History of the abacus

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A HISTORY OF THE ABACUS

Abstract: As the revolution in computing advances, it is appropriate to step back and look at the earliest practical aid to computation—the abacus. Its formal western origins lie with the Greeks and the expansion of trade in the seventh century BC, and its design and application showed remarkably little change over the following two thousand years. A measure of the usefulness of the abacus is seen by the fact that it survived the advent of algorism by some six centuries but its major significance for western culture lies in its perfect and seminal representation of the decimal system.

"Can you do Addition?" The White Queen asked.
"What's one and one and one and one and one and one and one and one and one and one?"
"I don't know," said Alice. "I lost count."
"She can't do Addition," the Red Queen interrupted.

Pure Science—like social science—is addicted to revolution and revolution usually implies the rejection and exclusion of all that came before. Subsequent reassessment, however, usually reveals this to be a short-sighted attitude. Familiarity with relevant historical developments can sharpen appreciation of a contemporary situation and, where appropriate, an assessment in context of past attainments can bring a recognition of the limitations of current achievements and provide the spur that prevents complacency. Therefore, as the latest generation of silicon chips carries the revolution in computation a stage further, it is appropriate to examine the early history of counting and calculating aids, and look at the use of the abacus.

As Conant has noted, we know of no language in which the suggestion of number does not appear, and the words which give expression to the number sense must be among the earliest words to be formed in any language. The need for calculation was as ubiquitous among ancient civilizations as our own; everyday intercourse required information as to distance, time, size and cost.
Finger counting is the most obvious starting point for calculating, as every parent and teacher observes from the instinctive reactions of children, and an elaborate system developed from it at an early date. The persistence of this simple but effective means of numbering is evidenced by the inclusion in the 1543 edition of Robert Recorde's *Ground of Artes* of a section entitled "The arte of nombrynge by the hande." It was certainly easier to express numbers than to perform complicated calculations, as is seen by the lengthy passage in Bede's *De Temporum Ratione*, composed 800 years earlier, but Nagl adds his authority to the use of the fingers for actual reckoning. The influence of finger counting is seen in the symbolic representation of the number "one" in major cultures—Babylonian, Egyptian, Greek, Roman and Indian each used "1," taken from the single lifted finger. However, a more notable legacy for the purpose of this present paper is the universal decimal system:

We must regard as an unquestionable hypothesis that the ten fingers of both hands provided the principle support for it [the abacus], and led to the decimal system.⁶

The needs of government and commerce in a sophisticated society call for the keeping of records, and it is natural that systems of written notation should develop. Before that was possible, however, the concept of group number must have been discovered and this too would have had its origins in finger counting, with the use of the whole hand as a group of 5 or both hands for 10. Conant mentions the following method used to determine the numbers of soldiers in a primitive society as late as the early nineteenth century. Each soldier was made to go through a passage in the presence of the principal chiefs, and as he went through a pebble was dropped to the ground. This continued until a heap of 10 was obtained, when one was set aside and a new heap begun. Upon the completion of 10 heaps a pebble was set aside to indicate 100 and so on, until the entire army had been numbered. Faced with similar problems, the herdsman of pre-classical antiquity must also have discovered the practical value of group numbers and, equally significant, the principle of place value, which was much later to reappear in the working of the abacus.

Records inscribed on clay tablets and written on papyrus demonstrate that a written notation using these concepts was firmly established 5,000 years ago. Archibald mentions the Oxford mace of 3100 BC, on which there is a record of 120,000 prisoners, 400,000...
captive oxen, and 1,422,000 captive goats. Of particular interest in the current context is the so-called Senkereh Tablet (2300-1600 BC), a clay tablet unearthed near Babylon in 1854 and now in the British Museum. It shows a list of the squares of numbers up to 24 (and beyond) and was used, presumably, as a kind of "ready reckoner" (Illustration 1). Pullan discusses the tablet and makes the significant observation that

_Nothing is known of the way in which the calculations were made_, but the tablet is proof that some method of multiplying numbers was known in Babylon about two thousand years BC [my emphasis].

