2011

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EXPERIMENTAL DAMAGE DIAGNOSIS
OF A MODEL THREE-STORY SPATIAL FRAME

A Thesis
presented in partial fulfillment of requirements
for the degree of Master of Science
in the Department of Civil Engineering
The University of Mississippi

by

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May 2011
ABSTRACT

Structural damage can be induced by a variety of events from short-term abnormal stresses to long-term natural aging. Detection of changes in a structure’s ability to withstand subsequent loads can aid decisions on safety, repair, rehabilitation, and demolition. Dynamic property shifts can show internal cracks and minor damage before propagation or failure occurs, but only if a proper indicator is selected.

In order to evaluate potential damage indices, a three-story metal frame building was constructed. Using Star Modal software, dynamic structural properties were obtained from modal decomposition on experimental tap test responses. The natural frequencies and mode shapes of the structure established an as-built baseline for comparison to ten other scenarios with removed bracing. Once modal properties for each case were determined, six unique damage indicators were applied to identical experimental data via twelve algorithms. The effectiveness of each damage detection technique was assessed, and final recommendations for the three-story model building were made.

Significant observations can be made about each mode shape based damage detection algorithm. Modal Assurance Criterion (MAC) is adequate for detecting incremental damage that affects the twisting motion of the structure. Although Coordinate Modal Assurance Criterion (COMAC) can predict general areas of damage, it has difficulty identifying exact damaged locations; however, it does effectively predict damage for the cumulative scenario. Modal curvature methods are not suitable for any case due to inaccuracy and occasional high frequency bias. Flexibility based methods are quite accurate for sequential damage but are not sensitive
enough to detect cumulative damage. A major shortcoming, the story stiffness method can predict which global floor but not local element is damaged. Of all the implemented algorithms, frequency response function (FRF) subtraction using the FRFs as a direct indicator is the most accurate damage detection scheme for the three-story test structure.
LIST OF ABBREVIATIONS AND SYMBOLS

**COMAC**  Coordinate Modal Assurance Criterion

**DC**  Damage case

**DLV** \(j\)  Damage location vector for the \(j^{th}\) DOF

**DOF**  Degree of freedom

**\(F_{ij}\)**  Flexibility matrix

**FRF**  Frequency response function

**FRF\(_d\)**  FRF division damage index

**FRF\(_s\)**  FRF subtraction damage index

**K\(_{ij}\)**  Stiffness matrix

**MAC**  Modal Assurance Criterion

**SSDI** \(_l\)  Story damage index of the \(l^{th}\) story

**\(\kappa_{ij}\)**  Curvature of the \(i^{th}\) mode shape at the \(j^{th}\) measurement coordinate

**\(\phi_i\)**  \(i^{th}\) mode shape vector

**\(\phi_{ij}\)**  \(i^{th}\) modal displacement at coordinate \(j\)

**\(\omega_i\)**  Natural frequency corresponding the \(i^{th}\) mode

*  Denotes properties of a damaged structure
ACKNOWLEDGEMENTS

I would first like to offer my utmost gratitude to my academic and research advisor, Dr. Elizabeth Ervin. With her encouragement, professional guidance, and support, I have successfully completed my Master's degree. I extend thanks to Drs. Chung Song and Cristiane Surbeck for their generous service as committee members; they contributed unique perspectives and ultimately improved the quality of this work. I am also indebted to my professors over the last two years. I would especially like to thank Drs. Ahmed Al-Ostaz, Christopher Mullen, James Chambers, and Xin Dang for their professional instruction in my coursework. Thanks to the University of Mississippi, the National Science Foundation, and the Mississippi State Space Grant Consortium for partially funding my research efforts.
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1. INTRODUCTION

1.1 Motivation

Structural health monitoring and evaluation is becoming an increasingly popular topic of research, especially within the last few decades. With applications to civil infrastructure such as bridges, buildings, dams, and tunnels, engineers need to assess the "health" state of structures and ultimately improve the overall safety and reliability of infrastructure. Damage can occur in a structure as various forms, including member cracks, loosened joints, metal fatigue, and temporal deterioration. Additionally, damage can be caused by an assortment of loadings, including but not limited to aging and abnormal, sudden stresses such as an earthquake or terrorist attack. One can compare an undamaged "healthy" structure to one that has been significantly altered via load (the "damaged" structure) in order to observe changes in material properties. Material properties (i.e. stiffness and density) as well as dynamic parameters (i.e. natural frequencies and mode shapes) directly affect how the structure moves and may significantly alter response data collected from a particular structure.

In order to detect damage within a structure, engineers instrument it with sensors in order to collect data that can be utilized to calculate its global dynamic parameters. Field data may be obtained from more conventional sensors, such as accelerometers, strain gauges, and displacement transducers, or even newer instrumentation involving piezoelectrics or optical measures. With any experimental data set, the preliminary step in structural health evaluation includes analyzing an undamaged as-built structure in order to establish the baseline structural dynamic properties. Once information is gathered and processed about the baseline case,
subsequent damage can theoretically be detected based solely on the collected data. If the structure is periodically monitored and data is collected on a regular basis, damage from both abnormal stresses and long-term aging effects can be detected. Once damage is identified and quantified within a structure, officials can decide to repair, rehabilitate, or demolish.

The main goal of structural health evaluation is to detect changes in the structure's dynamic properties that are produced by physical (usually not visible) damage. The aim is to detect cracks and minor damage before propagation or structural failure occurs. Ultimately, identifying and locating damage within a structure before failure happens could save both lives and money.

1.2 Literature

A large amount of literature has been published on the topics of structural health monitoring and evaluation. The first part of this section outlines and discusses well-documented damage indicators and methods. Note that these algorithms had not been compared and contrasted until the work herein. The second part explores applications of damage detection algorithms on real and laboratory-constructed structures. Both types of literature are imperative to review as they help researchers to better understand the intricacies of structural health evaluation.

1.2.1 Damage Indicators

The first reviewed group of damage indicators consists of those which incorporate mode shape differences. The experimental mode shapes for an undamaged and damaged structure are similarly compared, mathematically manipulated, and statistically correlated.
1.2.1.1 Modal Assurance Criterion (MAC)

The modal assurance criterion (MAC) is an analysis tool that compares the correlated mode pairs of the undamaged and damaged structures. It can detect a level of damage within a structure but cannot predict the location of the damage. The undamaged and damaged structure’s $i^{th}$ mode shape vector, $\phi_i$ and $\phi_i^*$, respectively, are correlated using

$$ MAC_i = \frac{|\phi_i \phi_i^*|^2}{(\phi_i \phi_i^T)(\phi_i^* \phi_i^*)} . $$

(Eqn. 1-1)

The value of $MAC_i$ ranges from 0 (no correlation) to 1 (full correlation) for every $i^{th}$ mode. If two sets of mode shapes are identical, then the value of MAC would be near 1 [20].

Several researchers examined and tested the effectiveness of MAC. Baghiee et al. [1] simulated progressive damage in reinforced concrete beams by incrementally applying a static load to several specimens. MAC values calculated from the data collected from these experiments verified that MAC values provide information about overall stiffness change due to damage but do not locate damage. Brasiliano et al. [2] numerically modeled a continuous concrete beam before and after two separate damage events and utilized MAC values to identify the health state of the beam. It was concluded from the results that MAC values are more sensitive to damage in higher modes than in lower modes. Heo et al. [12] applied MAC to measured mode shapes of a three-story frame structure and confirmed that joint stiffness reduction is detected by MAC values. Huth et al. [14] verified that MAC is not very sensitive to damage events that cause relatively small mode shape changes. This suggests that global damage is more easily recognized by MAC than local, small-scale damage. Similarly, Pandey et al. [20] found no significant changes in MAC values when numerically simulating reduction of modulus of elasticity in small sections along a cantilever and simply supported beam. These numerical simulations confirm that MAC is not sensitive enough to detect small, local damage.
1.2.1.2 Coordinate Modal Assurance Criterion (COMAC)

Similar to MAC, coordinate modal assurance criterion (COMAC) calculates mode shape correlation, but it incorporates multiple degrees of freedom (DOF) and can identify damage location. COMAC is a measure of the lack of correlation between two sets of mode shapes [20]. The undamaged and damaged structure’s $i^{th}$ modal displacement at coordinate $j$, $\phi_{ij}$ and $\phi^*_ij$, respectively, are the parameters used to calculate COMAC as

$$COMAC_j = \frac{\left(\sum_{i=1}^{m} |\phi_{ij} \cdot \phi^*_ij|\right)^2}{\sum_{i=1}^{m} \phi_{ij}^2 \cdot \sum_{i=1}^{m} \phi^*_{ij}^2}.$$  (Eqn. 1-2)

COMAC differs from MAC because it sums up all $m$ mode shapes at each measurement point, $j$. The value of $COMAC_j$ ranges from 0 (no correlation) to 1 (full correlation) for each $j^{th}$ DOF. If modal displacements at coordinate $j$ from two sets of mode shapes are identical, then the value of COMAC would be near 1 [12].

Baghiee et al. [1] accurately predicted damage location in reinforced concrete beams by analyzing COMAC using only the first three modes. Brasiliano et al. [2] calculated COMAC using difference mode shape combinations: COMAC was determined with modes 1 and 2 and then again with modes 3, 4, and 5. The COMAC value calculated with lower modes show a significant change at damaged DOFs; however, the COMAC value using higher modes do not detect damage locations. Lastly, Pandey et al. [20] demonstrated that COMAC is not sensitive enough to detect damage in its early stages because differences in dynamic response become averaged over all mode shapes.
1.2.1.3 Strain Energy

Although MAC and COMAC focus on the correlation between sets of mode shapes, numerous other methods are used to manipulate experimental mode shapes into a mathematical expression that predicts the presence of damage within a structure. For instance, Ndambi et al. [19] proposed a damage indicator that involves the values of strain energy within each member of a building. In order to determine strain energy, it is assumed that the structure is comprised entirely of Bernoulli-Euler beams. The strain energy of each beam is given by

\[ U_{ij} = \frac{1}{2} \int_0^{l_j} (EI) \left( \frac{\partial^2 \phi_i}{\partial x^2} \right)^2 dx \]  

(Eqn. 1-3)

where \( U_{ij} \) represents the strain energy associated with the \( i^{th} \) mode of the \( j^{th} \) beam, \( EI \) is the flexural rigidity of member \( j \), \( l_j \) is the length of member \( j \), and \( \phi_i \) is the \( i^{th} \) mode shape. In order to calculate the strain energy of the entire structure, the strain energy of each beam is summed over all \( n \) beams such that

\[ U_i = \frac{1}{2} \sum_{j=1}^{n} \int_0^{l_j} EI \left( \frac{\partial^2 \phi_i}{\partial x^2} \right)^2 dx \]  

(Eqn. 1-4)

where \( U_i \) is denoted as the strain energy of the entire structure associated with the \( i^{th} \) mode.

Next, the fractional strain energy, \( FSE_{ij} \), is defined as

\[ FSE_{ij} = \frac{U_{ij}}{U_i}, \text{ where } \sum_{j=1}^{n} FSE_{ij} = 1 \]  

(Eqn. 1-5)

The fractional strain energy can be calculated for both the undamaged and damaged structure and then compared in order to detect damage. A suitable damage indicator, \( \gamma_j \), is the ratio of fractional energies summed over all \( m \) modes of the undamaged structure to the fractional energies summed over all \( m \) modes of the damaged structures.
That is,

\[ \gamma_j = \frac{\sum_{i=1}^{m} FSE_{ij}^*}{\sum_{i=1}^{m} FSE_{ij}}. \]  
(Eqn. 1-6)

The value of the damage index, \( \gamma_j \), evaluates the health state of the damaged structure at member \( j \), thus identifying location. In order to determine the severity of damage, all damage indices are normalized. Outliers not within the bounds of \(-2 \leq Z_{\gamma_j} \leq 2\) indicate possible damage locations with a 95% confidence level. Note that

\[ Z_{\gamma_j} = \frac{\gamma_j - \mu_{\gamma_j}}{\sigma_{\gamma_j}} \]  
(Eqn. 1-7)

describes the normalized damage index value while \( \mu_{\gamma_j} \) and \( \sigma_{\gamma_j} \) represent the mean and standard deviation of the \( \gamma_j \) distribution [20]. Gutschmidt et al. [11] confirmed the reliability of the strain energy method utilizing the first six modes of a rectangular metal plate. The strain energy damage index identifies the general area of damage but cannot determine exact crack location.

