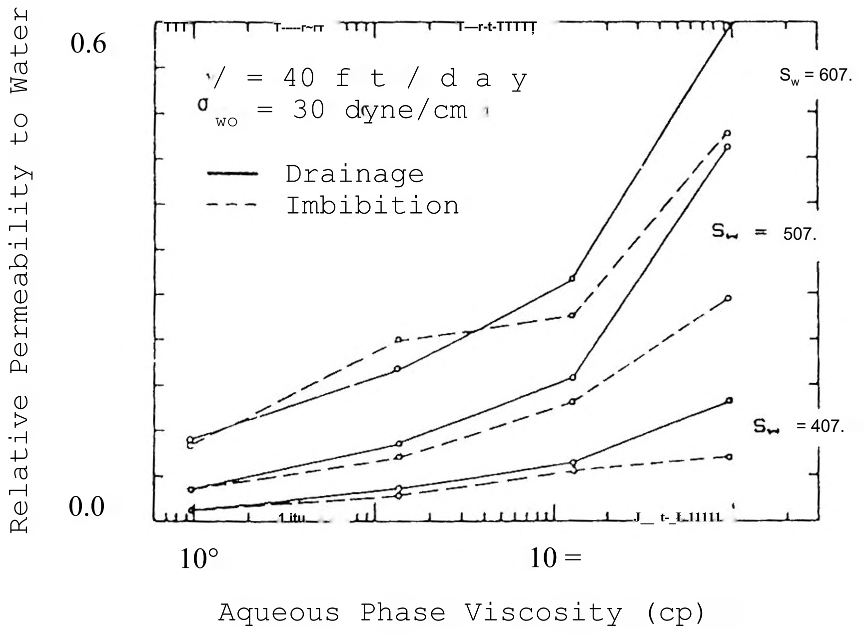


Figure 2.7 -The effects of viscosity on relative permeability data (Fulcher, et al.).

For imbibition floods the data from the reviewed investigations suggest the following:

(1) The effects of viscosity on flow properties are small for rocks with large absolute permeability (Leverett's[21] unconsolidated sands had permeability values between 3,200 and 6,200 md).
(2) As the viscosity ratio increases the residual non-wetting phase increases.
(3) As the viscosity of one of the phases increases the relative permeability to the other phase at a specific saturation point will decrease.
(4) As the wetting phase viscosity increases the wetting phase relative permeability at a specific saturation point will also increase.

For drainage floods initiated at 100 percent wetting phase saturation the data from the reviewed investigations suggest the following:

(1) As the wetting phase viscosity increases the irreducible wetting phase saturation increases.
(2) As the wetting phase viscosity increases the relative permeability to the non-wetting phase will decrease, and the relative permeability to the wetting phase will increase.

INTERFACIAL TENSION

In high interfacial tension systems ($\sigma_{wo} > 5$ dyne/cm) the effects of interfacial tension variations on flow properties are insignificant.[22-27] The effects of interfacial tension variations in low interfacial tension systems have been found to be dependent on other system properties. The results of most investigations reveal that the effects of interfacial tension are intricately linked with the components of the capillary number, which has been expressed in many different forms in the literature. All of the expressions measure the ratio of viscous to capillary forces; therefore, one of the more intuitive expressions for the number is represented as follows:

$$Nc = \frac{(V) \times (\mu)}{\sigma} \qquad (2.13)$$

where v is the Darcy velocity (q/A) and μ is the viscosity of the displacing phase. Comparing Equation 2.3 and Equation 2.13 brings to view the fact that in systems with constant microstructure and wetting properties the capillary number is truly a measure of the ratio of viscous to capillary forces.

To observe the effects of interfacial tension independent of other influences, interfacial tension must be changed while the other components of the capillary number are held constant. Experiments of this type have been reported by several investigators[24,26]. Figure 2.8 shows the effects of interfacial tension on relative permeability curves for both drainage (initated at 100 percent wetting phase saturation) and imbibition systems[24] (Note: the viscosity ratio is approximately one.). The data from the reviewed investigations suggest the following:

(1) The effects of interfacial tension on relative permeability curves for both drainage and imbibition systems are small in high interfacial tension systems .

(2) As interfacial tension decreases, the relative permeability of both phases at a specific saturation will increase for both drainage and imbibition systems.

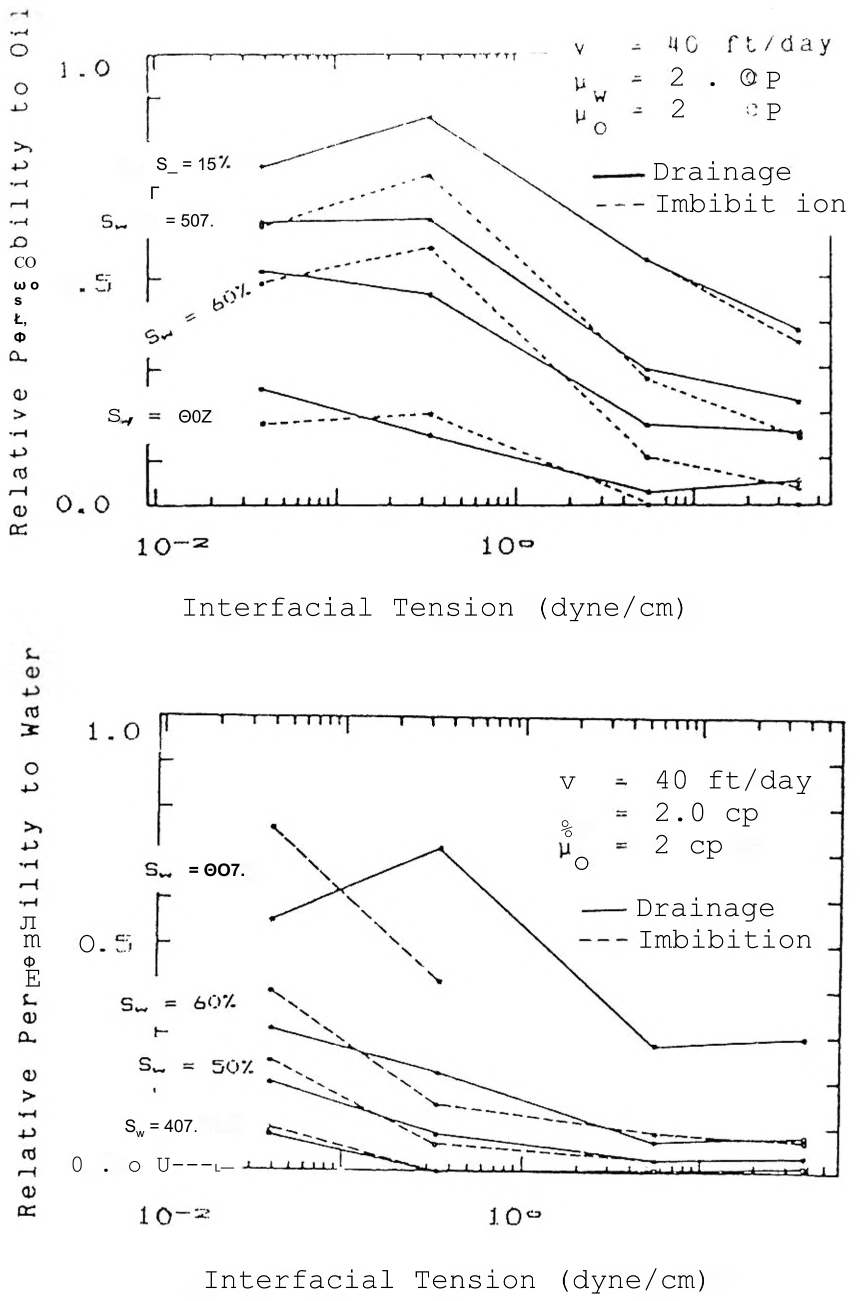


Figure 2.8 -Effects of interfacial tension on ^elative permeability curves (Fulcher, et al.).

There have also been multiple investigations of the effects of interfacial tension on residual oil saturation to waterfloods (in water wet systems).[22,24,25,27] The data indicates that reduction in interfacial tension causes a reduction in the residual oil saturation. Chatzis et al.[27] found a correlation between the capillary number, as a unit, and residual oil saturation; increasing the capillary number of a system above a critical point will cause a reduction in residual oil saturation. These results are consistent (decreasing interfacial tension will increase the capillary number). The effects of either the capillary number as a unit or the individual components of the capillary number on the irreducible wetting phase saturation are not clear. The data from Fulcher, et al. showed an increase in the irreducible wetting phase saturation as interfacial tension changed from 0.038 dyne/cm to 0.33^ dyne/cm; however, as the interfacial tension changed from .33^ dyne/cm to 5.5 dyne/cm the irreducible wetting phase saturation decreased. These results would be significant in evaluating the prospects for surfactant floods in oil wetting systems.

LABORATORY MEASUREMENTS

These sections examine some of the experimental problems of obtaining relative permeability data that is representative of the reservoir. The first section considers the natural problems that arise in making relative permeability measurements. In each additional section specific laboratory techniques will be considered, along with the problems unique to the specific technique.

GENERAL EXPERIMENTAL MEASUREMENT PROBLEMS

Outlet End Effects

When a wetting phase first arrives at the end-face of a sample there will exist a discontinuity in the wetting phase pressure across the outlet end of the sample.[28-31] From a microscopic view point this phenomena is expected by examination of equations 2.2 and 2.3. Figure 2.9 illustrates the last pore in a sample. At equilibrium within this pore space equations 2.2 and 2.3 must be satisfied; thus, there exist across the interface a pressure difference, $P\theta$, between the wetting and non-wetting fluid. This phenomena is important because it causes a saturation gradient to exist in the sample.

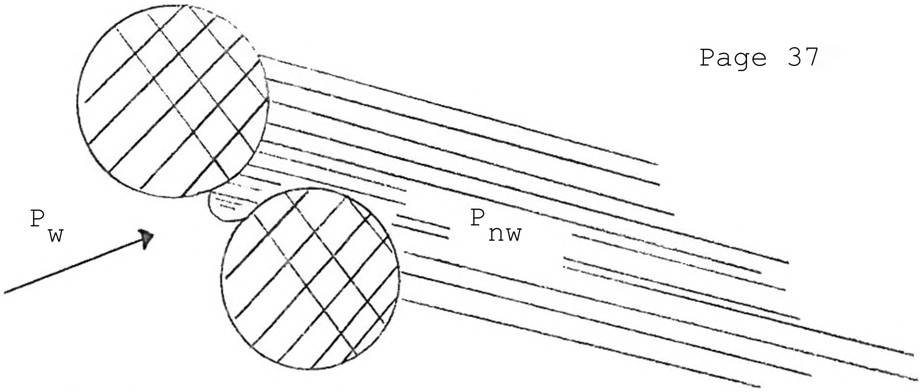
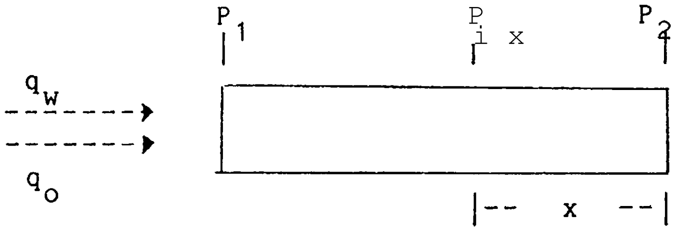


Figure 2.9 -Pore at the end of a sample when the wetting phase first arrives.



where $P_1 > P_x > P_2$

Figure 2.10 -Illustration used to present wetting phase build up argument.

The end-face will obtain an excess wetting phase saturation. The following argument demonstrates this fact: Consider a sample in which there is steady-state flow of both the wetting and non-wetting phases. At a distance x from the outlet end (as illustrated in Figure 2.10) Darcy's law may be expressed as follows:

$$\frac{q_w}{A} = v_w = \frac{k\,k_r}{\mu_w}\,\frac{(P_{wx} - P_{w2})}{x}$$

From equation 2.2 the capillary pressure at the end is

$P_{ce} = P_{nwe} - P_{we}$. Where the subscript "e" stands for values that exist at the very end of the media, and the subscript "x" stands for values at distance x from the end of the media. Therefore,

$$P_{wx} = P_{nwx} - P_{ex}$$

Assume that the outlet pressure is equal to the non-wetting phase pressure that exists at the very end of the media; that is,

$$P_{nwx} = P_{nwe} = P_2$$

$\lim x \to$ s.v. (where s.v. denotes "small value")

Since throughput is constant

$$\lim_{x \to s.v.} v_w = \text{constant}_1 = \lim_{x \to s.v.} \left[\frac{k\,k_{rw}}{\mu_w} \frac{(P_{wx} - P_2)}{x} \right]$$

Substituting gives,

$$\text{constant}_1 = \lim_{x \to s.v.} \left[\frac{K\,k_{rw}}{\mu_w} \frac{(P_{nwx} - P_{cx} - P_2)}{x} \right]$$

$$\text{constant } 1 = \lim_{x \to s.v.} \left[\frac{k\,k_{rw}}{\mu_w} \left[\frac{(P_{nwx} - P_2)}{x} - \frac{(P_{cx})}{x} \right] \right]$$

Since, $\lim_{x \to s.v.} \frac{P_{nwx} - P_2}{x} = \text{constants}$

then it must also be true that

$$\lim_{x \to s.v.} \frac{-P_{cx}}{x} = \frac{-P_{ce}}{x} = \text{constants}$$

And for the above equation to be true, P_{ce} must approach an arbitrarily small value as x approaches an arbitrarily small value. In a gross sense the capillary pressure will be zero if either the sample is completely saturated with one phase or at residual non-wetting phase saturation.[3,12] If both phases are flowing out the end-face then the end-face of the media must have a finite saturation of both phases; therefore, this saturation must be close to the non-wetting phase residual saturation in order for the capillary pressure to be a small value. The important result shown here and confirmed experimentally[1,2,28,30] follows: For imbibition processes the wetting phase cannot flow out the end until the capillary pressure at the end-face approaches zero. For drainage systems the non-wetting phase at the end does not obtain a saturation that is significantly different from the residual non-wetting phase saturation. For both of these cases the end effect causes a saturation gradient and a capillary pressure gradient to exist in the media. A typical saturation profile showing the saturation gradient is illustrated in figure 2.11.

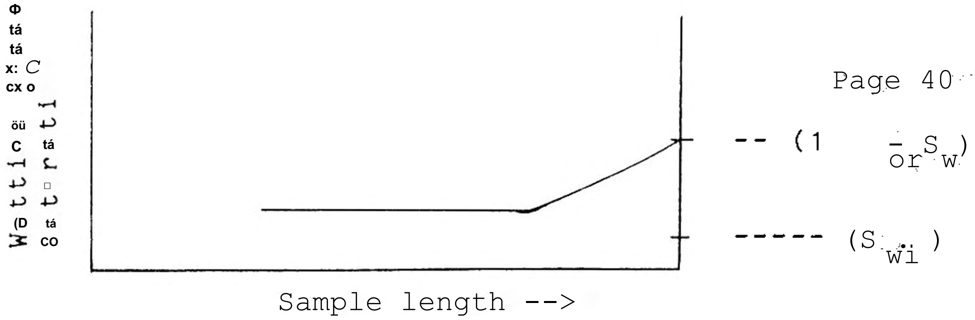


Figure 2.11 -Typical saturation gradient established in a core in which there is steady state flow of both wetting and non-wetting fluids.

Richardson, et al.[30] derived an equation that accounts for the end effects and makes predictions of the saturation profile that will exist in drainage systems. Kyte, et al.[32] suggested a scaling criterion for system properties that, if met, would reduce the end effects. The scaling criterion is listed as follows:

$$\frac{\mu L q}{A} > 1 \text{ to } 5 \qquad (2.14)$$

where L (cm) is the sample length, μ (cp) is the displacing phase viscosity, and q/A (cm/min) is the macroscopic fluid velocity. End effects become negligible in regard to relative permeability measurements and flood characteristics if the saturation and capillary pressure gradient are reduced to a small portion of the outlet end. Adherence to the scaling criterion localizes the saturation gradient to a small portion of the outlet end, and thus minimizes end effects. This criterion is used throughout the petroleum industry.

Inlet End Effects

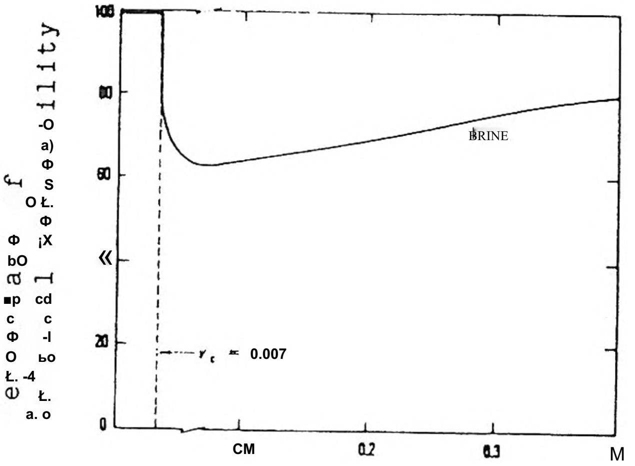
Kyte, et al.[2,3] also investigated inlet end effect problems. It has been observed that at the first of an imbibition flood the wetting phase may be spontaneously imbibed into a localized area causing counter-current flow of the non-wetting phase.[10,23] This is strictly a laboratory phenomena caused by the pressure discontinuity of the wetting phase (capillary pressure) as it moves through the inlet face. For some floods it has been observed that the injected wetting phase covers only a small portion of the inlet face throughout the entire flood. These effects can cause the flood front to become unstable. The unstable flood front can cause the wetting phase to breakthrough prematurely. The inlet effects are reduced for systems with the following properties: long cores, high oil to water viscosity ratios, low injection rates. Notice that some of these requirements directly conflict with those necessary to minimize outlet end effects. Outlet end effects cause larger errors than inlet end effects. Therefore, in cases of conflict the requirements to reduce outlet end effects are more closely adhered to than the requirements to eliminate the inlet end effects.

DYNAMIC DISPLACEMENT METHOD

Relative permeability values for both imbibition and drainage floods may be obtained with this technique. An initial saturation is established in the core; the displacing phase is then flooded into the core. Measurements of the pressure and fluid volumes of each phase produced as functions of time are the data needed to calculate relative permeability curves.[4]

When this technique is used the outlet end effects are reduced by adherence to the scaling criterion --equation 2.14. The major problem with this approach is that with samples of sizes typically available, the flow rates must be much larger than the flow rates that exist in the reservoir. High flow rates are hard to obtain with equipment that is typically available. Also, large flow rates may cause fine movements which will alter the permeability of the samples. Gabriel[33] found that if rates are above a critical velocity, v_c, fines movement may cause significant reduction of permeability. The results from a Berea sandstone are illustrated in Figure 2.12.

Of course, the magnitude of the permeability reduction varies from sample to sample. Rate effects have also been observed by investigators who have noticed that relative permeability data from the dynamic displacement method appears to be rate sensitive.[34]



Macroscopic fluid velocity (cm/sec)

Figure 2.12 -Effects of macroscopic fluid velocity on absolute permeability (Gabriel[33]).

Because of inlet effects, only rarely can the laboratory fluid viscosities be matched to those of the reservoir. Displacing to displaced fluid viscosity ratios are often set low in order to reduce inlet end effects, and increase the two phase flow region. Even with these precautions often only one or two data points will be obtained, and they will usually be at saturations of only limited interest --the rest of the relative permeability curve must be extrapolated.[10,12]

Heterogeneities of the sample may cause the flood front to become unstable. Heterogeneities may also cause relative permeability data obtained from the dynamic displacement method to have peculiar "hump shapes" which are not representative of the bulk properties of the reservoir.[13]

It should be mentioned that recently a promising calculation technique has emerged. This technique makes corrections for the end effects; therefore, slower flow rates can be used. This advancement has the potential to significantly increase the confidence level of the data obtained from the dynamic displacement method.[34]

HASSLER METHOD

Both inlet and outlet end effects are eliminated by placing the sample between two discs which are permeable only to the wetting phase. Radial grooves in the disc allow the non-wetting phase to be transmitted through the disc. Ideally the flow rate of one of the phases is adjusted so that the pressure drop of both phases across the sample is the same. Therefore, the pressure difference between the contiguous fluids, P_c, is held constant across the sample insuring no capillary pressure gradient or saturation gradient. In the past saturations have been determined gravimetrically.

It is hard to be sure the pressure drop of both phases across the sample is constant. It is easy to be certain that the pressure drop across the entire "sandwich" (discs and sample) is constant; however, the pressure drop of each phase through the discs may differ resulting with an unequal pressure drop across the sample. This will cause a capillary pressure gradient and saturation gradient to exist within the sample. This occurrence is illustrated in Figure 2.13, and was first investigated by Ross.[35] Ross showed that this effect causes significant experimental error.

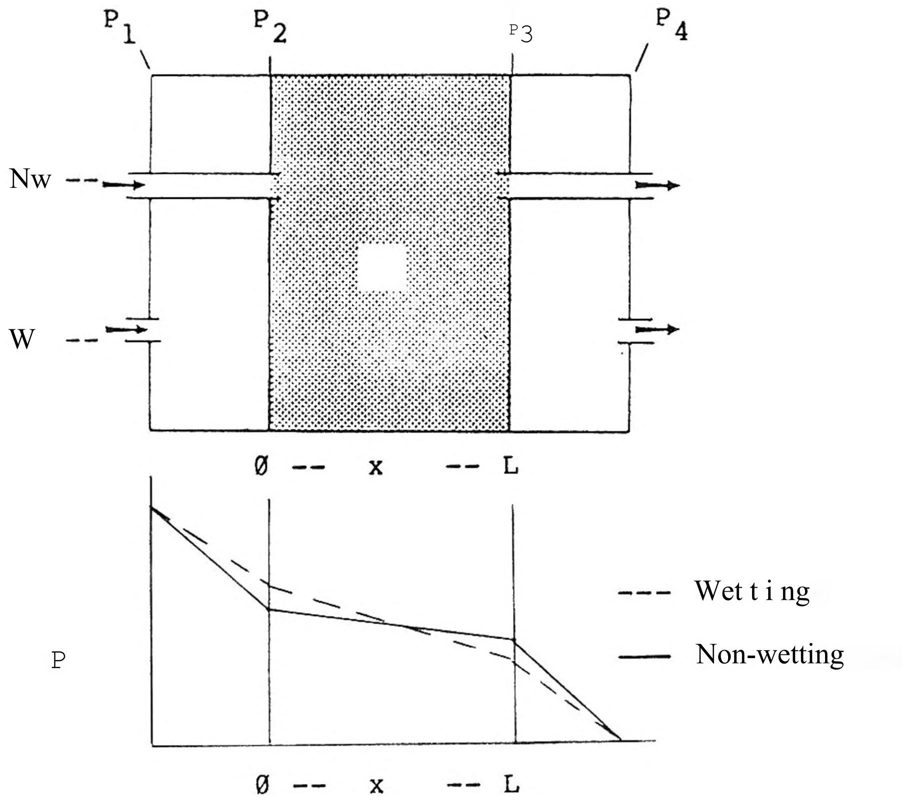


Figure 2.13 -Hassler system.

Saturations determined by weight require flow cessation. This encourages the fluid phases to redistribute to "no-flow" equilibrium positions. The resulting distributions may be un-characteristic of those existing in the reservoir at the same saturation. This effect is especially dramatic in gas/oil systems; when flow is stopped and pressure is removed, the gas will expand causing both oil and gas to be driven from the sample. No rigorous investigations of the magnitude of errors caused by these effects have been reported.