The fact is, as Smith has pointed out, the Babylonian notation, adapted to a combination of the numerical scales of 10 and 60, and limited by the paucity of basal forms imposed by the cuneiform characters, was ill-suited to calculation. It should not surprise us, therefore, either that a set of tablets was necessary, or that the methods of computation have nowhere come down to us, despite the efforts of the archaeologist.

A similar situation exists with Egyptian calculation, as the famous Rhind Papyrus makes clear. This document remains the principal source of knowledge of how the Egyptians counted, reckoned and measured. In his article, Newman refers to its sometimes cryptic style and the awkward calculating processes—when they are actually given. He notes that answers to problems are given first and then verified, not explained, and goes on:

_It may be, in truth, that the author had nothing to explain, that the problem was solved by trial and error—as, it has been suggested, the Egyptians solved all their mathematical problems._

It is certain that their arithmetic was essentially additive, that is they reduced multiplication and division to cumbersomely repeated additions and subtractions, with rare use of the multiplier 2 to double and halve, much as the Babylonians must have done in composing their tables. However, as Nagl carefully points out: "all calculation boils down to increasing or reducing one number in simple proportion to another."

Both Nagl and Pullan illustrate a dot diagram which appears on the back of a papyrus dating from 1500-1420 BC (Illustration 2). It seems to have no connection with the rest of the document and appears to be a casual aid to alleviate the tedium associated with
Illustration 1
Senkereh Tablet

Source: Department of Western Asiatic Antiquities, British Museum, Inv. no. 92680. Reproduced with the permission of the Trustees of the British Museum.
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simple calculation. It comprises 10 horizontal rows of 10 dots each, with a line drawn below the fifth row, and a separate vertical row to one side. Pullan notes\(^1\) that addition and subtraction of numbers below a hundred could be effected by counting forward or backward along the rows, and that the diagram could be used as a simple form of multiplication table. Only one other example is known of such a diagram, but, despite the paucity of evidence, it is difficult to avoid the conclusion that they represent the earliest form of practical reckoner, a stage beyond the accepted “table text” of Babylonia and a half-way house to the formal abacus which was to be developed under the Greeks.

The absence of evidence for the existence of a developed abacus in the Babylonian and ancient Egyptian civilisations could indicate that, in general, their societies did not need them. Less sophisti-

Illustration 2

Egyptian Dot Diagram

Source: Department of Egyptian Antiquities, British Museum, papyrus 10184 (Salier 4). Reproduced with the permission of the Trustees of the British Museum.
cated in trading terms and less expansionist in geographical terms than their Greek and Roman successors, their written notation, not perfected sufficiently to allow for ease in general computation, perhaps did not need to serve that purpose—all that was necessary was an occasional dot diagram to make simple calculations less laborious than would otherwise be the case. Moreover, as Pullan has pointed out,\(^\text{19}\) practical ways can make computational problem-solving unnecessary:

For example, the lengths of two pieces of wood can be added by laying them end to end, or their difference can be found by putting them side by side. Measuring ropes or tapes can be used in similar ways. Weights and money values can be compared, multiplied and divided by placing the actual weights or coins on a table, and other measures can often be treated similarly. It is possible to work out quite complicated tables of numbers, for example squares and square roots, with no more apparatus than a few pebbles or a measuring rod.

It could well be that the answers to many of the questions regarding Babylonian and ancient Egyptian computation should be sought in a straightforward combination of simple addition and subtraction, practical methods and repeated trial and error.