1.2.1.4 Mode Shape Curvatures

In addition to analyzing strain energies, mode shape curvatures can be examined and mathematically manipulated to produce more robust damage indicators. Sun et al. [30] examined mode shape curvature as an effective tool in predicting damage location within a structure. Modal curvature is defined as the second derivative of the modal displacement function and is generally quite sensitive to damage. These curvatures may be described as bending strains of bending mode shapes. The occurrence of damage reduces stiffness and ultimately increases the curvature. The change in mode shape curvature between two damage states of a structure can effectively estimate the level of damage present.
The curvature of the \(i^{th}\) mode shape at the \(j^{th}\) measurement coordinate of an undamaged structure, \(\kappa_{ij}\), is approximated by Pandey et al. [20] as

\[
\kappa_{ij} = \frac{\phi_i(j+1) - 2\phi_{ij} + \phi_i(j-1)}{h^2}
\]  
(Eqn. 1-8)

where \(h\) is the distance between the coordinates of the measurements and \(\phi_{ij}\) is the modal displacement of coordinate \(j\) at mode \(i\). Eqn. 1-8 can be similarly applied to the damaged structure in order to determine the mode shape curvature, \(\kappa_{ij}^*\), of the \(i^{th}\) mode shape at the \(j^{th}\) measurement coordinate of a damaged structure. The mode shapes used in this term are those obtained through experimental measurement of the damaged state.

A clear extension is one that incorporates the mode shape curvatures into a damage index that determines which DOFs possess the largest differences between the undamaged and damaged curvatures. This damage index is described mathematically as

\[
\Delta \kappa_{ij} = |\kappa_{ij}^* - \kappa_{ij}|
\]  
(Eqn. 1-9)

Entries in the \(\Delta \kappa_{ij}\) matrix with relatively high values may indicate locations where damage is present [20]. Another way to form a damage index is to calculate the ratio of damaged state to undamaged state curvatures. The following expression establishes a damage index that sums curvatures over all \(m\) modes \(\kappa_{ij}^*\)

\[
\Delta \kappa_j = \frac{\sum_{i=1}^{m} \kappa_{ij}^*}{\sum_{i=1}^{m} \kappa_{ij}}
\]  
(Eqn. 1-10)

An alternative method of manipulating mode shape curvatures was introduced in Baghiee et al. [1]. The COMAC of the curvatures,

\[
COMAC\ of\ \kappa(j) = \frac{|\sum_{j=1}^{m} \kappa_{ij}^*|^2}{\sum_{j=1}^{m} \kappa_{ij}^2 \cdot \sum_{j=1}^{m} \kappa_{ij}^*^2}
\]  
(Eqn. 1-11)
is used as a damage index for determining the level and location of damage within a structure. Multiple researchers focus on the use of modal curvatures as a means to predict damage severity and location within different types of structures. For example, Brasiliano et al. [2] showed that differences in mode shape curvatures correctly locate damage in two locations along a numerically modeled concrete beam. Additionally, Sun et al. [30] used the modal curvatures calculated from simulated data for a real-life cable-stayed bridge and proved that this method is useful in detecting damage close to pier and tower supports.

1.2.1.5 Mode Shape Area

Changes in mode shapes can be quantified in numerous ways, as the aforementioned methods using displacements, strain energy, and curvatures. The final reviewed mode shape based damage index involves calculating the area underneath the mode shape curves. Huth et al. [14] established the mode shape area method for damage indication. First, the structure is split into $Q$ parts in order to easily integrate the entire mode shape, $\phi_i$. Next, the area underneath the mode shape is calculated for each $q^{th}$ section. Then, each piece of the integrated mode shape is normalized with respect to the entire area underneath the mode shape curve for the healthy structure. This normalized value is denoted as $\hat{A}_q(\phi_i)$ for each $q^{th}$ piece and $i^{th}$ mode shape. The damage index formulated using this approach, $\hat{A}(\phi_i)$, sums the normalized mode shape areas over all $Q$ pieces as

$$\hat{A}(\phi_i) = \sum_{q=1}^{Q} \hat{A}_q(\phi_i) = \sum_{q=1}^{Q} \frac{\int_0^{l_q} |\phi_i(x)| \, dx}{\int_0^{L} |\phi_i(x)| \, dx}.$$  \hspace{1cm} \text{(Eqn. 1-12)}$$

In the preceding equation, $l_q$ represents the length of the $q^{th}$ piece and $L$ is the total length of the mode shape in the $x$ coordinate. Note that $\hat{A}(\phi_i)$ is calculated separately for each mode. If
\( \hat{A}(\phi_i) \) is unity, the structure is undamaged; however, if \( \hat{A}(\phi_i) \) significantly deviates from 1, the structure is possibly damaged. This method is utilized solely to determine if damage is present within a structure and does not attempt to quantify damage severity or location [14].

The second reviewed group of damage indicators employs a flexibility or stiffness change before and after damage. These damage indicators are constructed on the assumption that structural damage does not impose an appreciable change in mass or damping such that damage only causes fluctuations in the structural stiffness [15].

1.2.1.6 Modal Flexibility Index

The first flexibility-based damage prediction method is based upon direct flexibility changes. For initiation, the flexibility matrix must be constructed based upon experimental data. Huth et al. [14] proposed a way to calculate the flexibility matrix as

\[ F_{ij} = \sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_i \phi_i^T \]  

(Eqn. 1-13)

where \( \omega_i \) is the natural frequency corresponding the \( i^{th} \) mode and \( \phi_i \) is mode shape vector for the \( i^{th} \) mode. The flexibility matrix estimated by Eqn. 1-13 is calculated by summing over all \( m \) modes. Changes in flexibility can be demonstrated by examining the absolute difference between the undamaged structure's flexibility matrix, \( F_{ij} \), and the damaged structure's flexibility matrix, \( F_{ij}^* \), as

\[ \Delta F_{ij} = |F_{ij}^* - F_{ij}| \]  

(Eqn. 1-14)

The columns of the \( \Delta F \) matrix in Eqn. 1-14 represent the difference in displacements of the structure induced by a unit force acting at each \( j^{th} \) degree of freedom. The columns with the largest absolute changes in flexibility coincide with locations of damage.
Although this damage indicator can predict the location of structural weakness, it does not accurately detect severity of damage [14].

Numerous other works examine similar methods with differing results. For example, Gutschmidt et al. [11] utilized changes in the flexibility matrix in order to predict the presence of damage on a rectangular metal plate. Although the algorithm identified overall damage, it did not successfully locate damage. On the other hand, Sun et al. [30] showed that flexibility changes can accurately predict damage location on a cable-stayed bridge.

A more robust measure of damage is the Modal Flexibility Index. The Modal Flexibility Index, $MFI_j$, uses the diagonal terms of the undamaged structure's flexibility matrix, $F_{jj}$, and damaged structure's flexibility matrix, $F_{jj}^*$, and is defined by

$$MFI_j = \frac{|F_{jj}^* - F_{jj}|}{F_{jj}}.$$  \hspace{1cm} (Eqn. 1-15)

The Modal Flexibility Index will vary based on which $j^{th}$ DOF is examined. Values of $MFI_j$ that deviate from unity indicate the possibility of damage at the $j^{th}$ location. To indicate damage even more clearly, MFI values can be normalized to form Z-values that can be beneficial in determining location of damage within a structure.

$$Z_{MFI_j} = \frac{MFI_j - \mu_{MFI_j}}{\sigma_{MFI_j}}$$ \hspace{1cm} (Eqn. 1-16)

where $Z_{MFI_j}$ describes the normalized damage index value while $\mu_{MFI_j}$ and $\sigma_{MFI_j}$ represent the mean and standard deviation of the $MFI_j$ distribution. Outliers not within the bounds of $-2 \leq Z_{MFI_j} \leq 2$ indicate possible damage locations with a 95% confidence level [13].
1.2.1.7 Damage Localization Algorithm

Another flexibility/stiffness damage detection technique to consider is the Damage Localization Algorithm. This method is based upon changes in stiffness; however, unlike previously discussed stiffness damage detectors, this one assumes that the actual stiffness matrix is known. In experimental work, determining the exact stiffness matrix may prove difficult. Nevertheless, the Damage Localization Algorithm is quite innovative and is included in this review.

The damage index established by Park et al. [10], $D_{lij}$, is mathematically defined as

\[
D_{lij} = \frac{s_j}{s_j^*} = \left( \frac{\phi_i^* K_{ij} \phi_i^* + \phi_i^* K \phi_i^*}{\phi_i^* K_{ij0} \phi_i^*} \right) \frac{\phi_i^* K \phi_i^*}{\phi_i^* K_{ij0} \phi_i^*}.
\] (Eqn. 1-17)

$D_{lij}$ describes the damage indicator for the $i^{th}$ mode in the $j^{th}$ member of the structure, $s_j$ and $s_j^*$ represent stiffness estimations of the $j^{th}$ member of the undamaged and damaged structure, respectively, and $\phi_i$ and $\phi_i^*$ are the $i^{th}$ mode shape vectors of the undamaged and damaged structure, respectively. $K$ is the undamaged system stiffness matrix and $K_{ij0}$ represents the portion of the undamaged system stiffness matrix that contains only geometric quantities. The damage indicator in Eqn. 1-17 can be summed over all $m$ modes to produce a measure of damage in each $j$ member of the structure. That is,

\[
D_{lj} = \frac{\sum_{i=1}^{m} \left( \phi_i^* K_{ij0} \phi_i^* + \phi_i^* K \phi_i^* \right) \phi_i^* K \phi_i^*}{\sum_{i=1}^{m} \left( \phi_i^* K_{ij0} \phi_i^* + \phi_i^* K \phi_i^* \right) \phi_i^* K_{ij0} \phi_i^*}.
\] (Eqn. 1-18)

If $D_{lj} \leq 1$, member $j$ is undamaged. Conversely, if $D_{lj} > 1$, there may be some damage in member $j$. 

In order to truly determine if a member is damaged, the damage index values are
normalized to fit the Standard Normal curve. The normalized damage index for the \( j^{th} \) location
is given by
\[
Z_{DIj} = \frac{DI_j - \mu_{DIj}}{\sigma_{DIj}}.
\]  
(Eqn. 1-19)

\( Z_{DIj} \) describes the normalized damage index value while \( \mu_{DIj} \) and \( \sigma_{DIj} \) represent the mean and
standard deviation of the \( MFI_j \) distribution. Outliers not within the bounds of \(-2 \leq Z_{DIj} \leq 2\)
indicate possible damage locations with a 95\% confidence level.

Additionally, the relative magnitude of damage at a given location \( j, \alpha_j \), can be expressed
as the fractional change in stiffness of an element.
\[
\alpha_j = \frac{s_j^* - s_j}{s_j} = \frac{1}{DI_j} - 1
\]  
(Eqn. 1-20)

If \( \alpha_j = 0 \), the \( j^{th} \) element of the structure is undamaged; on the other hand, if \( \alpha_j < 0 \), the \( j^{th} \)
member is damaged. Also, if \( \alpha_j = -1 \), then stiffness capacity or residual strength is completely
lost in member \( j \) [10].

1.2.1.8 Story Stiffness

The next stiffness damage indicators examined are intended for use on multi-story
buildings. Each of the following damage indices takes into the account that local damage to a
real structure causes a reduction in story stiffness. Wang et al. [32] described a damage
detection technique that can quantify damage in every story of a building. The damage index of
the \( l^{th} \) story, \( SDI_l \), is expressed as
where \( \ast \) denotes parameters from the damaged state. \( k_l \) represents the story stiffness, \( m_l \) is the mass of the \( l^{th} \) story, \( \phi_{lj} \) is the value of the \( l^{th} \) mode shape at the \( j^{th} \) story, \( \omega_l \) describes the natural frequency corresponding to the \( l^{th} \) mode, and \( L \) is the total number of stories. The change in mode shape, \( \Delta \phi_{jl}^\ast \), is defined as

\[
\Delta \phi_{jl} = \begin{cases} 
\phi_{jl} - \phi_{j(l-1)}, & \text{for } l = 2, 3, \ldots, L \\
\phi_{jl}, & \text{for } l = 1
\end{cases}
\]  
(Eqn. 1-22)

The values of \( SDI_l \) range from 0 (no damage) and 1 (collapse) for each \( l^{th} \) story. For most buildings, the mass of each floor is uniform throughout the structure. Taking this into account, the approximate value of \( SDI_l \), denoted as \( ASDI_l \), becomes

\[
ASDI_l = 1 - \frac{\omega_l^2 \sum_{j=1}^{l} m_j \phi_{ij}^*}{\omega_l^2 \sum_{j=1}^{L} m_j \Delta \phi_{jl}^*}.
\]  
(Eqn. 1-23)

Alternatively to stiffness, the flexibility of each story can be calculated and utilized as a damage indicator. The diagonal terms of the modal flexibility matrix are expressed as

\[
F_l = \sum_{i=1}^{m} \frac{\phi_{il}^2}{\omega_i^2}.
\]  
(Eqn. 1-24)

where \( F_l \) is the static displacement due to a unit static load applied at the \( l^{th} \) story. In order to compare damage states, the modal flexibility damage index, \( MFDI_l \), is calculated for each story.
The $MFDI$ of the $l^{th}$ story is defined as

\[
MFDI_l = 1 - \frac{F_l^*}{F_l} = 1 - \frac{\sum_{j=1}^{L} \frac{\phi_{lj}^2}{\omega_l^2}}{\sum_{j=1}^{L} \frac{\phi_{lj}^2}{\omega_l^2}}.
\]  
(Eqn. 1-25)

A $MFDI_l$ value of 0 indicates no damage while a value of 1 would denote collapse of the $l^{th}$ story of the structure [15].

