1972

Regression estimates for accounting purposes

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Recommended Citation
Haskins & Sells Selected Papers, 1972, p. 172-194

REGRESSION ESTIMATES FOR ACCOUNTING PURPOSES

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Presented before the
New York Chapter, American
Statistical Association–
November 1972

There are many places in financial accounting where estimates can be used and should be used in place of the tedious 100 percent counts or calculations that are the general rule today. Some of the more obvious possibilities are estimates of inventories, accounts receivable, cost of sales, unearned discounts and various kinds of reserves. Accountants need to be convinced that satisfactory estimates can be obtained for these purposes, and that very large amounts of time can be saved through the use of statistical estimates.

In many companies today, the detailed accounting records are maintained on a computer. This means that there is an existing record that could produce a list of the book values associated with all the components of the asset, liability, revenue, or expense account to be estimated. To be more specific, let us talk in terms of an inventory of 2,500 different parts, maintained on a perpetual inventory record, by computer. Associated with each part number would be a quantity on hand for that part and a unit cost. An extension of the quantity on hand times cost for each part number would provide the book value of the inventory.

The purpose of this talk is to show how a satisfactory estimate of the physical value of this inventory may be obtained and, by extension, how estimates of other such accounting records may be obtained with a minimum of time and effort.

FEATURE OF SAMPLING INTEREST

The question at issue is whether the perpetual inventory record is a reliable indicator of the exact condition of the physical inventory maintained in the stock room or warehouse. If we could be sure that there was an exact correspondence, the perpetual record would be the best estimate of the inventory value. In a few situations, such as inventories of precious metals, the control procedures may be sufficiently good that such an exact correspondence will exist. In most cases, however, the possibilities of errors, mis-postings, losses, and pilferage are such that the expectation of an exact correspondence is unlikely. The feature of sampling interest thus becomes the relationship between the physical inventory and the perpetual record. We are interested not so much in the sample itself as in the relationship of the sample items to the corresponding items on the book record. This indicates the use of regression estimates and, in particular, stratified regression estimates, for reasons that will be developed later.

REGRESSION ESTIMATES

Most accountants know about a mean estimate, and a reasonable number of accountants would be familiar with a ratio estimate, but very few have had any experience with regression estimates. For this reason it may be helpful at this time to consider the characteristics of a regression estimate.

A regression estimate is more efficient than a mean estimate in that it makes use of known information about the book population, and it will be better than a ratio estimate in all but the unlikely case where the line of regression passes through the origin. It has been pointed out elsewhere that a mean estimate and a ratio estimate are special cases of a regression estimate. The principal reason that regression estimates have not been used more widely in those situations where they are applicable is that the necessary calculations are time-consuming and tedious. With a computer available, this reason becomes academic.

Regression calculations based on the sample data yield an estimate of the following formula:

$$\bar{y} = a + b\bar{x}$$

where b is the usual least squares estimate of the slope, $\bar{y}$ and $\bar{x}$ are the respective means of the sample, and a is the intercept. The rationale for this type of estimate is that if, in the sample, the physical values are somewhat less than the book values, it may be inferred that a similar relationship exists between the unknown physical inventory total and the known book inventory total.

As the total value of the book inventory is known, the mean of the book population $(\bar{X})$ is known. The sample provides a value $(\bar{x})$, for the book sample mean. The unknown mean $(\overline{Y})$ of the entire physical inventory may be estimated by correcting the mean of the physical sample $(\overline{y})$ for the difference between the mean of the book population and the mean of the book sample.

This is done by multiplying the difference in the means by the coefficient b and adding the physical sample mean in the following formula:

$$\overline{Y} = \overline{y} + b\,(\overline{X} - \overline{x})$$

The coefficient represents the expected change in the physical value as the book value is increased by one. The nature of the estimate is presented in graphic form in Exhibit 1.

- **Stratified Estimate** Accounting populations lend themselves readily to stratified sampling. In stratified sampling the population is first stratified into a given number of strata and each stratum is then treated as a sub-population. A random sample is drawn from each of these strata. An estimate is obtained for each stratum and the results are combined into a single overall estimate, with its associated variance.

The logical variate for stratification of an inventory would be the physical value. This, of course, is unknown but the book value is a reasonable approximation of this value and consequently is ideal for stratification purposes. The principal reason for stratification is to reduce the variance and thus tighten the precision surrounding the estimate. A stratified estimate based on inventory values will invariably be better than an unstratified estimate. The unstratified estimate contains variances between strata that are eliminated from a stratified estimate. The stratified variance must therefore be lower, and in most cases is considerably lower.

- **Stratified Regression Estimates** In any sampling situation a decision has to be made, at some point, as to which type of estimate is best. In most accounting situations the preference would be for the estimator with the lowest variance. The variance of a regression estimate will be smaller than that of a mean estimate unless the correlation between x and y is zero, and smaller than that of a ratio estimate unless the regression line passes through the origin. In these two special cases the variances will be equal.

There are gains from using stratification and there are gains from using regression estimates, and the combination makes a powerful tool in inventory estimation, due to the expected correspondence between the book and physical values.