PENN STATE METHOD

Both inlet and outlet effects are eliminated by placing the test sample between two similar materials. The inlet end piece disperses the fluids and the outlet end piece is of sufficient length to remove the effects of the saturation gradient and capillary pressure gradient from the sample. The pressure drop across the sample is measured. Care must be taken in establishing capillary contact between the end pieces and the sample. Richardson, et al.[30] found that large errors can occur if adequate capillary contact is not established.

Saturations in the past have been determined gravemetrically. As discussed in the above paragraph there are some uncertainties with this technique. Saturations may also be determined with X-rays; however, this technology has only recently been developed.

CHAPTER III

METHODOLOGY

DATA SELECTION AND DEFINITION OF SYSTEMS FOR WHICH MODELS ARE APPLICABLE

Clarity of project direction requires a precise statement of the project objectives. The main objective of this project was to develop models that predict multiphase flow characteristics of consolidated porous media, relative permeability curves, from easily obtainable rock and/or fluid properties. Ideally it is desirable to have models that make extremely accurate predictions of flow characteristics for ”all" systems that are of interest and can exist in hydrocarbon reservoirs. While a universal model which will accurately predict all reservoir systems was not obtained; models that predict within reasonable accuracy in several of the most common reservoir systems were developed. These models should be extremely valuable in many situations. The range of application of these models is listed in the INTRODUCTION.

It is apparent from the literature review that there are numerous variables that influence multiphase flow characteristics of porous media. These variables are

(1) Microstructure,
(2) Wettability,
(3) Fluid injection sequences,
(4) Fluid properties (interfacial tension, viscosity).

The literature review also revealed that each experimental technique is to some extent flawed. It is apparent that the accuracy and precision of the relative permeability data depends on the technique from which it was obtained.

Some of the variables that affect relative permeability curves cannot be completely described in a quantitative manner. Other variables that affect the curves were not recorded. The available experimental data was organized according to the experimental technique from which it was obtained. The data was packaged as sets of relative permeability curves and corresponding rock properties --porosity, absolute permeability, capillary pressure data. Interfacial tension was reported as being "high"; however, no specific values were reported. There were certainly some fluctuations of this variable. In gas/oil systems the viscosity ratio is always extremely low; thus, lack of information on the specific value of this variable does not present a problem when trying to develop models for gas/oii systems . In water/oil systems the viscosity ratio was near unity for all the data obtained with the Penn State method. When the dynamic displacement method was used the viscosity ratio was made low in order to reduce end effects and increase the two phase flow region; however, it was noted that the values of this ratio varied from test to test. There were no direct measurements of wettability; estimates of the quantity were determined indirectly. This determination was possible because both gas/oil and water/oil data was available for each sample (This will be discussed in detail in the next section.). Capillary pressure data was available; however, this data gives only a limited description of media microstructure --no information on connectivity.

At an early stage of the project a critical decision had to be made; that is, in view of the absence of information on some of the influencing variables and the magnitude of the experimental error in the existing relative permeability data could any of the project objectives be met? It was decided that models adequate for many situations could be developed. This decision was based on the results of the literature review. Recall from the literature review that in high interfacial tension systems, small changes in either the interfacial tension or in the viscosity ratio do not dramatically change the relative permeability curves;

thus, specific values of interfacial tension and viscosity are not mandatory for models to be developed and applied within this narrow range of systems. Therefore, models applying to the following systems were developed: water/oil systems where a waterflood is initiated at an initial non-flowable water saturation, the fluids have near unit viscosity ratios, high interfacial tension exists between the fluids; gas/oil systems where a gasflood is initiated at an initial non-flowable water saturation, high interfacial tension exists between the fluids, low viscosity ratios. Note: these systems are common in hydrocarbon reservoirs.

Because of the numerous flaws, as discussed in the LITERATURE REVIEW, with the dynamic displacement method and since the viscosity ratio fluctuated, data obtained with this technique was not used. Water/oil systems models were developed with data obtained exclusively from the Penn State method. Gas/oil systems models were developed with data obtained exclusively from the Hassler method. The experimental procedures and the details of these measurement techniques have been discussed by Brownscombe, et al.6

Since much of the existing experimental data was not usable recommendations as to the types of programs needed to meet these objectives (expand applicability of the models and improve their accuracy) are being formulated as part of an extension of this research project.

Figure 3-1 summarizes the process that has been described in this section.

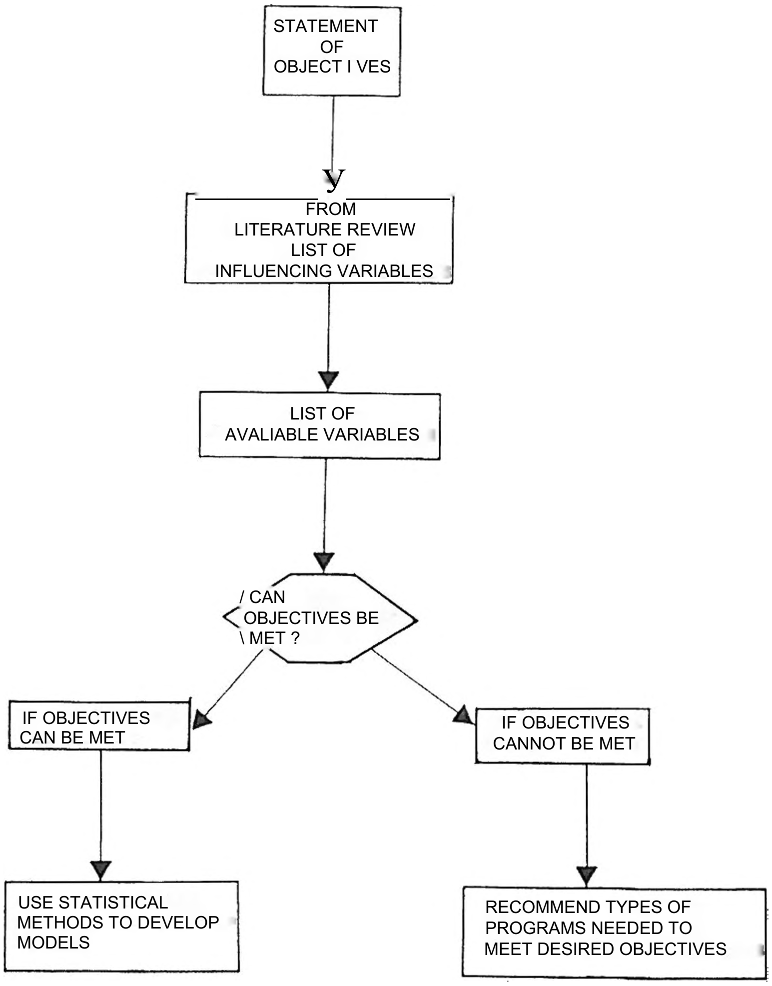


Figure 3-1 -This is a schematic of the project steps.

MODEL DEVELOPMENT

WETTABILITY ASSESMENT

Figure 2.5 illustrates that wettability dramatically affects relative permeability curves; thus, it is imperative to have an assesment of this variable. Ideally this assesment should be independent of the relative permeability curves, and describe in a complete sense the wettability of the system; that is, the assessment should measure any changes in wettability that result with changes of flow properties. One of the major problems in obtaining a complete measure of wettability is the heterogeneous wetting nature of individual pores. For example, contact angle measurements certainly do not give any assesment of the degree of wetting heterogeneit y. Because pore-level physics dominates phase locations within the pore-space[1,7], the inability to make sensitive measures of wettability will limit model accuracy.

Figure 2.5 also indicates that relative permeability is a continuous function of wettability. Because the available data contains no direct measure of wettability, this continuous function had to be approximated with discrete functions. Thus, the data was placed into categories of wettability. The categorization process is described as follows: It is well known that in liquid/gas/solid systems (specifically, systems of hydrocarbons, water and oil bearing rock) the liquid phase is strongly wetting.[1] Therefore, in the gas/oil systems the oil phase is always strongly wetting. For each sample both gas/oil and water/oil relative permeability data was available; an assesment of wettability was made by comparing the curves of water/oil systems to the curves of gas/oil systems. The water/oil systems were classified according to the following scheme :

WATER WET --if,

$$\left[\frac{[k_{ro}(S_{o1}) \text{ for water/oil system}]}{[k_{ro}(S_{o1}) \text{ for gas/oil system}]}\right] > 2 \qquad (3.1)$$

where $S_{o1} \approx 0.8 x (1 - S_{wi})$

and

$$\left[\frac{[k_{rg}(S_{o2}) \text{ for gas/oil system}]}{[k_{rw}(S_{o2}) \text{ for water/oil system}]}\right] > 5 \qquad (3.2)$$

where $S_{o2} \approx 0.2 x (1 - S_{wi})$ for most data; however, occasionally both curves did not have data in this region, in these cases $S_{o2} \approx 0.3 x (1 - S_{wi})$

OIL WET --if,

$$\left[\frac{\text{\^{}ro\^{}ol\^{} fOr water/oil system]}}{\text{Ck}_{ro}\text{(S}_{o1}\text{) for gas/oil system]}}\right] < 1 \qquad (3.3)$$

and

$$\left[\frac{\text{Ck\^{}tS\^{}) for gas/oil system]}}{\text{Lk}_{rw}\text{(8}_{o2}\text{) for water/oil system]}}\right] < 1 \qquad (3.4)$$

INTERMEDIATE WET --Systems that could not be classified as either oil wet or water wet.

The motivation for this type of classification scheme comes from observing Figure 2.5, which was obtained from Owens, et al.[20] The relative permeability curves in this figure were classified according to the above scheme, the systems that had a contact angle between 0 degrees and 35 degrees were classified as water wet; systems that had a contact angle between 135 degrees and 180 degrees were classified as oil wet. (Note: the gas/oil system relative permeability curve used to make these classifications is also presented by Owens et, al.[20]). The discussion in this paragraph is not attempting to imply that there exists a strict quantitative relationship between the above classification scheme and contact angle measurements; instead the discussion is only an attempt to validate, to some extent, this type of classification scheme.

There were initial concerns as to the validity of the above classification scheme. The gas/oil systems are composed of three immiscible fluid phases --oil phase, gas phase and water phase (the initial water saturation). The water/oil systems are composed of two immiscible fluid phases --oil phase and water phase. The exact manner in which these differences affect the above ratios is not clear; however, it was believed that as long as the curves are classified in a consistent manner, this factor will not lead to fallaciously classified systems. Additionally, this scheme adequately classified the data in Figure 2.5.

EVALUATION OF EXISTING MODELS

Numerous models have been proposed in the past.36137,38139140 The best results come from the models proposed by Corey Q V and Honarpourr o£ .

The BMDP Statistical Software Package was used to evaluate these models. This Package was also used in developing the "final models" proposed in this work. The statistical package performs multiple statistical operations including linear and non-linear maximum likelihood estimates. The package will also make a variety of plots that are needed for visual observations of the residuals or errors which are denoted by "E" and defined as follow:

E = (model prediction - actual value) (3.5)

Plots of residuals vs. predicted, relative frequency of residuals, and residuals vs. oil saturation were made for the more plausible existing models. From this analysis it was apparent that models for water/oil systems made predictions of an undesirable accuracy. Models presented by Honarpour made superior predictions for gas/oil systems; however, some of the terms in his displacing phase predicative equation are usually not available until the relative permeability curves have been determined. Discrimination among models requires more of a quantitative assesment than can be obtained from observing plots of the residuals. To make these assessments it is necessary to have an estimate of the "pure error" in the data.

DETERMINATION OF PURE ERROR AND WEIGHTING FUNCTIONS

"Pure error" refers to the random fluctuation of a measured quantity.[5] Errors of this type are intrinsic to all measured data. There were two factors dominating the magnitude of the pure error in the available data; they are experimental errors, and the random fluctuations of un-measured variables. Pure error estimates are usually obtained from data replication and may be computed with the following equation:[41]

$$s_e^{2} = \frac{\sum_{i=1}^{k} \sum_{j=1}^{n_i} \left[w_i \times (Y_{ij} - \bar{Y}_i)^2 \right]}{\sum_{i=1}^{k} \left[n_i - k \right]} \tag{3.6}$$

where Y_{ij} represents the j'th observation of the i'th level (a specific value of the independent variable), k represents the number of levels, n^ is the number of points at this level, $\bar{Y}$^ is the expected value of the random variable at the i'th level, and w^ is a weighting function. The weight is used to ensure that the errors at each level have equal and constant variance. In most cases instead of using a weighting function a transformation of the random variable will be made.

Pure Error in Water/Oil Systems

Replicates were not available. However, an estimate of pure error is necessary to make quantitative discriminations among models. Figure 2.5 illustrates that there is significant variation within a wettability classification. This variation can be viewed as the error due to approximating the continuous changes in wettability with discrete categories. Thus, the variance within a wettability classification caused only by random fluctuations of wettability within this category represents a greatest lower bound of the pure error. This greatest lower bound was used as an estimate of the pure error. Log transformations made the variance in the available data virtually constant for κ_{rw} and k_{ro} in oil and intermediate wetting systems (with $w^{\wedge} = 1$). In water wetting systems the Log transformation made the variance of κ_{rw} constant (with $w_i = 1$) however; for κ_{ro} the Log transformation did not make the variance constant. Therefore, a weighting function was used, $w_i = 1/k^{\wedge}_{o}$. The estimates of pure error for these systems are listed as follows :

Oil Wet

VHbogtk^)] = s_e^2 = 0.0609 with 12 d.F.

V(log(k_{pw})] > s_e^2 = 0.0094 with 17 d.F.

Intermediate Wet

V[Log(k_{ro})] - s_e^2 = 0.042 with 18 d.F.

V[Log(k_{rw})] = s_e^2 = 0.037 with 8 d.F.

Water Wet

V (k_{r0}) = s_e^2 = 0.125 with 7 d.F.

V[L°g(k_{rw})] = s_e^2 = 0.065 with 5 d.F.

where d.F. is the degrees of freedom associated with the relative permeability variable.[5]

Pure Error in Gas Oil Systems

In a rigorous sense there were no replicates available for this system. However, a semi-independent estimate was made from an algorithm presented by Sawyer.[5] The estimates are listed as follows:

V(Log[$_{ro}$ $_e$κ]) ^{2}= S = 0.04 with 5 d.F.

V(Log[$k_{*\sigma o\theta}$]) - $S_{\circ}^2$ = 0.09 with 7 d.F.

EVALUATION OF MODEL ADEQUACY

A statistical test called the F-test was used to evaluate the adequacy of the existing and proposed models. It can be shown that the ratio of two independent estimates of the variance of a normally distributed random variable has a probability density function called the F-distribution. This ratio, denoted by "f", may be computed as follows:

$$f = \left(\frac{\underline{\text{model mean square}}}{S_e^{\,2}} \right) = \left[\frac{\text{SSE/d.F.}}{S_e^{\,2}} \right] \qquad (3.7)$$

where SSE is the sum of the squares of the errors or residuals (defined by equation 3.5). Since the estimates of replication are dubious this test is not rigorous, but it does give a quantitative assesement that can be used for model discrimination. The mean square of models that are statistically adequate should approach the estimate of the pure error in the data. The F-distribution has been tabulated, and the probability that the mean square of a particular model and the pure error estimate are approximating the same value can be determined. The smaller the ratio, f, the higher the probability that the model mean square and the pure error estimate are approximating the same value. These probabilities were compared for model discrimination.

ORIGIN OF "FINAL" MODELS

Numerous existing models and original models were tested. The models determined to be the "best" (presented in APPENDIX A) were the models that had the lowest ratio, f, (thus the highest probability of making predictions within the pure error of the data) and met the following physical and conceptual requirements :

Water/oil systems

(1) $k_{ro}(S_{orw}) = 0$ (3.8)

(2) $k_{rw}(S_{wi}) = 0$ (3.9)

(3) $k_{ro}(1 - S_{wi}) \approx 1$ (3.10)

Gas/oil systems

(1) $k_{ro}(S_{org}) = 0$ (3.11)

(2) $k_{rg}(S_{gc}) = 0$ (3.12)

(3) $k_{ro}(1 - S_{wi}) \sim 1$ (3.13)

The functional forms of all of the displacing phase models presented here are similar to those presented by

Corey[37]. The functional form of equation AII1 is similar to an equation proposed by Honarpour[36]. The functional form of all the other models developed to predict the oil relative permeability was arrived at through the following conceptual considerations: Observations of several Semi-Log plots of relative permeability revealed that Log(k_{ro}) vs. S_o has non-linear tendencies. Thus the idea was to find models that met the requirements imposed by equations 3.Ö – 3-13 and have the Log of the predicted value vary in a non-linear manner with changes in displacing phase saturation. It turned out that based on the above model evaluation process these forms made better prediction for oil relative permeability in gas/oil systems and in water/oil systems which had oil and intermediate wettability.

The parameters of the models were determined with the BMPD non-linear curve fitting program. The weighted or transformed variance was minimized; thus, the parameters have been adjusted such that the models give maximum likelihood estimates. The model developement process is summarized with the following schematic.

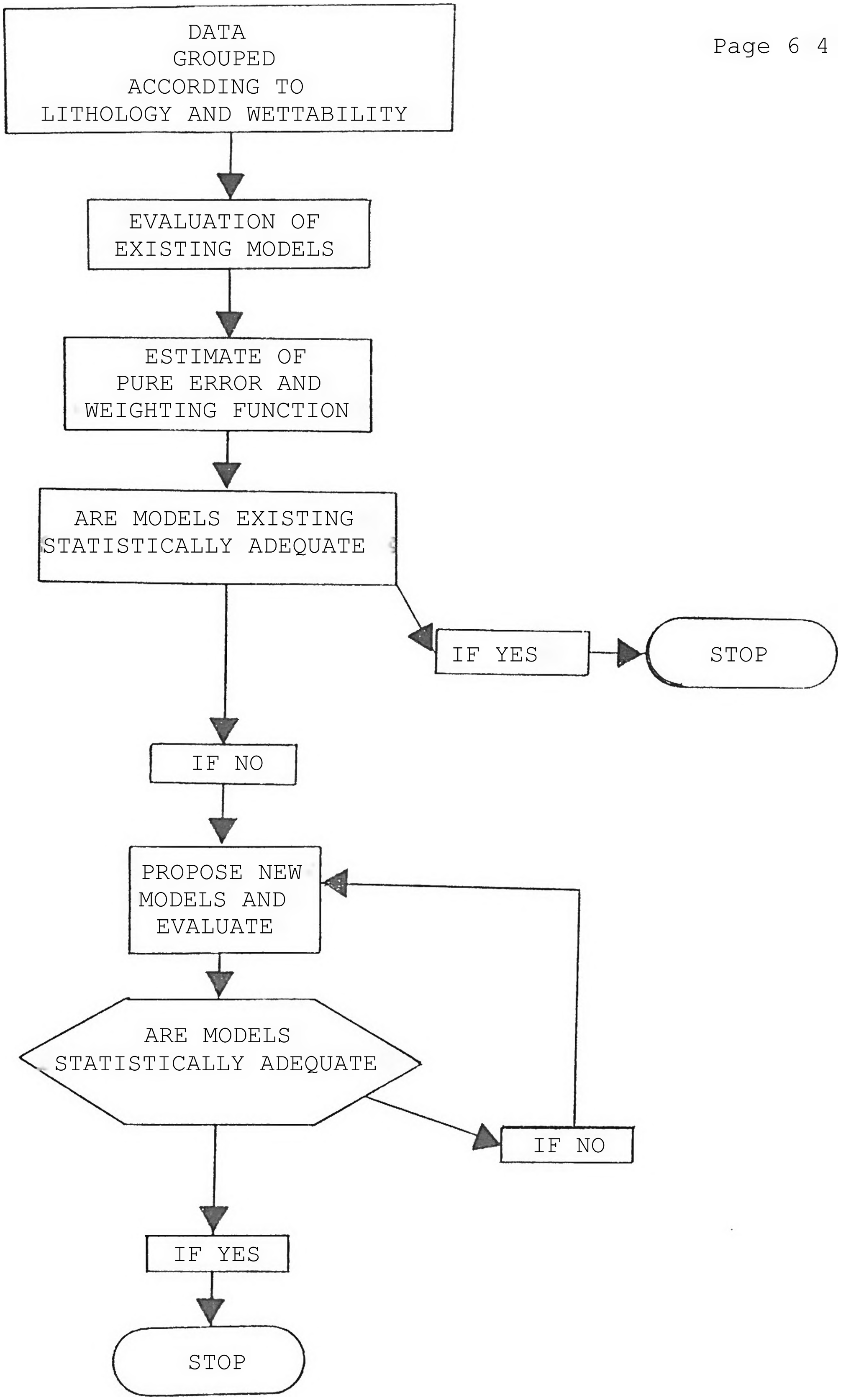


Figure 3.2 -Summary of model development process.

CHAPTER IV

RESULTS

The models which were developed with the methodology of chapter III are presented in APPENDIX A. Illustrations of typical predictions obtained from these models are presented in APPENDIX B. These models have the highest probability of predicting within the pure error of the experimental data used in their development. Table 4.1 lists the total number of data points and the number of relative permeability curves used to develop the models presented in APPENDIX A. The table also gives the system for which each model applies.

Table 4.1 -Summary of "best" models.

EQUATION	SYSTEM	NUMBER OF DATA POINTS	NUMBER OF RELATIVE PERMEABILITY CURVES	QUANTITY PREDICTED
A 1I1	water/oil- water wetting- sandstone	80	13	K_{r0}
ЛИЗ	water/oil - water wetting- sandstone	80	13	K_{rw}
AI 15	water/oil - water wetting- sandstone	80	13	k_{rw}
Alili	water/oil- Intermediate wetting- sandstone	132	22	K_{ro}
AI I I 3	water/oil- Intermediate wetting- sandstone	132	22	k_{rw}
A I 1 I 5	water/oil — intermediate wetting- sandstone	132	22	k_{rw}
AIV 1	water/oil - oil wetting- sandstone	51	8	K_{ro}
AIV3	water/oil- oil wetting- sandstone	51	8	K_{rw}

Table 4.1 -Summary of "best" models (continued) .

EQUATION	SYSTEM	NUMBER OF DATA POINTS	NUMBER OF RELATIVE PERMEABILITY CURVES	QUANTITY PREDICTED
AIV5	water/oil - oil wetting-sandatone	51	A	k_{rw}
AVI	water/oil-Intermediate wetting-carbonate	80	• 3	K_{ro}
AV3	water/oil Intermediate wetting-carbonate	80	13	k_{rw}
AV5	water/oil - Intermediate wetting-carbonate	80	13	K_{rw}
AVII	gas/oil-sandstone	89	400	K_{ro}
AVI3	gas/oil - sandatone	89	'too	K_{rg}
AV 15	gas/oil-sandatone	89	It 00	K_{cg}
AV I I 1	gas/oil - carbonate	18	110	K_{ro}
AV I 13	gas/oil-carbonate	18	ito	K_{rg}
AVI 15	gas/oil - carbonste	18	iio	k_{rg}

Two displacing phase models are presented for each system. If the displacing phase relative permeability at residual oil saturation ($K_{rd}[S_{ord}]$) is known, then the best predictions (for the system under consideration) are obtained from the equations that contain this term. Of course, if $K_{rd}[S_{ord}]$) is not known the equation without this term should be used.