It is certain that, although the written numerations of the Greeks represented a step forward from cuneiform or hieroglyphs, and great advances were made in the science of mathematics under the Greeks, calculation with large numbers was still a cumbersome affair. One merely needs to consider the fact that to be able to express any number below a million, our Arabic notation requires six places and ten symbols, the nine digits and the cypher. The most advanced Greek system (the alphabetic of the fifth century BC) required six places, no cypher, but fifty-four symbols; where one of our symbols—8, for example—can stand in any of the six places and take to itself a new value in each position, in the Greek alphabetic system, separate symbols were required for each of the values eight, eighty, eight hundred, eight thousand, eighty thousand, and eight hundred thousand.\(^\text{20}\) Despite the efforts of Heath,\(^\text{21}\) who (albeit as a modern mathematician) has skillfully demonstrated the ability of the alphabetical notation to answer relatively complex mathematical problems, it is difficult for a layman not to agree with writers such as Popp,\(^\text{22}\) Taton,\(^\text{23}\) Yeldham,\(^\text{24}\) and Smith,\(^\text{25}\) who would support Bell.\(^\text{26}\)
Sporadic attempts to rehabilitate the battered reputation of Greek logistic as a workable system appear to be based on misapprehensions of what the Greeks actually did, and the majority opinion remains that of the conservative and sympathetic historian of Greek mathematics who characterised Greek numeration as vile.

Whatever the talents of contemporary specialist mathematicians, it is also certain that, with the expansion of trade in the Mediterranean, particularly after the appearance of coinage in the seventh century BC, the ordinary Greeks had a more pressing need of extensive calculations for everyday commercial purposes than their predecessors had ever required. Furthermore, we tend to forget that cheap paper is a comparatively recent phenomenon—clay tablets and wax are quite unsuited to extensive numerical work; papyrus was expensive (the reason for its frequent reuse); and parchment was an invention—again expensive—of the fifth century BC. It is not surprising, therefore, that under these circumstances the Greeks should be the first to develop the abacus in the West.

Numerous literary references and the physical evidence provided by archaeology attest to the widespread use of an abacus in classical Greece. Of the latter, the most famous and complete is the so-called Salamis Table, discovered in 1846 and now in the Epigraphical Museum in Athens (Illustration 3). In her definitive study of the tablet, Lang notes a total of 12 extant Greek abaci of varying forms. These are naturally of stone and of a size which indicates a quasi-public use, but many casually converted roof tiles with rows of scratched numbers are also known, which would have served the ordinary trader in a similar way. A detail of the “Darius Vase” (c 300 BC), found at Canossa and now in the Naples Museum, shows Darius' treasurer (surely one of the earliest representations of an accountant) seated at a small, marked abacus table receiving tribute and, with the help of pebbles on the abacus, determining the amount to be written on a wax table held in his free hand (Illustration 4). The abacus clearly shows the sum 1231 drachms 4 obols. Lang concludes that two kinds of abacus were in use in ancient Greece—one with unlabelled and transected columns, and the other with labelled columns. The distinguishing features of both of them, however, are columns vertical to the user and the use of unmarked pebbles that may be moved from column to column. Both Lang and Nagl have shown how calculations would have been carried out on a Greek abacus, but detailed descriptions of the moves tend to give the impression that the processes are much
Illustration 3

Salamis Table

Source: Epigraphical Museum, Athens. Photo TAP.
**Illustration 4**

**Darius Vase**

*Source: Photo courtesy of the Deutsches Archaeologisches Institut, Rome. The relevant painting is at twenty five to.*
more complicated than they actually are in practice; the actual working time is but a fraction of the time taken to explain the moves. Apart from its possible origin in and obvious advantage for commercial purposes, Cajori claims the following achievement for the abacus:

Had the Greeks not possessed an abacus by the aid of which computation could be carried out independently of the numerical notation then in vogue, their accomplishment in arithmetic and algebra would have been less than it actually was.

The Roman notation was a considerable improvement over the Greek for simple calculations such as addition and subtraction, as the following example taken from Smith and Ginsburg demonstrates:

<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCCLXXVII (777)</td>
<td>DCCLXXVII (777)</td>
</tr>
<tr>
<td>CC X VI (216)</td>
<td>CC X VI (216)</td>
</tr>
<tr>
<td>DCCCCLXXXIII (993)</td>
<td>D L X I (561)</td>
</tr>
</tbody>
</table>

No mathematics needs to be learned to perform these calculations; simple rules such as 2 V's make an X are all that is necessary, and the entire operation is reduced to little more than counting. The only advantage of our Arabic notation in addition and subtraction is that it is easier (more compact) to write—it is certainly not easier to apply. However, for multiplication and division, where the Roman numerals are deficient, as well as for speedier performance of addition and subtraction, the abacus was in common use.