- **Optimum Stratification** The variance of a stratified estimate may be reduced even further by selecting the upper limits of each stratum in such a way that the variance in the stratum is minimized. The statistical theory for optimum stratification has been available for many years. While the optimum state can never be achieved absolutely, the goal of all optimization procedures is to reduce the variance to a minimum and a near minimum is ordinarily good enough.

In a two-stratum design, the upper cut-off of the lower stratum would lie between and would approximate the average of the two stratum means. Moving the stratum cut-off to the left or to the right would change both the stratum means and change the sum of the squared mean errors of each stratum. There will be one cut-off point, however, at which the sum of the squared mean errors of both strata would be at a minimum. Obviously this can be extended to any number of strata.

It has been shown that the optimum solution to minimizing the variance will be obtained when the expected contribution of each stratum to the total aggregate variance is made equal for each stratum.

- **Exponential Distribution** Most accounting populations follow an exponential distribution. The characteristics of this distribution are sufficiently well known that a close approximation to optimum stratification is relatively easy to obtain.

Consider the distribution that is to be divided into parts as a graph, with dollar values on the X or horizontal axis and the frequency, or the number of items for each dollar value, on the Y or vertical axis. Plotting the number for each dollar value will define the shape of the distribution. Multiplying the value by the number of items at that value and summing them would equal the total population value.

The usual approach to subdividing the area of an exponential population is to express the X or dollar values in terms of the mean. For this purpose, the mean may be approximate. A new X scale is developed with each X value being divided by the mean, in which case the mean value of the distribution is one. The Y scale is likewise scaled down by dividing the frequency by the largest number of items (Exhibit 2).

By the use of logarithms, or by reference to a table of the negative exponential function, the exponents may be found that will divide the area under the curve equally. For instance, if we want to divide the population into five strata, a negative exponent of .223 on the base line indicates the first cut-off point with an area of 80 percent remaining to the right of the cut-off point. The following points are .510, .916 and 1.6 respectively, leaving 60 percent, 40 percent and 20 percent of the area to the right. If vertical lines are drawn at these cut-off points, the appearance of the distribution will resemble a series of rectangles gradually decreasing in height and increasing in width.

A conservative assumption as to the variance of a rectangular distribution is 1/12 of the square of the width of the distribution. Since the height of each rectangle decreases as the width increases, we are moving fairly close to the goal of an equal variance for each stratum.

We know from the nature of an exponential distribution that the mean of each of these rectangles would lie slightly to the left of the midpoint. It would seem, intuitively, that this would produce a larger variance than if the mean were at the midpoint. This situation can be improved conceptually by constructing alongside the first stratum another stratum of equal width, with its height equal to the ordinate at the original first cut-off point. Considering these two rectangles as one stratum, the mean is now at the midpoint and the standard deviation is near to a minimum. If the second and subsequent strata are constructed in a like manner, it seems reasonable to assume that this is a close approximation to the cut-off point needed to minimize the variance.

This has been set forth in mathematical terms and proved to be a first approximation. As a practical matter, nearly all of the advantage to be gained from optimum stratification is obtained by using the first approximation. All that is then required to determine any cut-off point is to multiply the exponential value, in units of the mean, by twice the value of the mean. Thus, for a population whose mean value is $1,000, the first cut-off in a five-stratum design would be at $446.

ESTIMATION SAMPLING PROGRAMS

The statistical principles that have been discussed have been known for years and may be found in the standard textbooks or authoritative literature. The procedural problems of (1) achieving optimum stratification, (2) stratifying large populations, and (3) making regression estimates are sufficiently complicated that they were not likely to be used until high-speed data processing became available.

About eight years ago Haskins & Sells started the development of the

Auditape System, involving a collection of special programs designed to operate on a specified format under which data may be extracted from client records and other formats for further processing. It frees the accountant or auditor from the necessity of special programming to obtain data from existing computer records. Once the record is in the prescribed Auditape format, other programs may be used to accomplish specific purposes. Estimation sampling is one of these special purposes.

- **Sampling Procedure** The estimate from a statistical sample is simply an extrapolation or projection of the results obtained from a sample and is similar to the estimates that would ordinarily be made intuitively from a non-statistical sample. The unique feature of statistical sampling is that it provides a means for measuring the degree of assurance or, conversely, of uncertainty associated with the sample estimate. These measurements are based on the mathematical concepts of probability and are expressed in terms of reliability and precision.

The procedures for making a sampling application consist of the three stages of design, selection, and evaluation of the sample. We have one program that designs and/or selects a sample and another program that evaluates the sample. The unique characteristic of our programs is the way in which a number of known statistical concepts have been put together in an accounting oriented approach to obtain sufficiently precise estimates of asset or liability values for financial statement purposes.

DESIGN STAGE

To design a sample, it is necessary to identify the population to be sampled, define the sampling units and the related feature of sampling interest, specify the desired reliability and monetary precision, determine the basis for stratification, and compute the approximate sample size. The reliability and monetary precision to be used in designing the sample are matters for management decision, based on consideration of the relative degrees of assurance required and the related cost. Some indication of these relationships may be obtained by a simulation based on previous computer records.