The expected value of the parameters is that which minimizes the sum of the squares of the weighted residuals. These values were determined with a non-linear least squares curve fitting program. This program is part of the BMDP statistical software package.

A function of a random variable may be expanded in a Taylor series about the expected value of the random variable. If the variance of the terms in the expansion is known, then the variance of the series (thus, the variance of the function) may be computed. This type of expansion was used to obtain the variance of the models listed in Table 4.1. The variance of each model is presented in APPENDIX A. The assumptions needed to obtain these equations from the Taylor series expansion are the following:

(1) Second and higher order derivatives evaluated at the expected value of the parameters are negligible in numerical value.

(2) The parameters whose covariance is not known (or presented) are independent random variables.

These assumptions are reasonable at most values of oil saturation; however, in the neighborhood of the end-point saturations, the models, as functions of the parameters, become highly non-linear. Therefore, when the oil saturation is extremely close to the residual oil saturation $(S_o = \bar{S} +_{or\ w} .0001)$ the variance estimates will diverge.

The approximate 95% confidence interval for equation A1 is computed as follows:

$$\boxed{\begin{array}{l}\text{approximate}\\ \text{95\% confidence}\\ \text{i nterval}\end{array}} = \kappa_{r} \pm_{o} \{ 2 \ x \ V \ V_{ro}(\kappa \ j\sim \} \qquad (4.1)$$

The approximate 95% confidence interval for all of the other equation is computed as follows:

$$\boxed{\begin{array}{l}\text{approximate}\\ \text{95\% confidence}\\ \text{interval}\end{array}} - \kappa_{rs} \ x \ 10^{\pm[2} \ xVv\overline{(L0g(k_{rs}) \ ]} \qquad (4.2)$$

where the subscript "s" refers to the phase whose relative permeability is being predicted by the model under consider ation. The probability that the actual value will fall within the approximate 95% confidence interval is approximately 0.95.

CHAPTER V

CONCLUSIONS

(1) A suite of equations has been developed to predict relative permeability curves for the most common types of two phase systems that exist in hydrocarbon reservoirs. Limited input data is needed to use these equations -endpoint saturations only.

(2) The suite is superior to other proposed models because of the following reasons:

- (a) improved accuracy,
- (b) allows confidence interval estimates to be made ,
- (c) is designed to make more accurate predictions as more information becomes available.

(3) Numerous variables influence relative permeability curves. Some of these variables cannot be adequately measured on a quantitative basis.

(4) Lack of information on influencing variables and the experimental error in the available data limits the accuracy of the statistical models that can be developed with this data.

CHAPTER VI

RECOMMENDATION

(1) Investigations into methods of quantitatively measuring microstructure and wettability should be made .

(2) State of the art laboratory equipment should be obtained (collected and/or developed).

(3) State of the art laboratory equipment should be used to collect an extensive set of data (relative permeability curves, measures of all influencing variables) over a wide range of influencing variable values. Replicates should be obtained at each value of these influencing variables.

(4) After successful completion of recommendations (1) and (2), data analysis and statistical modeling techniques[5] should be used to develop statistical models.

APPENDIX A

TABLES OF RESULTS

(”Best” Equations and the Expected Value of Their Parameters)

AI . DEFINITION OF TERMS

Log -Base ten logarithm.

Ln -Base e logarithm.

S_{wI} - This is a non-flowable initial water saturation.

$$SOFG = \frac{(S_o - S_{org})}{(1 - S_{wI} - S_{org})}$$

$$SOFW = \frac{(1 - S_w - S_{orw})}{(1 - S_{wI} - S_{orw})}$$

$$S^* = \frac{(S_w - S_{wI})}{(1 - S_w - S_{orw})}$$

Definition of relative permeability for water/oil systems

$$K_{ro} = \begin{cases} \dfrac{K_o}{K_o(S_{wi})} , & S_{wi} \leqslant S_w \leqslant (1 - S_{orw}) \\ \text{undefined} , & \text{elsewhere} \end{cases}$$

$$K_{rw} = \begin{cases} \dfrac{K_w}{K_o(S_{wi})} , & S_{wi} \leqslant S_w \leqslant (1 - S_{orw}) \\ \text{undefined} , & \text{elsewhere} \end{cases}$$

Definition of relative permeability for gas/oil systems at an initial non-flowable water saturation

$$K_{ro} = \begin{cases} \dfrac{K_o}{k_o(S_{wi})} , & S_{org} < S_o \leqslant (1 - S_{wi}) \\ \text{undefined} , & \text{elsewhere} \end{cases}$$

$$K_{rg} = \begin{cases} \dfrac{K_g}{K_o(S_{wi})} , & S_{org} \leqslant S_o \leqslant (1 - S_{wi} - S_{go}) \\ 0 , & (1 - S_{wi} - S_{go}) < S_o \leqslant (1 - S_{wi}) \\ \text{undefined} , & \text{elsewhere} \end{cases}$$

AII. Water/oil systems -"Best" equations developed for water wetting consolidated porous media -sandstone lithology.

Table AII1 -Range of rock/fluid properties of samples used to develop equations AII1 - AII6.

Properties	Range of Properties
Porosity (Í)	6.7 - 30.2
Absolute permeability (air) (md)	18.7 - 880
Initial water saturation ($)	6.7 - 34.4
Residual oil saturation ($)	11.9 - 36.3

Table AII2 -Expected value of the parameters of equations AII1-AII6.

Parameter	Expected Value
P 1	1.894152
V(P1)	0.056900
P2	-0.884172
V (P2)	0.307000
Cov(P1,P2)	-0.128231
P3	5.788393
v(P3)	0.033789
P 4	2.674019
V (P4)	0.040698
Cov(P3,P4)	-0.023736
P5	2.920516
V(P5)	0.016207
$V(S_{orw})$	0.003000

EQUATION AII1
Relative Permeability to Oil

$$k_{ro}(SOFW) = P1 \times \left[\frac{1 - S_{w} + (S_{orw} \times S_{wi})}{1 - S_{wi} + (S_{orw} \times S_{wi})}\right]^{P2}$$

EQUATION AII2
Variance -Relative Permeability to Oil

$$V(k_{ro}) = \left(\left(\left[\frac{\partial k_{ro}}{\partial P1}\right]^{2} \times V(P1)\right) + \left(\left[\frac{\partial k_{ro}}{\partial P2}\right]^{2} \times V(P2)\right)\right)$$

$$+$$

$$\left(\left[\frac{\partial k_{ro}}{\partial S_{orw}}\right]^{2} \times V(S_{orw})\right)$$

$$+$$

$$\left(2 \times \left[\frac{\partial k_{ro}}{\partial P1}\right] \times \left[\frac{\partial k_{ro}}{\partial P2}\right] \times Cov(P1,P2)\right)$$

Where

$$\frac{\partial k_{ro}}{\partial P1} = (SOFW)^{P1} \times \left[\frac{1 - S_w + (S_{orw} \times S_{wI})}{1 - S_{wI} + (S_{orw} \times S_{wI})}\right]^{P2} \times Ln(SOFW)$$

$$\frac{\partial k_{ro}}{\partial P2} = \left(\Lambda M (SOFW) \times \left[\frac{1 - S_w + (S_{orw}\ \ x\ S_{wl})}{1 - S_{wl}\ (S_{orw} \times S_{wl})}\right]^{P2}\right)$$

$$\times \left(Ln\left[\frac{1 - S_w + (S_{orw} \times S_{wl})}{1 - S_{wl} + (S_{orw} \times S_{wl})}\right]\right)$$

$$\frac{\partial k_{ro}}{\partial S_{orw}} = \left\{\left(P1 \times (SOFW)^{[P1 - 1]} \times \left[\frac{1 - S_w + (S_{orw} \times S_{wI})}{1 - S_{wI} + (S_{orw} \times S_{wI})}\right]^{P2}\right)\right.$$

$$\left.\times \left(\left\langle\frac{(-1)}{(1 - S_{wI} - S_{orw})}\right\rangle + \left\langle\frac{(1 - S_w - S_{orw})}{(1 - S_{wI} - S_{orw})^2}\right\rangle\right)\right\}$$

$$+$$

$$\left\{\left(P2 \times (SOFW)^{P1} \times \left[\frac{1 - S_w + (S_{orw} \times S_{wI})}{1 - S_{wI} + (S_{orw} \times S_{wI})}\right]^{[P2 - 1]}\right)\right.$$

$$\times \left(\left\langle\frac{(S_{wI})}{1 - S_{wI} + (S_{orw} \times S_{wl})}\right\rangle\right.$$

$$+$$

$$\left.\left.\left\langle\frac{\{1 - s_w + (S_{orw} \times S_{wl})\} \times (-S_{wl})}{(1 - S_{wl} + (S_{orw} \times S_{wl}))\}^2}\right\rangle\right)\right\}$$

EQUATION AII3
Relative Permeability to Water

$$k_{rw} = \left[1 - \left\langle \frac{S_{or\ W}}{(1 - S_{wI})} \right\rangle \right]^{P3} \quad x\ (1 - \mathrm{SOFW})^{P\ 1|}$$

EQUATION AII4
Variance -Relative Permeability to Water

Let $F = \mathrm{Log}(k_{rw})$

$$V(F) = \left(\left\langle \left[\frac{\partial F}{\partial P3} \right]^2 x\ V(P3) \right\rangle + \left\langle \left[\frac{\partial F}{\partial P4} \right]^2 x\ V(P4) \right\rangle \right)$$

$$+$$

$$\left(\left[\frac{\partial F}{\partial S_{orw}} \right]^2 x\ V(S_{orw}) \right)$$

$$+$$

$$\left(2\ x \left[\frac{\partial F}{\partial P3} \right] x \left[\frac{\text{PSF}}{\text{ЭРД}} \right] x\ \mathrm{Cov}(P3, P4) \right)$$

Where

$$\frac{\partial F}{\partial P3} = \text{Log}\left[1 - \left\langle\frac{S_{orw}}{(1 - S_{wI})}\right\rangle\right]$$

$$\frac{\partial F}{\hat{O}P\ 4} = \text{Lógd - SOFW)}$$

$$\frac{\hat{O}F}{\hat{O}S_{or\ w}} = \left\{\left[\frac{\left(-P3 \times \text{Log}(e)\right)}{\left[1 - \left(\frac{S_{orw}}{(1 - S_{wI})}\right)\right]}\right] \times \left[\frac{1}{(1 - S_{wI})}\right]\right\}$$

$$+$$

$$\left\{\left[\frac{-P4 \times \text{Log}(e)}{\left(1 - \text{SOF,}\right)}\right] \times \left[\left\langle\frac{(1 - S_w - S_{orw})}{(1 - S_{wI} - S_{orw})^2}\right\rangle - \left\langle\frac{1}{(1 - S_{wI} - S_{orw})}\right\rangle\right]\right\}$$

EQUATION AII5
Relative Permeability to Water

$$k_{rw} = k_{rw}\,k_{or}(S_w) \times (1 - SOFW)^{P5}$$

EQUATION AII6
Variance -Relative Permeability to Water

Let $F = \mathrm{Log}(\kappa_{rw})$

$$V(F) = \left(\left\langle \left[\frac{\partial F}{\partial P5} \right]^2 \times V(P5) \right\rangle + \left\langle \left[\frac{\partial F}{\partial S_{orw}} \right]^2 \times V(S_{orw}) \right\rangle \right)$$

Where

$$\frac{\partial F}{\partial P5} = \mathrm{Log}(1 - SOFW)$$

$$\frac{\partial F}{\partial S_{orw}} = \left(\frac{-P1 \times \mathrm{Log}(e)}{(1 - SOFW)} \right)$$

$$\times \left(\left\langle \frac{(1 - S_w - S_{orw})}{(1 - S_{wI} - S_{orw})^2} \right\rangle - \left\langle \frac{1}{(1 - S_{wI} - S_{orw})} \right\rangle \right)$$

AHI. Water/oil systems -"Best" equations developed for intermediate wetting consolidated porous media -sandstone lithology.

Table AIII1 -Range of rock/fluid properties of samples used to develop equations AIII1 - AIII6.

Properties	Range of Properties
Porosity (%)	9-9 - 32.9
Absolute permeability (air) (md)	4.4 - 2,260
Initial water saturation (%)	6.1 - 44.7
Residual oil saturation ($)	20.4 - 47-3

Table AIII2 -Expected value of the parameters of equations Alili - AIII6.

Parameter	Expected Value
P6	2.004366
V (P6)	0.012278
P7	-0.071378
V(P7)	0.000436
Cov(P6 , P7)	-0.002039
P8	3.985186
V(P8)	0.078672
P9	0.325444
V (P9)	0.003694
Cov(P8 , P9)	0.014165
P10	0.298413
V(P10)	0.000632
$V(s_{or\,w})$	0.003000

EQUATION Alili
Relative Permeability to Oil

$$k_{ro} = \frac{C\ P6 \times (s^*)^{P7}\ J}{SOFW}$$

EQUATION AIII2
Variance -Relative Permeability to Oil

Let $F = \mathrm{Log}(k_{ro})$

$$V(F) = \left(\left\langle \left[\frac{\partial F}{\partial P6} \right]^2 \times V(P6) \right\rangle + \left\langle \left[\frac{\partial F}{\partial P7} \right]^2 \times V(7) \right\rangle \right)$$

$$+$$

$$\left(\left[\frac{\partial F}{\partial S_{orW}} \right]^2 \times v(S_{orw}) \right)$$

$$+$$

$$\left(2 \times \left[\frac{\partial F}{\partial P6} \right] \times \left[\frac{\partial F}{\partial P7} \right] \times \mathrm{Cov}(P6, P7) \right)$$

Where

$$\frac{\partial F}{\partial P6} = (S^*)^{P7} \times \mathrm{Log}(SOFW)$$

$$\frac{\partial F}{\partial P7} = P6 \times (S^*)^{P7} \times \mathrm{Log}(SOFW) \times \mathrm{Ln}(S^*)$$

$$\frac{\partial F}{\partial S_{orW}} = \left\{ \left[P6 \times P7 \times (S^*)^{[P7 - 1]} \right] \times \left[\mathrm{Log}(SOFW) \times \left\langle \frac{S_w - S_{wI}}{(1 - S_w - S_{orw})^2} \right\rangle \right] \right\}$$

$$+$$

$$\left\{ \left[\frac{\mathrm{Log}(e) \times P6 \times (S^*)^{P7}}{SOFW} \right] \times \left[\left\langle \frac{(1 - S_w - S_{orw})}{(1 - S_{wI} - S_{orw})^2} \right\rangle - \left\langle \frac{1}{(1 - S_{wI} - S_{orw})} \right\rangle \right] \right\}$$

EQUATION AIII3
Relative Permeability to Water

$$K_{rw} = \left[1 - \left\langle \frac{S_{orw}}{(1 - S_{wI})} \right\rangle \right]^{P8} \times \left[1 - (SOFW)^{P.9} \right]$$

EQUATION АШД
Variance -Relative Permeability to Water

Let F ’ L°8<k_{rw})

$$V(F) » \left(\left\langle \left[\frac{\partial F}{\partial P8} \right]^2 \times V(P8) \right\rangle + \left\langle \left[\frac{\partial F}{\partial P9} \right]^2 \times V(P9) \right\rangle \right)$$

$$+$$

$$\left(\left[\frac{8F}{\hat{O}S_{orw}} \right]^{'I\ 2} \times V(S_{orw}) \right)$$

$$+$$

$$\left(2 \times \left[\frac{\hat{O}F}{3P8} \right] \times \left[\frac{ðF}{3P9} \right] \times Cov(P8, P9) \right)$$

Where

$$\frac{\partial F}{\partial P8} = \text{Log}\left[1 - \left\langle\frac{S_{orw}}{(1 - S_{wI})}\right\rangle\right]$$

$$\frac{\partial F}{\partial P9} = \left[\frac{-\ \text{Log}(e) \times (\text{SOFW})^{P9} \times \text{Ln}(\text{SOFW})}{1 - (\text{SOFW})^{P9}}\right]$$

$$\frac{\partial F}{\partial S_{orw}} = \left\{\left[\frac{-\ P8 \times \text{Log}(e)}{\left[1 - \left(\frac{S_{orw}}{(1 - S_{wI})}\right)\right]}\right] \times \left\langle\frac{1}{(1 - S_{wI})}\right\rangle\right\}$$

$$+$$

$$\left\{\left(\frac{-P9 \times \text{Log}(e) \times (\text{SOFW})^{[P9 - 1]}}{1 - (\text{SOFW})^{P8}}\right)\right.$$

$$\left.\times \left(\left\langle\frac{(1 - S_w - S_{orw})}{(1 - S_{wI} - S_{orw})^2}\right\rangle - \left\langle\frac{1}{(1 - S_{wI} - S_{orw})^2}\right\rangle\right)\right\}$$

EQUATION AIII5
Relative Permeability to Water

$$k_{rw} \blacksquare {}^{k}r_{w}({}^{s}or_{w}{}' \,{}^{x}\, \Gamma' \,-O0FW)^{M0} \quad]$$

EQUATION AII16
Variance -Relative Permeability to Water

Let $F = \text{Lo g}\,(\kappa_{rw})$

$$V(F) = \left(\left[\frac{\tilde{O}F}{ЭP1o}\right]^{2} \times V(P10)\right) + \left(\left[\frac{\hat{O}F}{3S_{orw}}\right]^{2} \times V(S_{orw})\right)$$

Where

$$\frac{3F}{3P10} = \left(\frac{-\ \text{Log}(e) \times (SOFW)^{P1^{\circ}} \times \text{Ln}(SOFW)}{1 - (SOFW)^{P1^{\circ}}}\right)$$

$$\frac{\partial F}{3S_{orw}} = \left(\frac{-\ \text{Log}(e) \times PЮ \times (SOFW)[^{P1^{\circ}} - 1]}{1 - (SOFW)^{P1^{\circ}}}\right)$$

$$\times \left(\left\langle\frac{(1 - S_{w} - S_{orw})}{(1 - S_{wI} - S_{orw})^{2}}\right\rangle - \left\langle\frac{1}{(1 - S_{wI} - S_{orw})}\right\rangle\right)$$

AIV. Water/oil systems -"Best" equations developed for oil wetting consolidated porous media -sandstone lithology.

Table AIV1 -Range of rock/fluid properties of samples used to develop equations AIV1 - AIV6.

Properties	Range of Properties
Porosity (%)	12.0 - 30.2
Absolute permeability (air) (md)	12.5 - 590
Initial water saturation ($)	12.0 - 36.0
Residual oil saturation ($)	6.8 - 29.5

Table AIV2 -Expected value of the parameters of equations AIV1 - AIV6.

Parameter	Expected Value
P11	4.633225
V(P11)	0.049147
P12	-0.160940
V(P12)	0.004561
Cov(P11, P12)	-0.005039
P13	3.508615
V(P13)	0.403488
P14	0.388657
V(P14)	0.010850
Cov(P13,P14)	0.055532
P15	0.308159
V(P15)	0.001942
$V(S_{orw})$	0.003000

EQUATION AIV1
Relative Permeability to Oil

$$k_{ro} = SOFW \left[P11 \times (S^*)^{P12} \right]$$

EQUATION AIV2
Variance -Relative Permeability to Oil

Let $F = \mathrm{Log}(k_{ro})$

$$V(F) = \left(\left\langle \left[\frac{\partial F}{\partial P11} \right]^2 \times V(P11) \right\rangle + \left\langle \left[\frac{\partial F}{\partial P12} \right]^2 \times V(P12) \right\rangle \right)$$

$$+$$

$$\left(\left[\frac{\partial F}{\partial S_{orw}} \right]^2 \times V(S_{orw}) \right)$$

$$+$$

$$\left(2 \times \left[\frac{\partial F}{\partial P11} \right] \times \left[\frac{\partial F}{\partial P12} \right] \times \mathrm{Cov}(P11, P12) \right)$$

Where

$$\frac{\text{ÒF}}{\text{BP}\,1\,1} = (S^*)^{P12} \times \mathrm{Log}(SOFW)$$

$$\frac{\text{BF}}{\text{BP}\,1\,2} = P1\,1 \times (S^*)^{P12} \times \mathrm{Log}(SOFW) \times \mathrm{Ln}(S^*)$$

$$\frac{\text{BF}}{\text{Bs}_{orw}} = \left\{ \left[P1\,1 \times P12 \times (S^*)^{\wedge 12} - 1 \right] \right.$$

$$\left. \times \left[\mathrm{Log}(SOFW) \times \left\langle \frac{S_w - S_{wI}}{(1 - S_w - S_{orw})^2} \right\rangle \right] \right\}$$

$$+$$

$$\left\{ \left[\frac{\mathrm{Log}(e) \times P11 \times (S^*)^{P12}}{SOFW} \right] \right.$$

$$\left. \times \left[\left\langle \frac{(1 - S_w - S_{orw})}{(1 - S_{wI} - S_{orw})^2} \right\rangle - \left\langle \frac{1}{(1 - S_{wI} - S_{orw})} \right\rangle \right] \right\}$$

EQUATION AIV3
Relative Permeability to Water

$$k_{rw} = \left[1 - \left\langle \frac{S_{or\ W}}{(1 - S_{wI})} \right\rangle \right]^{P13} \quad x \quad \overset{-}{\text{[j}} - (\text{SOFW})\text{^^} \; \Big]$$

EQUATION AIV4
Variance -Relative Permeability to Oil

Let $F = \mathrm{L\,o\,g}\,(\kappa_{r\,w})$

$$V(F) = \left(\left\langle \left[\frac{\partial F}{\partial P13} \right]^2 x \; V(P13) \right\rangle + \left\langle \left[\frac{\partial F}{\partial P14} \right]^2 x \; V(P14) \right\rangle \right)$$

$$+$$

$$\left(\left[\frac{3F}{\partial s_{or\ w}} \right]^2 x \quad V(S_{or\ w}) \right)$$

$$+$$

$$\left(2 \; x \left[\frac{\text{ЭE}}{\text{BPI}_{\text{з}}} \right] x \left[\frac{\text{ЭE}}{\text{ЭP I}_{\text{д}}} \right] x \; \text{Cov(P1з,p 1 м)} \right)$$

Where

$$\frac{\partial F}{\partial P13} = \mathrm{Log}\left[1 - \left\langle\frac{S_{orw}}{(1 - S_{wI})}\right\rangle\right]$$

$$\frac{3F}{OP1\ M} \,\overset{"}{=}\, \left[\frac{-\ \mathrm{Log(e)}\ x\ (\mathrm{SOFwf}\hat{}\ x\ \mathrm{Ln(SOFW)}}{1 - (\mathrm{SOFW}\ )^{P14}}\right]$$

$$\frac{\grave{O}F}{\partial S_{orw}} = \left\{\left[\frac{-\ P13\ x\ \mathrm{Log(e)}}{\left[1 - \left(\frac{S_{or\ w}}{(1 - S_{wI})}\right)\right]}\right] x \left\langle\frac{1}{(1 - S_{wI})}\right\rangle\right\}$$

$$+$$

$$\left[\frac{-PI\ 4\ x\ \mathrm{Log(e)}\ x\ (\mathrm{SOFW})^{L\ \Gamma\ P\ 1\ 4\ -\ 1]}}{1 - (\mathrm{SOFW})^{P_{\circ}1\ ?}}\right]$$

$$x \left[\left\langle\frac{(1 - S_{w} - S_{orw})}{(1 - S_{wI} - S_{orw})^{2}}\right\rangle - \left\langle\frac{1}{(1 - S_{wI} - S_{orw})^{2}}\right\rangle\right]$$

EQUATION AIV5
Relative Permeability to Water

$$k_{rw} = k_{rw}(S_{orw}) \quad x \; (1 - <\text{SOFW})^{P15}\}$$

EQUATION AIV6
Variance -Relative Permeability to Water

Let F « Lo g (κ_{rw})

$$V(F) = \left\langle \left[\frac{\partial F}{\partial P15}\right]^2 \text{ x } V(P15) \right\rangle + \left\langle \left[\frac{3F}{3s_{or\,w}}\right]^2 \text{ x } V(S_{orw}) \right\rangle$$

Where

$$\frac{\partial F}{\partial P15} = \left(- \frac{\text{Log}(e) \text{ x } (\text{SOFW})^{pic_3} \text{ x } \text{Ln}(\text{SOFW})}{1 - (\text{SOFW})^{P15}} \right)$$

$$\frac{\partial F}{\partial S_{orw}} = \left[- \frac{\text{Log}(e) \text{ x } P15 \text{ x } (\text{SOFW})^{[P15 - 1]}}{1 - (\text{SOFW})^{P15}} \right]$$

$$\text{x} \left[\left\langle \frac{(1 - S_w - S_{orw})}{(1 - S_{wI} - S_{orw})^2} \right\rangle - \left\langle \frac{1}{(1 - S_{wI} - S_{orw})} \right\rangle \right]$$

AV. Water/oil systems -"Best" equations developed for intermediate wetting consolidated porous media -carbonate lithology.