The Romans were able to improve upon the Greek abacus by virtue of their notation itself. The importance of the intermediate point—5, 50, 500—in the Roman decimal system meant that the columns which had previously advanced in factors of ten, could now be subdivided in a way which gave a notational value to every counter that appeared on the abacus. In other words, a column did not need to be interpreted before its value could be expressed, as under the Greek system. The number of counters could be reproduced just as they appeared in the columns of the abacus, thus reducing the number of pebbles or counters necessary to express a number. An important corollary would be the resultant increase in the speed of operation, by removing the necessity to assimilate and move anything up to nine pebbles—perhaps haphazardly ar-
ranged—in a column; under the Roman system, there cannot be more than four stones for the brain to absorb and the finger to move during addition and subtraction, and five in multiplication and division.

One of the most frequently recurring adjectives used to describe the Romans is “practical,” and it is typical that they should afford the abacus, the practical reckoning instrument, a significant place in their education. The form of the modified abacus must have been similar to the Greek—smooth pebbles (“calculi” = “ΨΗΠΟΙ”) used on a suitably marked piece of stone or wooden table. Roman calculi have been found on numerous sites throughout the world and in such quantities as to put beyond doubt their nature. The existence of a few bead-frame abaci, small enough to be held in the hand and manipulated like a pocket calculator, has led some writers to believe that the Roman system was generally more sophisticated than it actually was. The paucity of extant specimens (only three survive, although four have been illustrated in the literature), as compared with the frequency with which calculi are met, can only indicate that their use was very restricted. Perhaps this was because the physical operation of an abacus with pebbles was more easily followed and therefore understood, and also because the sophistication of the bead-frame abacus hints at an expense beyond the pocket of the ordinary working man.

Nothing is known of the western abacus between the fall of Rome and post medieval days. The eleventh and twelfth centuries provided an interesting divertissement in this interregnum in the form of the abacists, of whom the best known was Gerbert of Aurillac, who became Pope Sylvester II in AD 999. “Abacists” is a misnomer in our context, inasmuch as the abacus described in the treatises of the period bore no resemblance to the abacus with which we are familiar from Roman times and the pages of later printed arithmetics. Its appearance gave rise to the term “arc abacus,” as the vertical columns—up to 27 divided into 9 sets of 3 columns each—were surmounted by connecting arcs. The Roman calculi were replaced by counters marked with a number utilising the newly discovered Arabic notation from 1-9, the 0 being redundant on an abacus, and the treatises which described its operation eschewed addition and subtraction, concentrating on multiplication and division (Illustration 5).

The abacists have been the subject of scholarly articles by Evans, in which she notes the tendency of Gerbert and others to give a list of straightforward rules and results without explanatory
details, a conclusion given eloquent testimony by the numerous treatises published by Bubnov, and the failure of many to give even the barest diagrammatic help to their readers. It would be difficult not to agree with the twelfth century historian William of Malmesbury’s comment that the rules of the abacus were “scarcely understood by the sweating abacists themselves.”47 As Evans has pointed out,48 it is possible to learn to calculate by learning a series of rules (as the Greeks and Romans had done with their abacus) and being conscientiously accurate in putting them into practice, but there comes a point when one must understand what one is doing. This point is particularly relevant when it is considered that the specific audience for the treatises was schools and scholars, and not commerce: “[the abacist movement] created a strongly scholastic method which had hardly any connection with the commercial world. . . .”49

The effective death-knell of the arc abacus was the influence exerted by Leonardo of Pisa’s Liber Abaci, written in 1202 and revised in 1228. Despite its (to our eyes) misleading title, this was a comprehensive study of arithmetic, elementary algebra, and geometry, which used the full arabic notation, including the zero, thereby making impossibly anachronistic any scholastic retention of the “arcus Pythagoreus.”