- **Sample Specifications** The sample size required is a function of the reliability and the precision specified, the stratification to be used, and the characteristics or condition of the population with respect to the feature of sampling interest.

Some preliminary estimates of the extent of variability to be expected in the sample need to be made in order to compute the approximate sample size required. This preliminary estimate of variability could be based on the results of a previous complete inventory if such is available or on the results of a previous sample. The next best thing is to make an analysis of the most recently available perpetual record and to introduce some conservatism into the precision parameter to provide for the unknown and perhaps unexpected relationship between the book inventory and the physical inventory. In this connection, general experience with other inventories believed to be roughly comparable to this one can be helpful in the design stage.

Since the preliminary estimate of variability to be expected is necessarily based on earlier, and possibly incomplete, data, the reliability and precision obtained the first time around in the evaluation of the actual sample is likely to differ somewhat from that specified in designing the sample and computing the sample size. In practice, until further experience is available with respect to the values being estimated, some deliberate conservatism in the preliminary precision parameter used for computing sample size ordinarily is desirable.

In the design stage, several specific operations are accomplished. To begin with, certain data must be specified, such as the number of items in the population, the actual or approximate total dollar value, the fields in which the necessary values may be found, the desired monetary precision, and the reliability level expressed as a percentage.

The initial data that has been specified is passed through a series of checks to be sure that the specifications are reasonable. If no errors are detected, the processing will continue. The mean of the book population is determined from the data as specified and the program proceeds to compute the upper cut-off for each stratum. The first or lowest stratum is for zero amounts and minus values. Unless specified otherwise, the remainder of the distribution will be divided into twenty strata. Stratum number 21, the highest stratum, will be sampled completely. Minus values greater than $100 will also be included in this upper stratum so that they will be counted 100 percent. This $100 value is arbitrary and may be changed to any other amount by specification.

The upper cut-off in dollars of the next to the highest stratum will be approximately six times the mean, unless another value is specified. If a higher or lower cut-off is specified, the remaining stratum cut-offs will be scaled up or down accordingly. In those situations where the exponential distribution does not appear to be satisfactory the desired stratum cut-offs may be entered by initial specification.

- **Stratification** The statistical frame on which the sample design is to be based is available to the computer in tape or disk form in Auditape format. The purpose of stratifying the population in advance of selecting the sample is to determine the number of items that fall in each stratum, the total dollar value of these items, the mean value and the standard deviation within each stratum. If two values, representing book and physical, are available as input to the computer, the standard deviation computed will be a standard deviation from the line of regression. If only one value is available, the standard deviation will be measured from the mean. In addition, totals and statistics are determined for the entire population without regard to stratum.

The stratification procedure is simple: Each item is taken as it appears on the input tape and is measured against the boundaries of the strata to determine the stratum to which it applies. The item is counted as an item in that particular stratum, and the dollar value is accumulated in a total for that stratum. The values are squared and cross-products are obtained for subsequent determination of the standard deviation. If any physical value exceeds the upper cut-off and is four times greater than the corresponding book value, that item will be included in the top stratum for design purposes. This is a precaution to prevent a few huge or unusual variances from upsetting the sample design.

Some inventories of parts and supplies may contain several hundred thousand separate items, and the stratification procedure may require more than an hour of computer time. In these cases a satisfactory sample design may be obtained by stratifying a sample from the population rather than stratifying every item. A sampling rate such as one in five may be specified, in which case the computer, with a random start, will select every fifth item thereafter and stratify in the usual way. When this sampling stratification procedure is completed, the results in turn are multiplied by five to approximate the result that would be obtained by stratifying the entire population. The standard deviations are increased by two standard errors so that there is not likely to be an underestimation of the sample size through use of this procedure.

The stratification stage may be by-passed completely if an assumption is made that the population is distributed exponentially. Assumptions will be made in regard to the number of items in each stratum and as to the total values of each stratum, based on the specified total number of items and the total value. Assumed means and standard deviations are determined and used in the same way as if a complete or partial stratification procedure had been followed. A sample selected on the basis of the exponential assumption will in most cases approximate the results otherwise obtained and eliminate the time requirement to pass a large tape through the stratification procedure.

■ **Size of Sample** The first step in the determination of the size of the sample is to find the weighted average standard deviation. This is obtained by multiplying the standard deviation in each stratum by the number of items in the stratum and summing over all strata, except for the first and last, and by dividing this sum by the total number of items in all but the first and last strata to arrive at a weighted average standard deviation. The first stratum is omitted because it often contains only zeros and, therefore, no standard deviation. The last stratum is omitted because it will be counted in its entirety and consequently there will be no variance.

The next step is to calculate the standard normal deviate for the prescribed percentage of reliability. This percentage will generally range from 90 to 99 percent with a reliability of 95 percent being used in most cases. Using the specified percentage of reliability, a subroutine produces the corresponding normal deviates.