Table AV1 -Range of rock/fluid properties of samples used to develop equations AV1 - AV 6.

Properties	Range of Properties
Porosity ($)	9.0 - 28.0
Absolute permeability (air) (md)	4.3 - 161
Initial water saturation (?)	7.7 - 19.5
Residual oil saturation (?)	8.1 - 41.1

Table AV2 -Expected value of the parameters of equations AV1 - AV6.

Parameter	Expected Value
P16	0.170081
V(P16)	0.036644
P17	5.362933
V(P17)	0.243111
Cov(P17,P16)	-0.084758
P18	1.779004
V(18)	0.044825
P19	2.326556
V(P19)	0.009289
Cov(P18,P19)	-0.011972
P20	2.287490
V(P20)	0.006932
$V(S_{or\,w})$	0.003000

EQUATION AV1
Relative Permeability to Oil

$$k_{ro} = (SOFW)^{P16} \times \left[\frac{1 - S_w + (S_{orw} \times S_{wl})}{1 - S_{wl} + (S_{orw} \times S_{wl})}\right]^{P17}$$

EQUATION AV2
Variance -Relative Permeability to Oil

Let $F = \mathrm{Log}(k_{ro})$

$$V(F) = \left(\left\langle\left[\frac{\partial F}{\partial P16}\right]^2 \times V(P16)\right\rangle + \left\langle\left[\frac{\partial F}{\partial P17}\right]^2 \times V(P17)\right\rangle\right)$$

$$+$$

$$\left(\left[\frac{\partial F}{\partial S_{orw}}\right]^2 \times V(S_{orw})\right)$$

$$+$$

$$\left(2 \times \frac{\partial F}{\partial P16} \times \frac{\partial F}{\partial P17} \times \mathrm{Cov}(P16, P17)\right)$$

Where

$$\frac{\partial F}{\partial P16} = \mathrm{Log(SOFW)}$$

$$\frac{\partial F}{\partial P17} = \mathrm{Log}\left[\frac{1 - S_w + (S_{orw} \times S_{wI})}{1 - S_{wI} + (S_{orw} \times S_{wI})}\right]$$

$$\frac{\partial F}{\partial S_{orw}} = \left\{ \left(\frac{P16 \times \mathrm{Log(SOFW)}}{(\mathrm{SOFW})}\right) \times \left(\left\langle\frac{(-1)}{(1 - S_{wI} - S_{orw})}\right\rangle + \left\langle\frac{(1 - S_w - S_{orw})}{(1 - S_{wI} - S_{orw})^2}\right\rangle\right)\right\}$$

$$+$$

$$\left\{\left[\frac{P17 \times \mathrm{Log}(e)}{\left[\dfrac{1 - S_w + (S_{orw} \times S_{wI})}{1 - S_{wI} + (S_{orw} \times S_{wI})}\right]}\right] \times \left[\left\langle\frac{(S_{wI})}{1 - S_{wI} + (S_{orw} \times S_{wI})}\right\rangle + \left\langle\frac{\{1 - S_w + (S_{orw} \times S_{wI})\} \times (-S_{wI})}{\{1 - S_{wI} + (S_{orw} \times S_{wI})\}^2}\right\rangle\right]\right\}$$

EQUATION AV3
Relative Permeability to Water

$$K_{rw} = \left[1 - \left\langle \frac{S_{orw}}{(1 - S_{wI})} \right\rangle\right]^{P18} \times (1 - SOFW)^{P19}$$

EQUATION AVM
Variance -Relative Permeability to Water

Let $F = \mathrm{Log}(k_{rw})$

$$V(F) = \left(\left\langle \left[\frac{\partial F}{\partial P18}\right]^2 \times V(P18) \right\rangle + \left\langle \left[\frac{\partial F}{\partial P19}\right]^2 \times V(P19) \right\rangle \right)$$

$$+$$

$$\left(\left[\frac{\partial F}{\partial S_{orw}}\right]^2 \times V(S_{orw}) \right)$$

$$+$$

$$\left(2 \times \left[\frac{\partial F}{\partial P18}\right] \times \left[\frac{\partial F}{\partial P19}\right] \times \mathrm{Cov}(P18,P19) \right)$$

Where

$$\frac{\text{ÒF}}{\text{ÒP 1 8}} = \text{Log}\left[1 - \left\langle \frac{S_{orw}}{(1 - S_{wI})} \right\rangle \right]$$

$$\frac{\text{8F}}{\text{ЭP 1 9}} = \text{Log}(1 - \text{SOFW})$$

$$\frac{\text{8F}}{\text{8s}_{orw}} = \left\{ \left[\frac{-P18 \text{ x } \text{Log}(e)}{\left[1 - \left(\frac{S_{orw}}{(1 - S_{wI})}\right)\right]} \right] \text{ x } \left\langle \frac{1}{(1 - S_{wI})} \right\rangle \right\} + \left\{ \left[\frac{-P19 \text{ x } \text{Log}(e)}{1 - \text{SOFW}}\right] \text{ x } \left[\left\langle \frac{(1 - S_{w} - S_{orw})}{(1 - S_{wI} - S_{orw})^{2}} \right\rangle - \left\langle \frac{1}{(1 - S_{wI} - S_{orw})} \right\rangle \right] \right\}$$

EQUATION AV5
Relative Permeability to Water

$$K_{rw} = k_{rw}(S_{orw}) \ x \ (1 - SOFW)^{P2^\circ}$$

EQUATION AV6
Variance -Relative Permeability to Water

Let F » Log(k^^)

$$V(F) = \left\langle \left[\frac{\partial F}{\partial P20}\right]^2 x \ V(P18) \right\rangle + \left\langle \left[\frac{3F}{\eth s_{orw}}\right]^2 x \ V(S_{orw}) \right\rangle$$

Where

$$\frac{3F}{3P20} \ » \ Log(1 - SOFW)$$

$$\frac{\partial F}{3s_{orw}} = \left(\frac{-P20 \ x \ Log(e)}{(1 - SOFW)}\right)$$

$$x \left(\left\langle \frac{(1 - S_w - S_{orw})}{(1 - S_{wI} - S_{orw})^2} \right\rangle - \left\langle \frac{1}{(1 - S_{wI} - S_{orw})} \right\rangle \right)$$

AVI. Gas/oil systems at initial non-flowable water saturation **-sandstone lithology.**

Table AVI1 -Range of rock/fluid properties of samples used to develop equations AVI1 - AVI6.

Proper ties	Range of Properties
Porosity (%)	8.9 - 30.6
Absolute permeability (air) (md)	4.4-2,260
Initial water saturation ($)	1.8 - 55.0
Residual oil saturation (%)	2.0 - 31.9

Table AVI2 -Expected value of the parameters of equations AVI1 - AVI6.

Parameter	Expected Value
P21	4.278842
V(P21)	0.002556
P22	-0.184391
V(P22)	0.000248
Cov (P21 , P22)	-0.000214
P23	0.845762
V(P23)	0.009560
P24	1.479469
V(P24)	0.028514
Cov(P23 , P24)	-0.015954
P25	0.866075
V(P25)	0.009949
P26	1.434929
V(26)	0.024656
Cov(P25 , P26)	-0.015116
$V(S_{org})$	0.003000
$V(S_{go})$	0.001250

EQUATION AVII
Relative Permeability to Oil

$$k_{ro} = SOFG^{\left\{ P21 \times \left[\frac{(1 - S_{wI} - S_o)}{S_o} \right]^{P22} \right\}}$$

EQUATION AVI2
Variance -Relative Permeability to Oil

Let $F = \mathrm{Log}(k_{ro})$

$$V(F) = \left(\left\langle \left[\frac{\partial F}{\partial P21} \right]^2 \times V(P21) \right\rangle + \left\langle \left[\frac{\partial F}{\partial P22} \right]^2 \times V(P22) \right\rangle \right)$$

$$+$$

$$\left(\left[\frac{\partial F}{\partial S_{org}} \right]^2 \times V(S_{org}) \right)$$

$$+$$

$$\left(2 \times \left[\frac{\partial F}{\partial P21} \right] \times \left[\frac{\partial F}{\partial P22} \right] \times \mathrm{Cov}(P21, P22) \right)$$

Where

$$\frac{\partial F}{\partial P21} = \left[\frac{(1 - S_{wI} - S_o)}{S_o}\right]^{P22} \times \mathrm{Log(SOFG)}$$

$$\frac{\partial F}{\partial P22} = \left(P21 \times \left[\frac{(1 - S_{wI} - S_o)}{S_o}\right]^{P22} \times \mathrm{Ln}\left[\frac{(1 - S_{wI} - S_o)}{S_o}\right]\right)$$

$$\times \left(\mathrm{Log(SOFG)}\right)$$

$$\frac{\partial F}{\partial S_{org}} = \left(P21 \times \left[\frac{(1 - S_{wI} - S_o)}{S_o}\right]^{P22} \times \left[\frac{\mathrm{Log(e)}}{\mathrm{SOFG}}\right]\right)$$

$$\times \left(\left\langle\frac{(-1)}{(1 - S_{wI} - S_{org})}\right\rangle + \left\langle\frac{(S_o - S_{org})}{(1 - S_{wI} - S_{org})^2}\right\rangle\right)$$

EQUATION AVI3
Relative Permeability to Gas

$$K_{rg} = \left[\frac{(1 - S_{o} - S_{wl} - S_{go} - S)}{(1 - S_{wI} - S_{gc})}\right]^{P23} \times (1 - \text{SOFG})^{\wedge\wedge}$$

EQUATION AVI4
Variance -Relative Permeability to Gas

Let F Log(k_{rg})

$$V(F) = \left(\left\langle\left[\frac{\partial F}{\partial P23}\right]^{2} \times V(P23)\right\rangle + \left\langle\left[\frac{dF}{3P24}\right]^{2} \times V(P24)\right\rangle\right)$$

$$+$$

$$\left(\left\langle\left[\frac{3F}{9s_{org}}\right]^{2} \times V(S_{org})\right\rangle + \left\langle\left[\frac{\partial F}{\partial S_{gc}}\right]^{2} \times V(S_{gc})\right\rangle\right)$$

$$+$$

$$\left(2 \times \left[\frac{\partial F}{\partial P23}\right] \times \left[\frac{\partial F}{8P24}\right] \times \text{Cov}(P23, P24)\right)$$

Where

$$\frac{\partial F}{\partial P23} = \text{Log}\left[\frac{(1 - S_o - S_{wI} - S_{gc})}{(1 - s_{wl} - S_{gc})}\right]$$

$$\frac{\hat{O}F}{8\ P42} = \text{Log}(1 - \text{SOFG})$$

$$\frac{\partial F}{8s_{or\,g}} = \left[\frac{-\ P24 \times \text{Log}(e)}{1 - \text{SOFG}}\right]$$

$$\times \left[\left\langle\frac{(S_o - S_{org})}{(1 - S_{wI} - S_{org})^2}\right\rangle - \left\langle\frac{1}{(1 - S_{wI} - S_{org})}\right\rangle\right]$$

$$\frac{\partial F}{\eth s_{go}} = \left[\frac{P23 \times \text{Log}(e)}{\left[\frac{(1 - S_o - S_{wI} - S_{gc})}{(1 - S_{wI} - S_{gc})}\right]}\right]$$

$$\times \left[\left\langle\frac{(1 - S_o - S_{wI} - \text{JW}}{(1 - S_{wI} - S_{gc})^2}\right\rangle - \left\langle\frac{1}{(1 - S_{wI} - S_{gc})}\right\rangle\right]$$

EQUATION AVI5
Relative Permeability to Gas

$$k_{rg} = K_{rg}(S_{or})_g \times \left[\frac{\left(1 - S_o - S_{wl} S_{gc} - \quad \right)}{\left(1 - S_{gc} - S_{wl} - S_{org}\right)} \right]^{P25}$$

$$\times \quad (1 - SOFG)^{P26}$$

EQUATION AVI6
Variance -Relative Permeability to Gas

Let F » $\mathrm{Log}(k_{rg})$

$$V(F) = \left(\left\langle \left[\frac{\partial F}{\partial P25} \right]^2 \times V(P25) \right\rangle + \left\langle \left[\frac{\partial F}{\partial P26} \right]^2 \times V(P26) \right\rangle \right)$$

$$+$$

$$\left(\left\langle \left[\frac{\partial F}{\partial S_{org}} \right]^2 \times V(S_{org}) \right\rangle + \left\langle \left[\frac{\partial F}{\partial S_{gc}} \right]^2 \times V(S_{gc}) \right\rangle \right)$$

$$\left(\left\langle 2 \times \left[\frac{3F}{3P25} \right] \left[\frac{3F}{3P26} \right] \times \mathrm{Cov}(P25, P26) \right\rangle \right)$$

Where

$$\frac{\partial F}{\partial P25} = \mathrm{Log}\left[\frac{(1 - S_o - S_{wI} - S_{gc})}{(1 - S_{gc} - S_{wI} - S_{org})}\right]$$

$$\frac{\partial F}{\partial P26} = \mathrm{Log}(1 - SOFG)$$

$$\frac{\partial F}{\partial S_{org}} = \left\{\left[\frac{P25 \times \mathrm{Log}(e)}{\left[\dfrac{(1 - S_o - S_{wI} - S_{gc})}{(1 - S_L - S_{wI} - S_{gc} - S_{org})}\right]}\right]\right.$$

$$\left.\times \left[\frac{(1 - S_o - S_{wI} - S_{gc})}{(1 - S_{wI} - S_{gc} - S_{org})^2}\right]\right\}$$

$$+$$

$$\left\{\left[\frac{-P26 \times \mathrm{Log}(e)}{(1 - SOFG)}\right]\right.$$

$$\left.\times \left[\left\langle\frac{(S_o - S_{org})}{(1 - S_{wI} - S_{org})^2}\right\rangle - \left\langle\frac{1}{(1 - S_{wI} - S_{org})}\right\rangle\right]\right\}$$

$$\frac{\partial F}{\partial S_{ge}} = \left[\frac{P25}{\left[\dfrac{(1 - S_{o} - S_{wI} - S_{gc})}{(1 - S_{ge} - S_{wI} - S_{org})}\right]} \right]$$

$$\times \left[\left\langle \frac{(1 - S_{o} - S_{wI} - S_{ge})}{(1 - S_{wI} - S_{ge} - S_{org})^{2}} \right\rangle , - \left\langle \frac{1}{(1 - S_{wI} - S_{ge} - S_{org})} \right\rangle \right]$$

AVII. Gas/oil systems at initial non-flowable water saturation -carbonate lithology.

Table AVIU -Range of rock/fluid properties of samples used to develop equations AVIU - AVII6.

Properties	Range of Properties
Porosity (X)	9.1 - 28.7
Absolute permeability (air) (md)	1.4 - 161
Initial water saturation (%)	7.6 - 19.5
Residual oil saturation (%)	7.7 - 29.2

Table AVII2 -Expected value of the parameters of equations AV 111 - AV 11 6 .

Parameter	Expected Value
P27	5.083073
V(P27)	0.012018
P28	-0.163363
V (P28)	0.000880
Cov(P27,P28)	0.001354
P29	2.065039
V (P29)	0.049295
P30	-1.424404
V(P30)	0.216461
Cov(P29,P30)	-0.099786
P31	1.740887
V(P31)	0.038246
P32	-0.552873
V (32)	0.135834
Cov(P31,P32)	-0.068783
$V(s_{org})$	0.003000
$V(s_{go})$	0.001250

EQUATION AVIU
Relative Permeability to Oil

$$k_{ro} = \text{SOFG} \left\{ \text{P27} \times \left[\frac{(1 - S_{wI} - S_o)}{S_o} \right]^{\text{P28}} \right\}$$

EQUATION AVII2
Variance -Relative Permeability to Oil

Let $F = \text{Log}(k_{ro})$

$$V(F) = \left(\left\langle \left[\frac{\partial F}{\partial \text{P27}} \right]^2 \times V(\text{P27}) \right\rangle + \left\langle \left[\frac{3F}{3\text{P28}} \right]^2 \times V(\text{P28}) \right\rangle \right)$$

$$+$$

$$\left(\left[\frac{\partial F}{3s_{or\,g}} \right]^2 \times V(S_{org}) \right)$$

$$+$$

$$\left(2 \times \left[\frac{\partial F}{3\text{P27}} \right] \times \left[\frac{3F}{3\text{P28}} \right] \times \text{Cov}(\text{P27},\text{P28}) \right)$$

Where

$$\frac{\hat{O}F}{BP27} = \left[\frac{(1 - S_{wI} - S_o)}{S_o}\right]^{P28} \times \mathrm{Log}(SOFG)$$

$$\frac{\partial F}{\partial P28} = \left(P27 \times \left[\frac{(1 - S_{wI} - S_o)}{S_o}\right]^{P28} \times \mathrm{Ln}\left[\frac{(1 - S_{wI} - S_o)}{S_o}\right]\right)$$

$$\times \left(\mathrm{Log}(SOFG)\right)$$

$$\frac{\tilde{O}F}{Bs_{or\ g}} = \left(P27 \times \left[\frac{(1 - S_{wI} - S_o)}{S_o}\right]^{P28} \times \left\langle\frac{\mathrm{Log}(e)}{SOFG}\right\rangle\right)$$

$$\times \left(\left\langle\frac{(-1)}{(1 - S_{wI} - S_{org})}\right\rangle + \left\langle\frac{(S_o - S_{org})}{(1 - S_{wI} - S_{org})^2}\right\rangle\right)$$

EQUATION AVII3
Relative Permeability to Gas

$$k_{rg} = \left[\frac{(1 - S_o - S_{wI} - S_{gc})}{(1 - S_{wI} - S_{gc})} \right]^{P29} \times (1 - SOEG)^{P30}$$

EQUATION AVII4
Variance -Relative Permeability to Gas

Let $F = \text{Log}\ (\kappa_{rg})$

$$V(F) = \left(\left\langle \left[\frac{\partial F}{\partial P29} \right]^2 \times V(P29) \right\rangle + \left\langle \left[\frac{\partial F}{\partial P30} \right]^2 \times V(P30) \right\rangle \right)$$

$$+$$

$$\left(\left\langle \left[\frac{\partial F}{\partial S_{org}} \right]^2 \times V(S_{org}) \right\rangle + \left\langle \left[\frac{\partial F}{ds_{gc}} \right]^2 \times V(S_{gc}) \right\rangle \right)$$

$$+$$

$$\left(2 \times \left[\frac{\partial F}{\partial P29} \right] \times \left[\frac{dF}{\partial P30} \right] \times Cov(P29, P30) \right)$$

Where

$$\frac{\partial F}{\partial P29} = \text{Log}\left[\frac{(1 - S_{o} - S_{wI} - S_{gc})}{(1 - S_{wI} - S_{gc})}\right]$$

$$\frac{\partial F}{\partial P30} = \text{Log}(1 - \text{SOFG})$$

$$\frac{\partial F}{\partial s_{org}} = \left[\frac{- \text{P30} \times \text{Log}(e)}{1 - \text{SOFG}}\right]$$

$$\times \left[\left\langle\frac{(S_{o} - S_{org})}{(1 - S_{wI} - S_{org})^{2}}\right\rangle - \left\langle\frac{1}{(1 - S_{wI} - S_{org})}\right\rangle\right]$$

$$\frac{\partial F}{\partial s_{go}} = \left[\frac{\text{P29} \times \text{Log}(e)}{\left[\frac{(1 - S_{o} - S_{wI} - S_{gc})}{(1 - S_{wI} - S_{go})}\right]}\right]$$

$$\times \left[\left\langle\frac{(1 - S_{o} - S_{wI} - S_{gc})}{(1 - S_{wI} - S_{gc})^{2}}\right\rangle - \left\langle\frac{1}{(1 - S_{wI} - S_{gc})}\right\rangle\right]$$

EQUATION AVII5
Relative Permeability to Gas

$$\kappa_{rg} = k_r(S_{g\,org}) \times \left[\frac{(1 - S_o - S_{wI} - S_{go})}{(1 - S_{ge} - S_{wI} - S_{org})}\right]^{P31}$$

$$\times \quad (1 - SOFG)^{P32}$$

EQUATION AVII6
Variance -Relative Permeability to GAS

Let F = Log(k_{pg})

$$V(F) = \left(\left\langle\left[\frac{\partial F}{\partial P31}\right]^2 \times V(P31)\right\rangle + \left\langle\left[\frac{\partial F}{\partial P32}\right]^2 \times V(P32)\right\rangle\right)$$

$$+$$

$$\left(\left\langle\left[\frac{\partial F}{\partial S_{org}}\right]^2 \times V(S_{org})\right\rangle + \left\langle\left[\frac{\partial F}{\partial S_{gc}}\right]^2 \times V(S_{gc})\right\rangle\right)$$

$$\left(2 \times \left[\frac{\partial F}{\partial P31}\right]\left[\frac{\partial F}{\partial P32}\right] \times \mathrm{Cov}(P31,P32)\right)$$

Where

$$\frac{\partial F}{\partial P31} = \operatorname{Log}\left[\frac{(1 - S_{o} - S_{wI} - S_{gc})}{(1 - S_{gc} - S_{wI} - S_{org})}\right]$$

$$\frac{\partial F}{\partial P32} = \operatorname{Log}(1 - SOFG)$$

$$\frac{\partial F}{\partial S_{org}} = \left\{\left[\frac{P31 \times \operatorname{Log}(e)}{\left[\frac{(1 - S_{o} - S_{wI} - S_{gc})}{(1 - S_{wI} - S_{gc} - S_{org})}\right]}\right] \times \left[\frac{(1 - S_{o} - S_{wI} - S_{gc})}{(1 - S_{wI} - S_{gc} - S_{org})^{2}}\right]\right\}$$

$$+$$

$$\left\{\left[\frac{-P32 \times \operatorname{Log}(e)}{(1 - SOFG)}\right] \times \left[\left\langle\frac{(S_{o} - S_{org})}{(1 - S_{wI} - S_{org})^{2}}\right\rangle - \left\langle\frac{1}{(1 - S_{wI} - S_{org})}\right\rangle\right]\right\}$$

$$\frac{\partial F}{\partial S_{go}} = \left[\frac{P31}{\left[\frac{(1 - S_o - S_{wI} - S_{go})}{(1 - S_{gc} - S_{wI} - S_{org})}\right]} \right] \times \left[\left\langle \frac{(1 - S_o - S_{wI} - S_{gc} - S \text{[illegible]})}{(1 - \text{[illegible]} S_{wI} - S_{go} - S_{org})^2} \right\rangle - \left\langle \frac{1}{(1 - S \text{[illegible]} S_{wI} - S_{gc} - S_{org})} \right\rangle \right]$$