We have no reason to believe that the more conventional form of Roman abacus did not continue in use by the ordinary people throughout this time. The simple, proven pattern, which included addition and subtraction—a requirement on which the abacists were silent—as its most obvious feature, would doubtless have been preferred even with greater availability of manuscripts and ability to read. Furthermore, as Yeldham pointed out,51 the large number of counters required (9 for each column), the cost of engraving them, and the concentration required for selecting the properly marked counter at each step of the computation would have mitigated against popular use of Gerbert’s abacus.

During this same period, the so-called Dialogus de Scaccario52 was written, which describes the workings of the English Exchequer. Despite its composition date—the late 1170s53—when algorism using the new Arabic notation was coming into use, we are told that the Exchequer system of reckoning “follows traditional rules and not those of the new arithmetic.”54 The Exchequer table is a peculiar variant of the conventional abacus in which the columns also contain multiples of the individual numbers in them. The form and working have been well illustrated in a monograph by Berry,55
and Haskins\textsuperscript{56} and Evans\textsuperscript{57} have both compared its operation to contemporary practice on the abacus.

Stone\textsuperscript{58} and Gardner\textsuperscript{59} describe the controversy between abacists (here meaning users of the traditional abacus with Roman notation) and algorists, the proponents of the new Arabic notation which made abacal calculation redundant, as an acrimonious struggle. It is certain that the new Arabic numerals were slow to spread and met with much resistance which continued to give support to the abacal form of computation. Pullan\textsuperscript{60} and others mention the Florentine edict of 1299, forbidding bankers to use the new figures—which were felt to be too susceptible to fraudulent alteration—and the edict of 1348, wherein the University of Padua directed that books should have their prices marked "non per cifras, sed per literas clara [i.e., Roman numerals]."

However, it must be recalled that manipulation of a new notation had to be mastered at a time when reading and writing were still either very basic or the privilege of a minority of the public (and those mostly men). The shapes of the 10 digits of the new notation did not become standardised until after the introduction of printing; and the normal writing medium, rag paper, was not introduced into Europe until the twelfth century, and was made by hand and consequently scarce and expensive. All of these factors mitigated in favour of the status quo and the abacus. As Pullan has pointed out,\textsuperscript{61} after centuries of familiarity and use, people were well satisfied with the traditional Roman system—with counters to perform the actual computations, its notation was perfectly adequate for them.

A strange mixture of old and new pervaded commercial records for several hundred years, reflecting the gradual change from Roman numerals to Arabic without affecting the continued use of the abacus in compiling accounting records in the interregnum. Pullan illustrates\textsuperscript{62} a number of account books from the sixteenth and seventeenth centuries which juxtapose Arabic and Roman, even to the extent of an Arabic total at the foot of a column of Roman entries. This did not cause confusion in contemporary minds, the answer being simply that the abacus was used to arrive at the totals, which were then recorded in whatever notation the clerk cared to use. Support for this is provided by the Exchequer records of the seventeenth century, which show many examples of Arabic totals with dot diagrams—abacal jottings—at the side.\textsuperscript{63} Secondary evidence for its continued use in commerce can be seen in the haphazard arrangement of pounds, shillings and pence for individual entries in many early records; the determination of
totals would have been a precarious undertaking without the assistance of counters. Conclusive proof lies in the pages of contemporary literature.

With the advent of printing, a great series of books on arithmetic poured forth from the press. Most of these texts had as their object the teaching and dissemination of the new science of algorism. Their authors, after all, were mathematicians who recognized the greater efficiency of the Arabic notation and had doubtless used it for computation for a considerable period before merchants and accountants understood and accepted it. However, a considerable number of writers took cognizance of the fact that the abacus was still in general use, and included a section on "line reckoning" or "counter casting" in their arithmetics, showing how the old method could still be used in conjunction with the new Arabic numerals. It is difficult to determine which of altruism or commercialism was the stronger motive; it is preferable to believe the former. Hain quotes from the introduction to the 1658 edition of Recorde's book (supra): "[Counter casting] would not only serve for them that cannot read, but also for them that can do both but have not at some time their pen or tables ready with them."