At this point, with all the necessary factors available, a first approximation to the sample size is obtained and a finite population correction is made for sampling without replacement. The total sample then must be allocated to strata.

■ **Optimum Allocation** The principle of optimum (Neyman) allocation is that the variance will be at a minimum if the total sample is allocated to strata in accordance with the weighted standard deviation within each stratum. As these factors have already been calculated to determine the overall sample size, it is a simple computation to allocate the total sample to each stratum according to this principle. Some adjustments are made to the sample size so that the sample required will not exceed the number in the stratum nor in any case will it exceed one thousand items. As the allocation factor may produce a fractional number, the sample size in each stratum is rounded up to the nearest whole number.

The sampling interval is determined by dividing the total number of items in each stratum by the sample required. This interval is computed and rounded down to two decimal places. This interval is limited as an arbitrary rule to no less than one and to no more than a thousand. The sampling interval for the upper stratum is set at one, so that each item in that stratum will be selected. The interval for the lower stratum is set at the same interval as the second stratum or at such other rate as may be specified by the user. If it is known or expected that a large number of the zero book balances will actually have physical values, it may be necessary to use a lower sampling interval to get a larger sample.

- **Collapsing Strata** Due to the inclusion of an $n-2$ term in the formula for calculation of the variance of a regression estimate, it is necessary that no one stratum have less than three items to be evaluated. This either means that a minimum of three items must be selected in each stratum or that one or more strata must be combined to insure that three or more items are selected. One way to select more items is to increase the reliability or reduce the monetary precision or both. This would increase the overall sample size and increase the number in each stratum. Another way would be to reduce the number of strata by specification. Twenty strata may be more than is required in many applications.

There may be reasons, however, why neither of these approaches may be taken, and so a collapsing feature has been built into the program. It rectifies to some degree the failures in design of using low reliability or too many strata. The procedure examines the intervals in each stratum successively, determines the size of the sample that is to be taken and decides whether the sample is sufficient for the type of estimate to be made. If it is insufficient, this stratum will be added to the next stratum and the calculations will be repeated. The process is continued until all strata have been examined. If the indicated sample in the next to last stratum is insufficient, this stratum will be combined with the previous one. The first and last strata are not considered in this process so that the full 21 strata can be reduced only as low as three by this process. In general, this type of reduction will only affect one or two strata.

At this point in the program the design stage is completed and the results are printed for each stratum and in total. The program may continue into the selection stage or an option may be exercised to terminate at this point. If this option to terminate is elected, the necessary information will be punched out into cards so that the selection procedure may be continued at another time and perhaps with another accounting record. For large inventories, it is highly desirable to design at some period prior to the actual selection, just to minimize the computer time required at the selection date if time is likely then to be at a premium.

CASE STUDY

Before proceeding to a discussion of the selection stage, it may be useful to present a case study showing how the design principles are used in an actual situation.

The test tape was prepared by a special program to produce an exponential distribution of 2,500 items amounting to $10,000,000. The tape was forced to include 30 zero values and 20 minus values in order to be more representative of the normal condition of most inventory records. This tape was considered to be the detailed record of the perpetual inventory.

In most inventories, about 60 percent of the recorded items will match the physical counts and the remaining 40 percent will disagree by one or more items. Using the exponential distribution developed previously, and with a random start, the book values were duplicated for six out of every ten items and other values computed for the remainder. This was accomplished by generating a random number as a percentage, and using it to determine a normal deviate. The normal deviate is scaled by a common factor to increase or decrease the variability, and applied to the book value to determine a random physical value. The common scaling factor used is representative of an average inventory record.

This constructed tape was used as input to the design and selection program with the output of the design stage shown in Exhibits 3 and 4.

SELECTION OF SAMPLE

Whether the selection procedure continues from the design stage or is picked up at another date, the selection procedure is the same, although the input data may be different.

- **Random Start** Beginning with a random number supplied at the start of the program, a new random number is computed by a subroutine for each of the strata in turn. This random number is divided by the interval and the remainder is used as the random start within that particular stratum. As the random numbers and the intervals are calculated in a completely different way, no particular pattern of selection could develop. The first item on the tape may be or may not be selected depending upon which stratum it falls in and upon the random start within that stratum. Similarly for the second, third, fourth and all subsequent items.

- **Systematic Selection** Given the random start within each stratum, the program systematically selects items within each stratum. The first item that appears on the tape is matched to the stratum limits and it is accumulated in totals for that stratum number and dollar value. When the number of items in each stratum equals or first exceeds the random start for that stratum, that item is selected and passed to the output tape. All items in the top stratum are selected without reference to counters or random numbers. The program proceeds sequentially through the tape, maintaining track of twenty random starts, twenty separate intervals, and twenty separate counters, so that each item is selected or rejected in a completely random fashion.

  As a practical matter the indicated sample may not be selected exactly in each stratum. In some cases it may be one more or less due to the random start. In cases where the selection procedure is designed on one population and used to select from another population the sample sizes in each stratum may differ more widely.