APPENDIX B

ILLUSTRATIONS OF TYPICAL PREDICTIONS

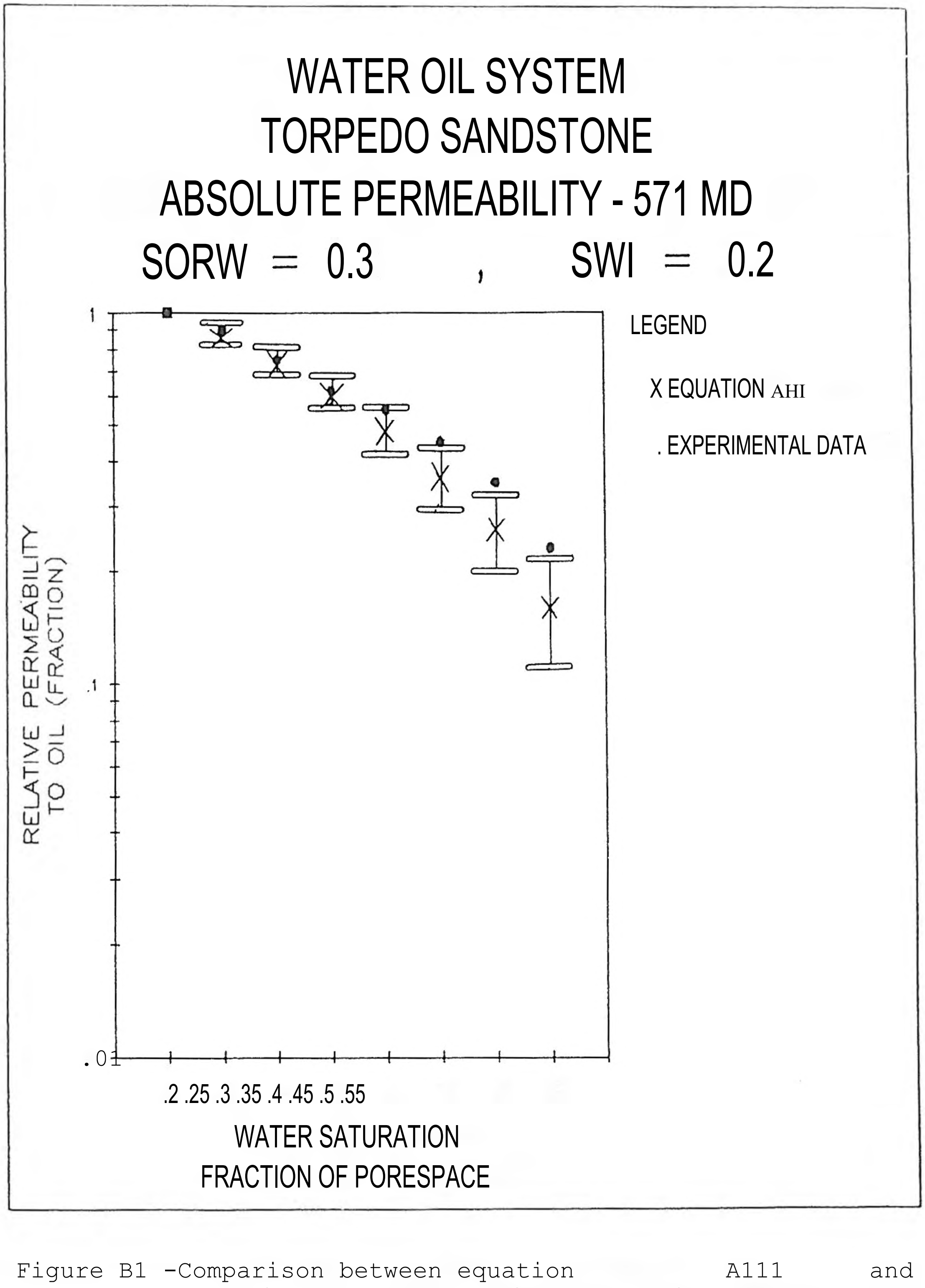


Figure B1 -Comparison between equation A111 and experimental data obtained from Figure 2.5 (contact angle 0 degrees).

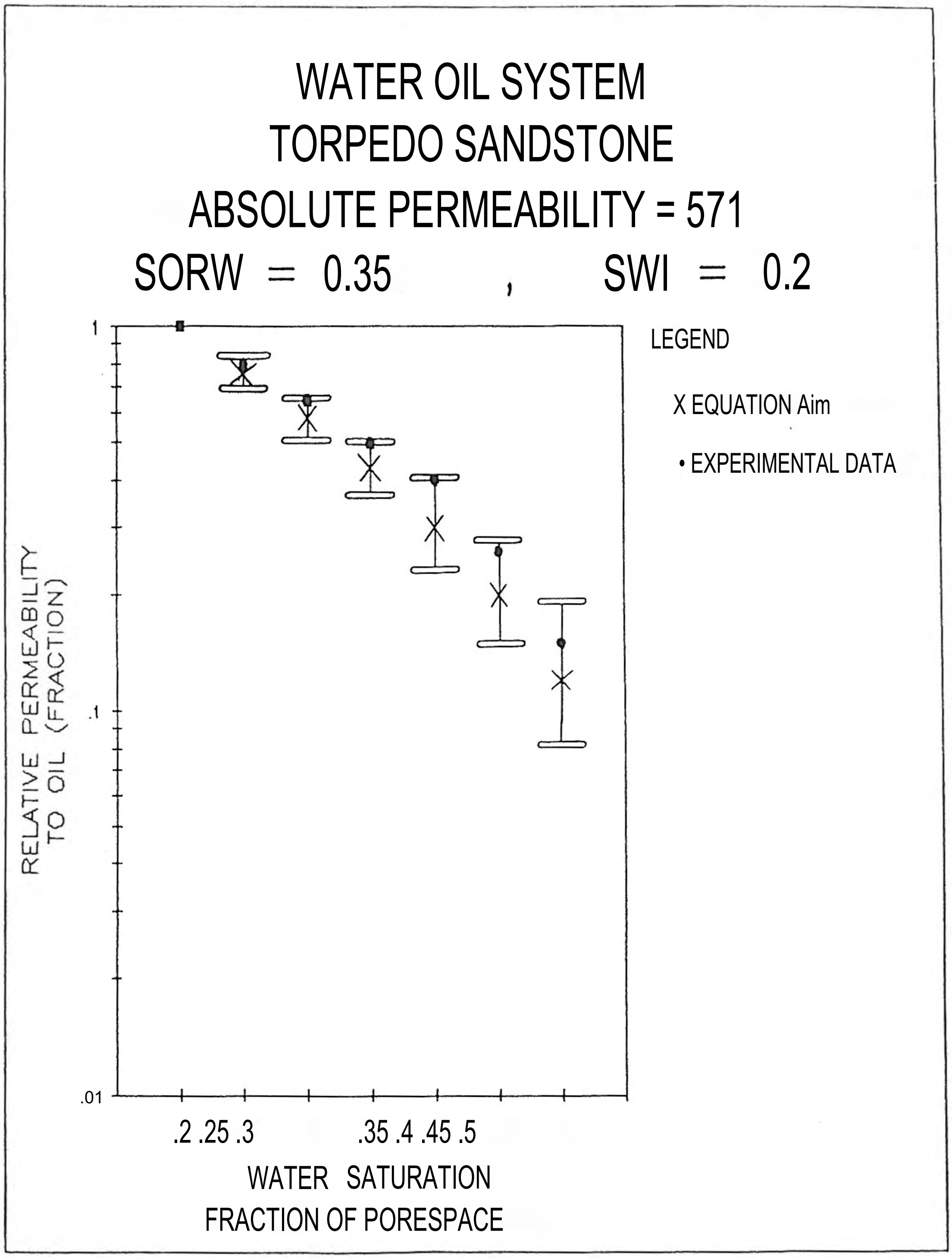


Figure B2 -Comparison between equation Alili and experimental data obtained from Figure 2.5 (contact angle 35 degrees).

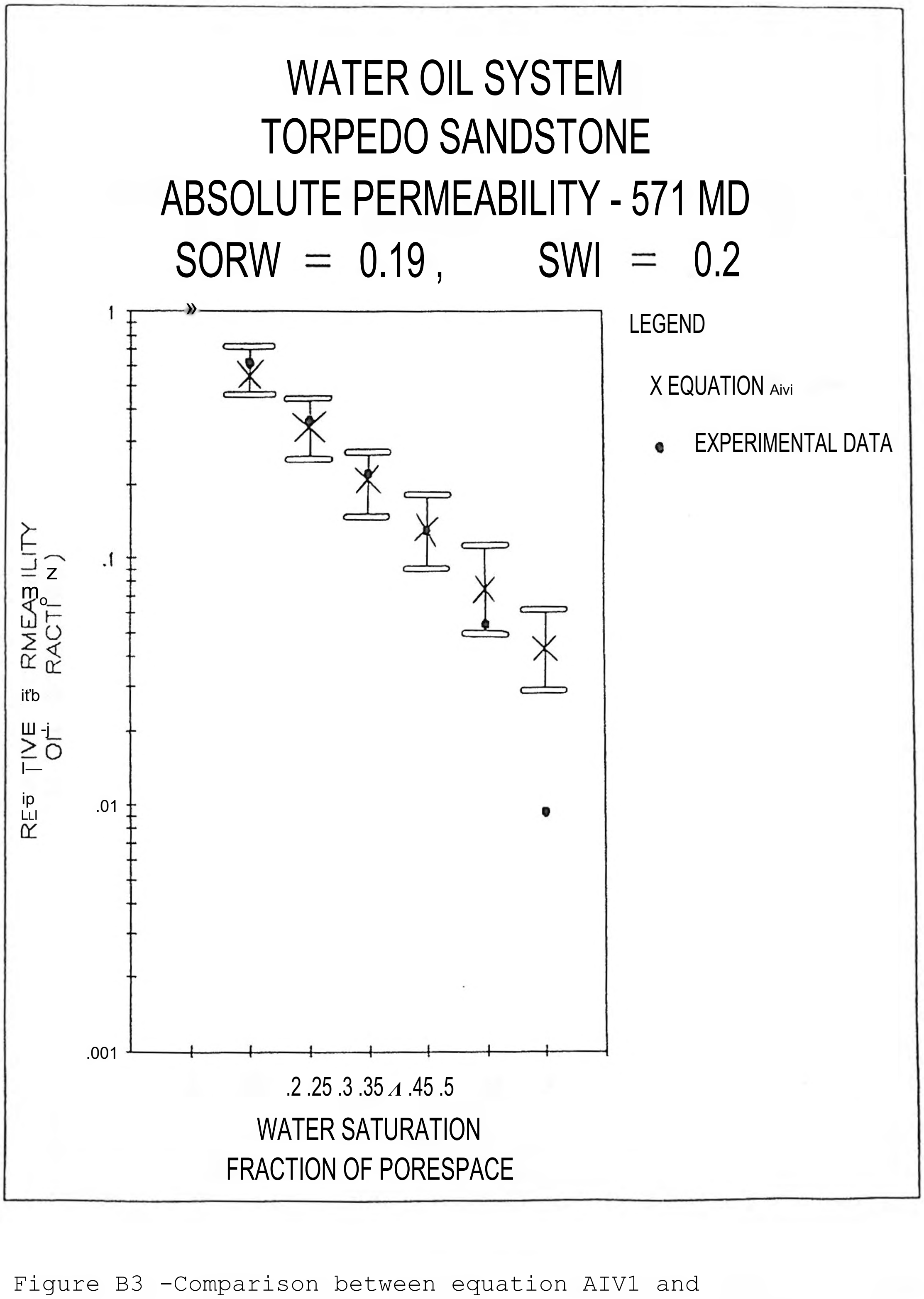


Figure B3 -Comparison between equation AIV1 and experimental data obtained from Figure 2.5 (contact angle 180 degrees).

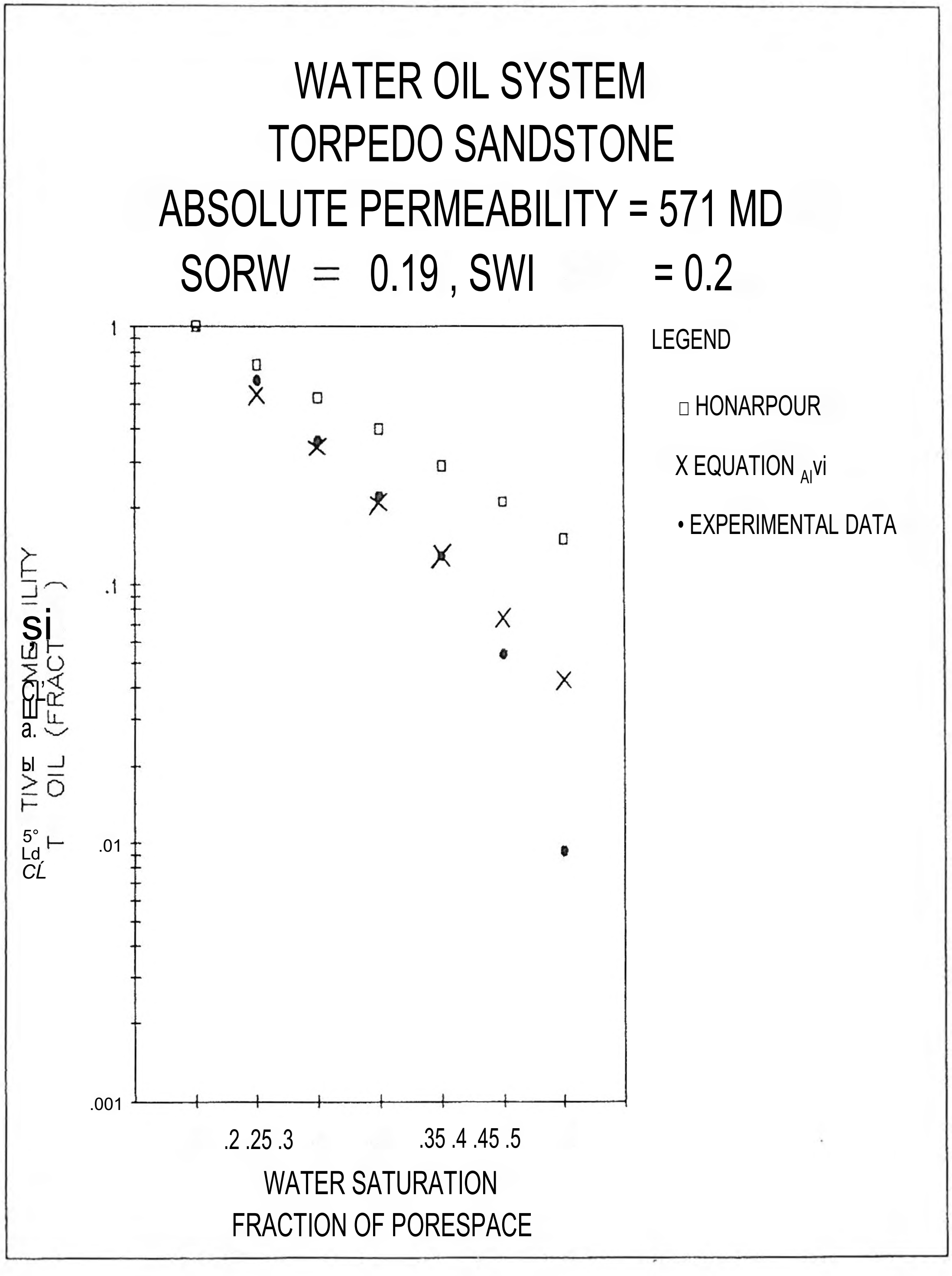


Figure B4 -Comparison of equation AIV1, experimental data obtained from Figure 2.5 (contact angle 180 degrees), and the model published by Honarpour.

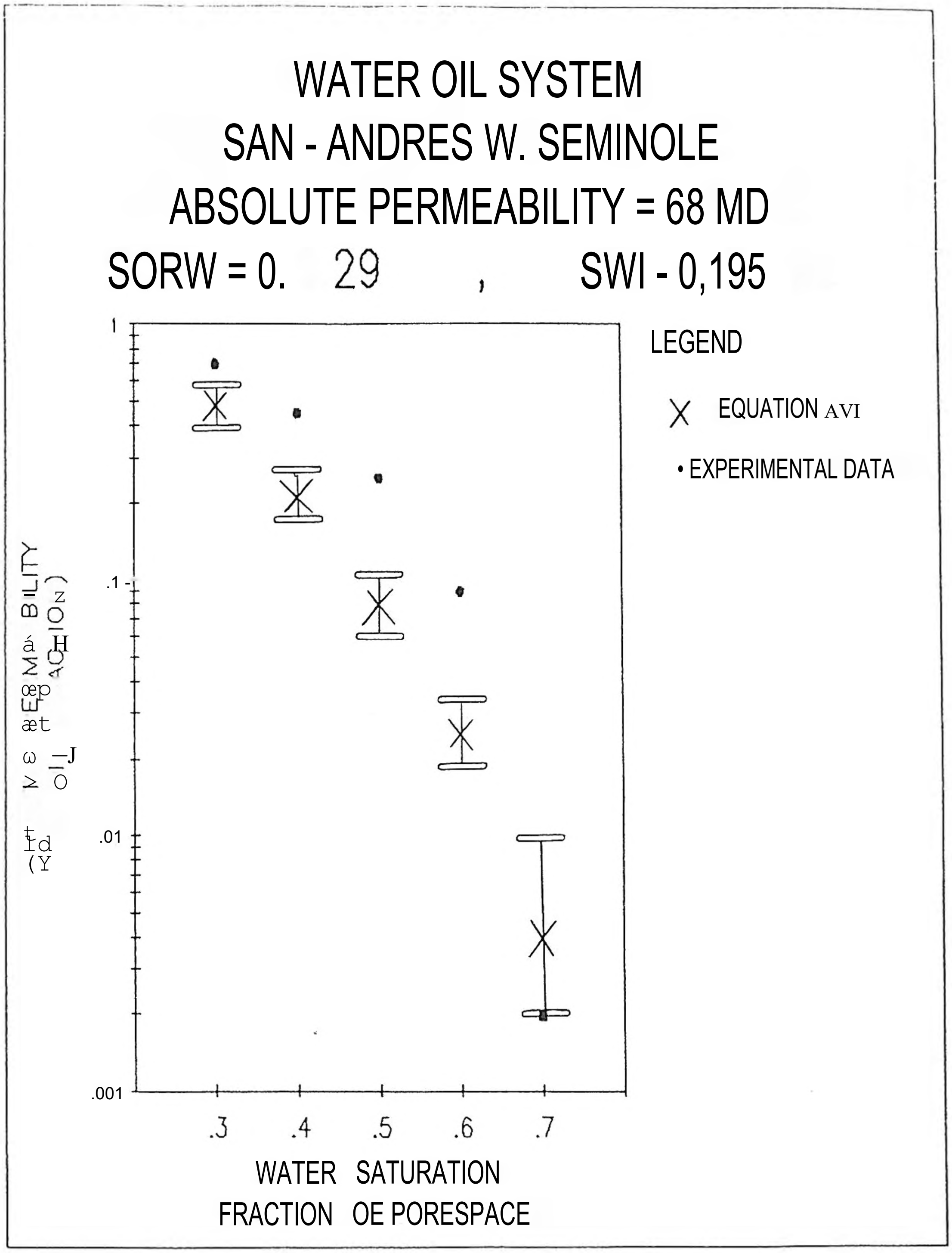


Figure B5 -Comparison between equation AV1 and experimental data.

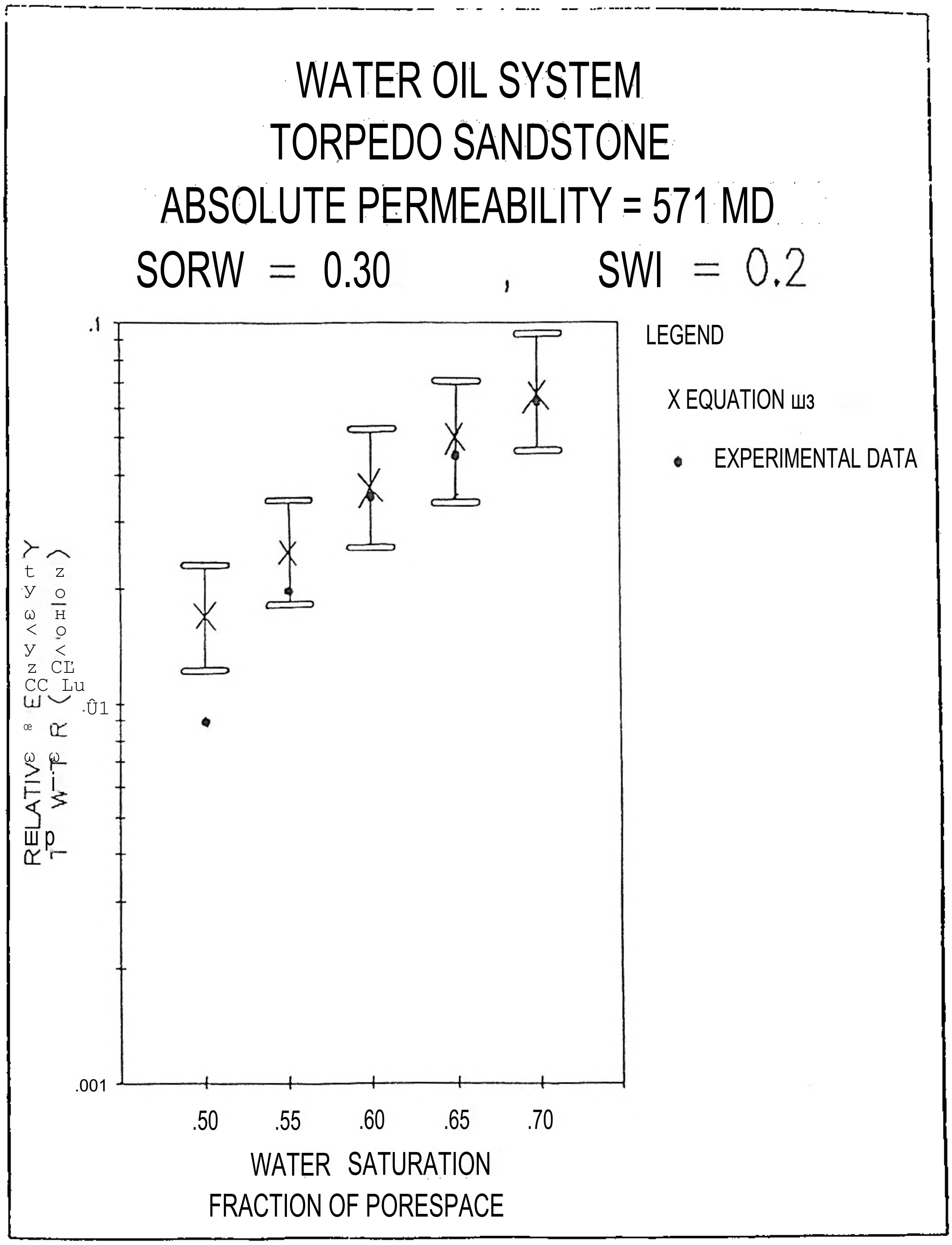


Figure B6 -Comparison between equation AII3 and experimental data obtained from Figure 2.5 (contact angle 0 degrees).