1.2.1.9 Damage Location Vector

Huynh et al. in 2005 [15] introduced the idea of the Damage Location Vector ($DLV$).  This unique damage indicator incorporates fluctuations in the frequency response functions (FRF) measured at each DOF.  The formula for the $DLV$ at any $j$ DOF is given by Eqn. 1-26.

\[
DLV_j = K \sum_{s=1}^{S} (FRF_{sj}^* - FRF_{sj})
\]  
(Eqn. 1-26)

The $K$ matrix represents the undamaged structure's stiffness matrix and can be obtained by either experiment and Eqn. 1-13 or finite element modeling.  $FRF_{sj}$ and $FRF_{sj}^*$ are the values of frequency response functions corresponding to the $s^{th}$ frequency for the undamaged and damaged structure, respectively.  The summation term symbolizes the difference between FRFs for any $j$ degree of freedom totaled over all frequencies.  A non-zero value of the $DLV_j$ denotes that damage is likely to have occurred in the $j^{th}$ DOF.  Conversely, a zero value of the $DLV_j$ indicates that there is no damage at the $j^{th}$ DOF.

The cumulative damage location plot (CDLP) is a tool used to display $DLV$ values in a convenient, useful manner.  The values on the independent axis of the plot are the numbered DOFs that comprise the structure and the dependent axis denote the absolute value of each $DLV_j$. 
The DLV usually works well in identifying single damage locations within a structure; however, it is not as reliable when attempting to detect multiple damage locations [15].

1.2.2 Analyzed Structures

Related literature also includes studies of similar structures, both in-situ and simulated. Excitation means vary from tap testing to ambient output, and diverse sensing methods were employed. For this work, a survey of built, laboratory, and numerical studies was performed and resulted in the following relevant analyses.

Numerical dynamic studies of damaged structures have been undertaken by many researchers. A two-span continuous beam is numerically simulated in [17] via fifty-one nodes. Ten damage cases were implemented by reducing the elastic modulus of various elements within the structure. The first three modes were identified and employed in three different modal stiffness-based damage algorithms. Overall, there was virtually no error in identifying damage location, but small errors were observed when calculating damage severity. Stubbs et al. estimate failure probabilities using a unique damage detection algorithm on an ABAQUS frame model [29]. Damage within a two-dimensional, 23-element truss structure was simulated and numerically analyzed in [35]. Two cases were considered: single element damage with a 30% stiffness loss and two damaged elements with an additional 20% stiffness reduction. The damage was detected by the damage localization vector, which relies on modal force error and the stiffness connectivity matrix. This damage detection technique was able to accurately locate and quantify stiffness loss even with 5% random noise introduced. Note that it also detected both single and multiple damage locations.
Finite element models have also been refined by modal updating using experimental data. Hu et al. examined the dynamics of an ultralight aircraft [13]. Finite element updating was employed using experimental modal analysis results. Experimental mode shapes and natural frequencies were obtained via MEscope; they used a multiple input, single output system where impacts were performed at several hundred points while response was measured at one fixed location. MAC values were calculated and used to refine the finite element model. Eleven experimental modes were determined via experimental modal analysis and verified with the finite element model. Pothisiri et al. developed a damage index that requires parameter estimation method using finite element modeling and modal response of a structure [24]. They estimate parameters by partitioning the mass matrix and minimizing the output error. They also propose a new element-group updating scheme utilized to locate damage in a systematic manner using spatially sparse data collected from a baseline and damaged structure. A two-span continuous bridge truss was numerically simulated incorporating thirty-five members. Damage is induced by reducing the axial stiffness of members within the truss by 75%. In this analysis, two members’ stiffnesses were reduced in one damage case while three members’ stiffnesses were reduced in another case. Noise was also introduced to the system and proved that the accuracy of damage detection with the objective function decreased with increased noise.

Farrar et al. establishes the four main stages of experimental structural health monitoring: 1) operational evaluation, 2) data acquisition and cleansing, 3) feature selection, and 4) statistical model development [8]. They used concrete columns to test the effectiveness of their damage detection methods and concluded that linear discriminant operators can identify the presence of damage. In three different ways, Curadelli et al. presented a linear scheme to detect structural damage via the instantaneous damping coefficient identification using a wavelet transform [7].
Both laboratory tests and numerical simulations were conducted in order to verify the success and accuracy of the method. First, a numerical simulation of the response of a two-dimensional, six-story, three-bay reinforced concrete frame was performed. The model was subjected to the 1997 Caucete, San Juan, Argentina, seismic acceleration time history, and damage was observed to increase as peak ground acceleration increased as expected. Next, the authors conducted an experiment on a 0.20 m by 0.10 m by 5.50 m reinforced concrete beam in flexure under two-point loading. Four piezoelectric accelerometers were used to measure the vertical acceleration at points on the structure. Natural frequencies shifted downward with increasing damage as expected. The authors performed a final experiment on an aluminum six-story single bay frame model. Damage was caused by subjecting the model to a horizontal unidirectional base motion on a shaking table. Again, as expected, their results show that increased shaking intensity also increases the amount of damage.

Fasel et al. built a laboratory test structure that is quite similar to the test structure examined in this work [10]. In order to simulate the dynamic response of a three-story frame structure, a scale model was constructed in the lab using Unistrut as columns and aluminum plates as floors. The entire structure measured 24” by 30” at the base and 61.125" in height. Two accelerometers were fastened to each joint, one on the column and one on the floor, in order to capture the relative motion of the joint. In order to induce variability, mass was added to the structure, the level of the shaker input was varied, and a small handheld shaker was employed. Utilizing a shaker table excitation input and experimental modal analysis, sixteen total global modes of the structure were successfully identified. The relative motion of each joint was fit to an autoregressive model, the residual errors between predicted actual time histories were analyzed, and finally, hypothesis testing was carried out in order to detect the presence of
damage. The authors proved that high excitation levels produced the most accurate damage indication results while low excitation levels significantly reduced the accuracy of the method.

Farrar et al. [9] subjected the same building model of [10] to damage by loosening or removing bolts in its foundation. A force transducer was mounted between the stinger and the base plate measuring the input excitation of the base of the structure. Nine total separate and unique damage cases are presented. Residual errors were the damage sensitive features developed for these studies. Statistical methods were applied to the residual errors to quantify when changes in this feature were significant. If the set of calculated damage indices for one joint contained 10% outliers, that specific joint was considered to be in control and undamaged. However, data sets with 10% to 80% outliers indicated a change in the operational conditions had taken place, but damage was not present. Finally, any damage indicator sets that contained over 80% outliers denoted that damage was present within the specific joint.

As Pavic et al. [23] point out, the massive size of civil structures makes artificial excitation very difficult. Additionally, the measurement of typically small responses requires ultra-sensitive transducers, especially when ambient vibrations are used to excite the structure. When conducting experimental modal analysis on buildings or bridges, closely spaced modes of vibration are difficult to identify from response only measurements. Due to closely spaced and highly coupled modes, a sledgehammer excitation is often used to induce an impulse load on a structure. Ideally, a structural health evaluation team has access to portable equipment and is trained to conduct relatively quick testing. Typically, a portable spectrum analyzer and curve fitting software are available on-site in order to calculate natural frequencies and mode shapes. The authors in [23] addressed the relationship between frequency resolution and sampling rate and determined that 0.125 Hz was a sound frequency resolution. A small frequency resolution
proved more important than a high sampling rate. They also made some conclusions on impact
techniques: too strong of a hammer blow can cause poor resolution and instable FRFs while too
gentle may not excite the structure enough for the transducers to measure the vibration signal.

Rezaei et al. [25] investigated damage detection through a set of experiments conducted
on a standard steel pipe commonly used in the oil and gas industry. This study examined the
effects of support flexibility on the damage index and its sensitivity. The damage indicator is
based upon the energy of the first intrinsic mode function obtained from experimentation. An
impulse hammer is used to excite the pipe (size 6 5/8 grade A), and the free vibration of the
undamaged pipe was recorded by five piezoceramic sensors. Damage was then induced by
manually grinding the outer surface of the pipe, simulating partial corrosion. Additionally, bolt
torques at the support were varied in order to measure sensitivity. It was found that the natural
frequencies were not sensitive to support bolt torques over a wide range. A discussion on impact
hammer tips was also presented, and the authors conclude that aluminum tips are especially
useful when exciting high frequencies. However, structural applications tend to have low natural
frequencies, and a plastic tip may produce the most consistent results.

Briand et al. [3] stressed the importance of early damage detection so that it will not
endanger lives. A novel damage indicator, the energy damage index was utilized on a
mechanical pipe joint on a condensation line. Self-loosening of bolts in conjunction with fatigue
failure is the most frequent cause of failure of dynamically loaded bolted joints. Eight
piezoceramic sensors measured joint response from an impulse hammer excitation at two
separate locations. Dynamic tests were conducted in order to assess the significance of impact
location, sensor location, frequency bandwidth, intrinsic mode function, and boundary condition.
Seven separate damage cases were implemented, each incorporating bolt loosening. A torque wrench was purposely not used, in order to follow actual real-world practice.

Numerous large-scale frequency studies have been performed with differing outcomes, even when accompanied by scaled laboratory verification. Yoshimoto et al. [36] first studied a four-story laboratory model with a pulse input applied to the base, and twenty seconds of response acceleration at each floor was measured and analyzed. The authors then applied their methods to an existing school building at Keio University in Japan. The seismically-isolated structure was comprised of seven stories, measuring a total of 31 m in height. Sixteen accelerometers and three displacement meters measured the structure's response to a simulated earthquake with the bottom of the structure excited in both the X and Y direction. The authors were only able to successfully identify the first two modes of the structure: the first bending modes in each sway direction. Cioara et al. [6] conducted experimental modal analysis on the Route 20 Bridge in Chittenango Creek, New York. Accelerations were measured with accelerometers while a portable modally tuned impact hammer designed by the authors was used to excite the structure. They proved that impulse loading is superior excitation versus ambient vibration from traffic; the higher order modes of the structure are extremely difficult to excite using ambient vibrations alone. Additionally, the impact hammer design allows for the exact same amount of force to be applied every single impulse excitation. Only the first two modes of vibration were successfully determined with frequencies below 5 Hz.

Damage detection is further complicated in real structures. Catbas et al. [4] closely studied the Seymour Bridge, a three-span, 130-foot long reinforced concrete deck-on-steel-stringer bridge that was constructed in 1953. Several damage scenarios were applied, and modal data was collected via multiple-input multiple-output testing techniques. A total of 32
accelerometers served as system outputs while 12 impulse impacts were considered be the inputs to the system. The impact tests were performed with a drop hammer that could generate a peak force of 1,300 lbf. 384 FRFs are obtained from the test within a 50 Hz frequency band. Damage was induced by removing one bearing support, which simulates the case of a floating bearing that has lost contact. Experimental modal decomposition identified fourteen modes for the structure both before and after damage. The complex mode indicator functions as well as enhanced frequency response functions were used in this analysis. Frequency shifts as well as MAC values were calculated and compared in order to analyze damage effects.

The modal flexibility-based deflection and curvature were the damage indicators utilized within the work of [5]. The indices are calculated directly using dynamic properties, and multiple-input, multiple-output systems accurately estimate the flexibility matrix of a structure. A steel grid was constructed in the laboratory that has structural response characteristics similar to those of a short to medium span bridge. While scour and settlement were simulated by removing a support, boundary damage was implemented by inserting bolts through the top and bottom support blocks, effectively restraining rotation and transition. Impact tests were performed at multiple locations along the structure and their response signals were measured by twelve strategically placed accelerometers. This work also examined the same Seymour Bridge, a three-span steel stringer highway bridge in Cincinnati, Ohio. Damage was induced by removing bearings and cutting steel elements. The first ten modes were determined utilizing experimental modal analysis of the response data obtained from thirty-two accelerometers. The modal curvature index proved accurate, but the authors concluded that dense spatial resolution is necessary.
Rodriguez et al.[26] developed a method to detect damage in a structure using only dynamic information from a post-damage event. The baseline stiffness method (BSM) was tested on a scale model of a four-story building. Damage was introduced to the model by removing bracing elements, and impact tests were performed in order to extract necessary modal information. Only the first two mode shapes were successfully identified. BSM worked well in identifying locations of removed braces, except when brace removal occurred within the first story. After the model analysis, experimental modal analysis was conducted on the Van Nuys building using Northridge earthquake data. The building consists of reinforced concrete and has a total of seven stories, and the first two modes were identified at 1 and 1.4 Hz, respectively. BSM correctly located damaged elements with evidence of cracking.