- **Sample Output** As each item is selected from the input tape, an output tape is created of only those items selected. At the time of selection, the stratum number is carried to the output tape and an option is available to carry a sequence number to the output tape. This is the sequential number or line number of the item as it appeared on the input tape. It is obtained by counting the items as they enter the program and is useful for tracing the sample back to the input tape if this should be necessary. At the end of the selection procedure, a printout (Exhibit 5) establishes the control over each stratum and shows the stratum cut-off, the population data, the selection interval and the sample data.

  The sample tape would normally be printed (Exhibit 6) as a record of the sample to be taken. This guards against unintentional destruction or erasure of the sample tape before the evaluation is completed.

  In order to facilitate the evaluation of the sample at a later date, the essential control information and stratum information is punched into cards.

It is also printed, in the event that the punched cards might be lost or mislaid. The use of the punched cards not only simplifies the evaluation but also protects against errors that might otherwise creep into the procedure.

Cards may be punched from the sample tape to provide a unit record for each item to be sampled. They also may be reproduced and interpreted to provide a basis for the actual count in the stockroom or warehouse when the actual count is undertaken. These cards would contain, at a minimum, the part number or other identification, the book value of the item selected, and the stratum number.

To permit simulation of samples from an input tape containing book and physical values such as a previous physical inventory, the physical values may be brought over to the cards as an option. These sample cards are then ready to be evaluated. It is a useful device for instructional purposes and to indicate how the selected sample will subsequently be evaluated.

EVALUATION OF SAMPLE

For each of the items selected in the sample, a value corresponding to the book value must be determined. In the case of a physical inventory, this would be done by counting, pricing, and extending to determine the physical value, or, in the case of reserves, it would be to determine the applicable reserve for that particular class. These procedures are essentially the same, whether the determination is made completely or by sample. The workload is sufficiently reduced with a sample, however, that more care can be exercised in determining these values properly the first time. Reliable results will be obtained at low cost and with reasonable speed. The stresses and irritations that are usually present when a lot of work must be done in a very short period of time will virtually disappear when the workload is reduced to a sample.

■ **Review Procedures** The precision surrounding the estimate will not be satisfactory if the sample contains a number of huge and unusual differences. If the difference is small enough so that the item will remain in the same stratum, even though it does not agree with the book value, the effect on the overall precision will be small. If the item moves to an adjacent stratum, the effect on the overall precision will still not be too large. It is when an item jumps more than one stratum in either direction that the effect on the precision becomes considerable. Some of these differences may be eliminated on closer inspection. Perhaps the wrong items have been counted, in which case a recount is necessary. Sometimes a mis-posting is discovered in the book records that should accordingly be adjusted.

When the sample data has been reviewed as carefully as possible, the evaluation should proceed. The program requires, as input, the specification cards that are punched out in the selection procedure, together with the sample data cards. The entire evaluation will be accomplished within a few minutes at the most.

- **Checking Procedures** The specification cards from the selection procedure that are introduced into the computer at the time of the evaluation contain the necessary data to make the evaluation and include the number of sample cards in each stratum. These cards are checked for input errors and the specifications are printed (Exhibit 7) before the evaluation process continues.

As the sample data cards are introduced, they are accumulated and checked against the predetermined totals carried forward in the specification cards. Error messages will be printed if one or more of the sample data cards are omitted. If a sample tape is used as input rather than data cards, the same check will be made on the information on the sample data tape.

- **Statistical Evaluation** As each sample data card is read in, the squares and cross products are calculated and the sums are accumulated by stratum. A determination is made to decide whether any stratum is too small for evaluation for the type of estimate specified. If such is the case, the strata will be collapsed as described earlier. The program then computes the means and the corrected sums of squares and cross products for each stratum that are required for the evaluation procedure. The reliability specified for the estimate is converted into its corresponding normal deviate by a subroutine.

The user has the choice of specifying a combined regression estimate or a separate regression estimate. The essential difference between them is whether the slope is determined over the entire sample or independently for each stratum. It has been our experience that the combined regression estimate works better in most accounting situations, even though a separate regression will often yield a slightly lower variance.

The evaluation is printed out stratum by stratum (Exhibit 8) and in total, together with a calculation of the upper and lower limits of the estimate.

- **Next Sample Design** Despite all the precautions that may have been taken in this design stage, the evaluated results may not meet the expectations. Accordingly, as a supplementary procedure, the computer has been programmed to continue, at this point, to compute the sample that should have been selected based on the sample results, in order to achieve the design precision. This may result in an increase or a decrease in the sample size. It shows, after the fact, what should have been done given present knowledge (Exhibit 9). This information is helpful in designing the next sample.

In those situations where a pilot sample may precede the selection of a full sample, the pilot sample would be evaluated and the results from this last stage of sample design may be used for selection purposes. An option permits cards to be punched at this point for use in the selection stage so that a small pilot sample may be used to assist in the design and selection of a larger sample.

SUMMARY

A sampling estimate taken on the basis described above is a reliable and acceptable alternative to the complete count for financial statement purposes. It is likely that more and more companies will want to follow this approach in the coming years. The speed with which the samples may be taken will reduce downtime in manufacturing or processing operations.