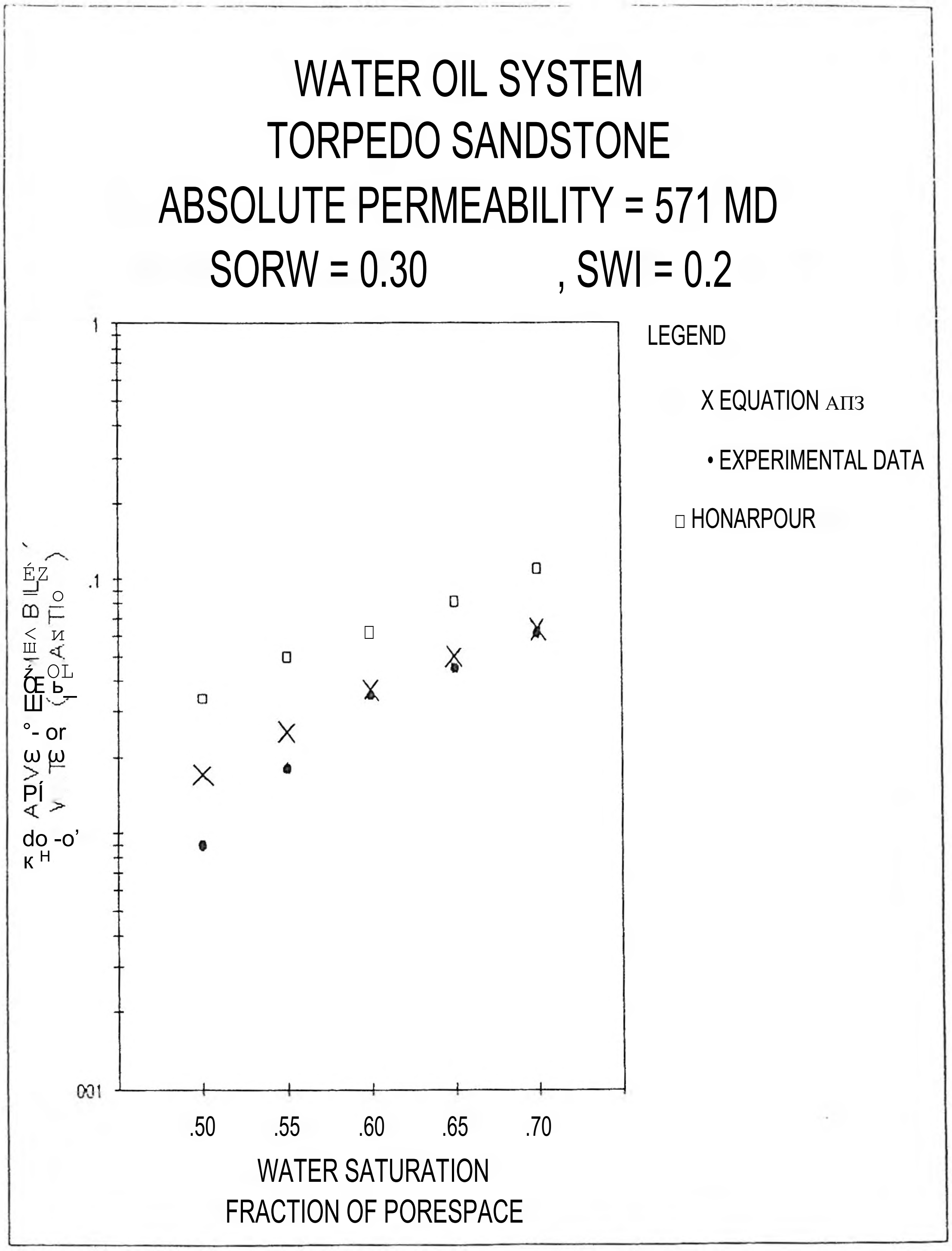


Figure B? -Comparison of equation AII3, experimental data obtained from Figure 2.5 (contact angle 0 degrees) and the model published by Honarpour.

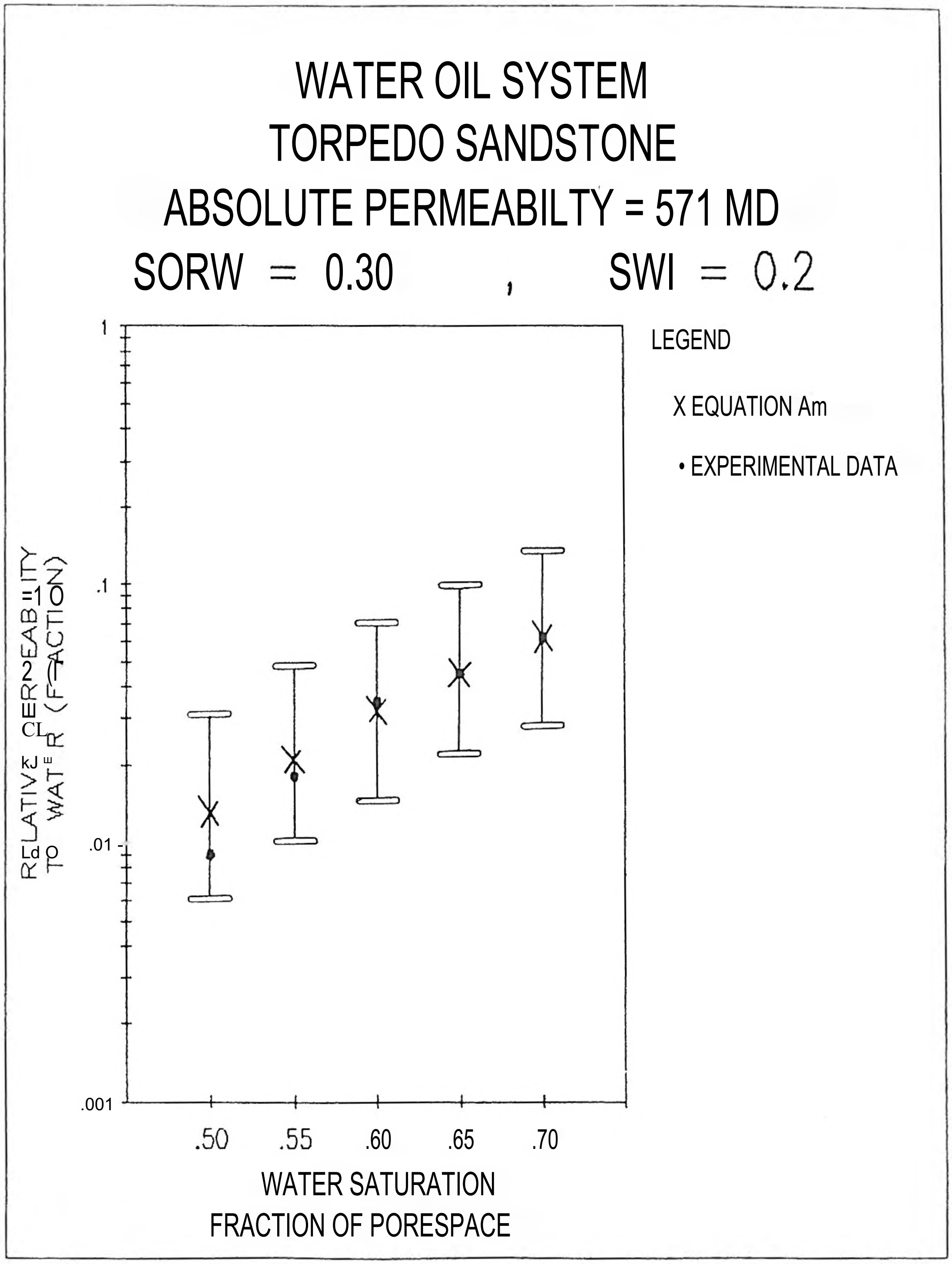


Figure B8 -Comparison between equation AII5 and experimental data obtained from Figure 2.5 (contact angle 0 degrees).

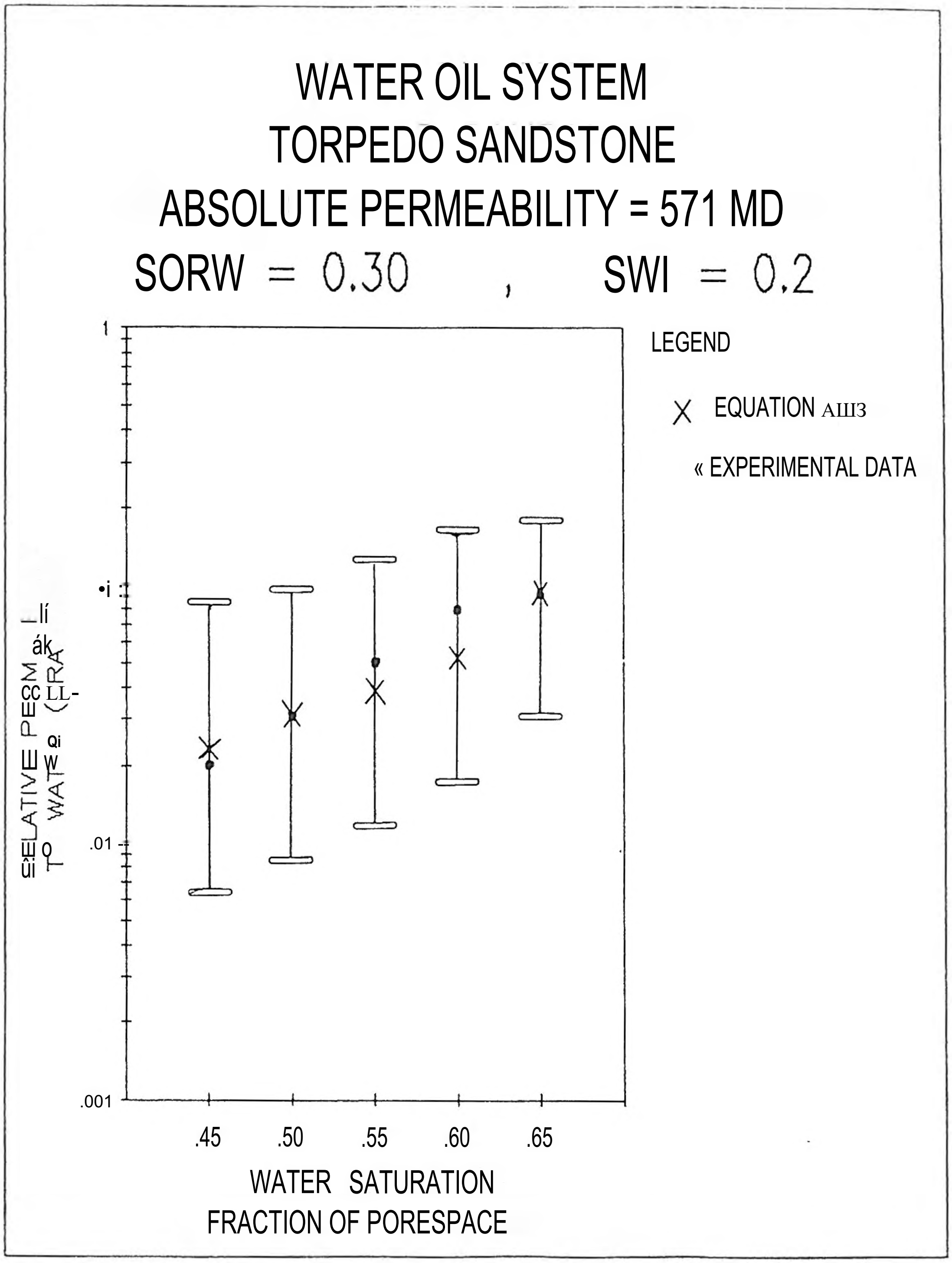


Figure B9 -Comparison between equation AIII3 and experimental data obtained from Figure 2.5 (contact angle 0 degrees).

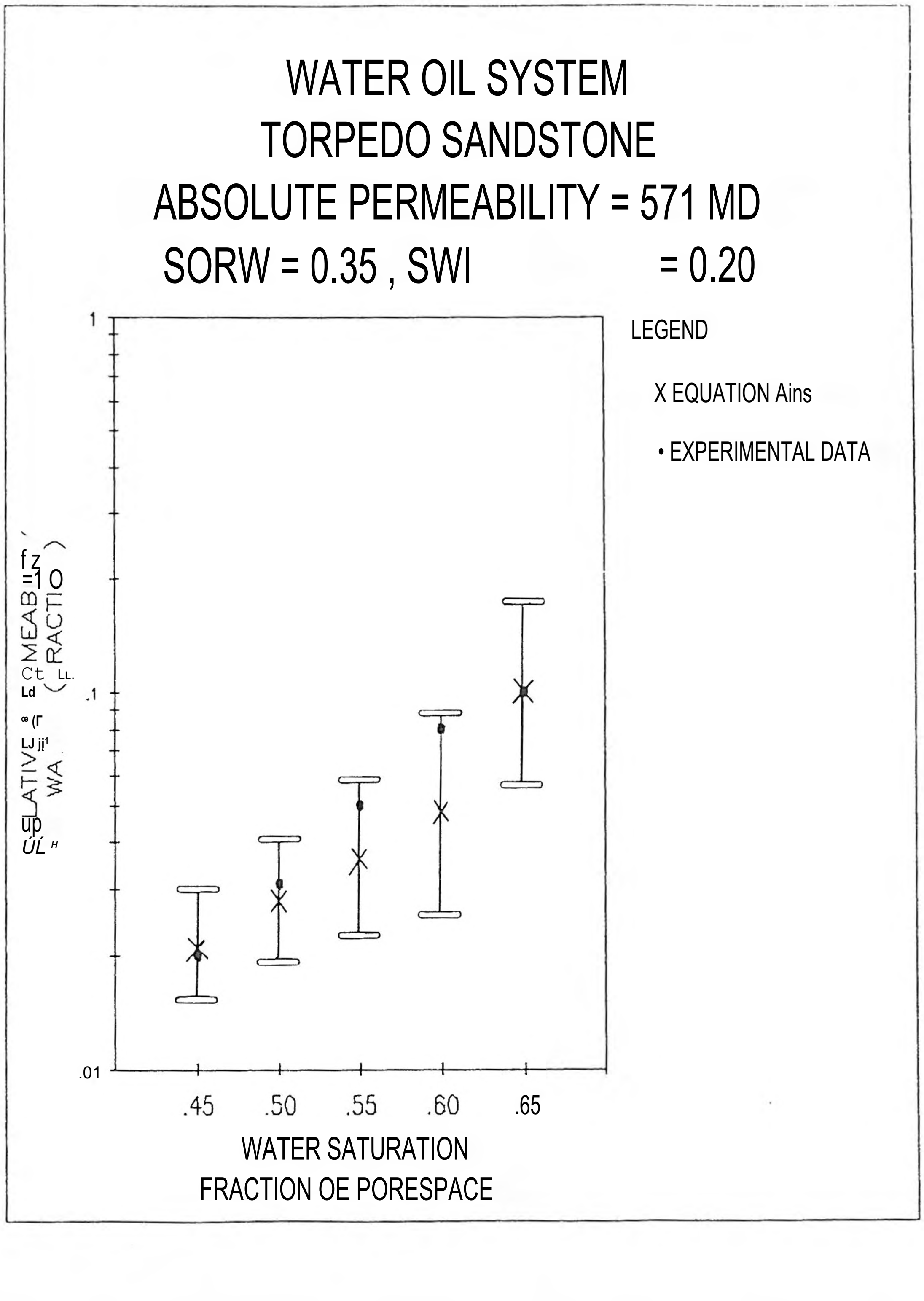


Figure B10 -Comparison between equation AIII5 and experimental data obtained from Figure 2.5 (contact angle 35 degrees).

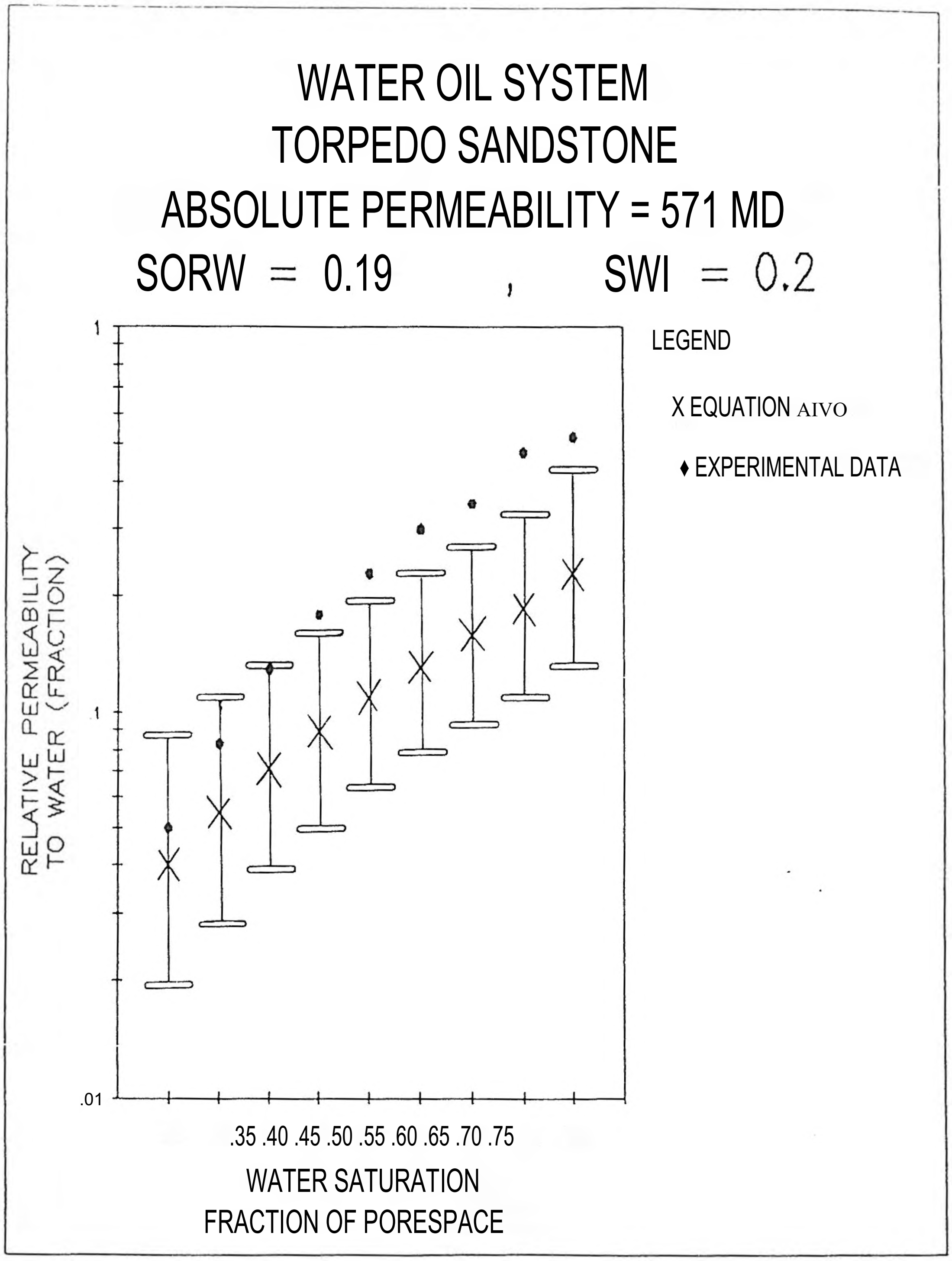


Figure B11 -Comparison between equation AIV3 and experimental data obatined from Figure 2.5 (contact angle 180 degrees).

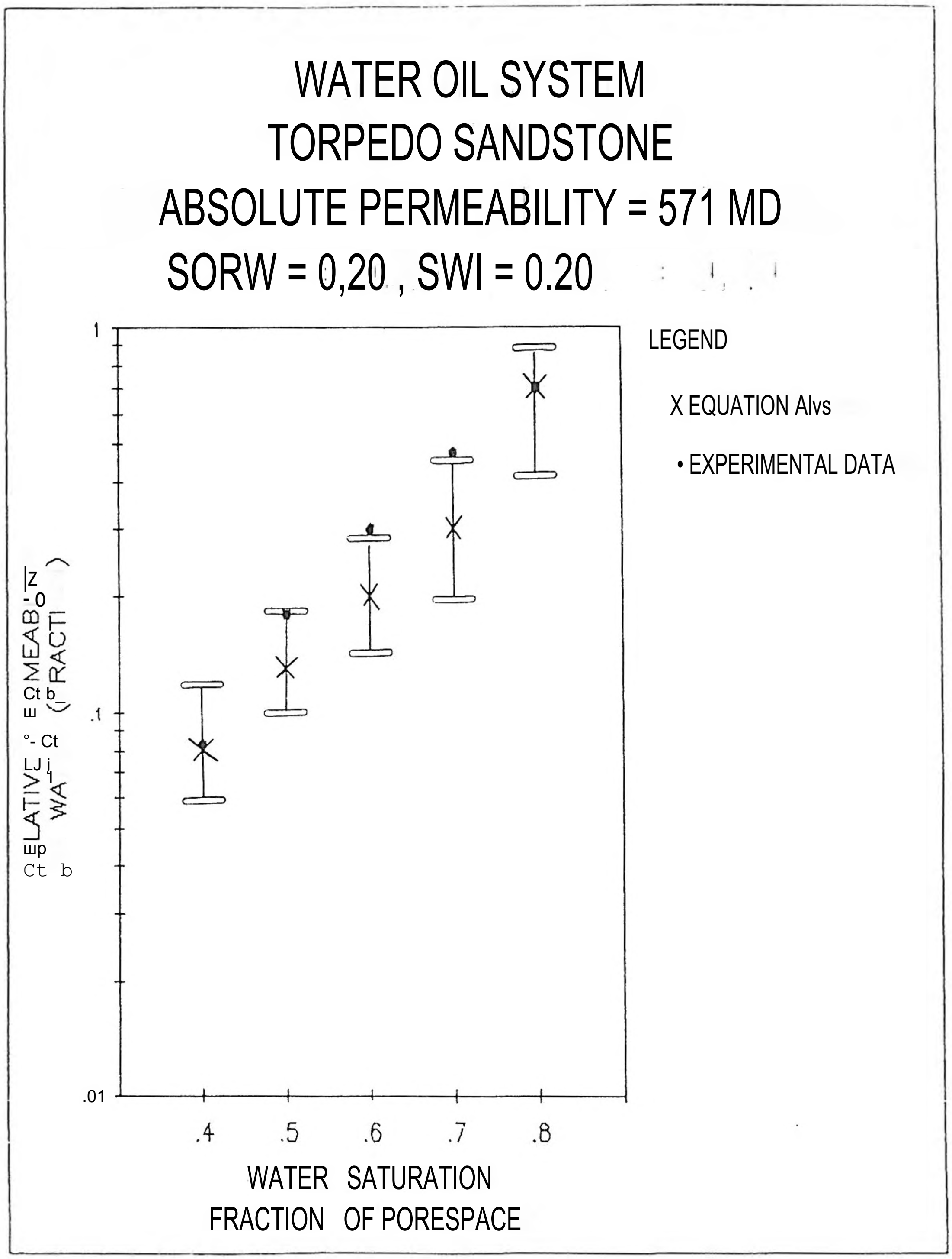


Figure B1 2 --Comparison between equation AIV5 and experimental data obtained from Figure 2.5 (contact angle 180 degrees).