Kim et al. [16] developed a curvature-based damage detection method for both location and severity of damage. They stressed the importance of using limited modal data, such as few lower frequency mode shapes, limited degrees of freedom, and spatially incomplete measurements. The damage indicator utilized was based upon modal flexure and uses noise-contaminated, output-only measured signals. The modal parameters were extracted solely using time-domain response data. The flexural damage index was employed on the Z24 Bridge on Swiss National Highway; this concrete box-girder bridge consists of three spans and is post-tensioned. Fifteen progressive damage cases that simulated pier settlement were implemented. For every case, the ambient accelerations of the bridge were recorded at 151 separate positions for nearly eleven minutes. After collecting response data, the first six modes of vibration were successfully identified. MAC values were also calculated, but the flexural damage index was most successful in determining damage location and extent.
Wang et al. [33] used principal component analysis, which is based upon a multivariate exponentially weighted formulation and ultimately calculates principal component coefficients (PCCs). An advantage, this damage detection method can be applied to either time domain or frequency domain responses. In order to assess the effectiveness of the damage indicator, a numerical analysis of a five-story shear frame was conducted. The frame was modeled as a five degree of freedom lumped mass system with a story mass of 250 kg and story stiffness of 10 MN/m. The following damaged cases were considered: 1) 20% reduction of stiffness in the fourth story, 2) 10% reduction in stiffness of the second and fourth stories, and 3) 5% reduction in story stiffness of first three stories. Utilizing the time and frequency domain data, damage is located and severity is successfully determined. The authors used another numerical example in order to test the damage indicator. A numerical example of a shear wall, 2 m wide by 6 m high by 250 mm thick, was modeled, and the dynamic response simulated via finite element theory. Three damage cases were presented for this specific example: Young's modulus was reduced by 30%, 20%, and 10% in the lower 1/6, 1/4 and 1/3 parts of the structure, respectively. Damage severity was accurately assessed between all damage cases. The final experimental study was the analysis of an I-40 Bridge; the bridge was instrumented with twenty-six accelerometers and four damaged states were induced to the steel plate girder section. A 2-foot web cut, 6-foot web cut, a 50% flange cut, and a 100% flange cut were made progressively to introduce damage. Forced vibration testing was conducted with the excitation generated by a hydraulic shaker located towards one end of the bridge. Overall, the damage indicator could detect damage but could not accurately assess severity within the I-40 Bridge.

The structural integrity of a concrete box-girder bridge was examined twice by [22]. Experimental field data were collected nine months apart on a two-span I-40 bridge. The modal
parameters were calculated from the measured FRFs, and the Damage Index Method was used to
detect damage within the bridge superstructure. Visual inspection was performed for
comparison, and surface cracks on the deck were recorded during both testing periods. A single-
input, multiple-output system was utilized in the field: an impact hammer was used at one
location to excite the bridge while acceleration was measured at thirty separate locations on the
bridge deck and four points along one column. They were able to identify the five lowest natural
frequencies, which were verified with a finite element model. A strong correlation exists
between the predicted damage locations and the observed damage locations in the bridge deck.

1.3 Objectives

In order to evaluate the effectiveness of potential damage indicators, a three-story spatial
frame structure made of steel and aluminum was constructed. Dynamic structural parameters
were obtained from the measured response of the test building. An accelerometer was placed at
the top of one of the columns comprising the structure in order to capture the building's response
to a roving hammer’s impulse signals. Twenty-six hammer hits were performed and the
responses were measured in two dimensions, resulting in a total of fifty-two measured frequency
response function (FRF) signals. Next, modal decomposition was utilized to obtain the natural
frequencies and mode shapes of the structure. Once all required data were acquired for the
structure, baseline structural dynamic properties of the test building were calculated.

After a baseline, “healthy,” or undamaged state of the structure was established, damage
was incrementally applied to the building by removing various members. A total of ten damage
scenarios with significant structural modifications were examined. Various types of damage
were simulated, including symmetric, asymmetric, single story, and multiple story damage. For
each damage case, impact tests were carried out, the resulting FRFs were used in modal
decomposition, and the dynamic parameters were calculated. Once all properties for each
damage case were determined, various damage indicators were applied. Finally, after the
application of several damage detection methods, the effectiveness of each technique was
evaluated and compared. Final recommendations for the three-story spatial frame were then
made based on the results of all damage indicators.

In short, specific objectives include the following:

- Experimentally evaluate the test structure’s baseline state as well as ten damaged cases
- Apply twelve damage detection algorithms to identical experimental data captured from
  the laboratory test structure
- Assess the effectiveness of six common damage indicators for the test structure
- Make general recommendations for damage detection on the test structure

1.4 Organization

This thesis is divided into four separate chapters. The first chapter serves as introduction
to structural health monitoring and evaluation. The second chapter outlines the data collection
and processing procedure and contains descriptions of each damage case. Once the necessary
structural dynamic parameters were established, natural frequencies and mode shapes were
utilized to calculate damage indicators. Chapter 3 discusses and compares the results of six
unique damage indices and twelve detection algorithms. Lastly, the final chapter includes
general conclusions on damage detection and makes specific recommendations for the three-
story spatial frame structure.
2. MODAL PARAMETERS

This chapter discusses the laboratory health evaluation of a model three-story building. Impact tests were performed on the baseline structure as well as ten different damage cases in order to collect the acceleration responses. The frequency responses of each impact case were then calculated and utilized to perform modal decomposition. The results of modal decomposition are natural frequencies and mode shapes for each damage case. The modal parameters obtained for all damage cases will be examined by assessing general trend behavior.

2.1 Test Structure

In order to evaluate the effectiveness of potential damage indicators, a three-story spatial frame, single bay structure was constructed from steel and aluminum (Figure 2-1).

![Figure 2-1: Three-story spatial frame structure (a) isometric view (b) side view.](image)
The columns are comprised of four continuous 1/8-inch aluminum angles with 1 inch flanges, measuring 24 inches in height. Figure 2-2 depicts the plan view of the structure; the columns are arranged approximately in a 4” by 4” square. The foundation of the structure consists of two stainless steel 5” flanges bolted to each column and fastened to the shaker slip table. Three 0.02” thick steel shim sheet squares measuring approximately 4” by 4” are fastened to the columns 6”, 14”, and 22” from the base. The shim sheets represent three floors and are bolted in each corner to each column with two small 0.5” flanges. Figure 2-3 depicts a zoomed-in view of a floor-column connection. Cross bracing is externally added within each story along all four faces. All 16 cross bracing members are made using 0.015” thick, 0.5” wide steel strips, each measuring approximately 12 inches in length. A wireframe model of the test structure with numeric labels assigned to each node is depicted in Figure 2-4. A list of materials used to construct the structure is contained in Table 2-1.

![Plan view of foundation.](image)

Figure 2-2: Plan view of foundation.
Figure 2-3: Typical column to floor corner connection.

Figure 2-4: Wireframe model.
Table 2-1: Materials list.

<table>
<thead>
<tr>
<th>Part of Structure</th>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Columns</td>
<td>• 4 aluminum 1&quot; wide, 1/8&quot; thick angle irons</td>
</tr>
<tr>
<td>Floors</td>
<td>• 3 stainless steel 0.02&quot; thick shim sheets measuring approximately 4&quot; by 4&quot;</td>
</tr>
<tr>
<td>Foundation</td>
<td>• 8 steel 1/8&quot; thick angle braces measuring 5&quot; by 1&quot;</td>
</tr>
<tr>
<td>Foundation connections</td>
<td>• 16 #10 machine screws and nuts, 32 washers</td>
</tr>
<tr>
<td></td>
<td>• 8 1/2&quot; bolts nuts, 16 washers</td>
</tr>
<tr>
<td>Cross bracing</td>
<td>• 16 stainless steel, 0.018&quot; thick, 1/2&quot; wide strips</td>
</tr>
<tr>
<td>Floor-to-column connections</td>
<td>• 36 #6 screws, 72 nuts and washers (12 screws, 24 nuts and washers per floor)</td>
</tr>
<tr>
<td></td>
<td>• 24 1/2&quot; wide angles (8 per floor)</td>
</tr>
</tbody>
</table>

As depicted in Figure 2-3, a typical floor-to-column connection consists of several small angles and #6 machine screws. In each corner of every floor, two 1/2" wide angles are carefully placed so that they overlap each other and a single machine screw is used to fasten the two braces to the bottom of the floor. The two holes on each column located at every floor-to-column connection are not drilled precisely on the as-built structure. This human source of error may have caused extra stress to be placed on the floor-to-column connections, resulting in stripping of multiple machine screws. Additionally, a uniform amount of torque was not applied to each screw when tightening the bolts, which may have resulted in varying strengths of each connection. These slight variations in connections may have caused rattling that could contribute to extra noise within the captured acceleration signals.

2.2 Experimental Setup

A schematic of the experimental set up for impact hammer tap tests is shown in Figure 2-5. A PCB-086C03 modally tuned impact hammer with 10 mV/lbf sensitivity is connected via a BNC cable to the National Instruments (NI) cDAQ-9172 data acquisition system. A Dytran
3055B1 triaxial accelerometer with the following sensitivities: 100.7 mV/g (X), 99.1 mV/g (Y), and 101.9 mV/g (Z), is also connected to the DAQ system with BNC cables. The DAQ is connected via USB to a desktop PC that runs Labview. Labview captures the signals created by the impact hammer and the response of the tap at the accelerometer [18]. All signals are captured in Volts. Additionally, various cords in the experimental setup are secured using blue and orange medical tape.

The triaxial accelerometer was placed at point 28 on the structure and acceleration was measured in both the X and Y directions, as indicated in Figure 2-5. The impact hammer was used to excite the building at 26 different locations, including the corners of all floors as well as the driving point 28 (location of accelerometer). Overall, thirteen separate points were tapped in two directions. More specifically, taps were performed on the following points in both the X and Y directions: 5, 6, 7, 8, 13, 14, 15, 16, 21, 22, 23, 24, and 28. Special care was taken to capture
an acceptable hammer hit, which comprises of a half sine wave in a very short duration. Impacts that included double hits or ones that did not produce a well-formed sine pulse were discarded. Acceleration data was recorded for a duration of 1 second and was sampled at 50,000 samples/second for each impact.

2.3 Data Processing

The DAQ system is directly integrated into Labview via Program A. The signals from the impact hammer and accelerometer were recorded and written to an .lvm file. A custom-built Labview VI was created specifically for this experiment; the front panel and block diagram are displayed in Figure 2-8 and Figure 2-9, respectively. The front panel contains nine graphs. The three plots on the right-hand side of the front panel display the un-triggered signals from the impact hammer and both acceleration directions. The three graphs in the middle column on the front panel contain triggered signals from the impact hammer and accelerometer. The top two graphs on the left portion of the front panel contain frequency response plots for both the X and Y directions. The frequency resolution on these plots is not small enough for analysis purposes; this problem is remedied later by using Matlab to compute the frequency response function [14]. The final graph in the lower left hand corner of the front panel contains a zoomed-in view of the impact hammer hit on a very small time scale. Control boxes for the sampling rate and number of samples to be captured are located in the center, towards the bottom of the front panel. Finally, the large panel in the lower right hand corner of Figure 2-8 controls the triggering settings. More specifically, the triggering threshold controls the point at which Labview begins to capture data. In this case, all signals are captured immediately after the voltage in the impact hammer exceeds 0.003 Volts.
The block diagram in Figure 2-9 contains the Labview code that operates the front panel in Figure 2-8. Loop 1, the leftmost loop in the block diagram controls the data acquisition, triggering and windowing. Three signals are continuously captured and plotted on the front panel in Loop 1: the impact hammer voltage, the X direction acceleration at point 28, and the Y direction acceleration at point 28. Once the trigger threshold is exceeded, indicating impact, Labview immediately begins recording the signals with an exponential window. After one second of data has been captured, the first loop is exited and the signals move to Loop 2, the center loop of Program A. In this loop structure, the frequency response functions (FRF) are calculated for both the X and Y direction accelerations. The FRFs are also plotted to the front panel and used as a check. If a hammer hit is performed and the FRFs look particularly "curvy" with no distinct peaks and valleys, it is indication of a subpar hit that must be redone. Once the FRFs are calculated, the program moves to the final, rightmost loop (Loop 3) in the block diagram. This loop asks the user if the data should be saved or not. Loop 3 allows the user to select "Yes" to save the data or "No" to try another hammer hit. Finally, if the user selects "Yes", the data is written to an .lvm file with a particular format.