It is evident that there can be substantial savings in non-productive labor through a sampling approach. It is also evident that customer service can be improved where shipping operations might otherwise be delayed by a complete shut down of several days to take inventory. Furthermore, there is good reason to believe that statistical estimates of this nature may in fact be closer to the true value than some complete counts have been in the past. •

EXHIBIT 1

GRAPHIC PORTRAYAL OF A REGRESSION ESTIMATE

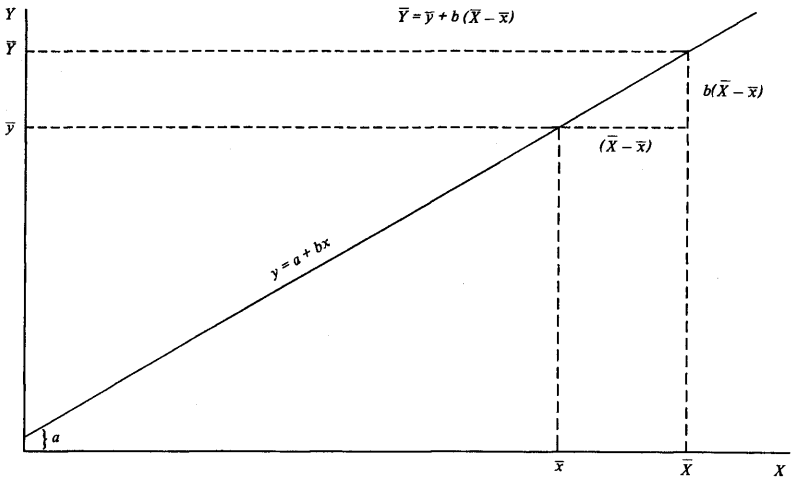

EXHIBIT 2

EXPONENTIAL DISTRIBUTION SUBDIVIDED INTO FIVE STRATA

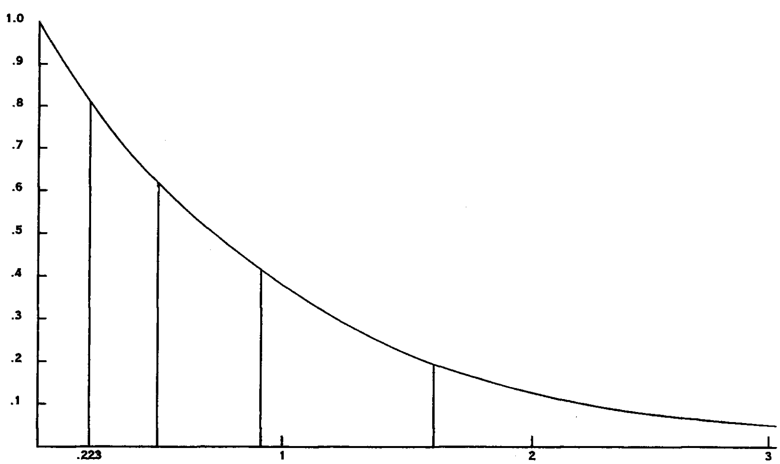

EXHIBIT 3

PRINTOUT OF DESIGN SPECIFICATIONS

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AUDITAPE SYSTEM  -  COPYRIGHT 1967-1971
ESTIMATION SAMPLE DESIGN AND SELECTION ROUTINE
EXPONENTIAL    1

SPECIFICATION CARD DATA
   ROUTINE TO BE USED FOR                                 DESIGN AND SELECTION
   INPUT FILE
      INPUT MEDIUM                                                   DISK 2311
      VARIABLE X
         DESCRIPTION                                                BOOK VALUE
         NUMBER OF ITEMS                                                  2500
         ACTUAL OR APPROXIMATE TOTAL                                  10000000
         AUDITAPE FIELD ASSIGNMENT                                           7
      VARIABLE Y
         DESCRIPTION                                               PHYS. VALUE
         AUDITAPE FIELD ASSIGNMENT                                           9
   SAMPLE DESIGN SPECIFICATIONS
      MONETARY PRECISION                                                200000
      RELIABILITY LEVEL (PERCENTAGE)                                     95.00
   RANDOM NUMBER                                                        585789
   OUTPUT FILE
      OUTPUT MEDIUM                                                       TAPE
      AUDITAPE FIELD ASSIGNED TO STRATUM NUMBER                              8
      AUDITAPE FIELD ASSIGNED TO SEQUENCE NUMBER                            12

NO ERRORS HAVE BEEN DETECTED IN SPECIFICATION CARDS.  PROCESSING IS CONTINUING.
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EXHIBIT 4