The earliest printed description of abacal arithmetic is Johann Widman's Algorithmus Linealis of 1488, a work devoted entirely to the counting board. It is significant that texts of this nature—mostly written by the so-called Rechenmeister, whose professional instrument was the abacus—disappeared long before those which merely treated it as an aberrant, though still commonly used, alternative to algorism. Smith's seminal Rara Arithmetica (supra) gives a comprehensive survey of early arithmetics and commercial arithmetics and provides ample evidence for the great variety of treatises, published throughout Europe, which contain explanations of the abacus. Grosse had earlier contributed a more specific work. In the Rara, Smith refers to the "father" of double-entry bookkeeping, Luca Paciolo, as having written a tract entitled Libro da Abacho, but gives no reference (it was not in the Plimpton library); there is no copy mentioned in the catalogues of the British Library, the Bibliothèque Nationale or the National Union. As it is also not noted in the bibliographies of his work in the major directories, it perhaps does not exist.

Barnard has made a major contribution to our knowledge of the form of the abacus during the last centuries of its life. Its physical shape had changed in line with the fashions and requirements of the times: it could be a complete table, sometimes folding,
and sometimes possessing tills or drawers; a counter combined with a cupboard; a board to be used on a table or the knee; or an ordinary table on which a counter-cloth (to act as the abacus) was placed.\(^6\) From his search of contemporary inventories, mostly in wills, Barnard found the counter in public establishments, both civil and ecclesiastical, as well as in the houses and places of business of all classes and callings: tradesmen, merchants, lawyers, clergy, and titled and untitled gentry, including royalty:

Manifestly during the centuries covered by these records and authorities it was practically ubiquitous, and one wonders how many of the older oak tables in our curiosity and secondhand furniture shops were once counters that have been converted, where necessary, by the later top so commonly seen on them, into ordinary tables. Such an alteration would help to account for the apparently total disappearance from amongst us of the reckoning-board.\(^6\)

The literature shows that the main principles of the abacus—that is, the values of the lines and spaces, or, in their absence, the values of the positions which they represented—are unvarying and do not differ from the Graeco-Roman pattern, except for the occasional non-decimal units of currency, weights, and measurements used in specialized cases. The counters were manipulated in many different ways during operation, however, as Recorde points out—"for the dyvers wyttes of men have invented divers and sundry wayes almost unnumerable."\(^7\)

Barnard summarizes the methods of operation found in his search of 120 arithmetical treatises (including different editions) as follows:\(^7\)

A. Methods with lines
   a. Process in which lines only were used.
   b. Process in which spaces only were used.
   c. Process in which both lines and spaces were used.
   d. Process in which both lines and spaces were used, and also 'Lyers,' or stationary counters, i.e., the 'Tree of Numeration.'

B. Methods without lines
   e. Process in which 'Lyers' were used, forming the 'Tree of Numeration.'
   f. Process in which no guides to help the eye were used.

The most common practice was c.
The abacus fell into disuse in England sometime between 1668, when the chapter on counter casting is still in the edition of Recorde's *Ground of Artes*, and 1699, when it is omitted with the following prefatory remark from the editor: "I have taken all the care I could to do the Author justice by Expunging what is now useless." Barnard remarks that it had possibly become discredited with the better-educated classes as early as 1610, but lingered on with the old-fashioned or unlearned, and especially with women in their housekeeping, only gradually to disappear during the last three quarters of the century. A similar pattern prevailed in the Protestant Netherlands, but in France it lingered on until the Revolution, to be given the coup-de-grâce by the new order, and there is evidence to support its existence in Germany up to the same period. The supposedly antediluvian abacus, despite the frequent derision heaped on it, thus managed to survive the advent of algorism by some six centuries.