The resulting .lvm file is an ASCII text file and can be opened and edited in MS Word or Notepad. Contained within the .lvm file are six columns: three columns are identical time data and the other three consist of one impact and two acceleration data sets. Each file was named in a systematic fashion that identifies impact and sensor location for each impact/response. For example, an impact at point 8 in the positive X direction and its measured acceleration at point 28 would have the following filename: +008x028.lvm. However, in all impact tests the accelerometer was placed at point 28 and therefore, the end of the filename including the sensor location was omitted in this step, reducing the file name to simply +008x.lvm.
In total, twenty-six impacts were performed on the undamaged structure, resulting in twenty-six Labview .lvm files.

After all twenty-six impact data sets were collected, the files were fed through several data processing software programs that correctly manipulated the data into a form accepted by Star Modal. Star Modal is a powerful modal analysis/decomposition software package that matches collected data to a 3-D model, taking structural geometry into account [12]. However, it requires specific formatting as described in the flowchart in Figure 2-6. The first step of processing begins with Mathematica. Mathematica is a symbolic-based mathematical software that can easily manipulate groups of files, efficiently create new files, and effectively format them [16]. Displayed in Figure 2-10, Mathematica (via Program B) reads the .lvm files, each one containing one impact and two acceleration time histories, and separates the data contained in each .lvm file into two .txt files: one containing impact and X acceleration information and one containing impact and Y acceleration information. Finally, the new files are renamed in order to reflect the data within; each filename consists of impact location and direction, as well as response location and direction. For example, data for impact in the X direction at point 8 and its measured acceleration at 28 in the X direction would have the file name of +008x+028x.txt.

A flowchart containing filename conventions for each program in data processing is displayed in Figure 2-7. Consistent file-naming methods ensure that data at all locations is accounted for and successfully post-processed.

After the Mathematica code (Program B) properly names and formats all fifty-two text files (twenty-six impacts in two directions), the data is imported into Matlab. Matlab is an engineering software package that can perform powerful signal processing calculations on long data sets. The function of Matlab in this process is to compute the frequency response function
(FRF) with a small frequency resolution (0.061 Hz). In Program C, FRFs are calculated for each set of impact/acceleration measurements by performing a Fast Fourier Transform (FFT) with a reduced data set (every 25th point). This transform allows the acceleration data to be represented in the frequency domain and separates the magnitude into real and imaginary parts using sinusoidal functions. A Matlab program was written in order to carry out the FFT using $2^{15}$ lines or points (Figure 2-11). The data is converted to the frequency domain and each resulting FRF is exported into a .txt file with the original file name but with a "p_" (for processed) appended to the beginning of the file name. These new .txt files each contain real and imaginary parts of the FRF corresponding to one impact/acceleration combination.

Next, all fifty-two files are imported back into Mathematica where a header is formatted and attached to the tops of all the data sets. The Mathematica code, Program D, that performs this step of the process is contained within Figure 2-12. The header contains information about the two signal channels involved, units, and length of the data set, as well as impact and acceleration locations and directions. The header is arranged in a specific format as dictated by Spectral Dynamics, the manufacturer of Star Modal. Once in the proper format with three columns of data (frequency and the real and imaginary portions of the FRF magnitude) and the correct information contained within the header, the files can be fed through Disk2Star (Figure 2-13). Disk2Star is a stand-alone software program made by Spectral Dynamics that is used to convert .txt files into binary .frf files which can ultimately be read by Star Modal [12].

Once the entire data set is loaded into Star Modal, modal decomposition is performed by the software. As detailed in Appendix B, the first five modes are selected by the user and are based on several parameters, including the relative size of its modal peak as well as frequency and damping stability. Additionally, mode shape animations are closely analyzed by the user to
ensure that the selected modes are true global modes. Once identified, the mode shapes, their corresponding frequencies, and the dynamic properties associated with the structure can be exported for use within mathematical damage indicators.

Figure 2-6: Data processing flowchart.

Figure 2-7: Example filename conventions for impact at point 8 in the X direction.
Figure 2-8: Program A, Labview front panel.
Reads signals from DAQ system

Plots untriggered signals

Trigger VI

Loop 1

Plots triggered signals

Plots triggered signals

Loop 2

VI that calculates FRFs

Loop 3

Popup: save data?

VI that writes data to a file

Figure 2-9: Program A, Labview block diagram.
Figure 2-10: Program B, Mathematica code that extracts essential data from each Labview .lvm and formats it for Matlab input.
Figure 2-11: Program C, Matlab code that calculates the real and imaginary parts of the frequency response function.

```matlab
%% FFT to find the participated frequencies
clear;
cif;
clo;
close m1

filename=uigetfile('.txt','MultiSelect', 'on');  %% choose the file you want to analyze
num=size(filename);  %% number of files

for i=1:num(:,1),
    name=filename(1,i);  %% obtain the file name of the ith file
    FN=filename(1,i);
    FN=FN(i);
    file=importdata(FN);  %% import the data to the variable "file"
    t=file(1:25:end,1);  %% time data
    disp=file(1:25:end,2);  %% data in the time domain
    disp_prd=dctrend(disp);  %% remove the linear trend from the data
    N=2^15;  %% number of points for the FFT
    nhalf=N/2+1;
    tdiv=t(2)-t(1);  %% time interval in the saved data
    fs=1/tdiv;  %% sample frequency
    disp_final=fft(disp_prd,N);  %% doing FFT change
    disp_real=real(disp_final);  %% real part of the FFT
    disp_imag=imag(disp_final);  %% imaginary part of the FFT
    amplitude=disp_final.*conj(disp_final)/N;
    frequency=fs*(0:N/2)/N;

    output=[frequency, disp_real(1:nhalf), disp_imag(1:nhalf)];  %% format of the output is: frequency real imaginary
    % use format of the output
    % name=[pname, name];  %% add "p_" in the names of the processed files, you can change it to whatever you want
    name=[pname, name(1:end-1), numel(output)];  %% output file name
    fprintf(fid,'%f %f %f
',output);
    fclose(fid);
end
```
SetDirectory["C:\Users\desktop\Desktop\Sim Testing 8.16.18\Tap Tests\Damage Case 7\Ready For Mathematica"]; outputfoldername = "readyfordisk2star";
totalnodes = 30;
typeimport = ".txt";
typeexport = ".csv";
typeexport2 = ".txt";

axis = {"x", "y", "z"};
direction = {"z", "x", "y"};

ExPoint[remode, examx, examy, examz, reasx, reasy, reasz, reasdir_] := direction[reasdir] /. ToString[NumberForm[examx, 2, NumberPadding = {"0", ""}, 2]] /. axis[examx];

ResPoint[remode, examx, examy, examz, reasx, reasy, reasz, reasdir_] := direction[reasdir] /. ToString[NumberForm[examx, 2, NumberPadding = {"0", ""}, 2]] /. axis[reasx];

Importfilename[remode, examx, examy, examz, reasx, reasy, reasz, reasdir_] := "p_" <> ExPoint[remode, examx, examy, examz, reasx, reasy, reasz, reasdir] <> typeimport;

Exportfilename[remode, examx, examy, examz, reasx, reasy, reasz, reasdir_] := StringTake[ResPoint[remode, examx, examy, examz, reasx, reasy, reasz, reasdir], {2, 5}] <> typeexport;

Rawdata[remode, examx, examy, examz, reasx, reasy, reasz, reasdir_] := Import[Importfilename[remode, examx, examy, examz, reasx, reasy, reasz, reasdir], "Table"];

Data[remode, examx, examy, examz, reasx, reasy, reasz, reasdir_] :=
  Transpose[{Rawdata[remode, examx, examy, examz, reasx, reasy, reasz, reasdir][[1, 2]], Rawdata[remode, examx, examy, examz, reasx, reasy, reasz, reasdir][[1, 3]], Rawdata[remode, examx, examy, examz, reasx, reasy, reasz, reasdir][[1, 4]], Rawdata[remode, examx, examy, examz, reasx, reasy, reasz, reasdir][[1, 5]]}];

Reader[remode, examx, examy, examz, reasx, reasy, reasz, reasdir_] :=
  {"Datatype": "FRF", "BlockSize": "<ToString[Length[Data[remode, examx, examy, examz, reasx, reasy, reasz, reasdir]]]", "Measurement ID": MeasurementID", "Analyzer ID": AnalyzerID", "Channel 1": "<ToString[Min[Data[remode, examx, examy, examz, reasx, reasy, reasz, reasdir][[1, 1]]]], "Channel 2": "<ToString[Min[Data[remode, examx, examy, examz, reasx, reasy, reasz, reasdir][[1, 2]]]], "Units": "VOLTS", "Transducer ID": "EXCITATION", "Frequency": "REAL", "INAG"};

Constructfile[remode, examx, examy, examz, reasx, reasy, reasz, reasdir_] :=
  Join[Reader[remode, examx, examy, examz, reasx, reasy, reasz, reasdir], Data[remode, examx, examy, examz, reasx, reasy, reasz, reasdir], {"Folder": Import[Importfilename[remode, examx, examy, examz, reasx, reasy, reasz, reasdir], "Table"]};

Renamefilename[remode, examx, examy, examz, reasx, reasy, reasz, reasdir_] :=
  StringTake[ResPoint[remode, examx, examy, examz, reasx, reasy, reasz, reasdir], {2, 5}] <> typeexport2;

Do[If[FileExistsQ[Importfilename[remode, examx, examy, examz, reasx, reasy, reasz, reasdir]], True, Rawdata[remode, examx, examy, examz, reasx, reasy, reasz, reasdir]; Data[remode, examx, examy, examz, reasx, reasy, reasz, reasdir]; Reader[remode, examx, examy, examz, reasx, reasy, reasz, reasdir]; Constructfile[remode, examx, examy, examz, reasx, reasy, reasz, reasdir]; CreateDirectory[outputfoldername]; Export[FileReplaceJoin[{outputfoldername, Exportfilename[remode, examx, examy, examz, reasx, reasy, reasz, reasdir]}], Constructfile[remode, examx, examy, examz, reasx, reasy, reasz, reasdir]]; RenameFile[FileReplaceJoin[{outputfoldername, Exportfilename[remode, examx, examy, examz, reasx, reasy, reasz, reasdir]}], FileReplaceJoin[{outputfoldername, Renamefilename[remode, examx, examy, examz, reasx, reasy, reasz, reasdir]}], {remode, 1, totalnodes}, {examx, 1, 2}, {reasx, 1, 3}, {reasdir, 1, 2}]];

Figure 2-12: Program D, Mathematica code that formats the FRF data properly for Disk2Star input.
2.4 Description of Damage Cases

Several damage scenarios were implemented in order to simulate real-world stiffness losses, environmental occurrences, and structural aging. The same data collection and data processing methods already discussed in Section 2.3 were carried on all remaining damage cases.

Table 2-2 summarizes all damage scenarios. Cross braces are labeled by the two nodes from Figure 2-4 that each member spans. Bold typeface represents a member that was removed between two consecutive damage cases. For example, member 8-15 is bold for DC 1 and not for DC 2; this is because member 8-15 was removed for DC 1 and remained off in DC 2. Figure 2-14 through Figure 2-24 pictorially represent each damage case using wireframe sketches, and some photographs of the model are provided.
### Table 2-2: Damage summary of scenarios

<table>
<thead>
<tr>
<th>Damage Case (DC)</th>
<th>Members Removed (node-to-node)</th>
<th>Short Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC 0</td>
<td>None</td>
<td>Undamaged (baseline)</td>
</tr>
<tr>
<td>DC 1</td>
<td><strong>8-15</strong></td>
<td>Single story asymmetric damage with one brace removed</td>
</tr>
<tr>
<td>DC 2</td>
<td><strong>8-15, 6-13</strong></td>
<td>Single story symmetric damage with one brace removed</td>
</tr>
<tr>
<td>DC 3</td>
<td><strong>8-15, 7-16</strong></td>
<td>Single story asymmetric damage with two braces removed</td>
</tr>
<tr>
<td>DC 4</td>
<td><strong>8-15, 7-16, 5-14, 6-13</strong></td>
<td>Single story symmetric damage with two braces removed</td>
</tr>
<tr>
<td>DC 5</td>
<td><strong>8-15, 5-14, 6-13, 7-16, 6-15, 7-14, 8-13, 5-16</strong></td>
<td>Single story symmetric damage with all first level cross bracing removed</td>
</tr>
<tr>
<td>DC 6</td>
<td><strong>8-15, 5-14, 6-13, 7-16, 6-15, 7-14, 8-13, 5-16, 15-24</strong></td>
<td>Multiple story asymmetric damage with one brace removed</td>
</tr>
<tr>
<td>DC 7</td>
<td><strong>8-15, 5-14, 6-13, 7-16, 6-15, 7-14, 8-13, 5-16, 15-24, 14-21</strong></td>
<td>Multiple story symmetric damage with one brace removed</td>
</tr>
<tr>
<td>DC 8</td>
<td><strong>8-15, 5-14, 6-13, 7-16, 6-15, 7-14, 8-13, 5-16, 15-24, 14-21, 13-22, 16-23</strong></td>
<td>Multiple story symmetric damage with two braces removed</td>
</tr>
<tr>
<td>DC 9</td>
<td><strong>8-15, 5-14, 6-13, 7-16, 6-15, 7-14, 8-13, 5-16, 15-24, 14-21, 13-22, 16-23, 14-23, 15-22</strong></td>
<td>Multiple story asymmetric damage with two braces removed</td>
</tr>
<tr>
<td>DC 10</td>
<td><strong>8-15, 5-14, 6-13, 7-16, 6-15, 7-14, 8-13, 5-16, 15-24, 14-21, 13-22, 16-23, 14-23, 15-22, 13-24, 16-21</strong></td>
<td>Multiple story symmetric damage with all cross bracing removed</td>
</tr>
</tbody>
</table>
2.4.1 Damage Case 0 (DC 0)

Damage Case 0 is considered the undamaged, baseline structure. All parts from Table 2-2 are attached to the structure. Data was collected for this case in order to extract dynamic parameters for the as-built structure and to compare to all subsequent damage scenarios.