PRINTOUT OF STRATIFICATION AND SAMPLE DESIGN

AUDITAPE SYSTEM - COPYRIGHT 1967-1971

ESTIMATION SAMPLE DESIGN AND SELECTION ROUTINE

EXPONENTIAL 1

CALCULATED DESIGN AND SELECTION DATA

STRATUM NUMBER	UPPER CUT-OFF	POPULATION DATA ITEMS	BOOK VALUE	PHYS. VALUE	INDICATED SAMPLE INTERVAL	SIZE	STANDARD DEVIATION OF REGRESSION
1	0	31	-28.63	0.0	10.33	3	0.0
2	410	233	46973.89	45975.71	233.00	1	34.28
3	842	223	138694.21	137972.52	105.00	2	84.83
4	1300	211	225059.37	224728.17	67.98	3	131.04
5	1785	200	307537.97	312499.91	42.12	5	211.47
6	2301	188	383237.56	378398.47	34.38	5	259.12
7	2853	176	452612.76	451596.29	24.21	7	367.96
8	3446	165	518668.93	530799.51	22.60	7	394.13
9	4086	152	571196.65	566820.34	19.96	8	446.20
10	4782	142	628281.60	628073.61	13.03	11	683.44
11	5545	129	664745.51	658291.14	14.49	9	614.67
12	6387	117	696154.02	701106.00	11.71	10	760.86
13	7330	106	724800.81	744887.74	9.13	12	975.90
14	8398	94	737005.53	745780.40	9.73	10	915.10
15	9631	82	736987.56	744189.66	8.74	9	1019.30
16	11090	69	711984.74	695869.96	8.61	8	1035.17
17	12875	58	691344.23	703728.14	5.61	10	1588.64
18	15176	45	626880.55	633308.62	6.00	7	1484.60
19	18420	33	548469.54	532863.54	3.89	8	2288.52
20	23965	21	433920.47	421385.09	3.67	6	2425.58
21		25	155472.72	161574.88	1.00	25	422.23
TOTAL		2500	9999999.99	10019849.70		166	685.51

THE NUMBER OF STRATA HAS BEEN REDUCED TO 20
BY COMBINING WITH THE NEXT STRATUM ANY STRATUM
WHERE SAMPLE SIZE WOULD BE TOO SMALL FOR A PROPER
EVALUATION.

EXHIBIT 5

CONTROL TOTALS FOR SAMPLE SELECTED

AUDITAPE SYSTEM - COPYRIGHT 1967-1971

ESTIMATION SAMPLE DESIGN AND SELECTION ROUTINE

EXPONENTIAL 1

SUMMARY OF POPULATION AND SAMPLE SELECTED

		POPULATION		SAMPLE		
STRATUM NUMBER	UPPER CUT-OFF	ITEMS	BOOK VALUE	INTERVAL	ITEMS	BOOK VALUE
1	0	31	-28.63	10.33	3	0.0
2	842	456	185668.10	152.00	3	1422.78
3	1300	211	225059.37	70.33	3	3192.31
4	1785	200	307537.97	50.00	4	6152.36
5	2301	188	383237.56	37.60	5	10215.86
6	2853	176	452612.76	25.14	7	17938.61
7	3446	165	518668.93	20.63	8	25185.24
8	4086	152	571196.65	19.00	8	29982.32
9	4782	142	628281.60	14.20	10	44236.67
10	5545	129	664745.51	14.33	9	46549.52
11	6387	117	696154.02	11.70	10	59500.18
12	7330	106	724800.81	8.83	12	82327.59
13	8398	94	737005.53	10.44	9	70520.59
14	9631	82	736987.56	9.11	9	80958.97
15	11090	69	711984.74	8.63	8	82715.04
16	12875	58	691344.23	5.27	11	131113.14
17	15176	45	626880.55	6.43	7	97105.55
18	18420	33	548469.54	4.13	8	132429.22
19	23965	21	433920.47	3.50	6	125752.35
20		25	155472.72	1.00	25	155472.72
TOTAL		2500	9999999.99		165	1202771.02

EXHIBIT 6

EXCERPTED PAGE FROM SAMPLE PRINTOUT

LINE NO.	BOOK VALUE	PHYSICAL VALUE	STRATUM NUMBER	SEQUENCE NUMBER
1	0.00	0.00	1	5
2	0.00	0.00	1	15
3	0.00	0.00	1	25
4	211.46 CR	0.00	20	31
5	425.97 CR	0.00	20	32
6	251.48 CR	0.00	20	33
7	267.58 CR	0.00	20	34
8	213.88 CR	0.00	20	35
9	669.97 CR	0.00	20	36
10	535.46 CR	0.00	20	37
11	389.95 CR	0.00	20	38
12	813.03 CR	0.00	20	39
13	704.31 CR	0.00	20	40
14	643.38 CR	0.00	20	41
15	369.85 CR	0.00	20	42
16	463.32 CR	0.00	20	43
17	663.38 CR	0.00	20	44
18	709.64 CR	0.00	20	45
19	341.69 CR	0.00	20	46
20	139.14 CR	0.00	20	47
21	681.59 CR	0.00	20	48
22	439.87 CR	0.00	20	50
23	192.91	183.58	2	163
24	467.65	467.65	2	315
25	762.22	762.22	2	467
26	917.86	529.62	3	543
27	1,062.34	886.87	3	611