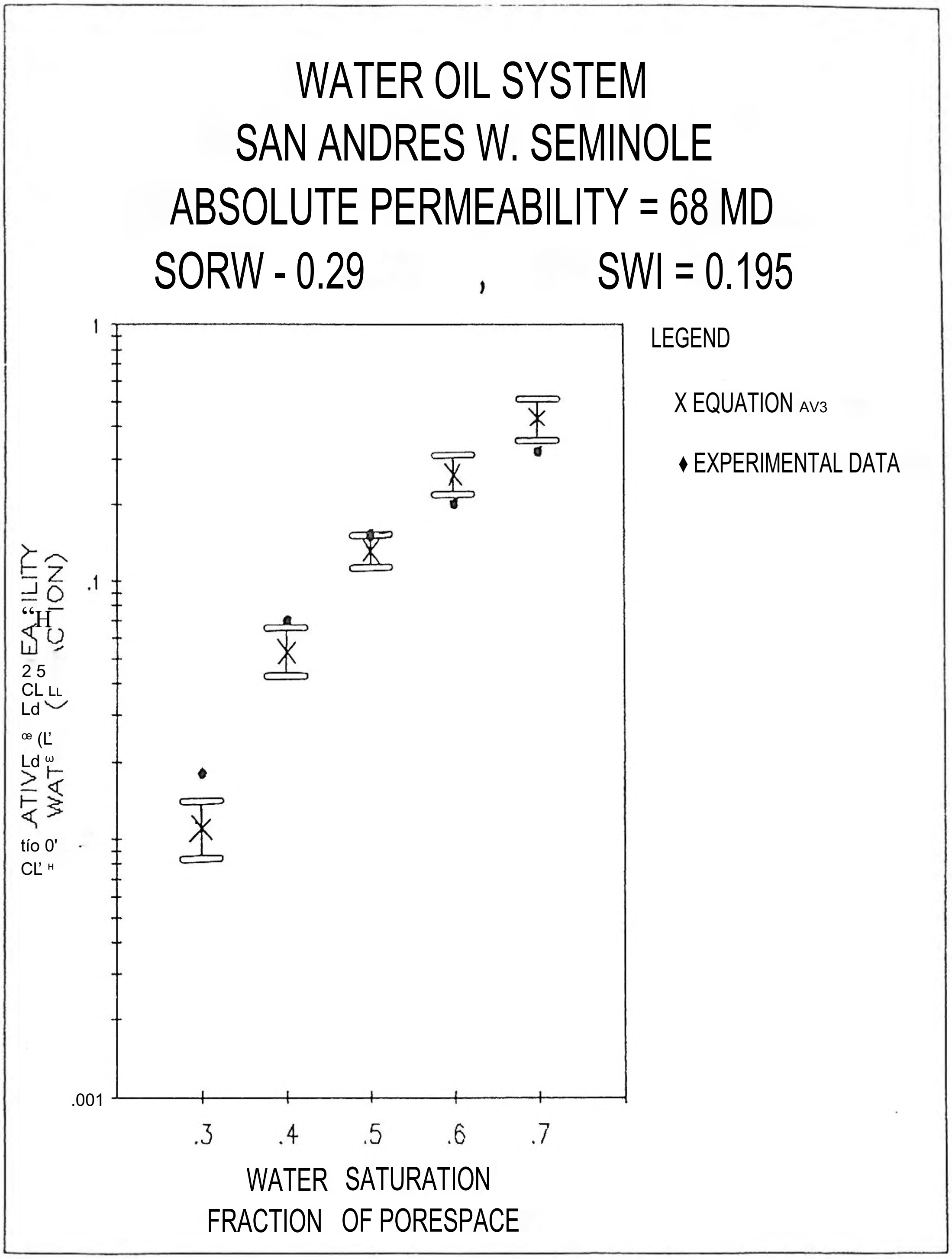


Figure B13 -Comparison between equation AV3 and experimental data.

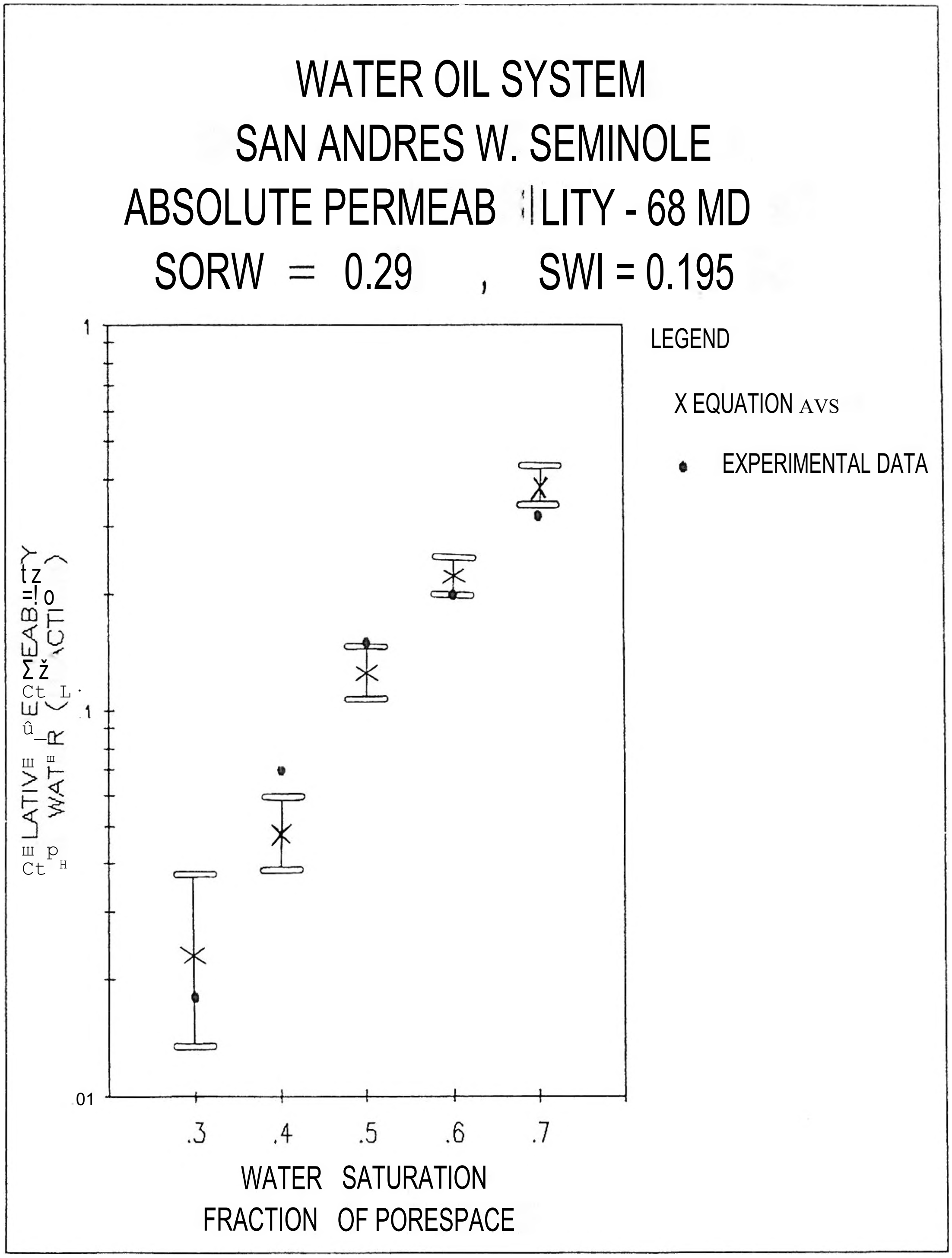


Figure B14 -Comparison between equation AV5 and experimental data.

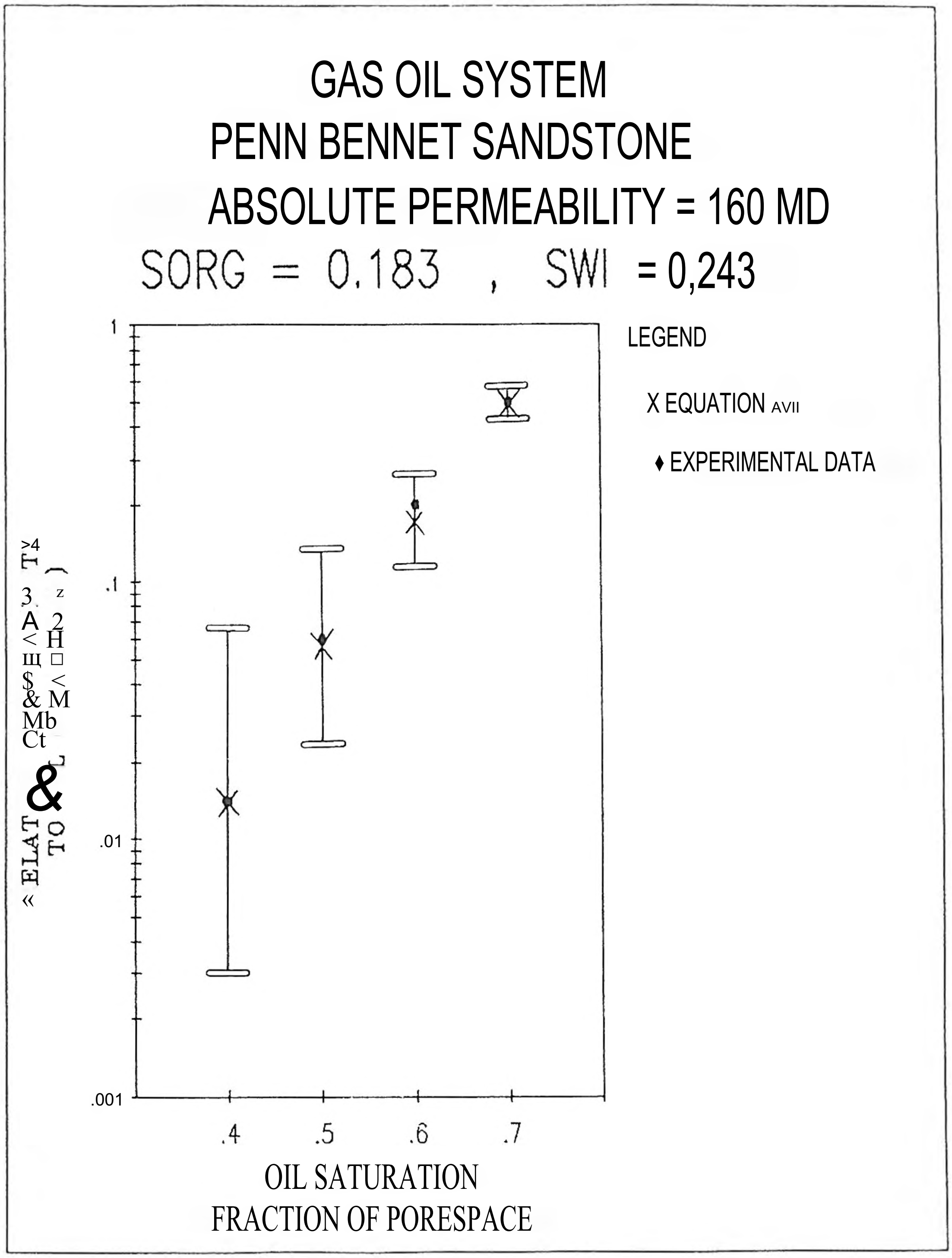


Figure BI 5 -Comparison between equation AVI1 and experimental data.

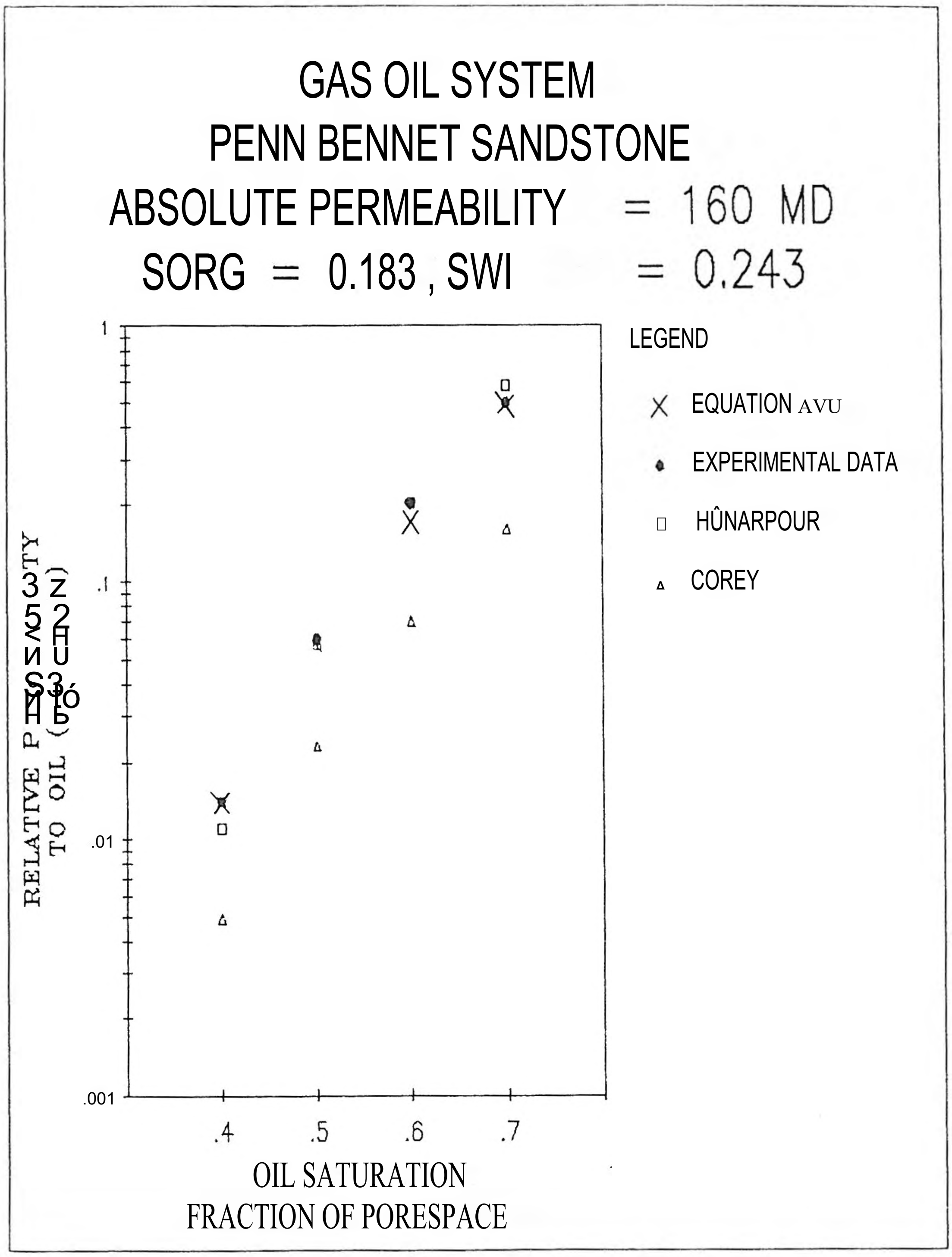


Figure BI6 -Comparison of equation AVI1, experimental data, and models published by Honarpour and Corey.

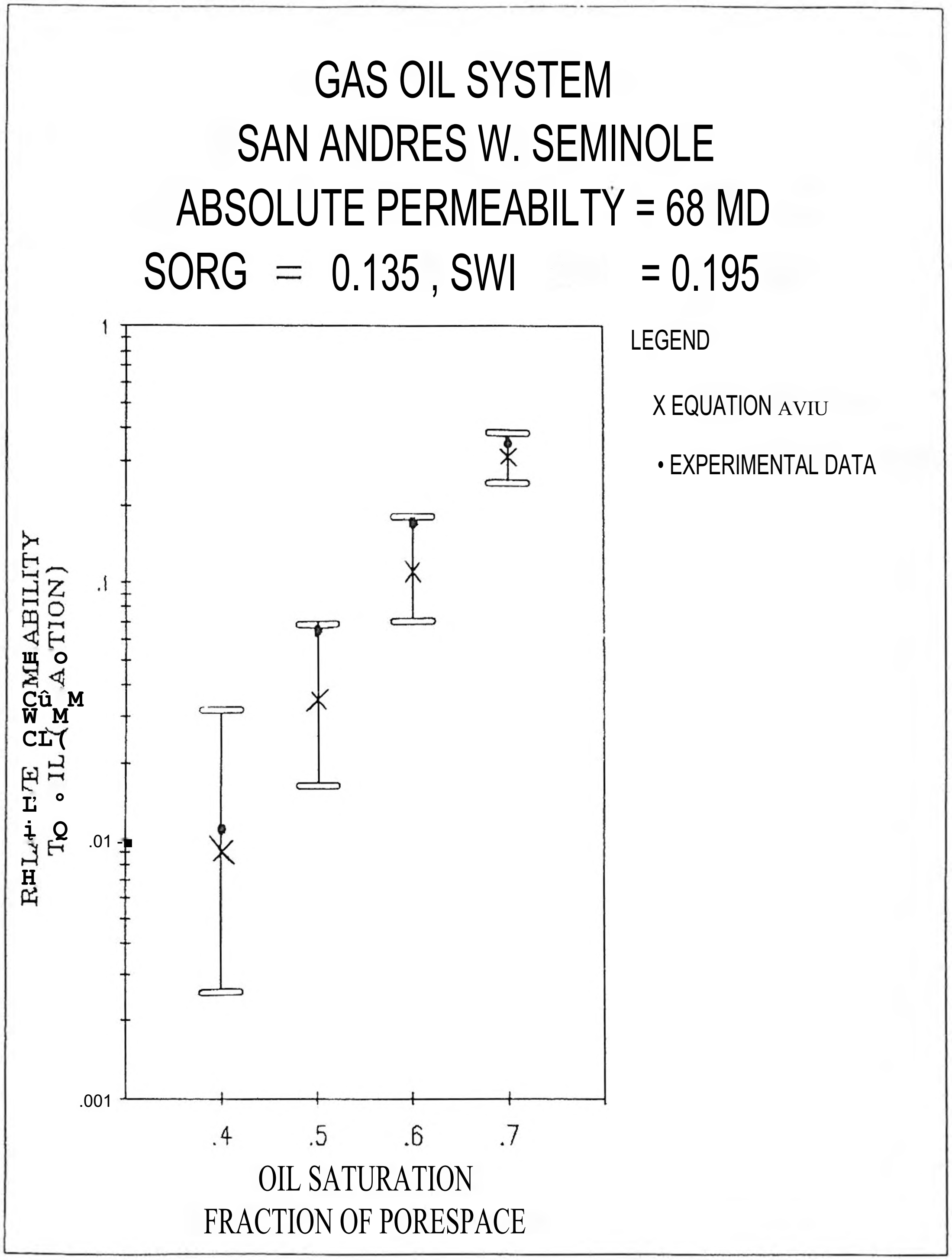


Figure B17 -Comparison between equation AVIU and experimental data.

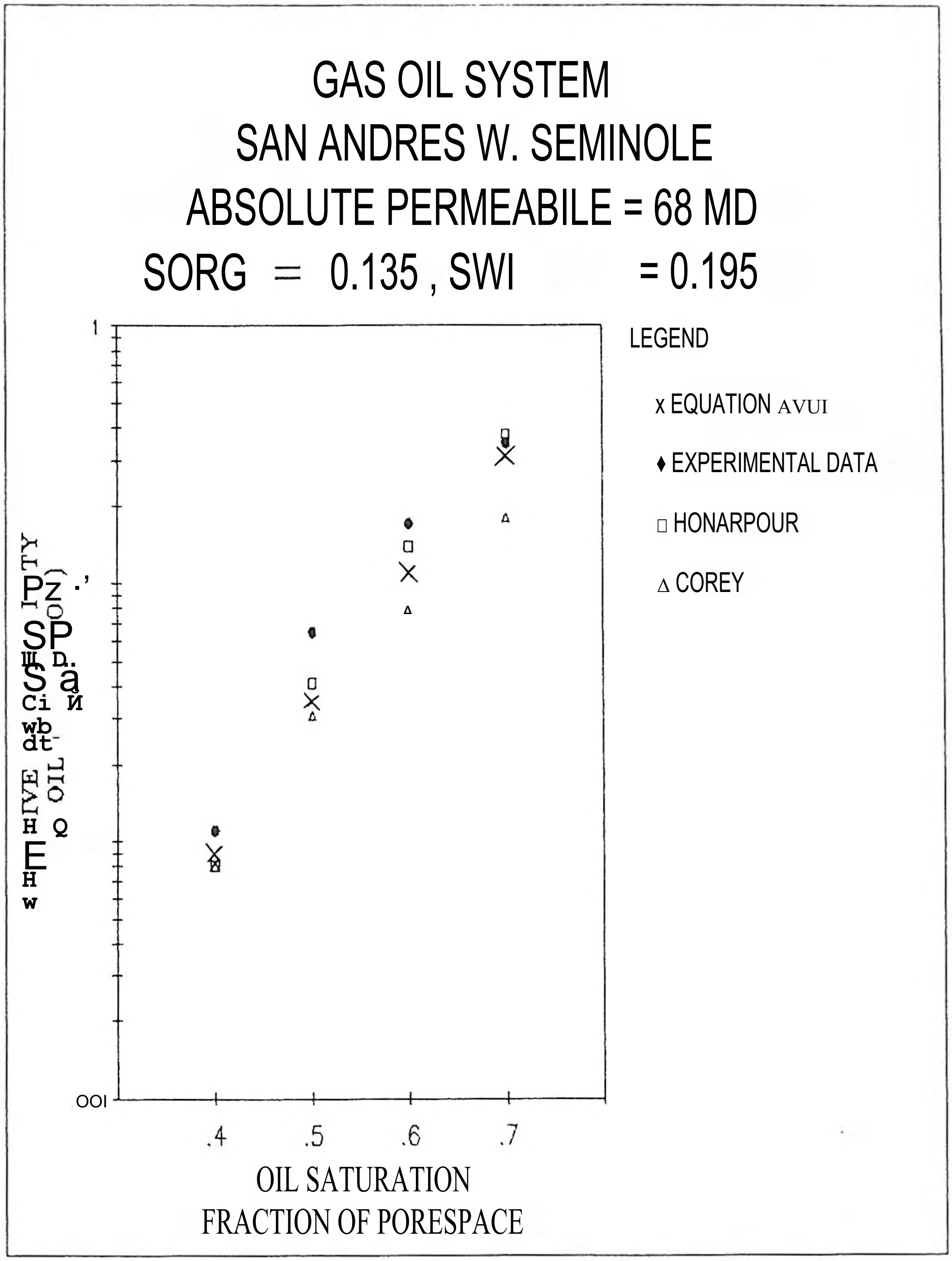


Figure BI 8 -Comparison of equation AVII1, experimental data, and models published by Honarpour and Corey.