At the lowest level, the abacus supplied the common man with a means of applying a reliable quantitative method long before education was a universal reality, and at the highest level:

> We shall always be able to claim for the abacus that it was the first completely successful representation of the decimal system, a representation whose perfection has still not been equalled.

**FOOTNOTES**

2. Conant, p. 433.
3. This text was a landmark in the long series of English publications on arithmetic and commercial arithmetic. The first edition appeared in 1540, and Smith (1908) notes no fewer than 52 editions over a period of 159 years. For comparative purposes, illustrations of finger numerals from a thirteenth century Spanish manuscript and a sixteenth century German arithmetic can be found in Smith and Ginsburg, pp. 455-456.
4. The Venerable Bede (AD 673-735). An English translation of this passage appears in Yeidham, (1926), pp. 30-34.
5. "Together (with the use of the abacus in late Roman times) the old form of hand reckoning remained in use for calculations of the simplest kind, and indeed for the actual process of calculating itself and not only for the expression of a number, as is commonly thought."

Original text:

> "Daneben (the use of the abacus in late Roman times) war das alte Handrecken für Rechnungen einfachster Art in dauernder Anwendung verblieben, und zwar auch für das Rechnen selbst, nicht bloß, wie man vielfach angenommen hat, für das ruhige Festhalten einer Zahl." (Nagl, p. 40.)
"Es ist eine alte, allgemeine und unbedenkliche Annahme, daß die zehn Finger beider Hände ihn (the abacus) dabei in vornehmlicher Weise unterstützten und zu den dekadischen Zahlensystem geführt haben." (Nagl, p. 5.)

Conant, pp. 435-436.
Archibald, p. 7.
Pullan, pp. 2-4.
Smith, (1921), p. 3.

Archibald, p. 58, notes that the 300 small mathematical tablets in the University of Pennsylvania collection, together with 200 in the Istanbul Museum, are nearly all "table texts."

With the exception of the Golenischev Papyrus in Moscow, which predates it by some 200 years, the Rhind Papyrus (discovered in 1858, now in the British Museum BM 10057 and 10058) is the oldest mathematical document in existence, dating from the twelfth dynasty (2000-1788 BC).

Newman, pp. 170-178 passim.

Original text:
"... alles Rechnen nichts anders ist als die Vermehrung oder die Verminderung einer Zahl nach Maßgabe einer anderen." (Nagl, p. 41.)

Yeldham, (1926), pp. 24-25.
Heath, p. 28 et seq.
Popp, p. 6.
Taton, p. 287.
Yeldham, (1926), p. 25.
Smith, (1921), p. 4.
Bell, p. 51.
Smith, (1921), p. 5.

The most famous amongst these are Herodotos (II, 36, 4); Aristophanes in his Vespae (332-333 and 656-657); and Diogenes Laertios (I, 59) who attributes to Solon the famous analogy between tyrants' men and the pebbles (ΨΗΦΟΙ) of an abacus.

Lang, pp. 275-276.
Heath, p. 27; Pullan, pp. 25-26.
Lang, p. 282.
Lang, pp. 275-276.
Lang, p. 279 et seq.
Nagl, pp. 62-63.

In this context, it is interesting to note that Archimedes was reputedly using an abacus when he was murdered in 212 BC.

Consider, for example, the expression of the number sixteen—with columns of "10," 7 pebbles would be needed; with the Roman system of "5s" only 3 stones are needed.

Both Horace (Satires I, vi) and Juvenal (Satire IX, 40) refer to this.

Pullan, pp. 20-21, discusses the frequent description of calculi as "gaming counters," the Romans being particularly fond of board-games: "Mere pastimes, however, would not account for the presence of calculi on so many Roman sites, or of other kinds of counters on so many earlier sites, whereas a certain amount of reckoning would occur wherever there was any kind of trade or industry involving the use of money and measures."