EXHIBIT 7

PRINTOUT OF EVALUATION SPECIFICATIONS

```
AUDITAPE SYSTEM  -  COPYRIGHT 1967-1971
ESTIMATION SAMPLE EVALUATION ROUTINE
EXPONENTIAL    1

SPECIFICATION CARD DATA
   INPUT SPECIFICATIONS
      INPUT MEDIUM                                             TAPE
      VARIABLE X
         DESCRIPTION                                     BOOK VALUE
         AUDITAPE FIELD                                           7
      VARIABLE Y
         DESCRIPTION                                    PHYS. VALUE
         AUDITAPE FIELD                                           9
      STRATUM NUMBER - AUDITAPE FIELD                             8
   TYPE OF ESTIMATE TO BE MADE                  COMBINED REGRESSION
   RELIABILITY LEVEL (PERCENTAGE)                             95.00
   TEST FOR UNUSUAL DIFFERENCES                                 YES
   TENTATIVE DESIGN DATA FOR NEXT SAMPLE
      MONETARY PRECISION                                     200000
      RELIABILITY LEVEL (PERCENTAGE)                          95.00
   STRATIFICATION SPECIFICATIONS
      NUMBER OF STRATA                                           20

NO ERRORS HAVE BEEN DETECTED IN SPECIFICATION CARDS.  PROCESSING IS CONTINUING.
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EXHIBIT 8

PRINTOUT OF ESTIMATE AND PRECISION AT SPECIFIED RELIABILITY

AUDITAPE SYSTEM - COPYRIGHT 1967-1971

ESTIMATION SAMPLE EVALUATION ROUTINE

EXPONENTIAL 1

SAMPLE EVALUATED AS A COMBINED REGRESSION ESTIMATE

STRATUM NUMBER	UPPER CUT-OFF	POPULATION ITEMS	POPULATION BOOK VALUE	SAMPLE INTERVAL	SAMPLE ITEMS	SAMPLE BOOK VALUE	SAMPLE PHYS. VALUE	ESTIMATED PHYS. VALUE
1	0	31	-29	10.33	3	0.0	0.0	-27.86
2	842	456	185668	152.00	3	1422.78	1413.45	185454.65
3	1300	211	225059	70.33	3	3192.31	2628.60	185390.40
4	1785	200	307538	50.00	4	6152.36	6874.12	343629.15
5	2301	188	383238	37.60	5	10215.86	10115.00	379480.25
6	2853	176	452613	25.14	7	17938.61	18246.38	460288.80
7	3446	165	518669	20.62	8	25185.24	25349.83	522094.25
8	4086	152	571197	19.00	8	29982.32	29982.32	571136.63
9	4782	142	628282	14.20	10	44236.67	46038.04	653856.68
10	5545	129	664746	14.33	9	46549.52	45837.40	654635.97
11	6387	117	696154	11.70	10	59500.18	56069.27	656012.28
12	7330	106	724801	8.83	12	82327.59	82106.46	722943.22
13	8398	94	737006	10.44	9	70520.59	71479.14	746999.50
14	9631	82	736988	9.11	9	80958.97	86187.99	784655.31
15	11090	69	711985	8.62	8	82715.04	83382.54	717798.59
16	12875	58	691344	5.27	11	131113.14	132124.43	696675.46
17	15176	45	626881	6.42	7	97105.55	95913.54	619114.47
18	18420	33	548470	4.12	8	132429.22	121398.87	502883.19
19	23965	21	433920	3.50	6	125752.35	122041.10	421175.30
20		25	155473	1.00	25	155472.72	161574.88	161574.88
TOTAL		2500	10000003		165	1202771.02	1198763.36	9985771.13

PRECISION AT 95.00 PERCENT RELIABILITY LEVEL

PRECISION RANGE - PLUS OR MINUS	240938.09
UPPER PRECISION LIMIT	10226709.22
LOWER PRECISION LIMIT	9744833.03

EXHIBIT 9

DESIGN INFORMATION FOR NEXT SAMPLE

AUDITAPE SYSTEM - COPYRIGHT 1967-1971

ESTIMATION SAMPLE EVALUATION ROUTINE

EXPONENTIAL 1

TENTATIVE DESIGN DATA FOR NEXT SAMPLE AS COMBINED REGRESSION ESTIMATE

STRATUM NUMBER	UPPER CUT-OFF	INDICATED SAMPLE INTERVAL	INDICATED SAMPLE ITEMS
1	0	10.33	3
2	842	218.06	2
3	1300	100.64	2
4	1785	24.74	8
5	2301	23.55	7
6	2853	18.33	9
7	3446	15.61	10
8	4086	14.43	10
9	4782	11.35	12
10	5545	11.28	11
11	6387	9.48	12
12	7330	7.50	14
13	8398	8.41	11
14	9631	7.44	11
15	11090	6.94	9
16	12875	4.83	12
17	15176	5.23	8
18	18420	3.86	8
19	23965	3.22	6
20		1.00	25
TOTAL			190

EXPECTED MONETARY PRECISION

AT RELIABILITY OF 95.00 PERCENT

WOULD BE $ 200000.