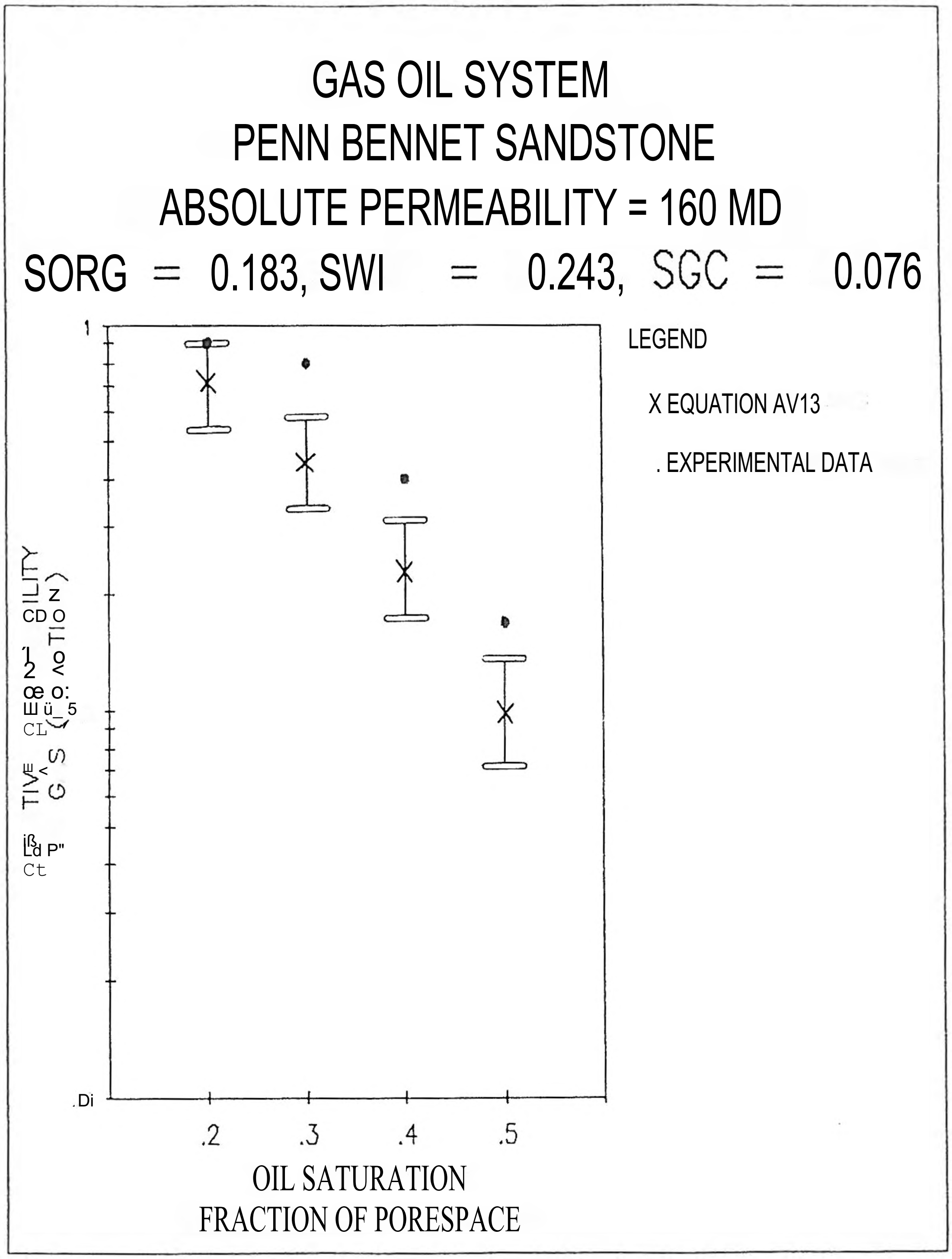


Figure D'19 -Comparison between equation AVIS and experimental data.

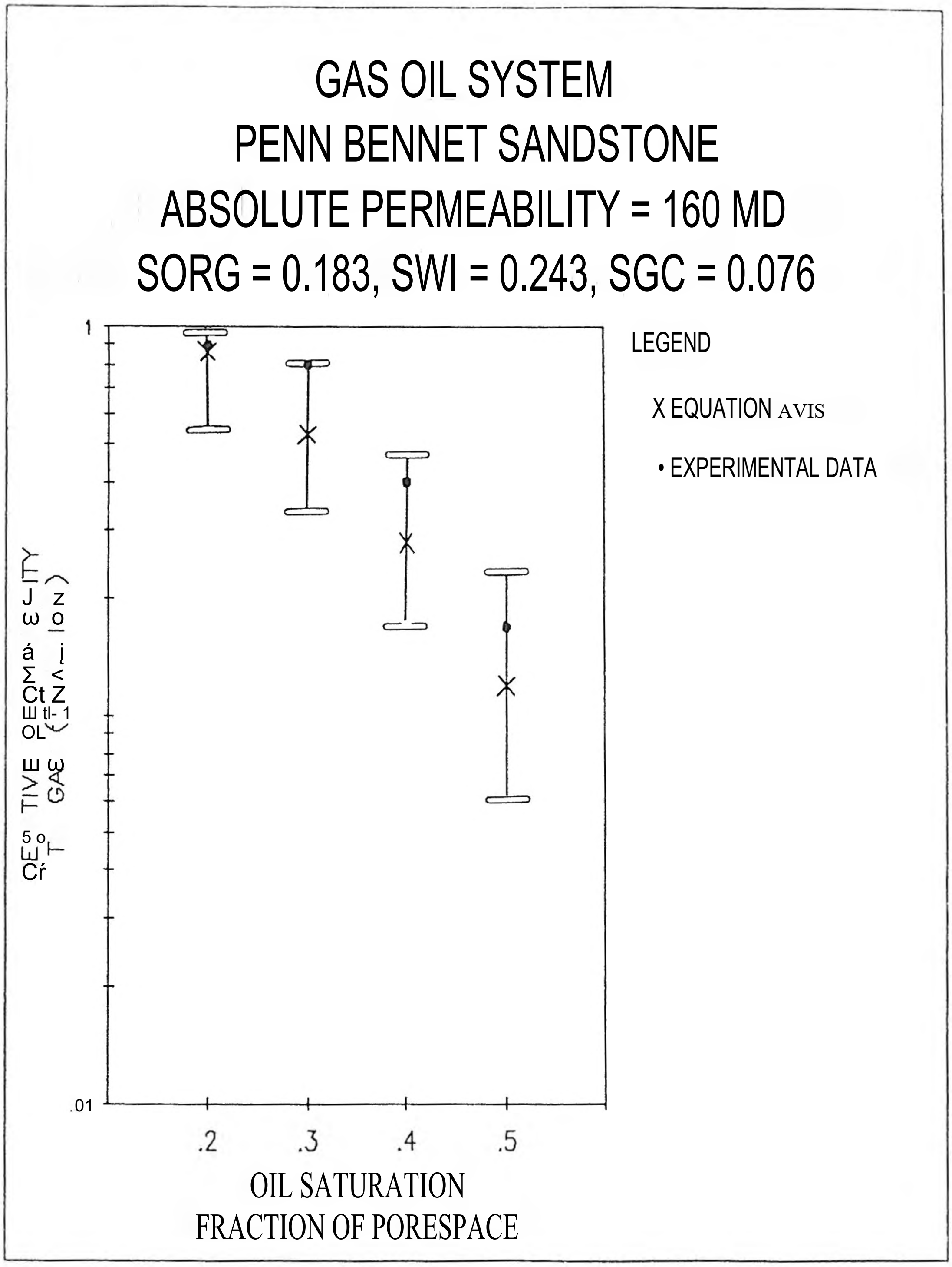


Figure B 2 O -Comparison between equation AVI5 and experimental data.

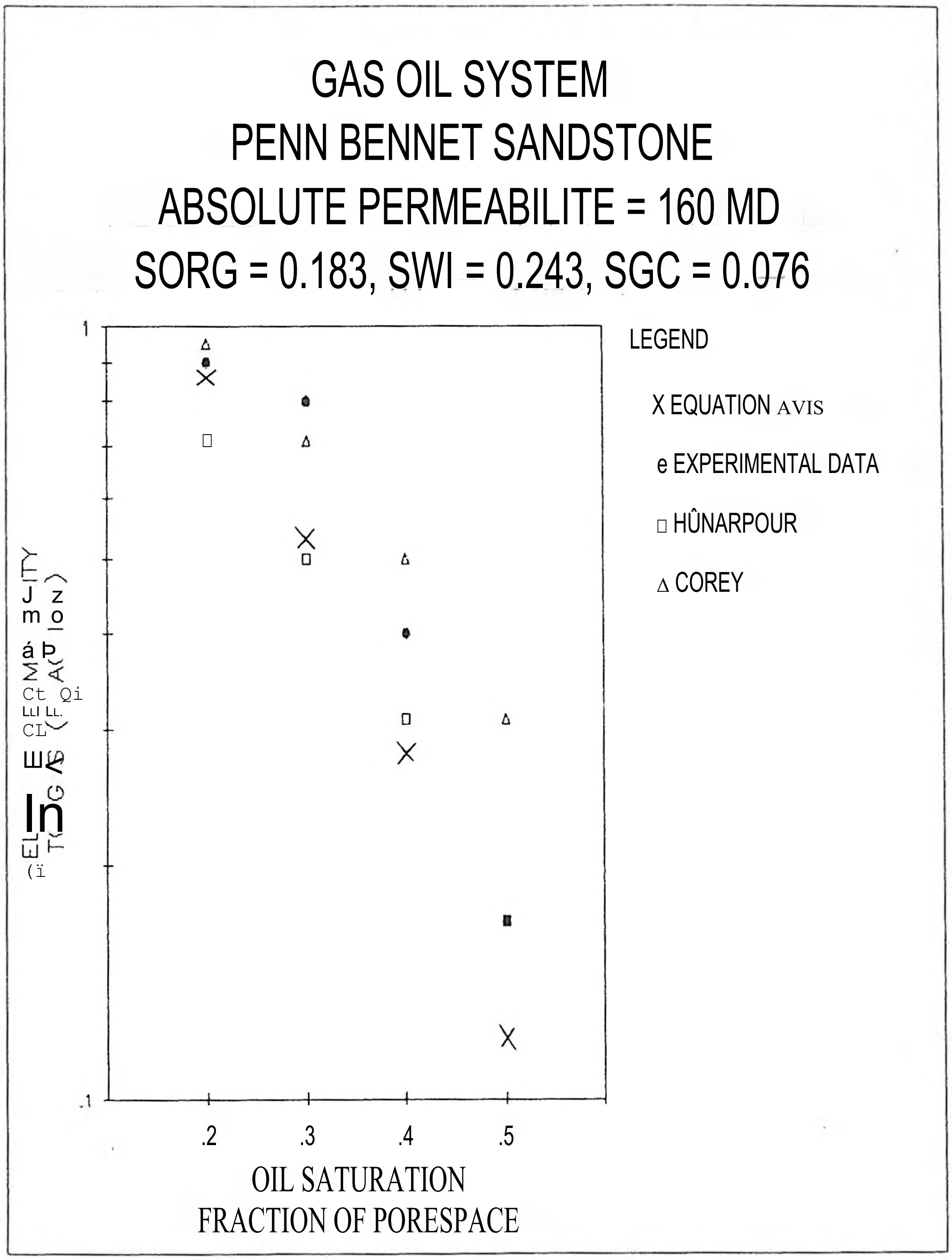


Figure B21 -Comparison of equation AVI5, experimental data, and models published by Honarpour and Corey.

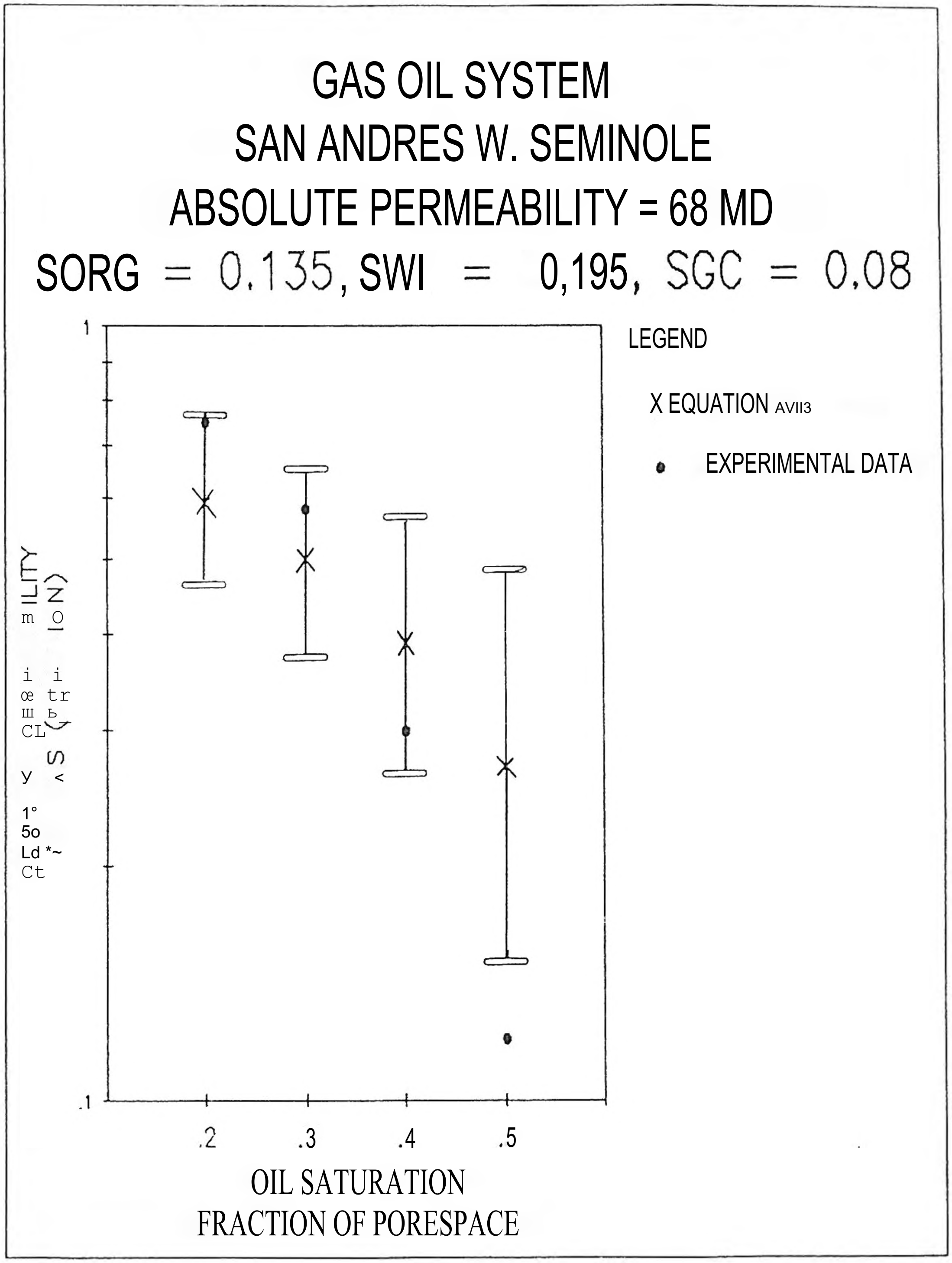


Figure B22 -Comparison between equation AVII3 and experimental data.

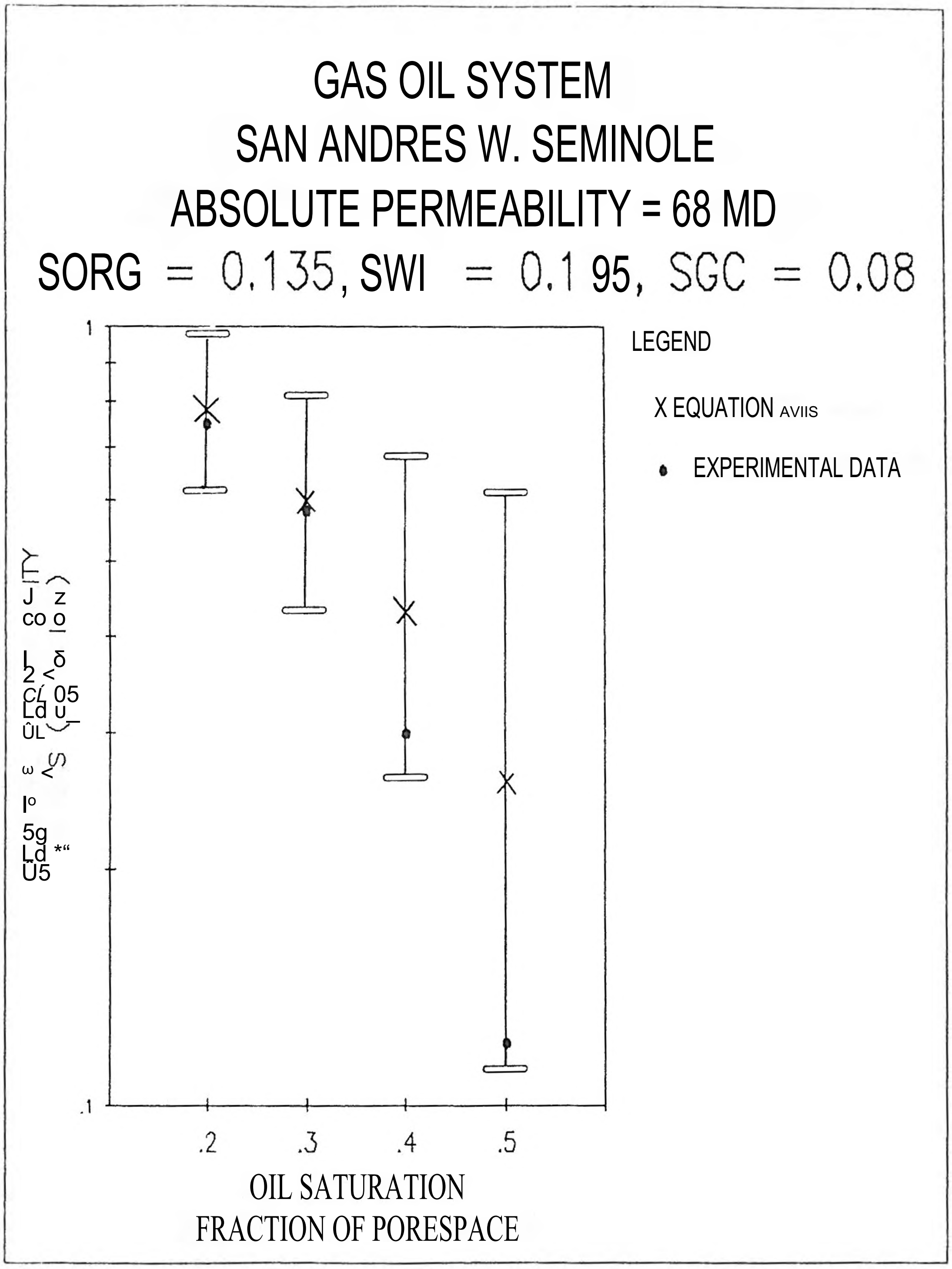


Figure B23 -Comparison between equation AVII5 and experimental data.

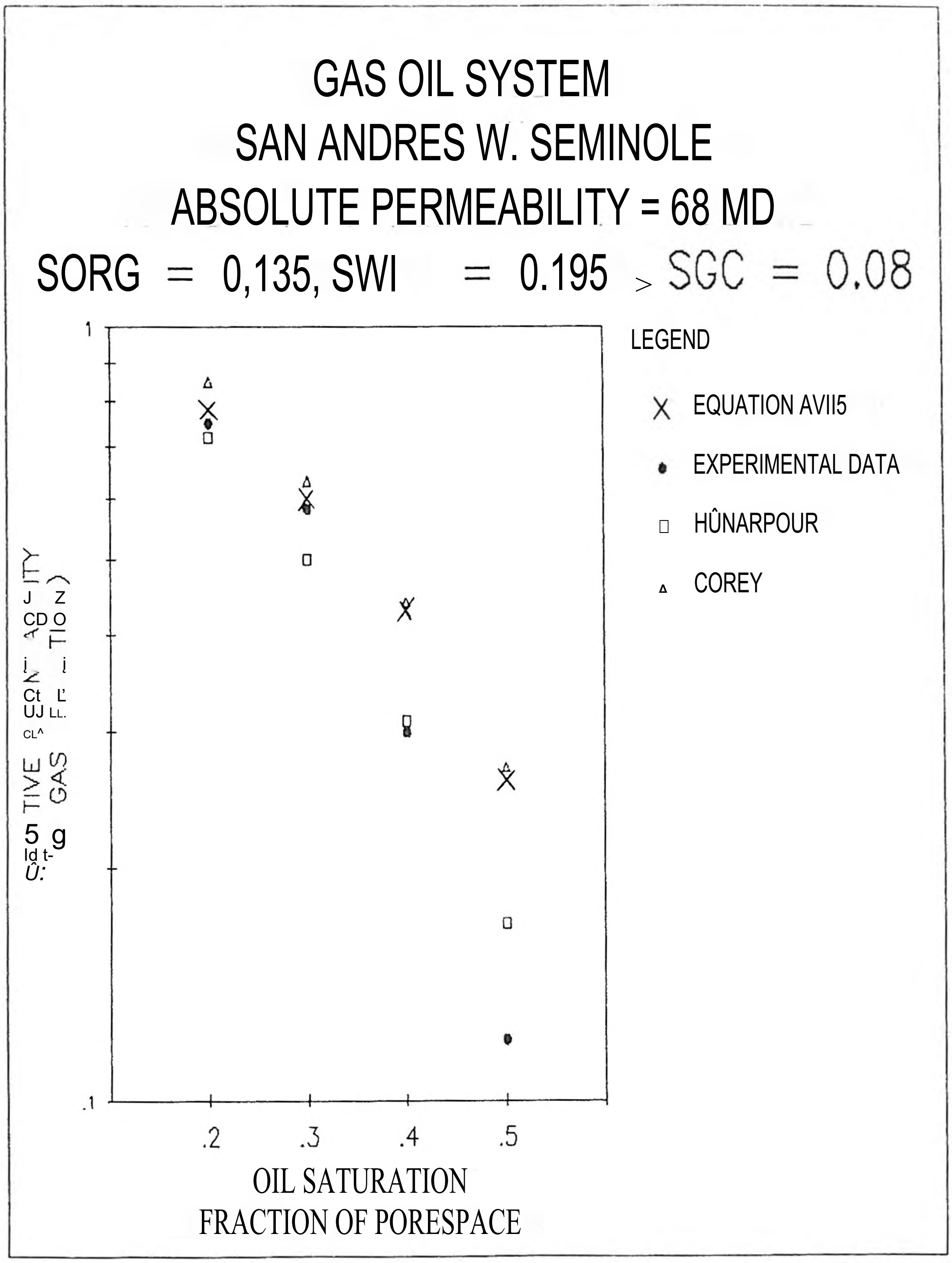


Figure B2Д -Comparison of equation AVII5, experimental data, and models published by Honarpour and Corey.

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