Figure 2-14: Damage Case 0.
2.4.2 Damage Case 1 (DC 1)

Damage Case 1 is the first incremental damage scenario which incorporates a missing member. DC 1 is considered to have single-story, asymmetric damage since only one member is removed on one face in one story. The member removed spans points 8 and 15 between floors 1 and 2 on side 3.

Figure 2-15: Damage Case 1 with member 8-15 removed.
2.4.3 Damage Case 2 (DC 2)

Damage Case 2 is the second incremental damage scenario and incorporates two total missing members. In order to move from DC 1 to DC 2, the member that spans points 6 and 13 is removed. DC 2 is a single-story, symmetric damage case; two symmetric members on opposite faces (sides 1 and 3) within the first story are removed from the structure.

Figure 2-16: Damage Case 2 with member 6-13 also removed.
2.4.4 Damage Case 3 (DC 3)

Damage Case 3 incorporates single-story, asymmetric damage. In order to move from DC 2 to DC 3, one brace is removed and another one is added. Due to the removal of the brace that spans points 7 and 16 paired with the addition of the member that connects points 6 and 13, DC 3 is a unique damage case within this experimental series. All other damage cases incorporate incremental damage by removing braces, but DC 3 is the only one that attempts to quantify the effects of re-attaching a member to the structure. The result is no cross bracing between the first and second floors on side 3.

Figure 2-17: Damage Case 3 with 6-13 replaced and 7-16 removed.
2.4.5 Damage Case 4 (DC 4)

A total of four members are removed in DC 4, making it a single-story, symmetric damage scenario. To reach DC 4 from DC 3, both members on side 1 between the first and second floors are removed in order to mirror side 3's configuration. The two removed members are 6-13 and 5-14.

Figure 2-18: Damage Case 4 with 6-13 and 5-14 removed.
2.4.6 Damage Case 5 (DC 5)

Damage Case 5 is the final single-story damage scenario. Members 6-15, 7-14, 5-16, and 8-13 are removed when transitioning from DC 4 to DC 5. All members in between the first and second floor on all four faces are removed, which makes DC 5 a single-story, symmetric damage case.

Figure 2-19: Damage Case 5 with no cross bracing between the first and second floors.
2.4.7 Damage Case 6 (DC 6)

Damage Case 6 has the first multiple-story damage. For this case, there are nine total braces not attached to the structure, including a new member that is removed moving from DC 5 to DC 6. On side 3, between the second and third floors, the member that spans points 15 and 24 is now removed, making DC 6 a multiple-story, asymmetric damage scenario.

![Figure 2-20: Damage Case 6 with 15-24 removed.](image-url)
2.4.8 Damage Case 7 (DC 7)

Damage Case 7 incorporates multiple-story, symmetric damage by having a total of 10 members detached. When transitioning from DC 6 to DC 7, only brace 14-21 is removed from side 1 in between the second and third floors.

Figure 2-21: Damage Case 7 with member 14-21 also removed.
2.4.9 Damage Case 8 (DC 8)

Damage Case 8 is the next incremental damage scenario. Similar to DC 7, DC 8 is considered to be a multiple-story, symmetrical damage case. In DC 7, only one brace is removed from both sides 1 and 3 in between the second and third floors. However, in DC 8, both braces are removed on sides 1 and 3 in between the second and third floors. When moving from DC 7 to DC 8, members 13-22 and 16-23 are removed. The result is four remaining cross bracing members on sides 2 and 4 in between the second and third floors.

Figure 2-22: Damage Case 8 with both 13-22 and 16-23 removed.
2.4.10 Damage Case 9 (DC 9)

Damage Case 9 incorporates multiple-story asymmetric damage by leaving only two cross bracing members on the structure, 13-24 and 16-21. During the transition from DC 8 to DC 9, the two braces removed from side 2 are 14-22 and 15-23. Thus, only the bay between the second and third floors on side 4 is braced.

Figure 2-23: Damage Case 9 with members 15-24 and 16-23 removed.
2.4.11 Damage Case 10 (DC 10)

Damage Case 10 is the final damage case in the series and is considered to be the “most damaged.” When moving from DC 9 to DC 10, the last two cross braces are removed. When members 13-24 and 16-21 are removed, the structure consists solely of columns and floors. That is, this case is basically a metal frame with no sway bracing.

Figure 2-24: Damage Case 10 with no cross bracing.
2.5 Calculating Frequency Response Functions

Figure 2-25a displays a typical hammer impact time history. In this example, the undamaged structure, DC 0, was tapped at point 8 in the X direction. The signal was measured by Labview (Figures 8 and 9) in Volts and then converted to pounds force (lbf) using the hammer's sensitivity of 10 mV/lbf. Although the signal was recorded for one second, the actual impact is much shorter in duration. Figure 2-25b displays the hammer hit on a time scale a thousand times smaller; the hammer hit lasts less than 0.001 second. Figure 2-25b also shows the half sine wave shape of an ideal hammer hit. If the hit is performed perfectly each time, it can be assumed that no force is applied to the structure while acceleration data is collected. This assumption permits one to calculate FRFs based solely on acceleration response data.

Figure 2-25: (a) Impact time history for tap at point 8 in the X direction for DC 0. (b) Zoomed-in view.

Figure 2-26 contains a plot of a typical acceleration response which is the result for the hit in Figure 2-25. This particular time history was recorded at point 28 in the X direction after the structure was hit at point 8 in the X direction. The original signal was measured in Volts by
Program A (Figure 2-8 and Figure 2-9) and was converted to acceleration units via sensor sensitivities of 100.7 mV/g in the X direction and 99.1 mV/g in the Y direction. Immediately after impact, the acceleration is almost 150 g's; however, the signal quickly dies out, becoming essentially zero after 0.5 seconds. For each damage case, fifty-two total acceleration data sets were captured in order to calculate FRFs for each impact/acceleration combination.

Once all acceleration data was collected for DC 0, Matlab (Figure 2-11) was utilized to perform a Fast Fourier Transform and produce an FRF for each impact/acceleration data set. The magnitude of the FRF is calculated by taking the square root of the sum of the real and imaginary parts squared. A sample FRF for impact at point 8 in the X direction and acceleration response at point 28 in the X direction over the range of 0 to 1000 Hz is displayed in Figure 2-26: Acceleration time history for point 28 in the X direction with hit at point 8 in the X direction for DC 0.

Once all acceleration data was collected for DC 0, Matlab (Figure 2-11) was utilized to perform a Fast Fourier Transform and produce an FRF for each impact/acceleration data set. The magnitude of the FRF is calculated by taking the square root of the sum of the real and imaginary parts squared. A sample FRF for impact at point 8 in the X direction and acceleration response at point 28 in the X direction over the range of 0 to 1000 Hz is displayed in Figure 2-26: Acceleration time history for point 28 in the X direction with hit at point 8 in the X direction for DC 0.
2-27. The normalized FRF magnitude was calculated by dividing all points by the maximum FRF magnitude value for each impact/acceleration combination. As evidenced by peaks, some resonant frequencies can be estimated as 100 Hz, 175 Hz, and 215 Hz; there is a clear peak at around 100 Hz and two peaks centered at 200 Hz. However, only estimations and observations can be made from this plot alone; in order to correctly identify resonant frequencies, FRFs for each impact/acceleration combination need to be analyzed together.

![Normalized FRF Magnitude vs Frequency](image)

Figure 2-27: Frequency response function for impact at point 8 in the X direction and acceleration at point 28 in the X direction for DC 0

2.6 Compare FRFs of one point for all DCs

Figure 2-27 is a sample of the frequency response function (FRF) data that was calculated via Matlab in Program C (Figure 2-11). The structure was impacted at point 8 in the X direction and the acceleration response in the X direction at point 28 was captured. The output acceleration data was used to calculate the FRF for each damage case. Figure 2-28 shows an overlay of the same impact/response FRF for each damage case. There are large peaks near 200
Hz for each FRF, which indicates a potential X direction vibration mode in most of the damage cases. However, no quantitative trend can be assessed by solely observing this plot without Star Modal analysis.

Similarly, the same type of plot can be analyzed for impact at point 8 in the Y direction and capturing acceleration in the Y direction at point 28 (Figure 2-29); two regions of interest are identified at 90 Hz and 200 Hz, which may indicate Y direction modes. Coupled modes may also be preliminarily examined by viewing the cross-correlated X direction impact versus Y direction acceleration, or vice versa.
Although qualitative observations can be made from analyzing a single point, the FRFs at point 8 for each damage scenario are insufficient for complete dynamic system analysis. A cumulative FRF must be produced from all fifty-two impact/acceleration measurements. As described in the Appendix, the calculations necessary to produce a modal peaks plot for each damage scenario are performed using Star Modal. The modal peaks plot is essentially a cumulative FRF that incorporates geometry of the structure as well as measured data. In general, the frequency domain data for each damage case was quite noisy for frequencies above 250 Hz. The higher frequencies detected usually correspond to acoustic disturbances in the structure, such as contacting of the cross bracing or angles connecting the floors to columns. Since the global motion of the structure is desired, the frequency range from 0 to 250 Hz is analyzed herein. This range of frequencies includes the first five global modes of the structure for all damage cases.
2.7 Results for each DC

2.7.1 DC 0

Figure 2-30 contains a plot of all FRFs for each impact/acceleration combination for the undamaged structure, DC 0. There are certain frequency ranges that may indicate resonance, such as 75 to 100 Hz, the peaks centered around 175 Hz, and two peaks between 200 and 250 Hz. There is much noise and discrepancy between each FRF in DC 0; peaks are not well formed nor in unison. This discrepancy could be attributed to the large number of small pieces attached to the structure such as the relatively thin cross bracing strips, as well as the small angles that fasten the floors to the columns. Although there are quasi-clear peaks, the cumulative FRF as well as mode shape animations must be analyzed in order to properly pick out glob modes of the baseline structure.

Figure 2-30: Overlaid FRFs for impact at all points for DC 0.

Figure 2-31 displays the modal peaks plot for DC 0. Figure 2-33 through Figure 2-41 as well as Figure 2-43 are similar plots for the ten remaining damage cases. The modal peaks plot
is calculated via Star Modal and incorporates model geometry as well as all fifty-two FRFs for each impact/acceleration combination. The process utilized to determine the true global modes of the structure is outlined in detail in Appendix B. The stability diagram, along with the modal peaks plot and mode shape animations are closely examined in order to find the first five experimental modes of each damage case. In Figure 2-31 there are clear resonance peaks at around 90, 175, 210, and 240 Hz. After closely investigating the mode shape animations around these frequencies as well as other peaks with relatively less magnitudes, the global modes of the structure were determined. The true global modes of the structure are highlighted in Figure 2-31 as grey dots and are described in Table 2-3. Modal properties for damage cases 1 through 10 are contained in Table 2-4 through Table 2-13.

![Figure 2-31: Modal peaks produced by Star Modal for DC 0.](image)
Table 2-3: Mode shapes and natural frequencies for DC 0.