Two are illustrated by Nagl, pp. 16-17, and a third, the British Museum specimen, by Pullan p. 19. All three are virtually identical.

"Arcus Pythagoreus" as compared with the normal "mensa Pythagorica"—the abacus, through an apocryphal version of the Geometria of Boethius (obs. AD 524), was popularly thought of as being an invention of Pythagoras; thus Adelard of Bath (c. AD 1120):

"The Pythagoreans invented it (the abacus) in order better to retain the subject matter which their master Pythagoras had taught them in his lectures. They gave the name "mensa Pythagorica" to it out of regard for their master; those that came later called it the abacus."

Original text:

"Pythagorici hoc opus [the abacus] composuerunt, ut ea, quae magistro suo Pythagora docente audierant, oculus subjecta retinerent et firmius custodirent. Quod ipsis quidem Mensam Pythagoriam ob magistri sui reverentiam vocaverunt; sed posteri tamen Abacum dixerunt." (Bubnov, p. 157, note 17.)

A rather greater controversy has raged around Boethius' supposed (and on balance refuted) claim that the Arabic numerals from 1-9—the zero is not mentioned in the text—were a legacy of the Pythagoreans (see Cajori, p. 68; for an opposite view see Gandz, p. 411).

Gerbert is popularly credited with being the first man to introduce Arabic numerals into the West—see Taton, pp. 472-473.

Our symbol for zero was occasionally employed by the abacists, notably by Ralph of Laon, but the implication in Ralph's treatise seems to be that it was not meant to denote a zero, but rather a blank space, a locus tenens in one of the fixed columns, to hold open a place that might otherwise be overlooked in the calculation. As Smith has pointed out (1921, p. 16) the abacists known the zero they would not have needed counters at all. Even a supporter of the zero theory is forced to admit "Quoique le texte [one in which he claims a 'proper' zero] ne soit pas très clair de lui-même, ..." (Bubnov, p. 275, note 9.)

A good illustration, taken from St John's College MS 17, appears in Yeldham, (1926), pp. 38-39. Much of the original Latin of this same abacus manuscript, written by a monk of Ramsey c. AD 1111, is translated into English in Yeldham, (Extracts).

Evans, (Difficultilia et Ardua); Evans, (From Abacus to Algorism); Evans, (Schools and Scholars).


Original text:

"a sudantibus abacistis vix intelligentur." Cited in Evans, (Schools and Scholars), p. 72
48 Evans, (Dificililma et Ardua), p. 37.
49 Original text:
"[the abacist movement] hat eine streng scholastische, mit dem Verkehrs-
leben kaum je in nähere Berührung getretene Methode herausgebildet . . . ." (Nagl, p. 45.)
50 Leonardo Fibonacci (son of Bonacci) was held to be one of the greatest and
most productive of medieval mathematicians.
51 Yeldham, (1926), p. 46.
52 Richard.
53 Richard, p. 11.
54 Original text:
"secundum consuetum currsum scaccarii non legibus arismeticis." (Rich-
ard, p. 75, lines 16-17).
55 Berry.
56 Haskins.
57 Evans, (Schools and Scholars), p. 79.
58 Stone.
59 Gardner, p. 125.
60 Pullan, p. 34.
61 Pullan, p. 35.
63 Pullan, passim; also Berry, p. 27.
64 Hain, p. 154.
65 Grosse.
66 Smith, (1908), Addenda, p. 4.
67 Barnard.
68 Barnard, pp. 252-253.
69 Barnard, p. 253.
70 Barnard, p. 265.
71 Barnard, p. 254.
72 Barnard, p. 87.
73 Barnard, p. 88.
74 Barnard, pp. 89-90.
75 Hain, p. 163.
76 Original text:
"Für alle Zeiten bleibt der Rechentafel der Ruhm gewahrt, daß sie die
erste vollständig gelungene Darstellung des dekadischen Stellensystems
war, eine Darstellung, die in ihrer Vollkommenheit bis heute nicht über-
troffen ist." (Nagl, p. 14.)

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