<table>
<thead>
<tr>
<th>Mode shape description</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>73.55</td>
<td>91.77</td>
<td>177.59</td>
<td>211.89</td>
<td>236.54</td>
</tr>
<tr>
<td>Normalized modal peaks magnitude</td>
<td>0.175191</td>
<td>0.883366</td>
<td>0.864976</td>
<td>0.448619</td>
<td>0.338431</td>
</tr>
</tbody>
</table>

The first experimental global mode occurs at 73.55 Hz with a normalized cumulative FRF magnitude of 0.175. This mode shape depicts the first bending mode in the X direction; the structure is generally swaying along the X direction with a small displacement. The columns spanning points 3-17 and 4-18 show the most motion along the X direction. The second mode shape is a Y direction sway and occurs at 91.77 Hz with a relative modal peaks magnitude of 0.883. Since the magnitude value is close to 1 and the mode shape animation clearly shows motion in only the Y direction, this is definitely the first bending mode in the Y direction. At 177.59 Hz, the third experimental mode occurs with a magnitude of 0.865. This mode is clearly the first torsion mode due to the relatively high magnitude value and the clear twisting of the structure depicted by model animation. The fourth mode has a slightly lower magnitude of 0.449 and occurs at 211.89 Hz. The model animation shows the second bending mode in the X direction. The last mode shape found appears at 236.54 with a magnitude of 0.338. This mode shape animation depicts the second bending mode in the Y direction with point 14 moving out of phase from the rest of the structure.
Figure 2-32 pictorially displays the experimental mode shapes listed in Table 2-3. A snapshot of each mode shape animation was captured at the maximum displacement. The blue lines and surfaces constitute the actual structure while the grey lines indicate the axial projections of the structure's movement. The grey projections below the structure give the best indication as to how the entire structure moves as well as clearly depicting torsion.
2.7.2 DC 1

Figure 2-33: Modal peaks produced by Star Modal for DC 1.

Table 2-4: Mode shapes and natural frequencies for DC 1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>51.83</td>
<td>85.28</td>
<td>162.46</td>
<td>209.82</td>
<td>237.57</td>
</tr>
<tr>
<td>Normalized modal peaks magnitude</td>
<td>0.183535</td>
<td>0.699565</td>
<td>0.090907</td>
<td>0.950644</td>
<td>0.120725</td>
</tr>
<tr>
<td>Mode shape description</td>
<td>1st bending mode in X direction</td>
<td>1st bending mode in Y direction</td>
<td>1st torsion mode</td>
<td>2nd bending mode along the axis that spans points 1 and 3</td>
<td>2nd bending mode along the axis that spans points 2 and 4</td>
</tr>
</tbody>
</table>

The first mode for DC 1 occurs at 51.83 Hz with a magnitude of 0.1854. The mode shape animation depicts the structure swaying in the X direction, indicating the first bending mode in the X direction. The first bending mode in the Y direction, mode 2, appears at 85.28 Hz with a magnitude of 0.186. Next, the third mode, which occurs at 162.46 Hz with a normalized magnitude of 0.091, has a shape animation that indicates torsion. Additionally, point 16 twists...
out of sync with the rest of the structure in this mode. The second bending mode along the axis that spans points 1 and 3 (along the diagonal between the X and Y direction) appears at 209.82 Hz with a magnitude of 0.950. The fifth mode occurs at 237.57 Hz with a modal peaks magnitude of 0.121. The mode shape animation shows the second bending mode along the axis that spans points 2 and 4 with some twisting throughout the structure.

2.7.3 DC 2

![Modal peaks produced by Star Modal for DC 2.](image)

Figure 2-34: Modal peaks produced by Star Modal for DC 2.

<table>
<thead>
<tr>
<th>Mode shape description</th>
<th>1st bending mode in X direction</th>
<th>1st bending mode in Y direction</th>
<th>1st torsion mode</th>
<th>2nd bending mode along the axis that spans points 1 and 3</th>
<th>2nd bending mode along the axis that spans points 2 and 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>49.27</td>
<td>87.09</td>
<td>163.79</td>
<td>210.3</td>
<td>233.99</td>
</tr>
<tr>
<td>Normalized modal peaks magnitude</td>
<td>0.189701</td>
<td>0.400985</td>
<td>0.107365</td>
<td>0.91059</td>
<td>0.256025</td>
</tr>
</tbody>
</table>

Table 2-5: Mode shapes and natural frequencies for DC 2.
The first mode occurs at 49.7 Hz with a normalized modal peaks magnitude of 0.190. The mode shape animation within Star Modal indicates the first bending mode in the X direction with points 5 and 8 moving out of phase from the rest of the structure in the Y direction. At 87.09 Hz, the second mode appears with a magnitude of 0.401. Although the modal peaks value is not close to one, the mode shape animation clearly indicates the first bending mode in the Y direction. The third mode occurs at 163.79 Hz with a magnitude of 0.1074. The animation depicts the first torsion mode with points 22 and 23 moving out of sync with the rest of the structure. The second bending mode along the axis that spans the points 1 and 3 appears at 210.30 Hz with a modal peaks magnitude of 0.911. The animation indicates the second bending mode along the 1-3 diagonal with point 23 twisting out of phase with respect to rest of the model. The final experimental mode for DC 2 occurs at 233.99 Hz and has a magnitude of 0.256. The animation depicts the second bending mode along the axis that spans points 2 and 4. This particular mode shape is not particularly convincing due to the relatively low magnitude value as well as an animation that is not too clear.

2.7.4 DC 3

![Modal peaks produced by Star Modal for DC 3.](image-url)
As indicated in Table 2-6, the first global mode of DC 3 appears at 61.22 Hz with a normalized modal peaks magnitude of 0.702. The animation for this mode clearly indicates the first bending mode in the X direction. The second mode occurs at 86.23 Hz with a magnitude of 0.989. Due to the relatively high magnitude as well as the clear mode shape animation, this is considered the first bending mode in the Y direction. At 161.40 Hz with a modal peaks magnitude of 0.150, mode 4 appears with a shape animation that clearly depicts torsion. The fourth mode of DC 4 occurs at 213.16 Hz with a magnitude of 0.796; the mode shape animation shows the second bending mode along the diagonal that spans points 1 and 3. The fifth and final mode of DC 4 appears at 230.10 Hz with a magnitude of 0.261. Despite the relatively low magnitude, the mode shape animation depicts the second bending mode along the axis that spans points 2 and 4 with the second floor of the structure expanding and contracting.
2.7.5 DC 4

Figure 2-36: Modal peaks produced by Star Modal for DC 4.

Table 2-7: Mode shapes and natural frequencies for DC 4.

<table>
<thead>
<tr>
<th>Mode shape description</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Mode 4</th>
<th>Mode 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>42.15</td>
<td>--</td>
<td>126.92</td>
<td>213.64</td>
<td>229.71</td>
</tr>
<tr>
<td>Normalized modal peaks magnitude</td>
<td>0.254181</td>
<td>--</td>
<td>0.054735</td>
<td>0.798946</td>
<td>0.295798</td>
</tr>
</tbody>
</table>

The first global mode occurs at 42.15 Hz and has a modal peaks magnitude of 0.254. The resulting mode shape clearly shows the first bending mode in the X direction. The first bending mode in the Y direction was not found for DC 4 and is therefore, completely omitted from proceeding analyses. The third mode appears at 129.92 Hz with a modal peaks magnitude of 0.055. Although the magnitude value is quite low, the mode shape animation depicts the first
torsion mode with side 4 of the structure moving with greatest displacement. Occurring at 213.64 Hz with a magnitude of 0.799, the fourth mode's shape shows the second bending mode in the X direction with the first and second floors of the structure twisting a bit. The final and fifth mode for DC 4 appears at 229.71 Hz and has a modal peaks magnitude of 0.296. The shape animation depicts a weak second bending mode in the Y direction with the first and second floors of the structure slightly twisting.

2.7.6 DC 5

![Figure 2-37: Modal peaks produced by Star Modal for DC 5.](image)

<table>
<thead>
<tr>
<th>Mode</th>
<th>1st bending mode in X direction</th>
<th>1st bending mode in Y direction</th>
<th>1st torsion mode</th>
<th>2nd bending mode in X direction</th>
<th>2nd bending mode in Y direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>38.98</td>
<td>52.76</td>
<td>127.43</td>
<td>216.66</td>
<td>234.07</td>
</tr>
<tr>
<td>Normalized modal peaks magnitude</td>
<td>0.163687</td>
<td>0.119061</td>
<td>0.109277</td>
<td>0.929742</td>
<td>0.626939</td>
</tr>
<tr>
<td>Mode shape description</td>
<td>1st bending mode in X direction</td>
<td>1st bending mode in Y direction</td>
<td>1st torsion mode</td>
<td>2nd bending mode in X direction</td>
<td>2nd bending mode in Y direction</td>
</tr>
</tbody>
</table>
As indicated by Table 2-8, the first mode for DC 5 appears at 38.98 Hz with a normalized magnitude of 0.1637 and an animation that displays the first bending mode in the X direction with point 24 moving out of phase with the rest of the model. The second mode occurs at 52.76 Hz and has a normalized modal peaks magnitude of 0.119. The corresponding mode shape indicates the first bending mode in the Y direction with point 5 twisting on its own. Next, the third mode shape clearly depicts the first torsion mode. The torsion mode occurs at 127.43 Hz and has a normalized magnitude of 0.109. The fourth mode for DC 5 appears at 216.66 Hz and has a modal peaks magnitude of 0.930. The resulting mode shape animation shows the second bending mode in the X direction with the column that spans points 4 and 28 twisting a small amount and point 21 moving solely in the Y direction. The fifth mode occurs at 234.07 Hz with a normalized magnitude of 0.627. The animation depicts a clear second bending mode in the Y direction and can be considered a strong mode.

2.7.7 DC 6

![Figure 2-38: Modal peaks produced by Star Modal for DC 6.](image-url)
The first experimental mode for DC 6 occurs at 37.16 Hz and has a magnitude of 0.14587. The corresponding mode shape animation shows a first bending in the X direction. The structure is swaying along the X axis as a whole except point 6 which moves out of phase.

The second mode appears at 53.12 Hz with a magnitude of 0.089. Despite the relatively small cumulative FRF magnitude, the mode shape animations shows a clear first bending in the Y direction; the structure is swaying along the Y axis with no motion in the X direction. Occurring at 74.91 Hz, the third mode has a modal peaks magnitude of 0.240. The mode shape animation depicts the model twisting uniformly except for the column that spans points 1 and 25 which moves with more displacement than the other three columns. Due to the dominant twisting motion, the third mode is denoted as the first torsion mode. The fourth mode appears at 195.15 Hz with a magnitude of 0.829. The resulting shape animation clearly depicts the structure's second bending mode in the X direction. The fifth mode, occurring at 231.23 Hz, has a magnitude of 0.431. The mode shape animation shows the structure's second bending mode along the axis that spans points 1 and 3.
Table 2-10: Mode shapes and natural frequencies for DC 7.

<table>
<thead>
<tr>
<th>Mode shape description</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st bending mode in X</td>
<td>23.04</td>
<td>52.42</td>
<td>70.44</td>
<td>179.16</td>
<td>233.07</td>
</tr>
<tr>
<td>direction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st bending mode in Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>direction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st torsion mode</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd bending mode in X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>direction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd bending mode</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>along the axis</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>that spans points</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 and 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The first mode detected occurs at 23.04 Hz and has a cumulative FRF magnitude of 0.493. The corresponding shape animation clearly shows the model swaying in the X direction, indicating that this is the structure's first bending mode shape in the X direction. At 52.42 Hz, the second mode occurs with a very low magnitude of 0.026. The mode shape animation shows motion everywhere on the structure with a slight Y sway. This mode is deemed the first bending
mode in the Y direction. Next, the third mode is found at 70.44 Hz with a high magnitude of 0.893. The corresponding mode shape animation indicates clear, uniform twisting; this is the structure's first torsion mode. The fourth mode appears at 179.16 Hz with a magnitude of 0.566. By analyzing the mode shape, it is clear that this is the building's second bending mode along the X axis. The fifth mode detected occurs at 233.07 Hz with a cumulative FRF magnitude of 0.077. The mode shape animation shows the structure moving along the diagonal that spans points 1 and 3 in the second mode of vibration.

2.7.9 DC 8

![Figure 2-40: Modal peaks produced by Star Modal for DC 8.](image)
Table 2-11: Mode shapes and natural frequencies for DC 8.

<table>
<thead>
<tr>
<th>Mode shape description</th>
<th>1st bending mode in X direction</th>
<th>1st bending mode in Y direction</th>
<th>1st torsion mode</th>
<th>2nd bending mode in X direction</th>
<th>2nd bending mode in Y direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>24.28</td>
<td>51.51</td>
<td>73.26</td>
<td>204.26</td>
<td>228.03</td>
</tr>
<tr>
<td>Normalized modal peaks magnitude</td>
<td>0.339778</td>
<td>0.012298</td>
<td>0.398064</td>
<td>0.99855</td>
<td>0.052416</td>
</tr>
</tbody>
</table>

The first mode in DC 8 is detected at 24.28 Hz with a modal peaks magnitude of 0.340. The mode shape animation shows the structure swaying in the X direction, indicating the first bending mode in the X direction. The second mode occurs at 51.51 Hz and has a cumulative FRF magnitude of 0.012. The mode shape animation depicts the structure generally swaying in the Y direction with the second floor expanding and contracting a small amount. The general sway makes this mode the first bending mode in the Y direction. The next mode appears at 73.26 Hz with a normalized magnitude of 0.400. The third mode shape animation displays a clear uniform torsion movement. The fourth mode occurs at 204.26 Hz with a large magnitude of 0.999. Due to the relatively large modal peaks magnitude and the clear mode shape animation, this mode is denoted as the structure's second bending mode in the X direction. The final mode appears at 228.03 Hz with a magnitude of 0.052. The fifth mode's animation shows the structure moving along the Y direction in the second mode of vibration with some torsion throughout.
2.7.10 DC 9

Figure 2-41: Modal peaks produced by Star Modal for DC 9.

<table>
<thead>
<tr>
<th>Mode</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>26.56</td>
<td>31.40</td>
<td>71.62</td>
<td>204.57</td>
<td>219.66</td>
</tr>
<tr>
<td>Normalized modal peaks magnitude</td>
<td>0.258068</td>
<td>0.198975</td>
<td>0.639847</td>
<td>0.911606</td>
<td>0.106719</td>
</tr>
<tr>
<td>Mode shape description</td>
<td>1st bending mode in X direction</td>
<td>1st bending mode in Y direction</td>
<td>1st torsion mode</td>
<td>2nd bending mode in X direction</td>
<td>2nd bending mode in Y direction</td>
</tr>
</tbody>
</table>

The first mode occurs at 26.56 Hz and has a cumulative FRF magnitude of 0.259. The corresponding mode shape animation shows a clear sway in the X direction with points 13 and 21 moving out of sync and with much more displacement than the rest of the structure. The second mode appears at 31.40 Hz and has a normalized magnitude of 0.199. The structure is generally swaying in the Y direction with point 21 twisting a small amount. After analyzing the mode shape animation, the second mode can be denoted as the first bending mode in the Y direction.
direction. The third mode is found at 71.62 Hz and has a magnitude of 0.640. Although the animation shows that side 4 is moving with the most displacement, the structure is generally twisting. This mode can be described as the first torsion mode of the structure. The next mode occurs at 204.57 Hz with a magnitude of 0.917. The animation depicts the structure bending in the second mode of vibration in the X direction with the structure twisting slightly throughout. The fifth mode appears at 219.66 Hz and has a magnitude of 0.107. The resulting mode shape animation displays the second bending mode in the Y direction.

2.7.11 DC 10

![Overlaid FRFs for impact at all points for DC 10.](image)

Figure 2-42: Overlaid FRFs for impact at all points for DC 10.
Figure 2-43: Modal peaks produced by Star Modal for DC 10.

Table 2-13: Mode shapes and natural frequencies for DC 10.

<table>
<thead>
<tr>
<th>Mode</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>18.77</td>
<td>27.30</td>
<td>53.64</td>
<td>197.09</td>
<td>211.48</td>
</tr>
<tr>
<td>Normalized modal peaks magnitude</td>
<td>0.066451</td>
<td>0.34433</td>
<td>0.142695</td>
<td>0.988794</td>
<td>0.541025</td>
</tr>
<tr>
<td>Mode shape description</td>
<td>1st bending mode in Y direction</td>
<td>1st bending mode along the axis that spans points 1 and 3</td>
<td>1st torsion mode</td>
<td>2nd bending mode along the axis that spans points 2 and 4</td>
<td>2nd bending mode along the axis that spans points 1 and 3</td>
</tr>
</tbody>
</table>

The first experimental mode is found at 18.77 Hz and has a normalized modal peaks magnitude of 0.0665. The mode shape animation shows some sway in the X direction. The next mode appears at 27.30 Hz with a magnitude of 0.344. The corresponding mode shape animation shows the structure's first bending mode along the axis that spans points 1 and 3. The third mode occurs at 53.64 Hz and has a normalized magnitude of 0.143. At 197.09 Hz the fourth mode appears with a normalized cumulative FRF magnitude of 0.989. The corresponding mode shape
animation depicts the structure's second bending mode along the axis that spans points 2 and 4. The final and fifth mode occurs at 211.48 Hz and has a magnitude of 0.541. The resulting mode shape animation shows the structure's second bending mode along the axis that spans points 1 and 3.

2.8 Modal Peaks Trends and Conclusions

In order to properly summarize this chapter's work, a modal peaks map is constructed and displayed in Figure 2-45. The independent axis in these waterfall plots is frequency in Hertz while the dependent axis represents the normalized cumulative FRF magnitude of each damage case. Each of the eleven modal peaks plots contained in Figure 2-45 can also be found in Section 2.7 of this work. The red arrows represent the first mode with the lowest natural frequency. The shape corresponding to the first mode is sway within the X direction. The second mode is labeled in orange and represents sway motion within the Y direction. The first torsion mode is indicated by green arrows and is considered to be the third experimental mode. The fourth mode, whose shape is the second bending mode in the X direction, is represented by
blue arrows. The fifth and final experimental mode is identified as the second bending mode in the Y direction and is labeled in purple. Each mode can be tracked throughout all damage cases (except mode 2 in DC 4) and general trends and behaviors can be analyzed.

Figure 2-45 clearly illustrates a decrease in natural frequencies as the structure becomes more damaged. As one progresses from DC 0 to DC 10, damage increases and the structure becomes more flexible due to the removal of cross-bracing members. In general, when stiffness decreases within a system, the natural frequencies will decrease as a result. This phenomenon is observed when comparing all eleven damage cases and validates the results of modal decomposition for incremental damage. In addition to the shift in natural frequencies, modal peak splitting also occurs within Figure 2-45. The dashed arrows point to peaks that represent a weaker form of the mode indicated by the solid arrow of the same color. For example, mode 3 in DC 1 has two possible locations, one at approximately 160 Hz that was identified as the stronger form of torsion, and one at 150 Hz whose mode shape is not as clear as the mode at 160 Hz. It is likely that these are not separate modes, just the one torsion mode at 175 Hz in DC 0 splitting into two modes in DC 1. Peak splitting is a direct result of damage and can ultimately help identify structural weakness within a system.

Similar to the one displayed in Figure 2-45, mode mapping waterfall plots are created for symmetric and asymmetric damage, as well as single-story and multiple-story damage. All four of these plots are contained within Appendix A of this work. Trends observed within Figure 2-45 can also be found within every modal peaks map within Appendix A; frequency shifts and modal peak splitting are witnessed within all categories of damage.
Figure 2-45: Mode map for all damage cases.
In order to move forward with the analysis of several damage indicators, the modal parameters determined from the process outlined in this chapter are assumed to be correct for the three-story spatial frame. The next chapter of this work includes mathematically manipulating the true global mode shapes and natural frequencies of each damage case; it is imperative that the correct values for dynamic properties be used. Overall, the results obtained via experimental modal analysis are an accurate portrayal of the dynamic response of each damage configuration.
3. DAMAGE DETECTION METHODS AND RESULTS

In order to implement damage detection algorithms and assess their relative effectiveness, dynamic structural properties, such as natural frequencies and corresponding mode shapes, must be obtained via measurement for both baseline and damaged configurations. Since natural frequencies and mode shapes were successfully determined within Chapter 2, changes in the dynamic properties of a structure can now be manipulated to form indices that may identify the level and/or location of structural weakness. Six unique damage detection methods were implemented on the three-story spatial frame; variations on three methods were further examined.

The damage indicators analyzed within this chapter include MAC, COMAC, methods involving modal curvatures, flexibility-based algorithms, story stiffness approximations, and direct FRF comparisons. Each of these damage indicators were carried out for both sequential and cumulative damage. Sequential damage scenarios include incremental modifications that result in small damage steps. DC 5 vs. CD 6 is utilized throughout this chapter as the main example of sequential damage. Conversely, cumulative damage scenarios are ones that incorporate a large amount of stiffness loss, i.e. DC 0 vs. DC 9. For this particular case DC 0 and DC 9 would be considered the baseline and damaged structures, respectively.

Some suspect experimental data needs to be eliminated before implementing damage detection methods. Since the second mode for DC 4 was unable to be identified, DC 4 is eliminated as a suitable damage case. A complete set of damage indices cannot be formed since DC 4 is missing critical mode shape data for the first bending mode in the Y direction.
Additionally, DC 10 is removed from the damage diagnosis due to poor data fidelity. Despite well-defined mode shapes, initial damage indicator testing showed that DC 10 produces undesirable results with a relatively high amount of noise and false positives. This is due to the significant structural flexibility and weak column coupling. With DC 4 and DC 10 removed from the damage indicator analysis, there are a total of seven sequential and one cumulative damage scenarios examined within this chapter.

3.1 MAC

The first damage indicator employed on the three-story test structure is the modal assurance criterion (MAC). MAC correlates mode pairs of the undamaged and damaged structures. It is a well-documented statistical quantity that is also available directly in Star Modal. The undamaged and damaged structure’s $i^{th}$ mode shape vector, $\phi_i$ and $\phi_i^*$, respectively, are correlated using

$$MAC_i = \frac{|\phi_i \phi_i^*|^2}{(\phi_i^T \phi_i^*) (\phi_i^{*T} \phi_i^*)}.$$  

(Eqn. 1-1)

where $i$ spans from 1 to 5, corresponding to the experimentally obtained mode shapes [20]. For this damage indicator and all subsequent damage indices, the normalized mode shapes will be utilized in all calculations. The normalization method includes dividing each mode shape vector by the maximum quantity contained in that particular vector.
3.1.1 Initial Investigation of MAC

Figure 3-1 graphically compares MAC values calculated utilizing three different portions of the mode shapes (real, imaginary, and magnitude) and those obtained from Star Modal for DC 5 vs. DC 6. In this case, DC 5 is considered the baseline structure and DC 6 the damaged structure.

![Graph of MAC values](image)

Figure 3-1: Calculated MAC values for DC 5 vs. DC 6 using real (blue), imaginary (red), and magnitudes (green) of mode shapes as compared to Star Modal output (purple).

In general, the MAC values obtained from Star Modal are greater than the calculated MAC values using imaginary portions of the mode shapes and less than the MAC values calculated using the mode shape magnitudes. It is evident that MAC values obtained from Star Modal are most similar to the MAC values calculated using mode shape magnitudes. MAC values from Star Modal and mode shape magnitudes both follow a general pattern with mode 3 having the lowest MAC values and modes 2 and 4 possessing the largest MAC values.
The imaginary portions of the mode shapes lead to the greatest range of MAC values, spanning from 0% correlated at mode 3 to 65.3% correlated and mode 4. Additionally, when comparing DC 5 and 6, MAC calculated using imaginary portions of the mode shapes are, on average, 39.7% less than Star Modal's MAC values. Due to extreme MAC values, the imaginary parts of mode shapes should not be used when attempting to detect mode shape changes between damage cases. Additionally, the real portions of the mode shapes are not ideal when calculating MAC values since they produce values that are, on average, 24.3% different from values obtained from Star Modal. Utilizing mode shapes magnitudes in calculations in the most efficient way to determine MAC values. The values calculated with mode shape magnitudes are, on average, only 29% greater than the values obtained by Star Modal. Due to the similar trend behavior as Star Modal's values, MAC should be calculated using mode shape magnitudes.

Similar types of plots as the one contained in Figure 3-1 can be generated for all damage scenarios. Moreover, trends found in MAC values for DC 5 vs. DC 6 are also observed in all damage scenarios. Additionally, the real portions of mode shapes produce MAC values that fluctuate often while the imaginary parts of mode shapes lead to MAC values that are quite extreme with a large range. In all damage scenarios, the MAC values calculated utilizing mode shape magnitudes best matched values obtained by Star Modal. Due to the similarity between Star Modal's MAC values and trends, MAC values calculated for every damage scenario will incorporate mode shape magnitudes.

The Star Modal MAC values can serve as both verification of calculated values and direct use in application. Figure 3-2 compares MAC values for all sequential damage scenarios. Figure 3-2a depicts MAC values calculated utilizing mode shape magnitudes and Figure 3-2b contains Star Modal's MAC values.
When comparing calculated MAC values to those produced within Star Modal, it can be seen that calculated values in Figure 3-2a are generally greater than those in Figure 3-2b. However, the overall trend of the data matches quite well between both parts a and b of Figure 3-2. Fluctuations of MAC values with respect to each mode are similar in each plot. Overall the calculated MAC values depicted in Figure 3-2a are 33% larger than the values obtained by Star Modal. Although damage indicator values differ so greatly as a whole when compared to each other, due to similar data patterns, the calculated MAC values are verified as a reasonable measure of correlation between mode shape sets.

3.1.2 Sequential Damage

Table 3-1 contains the MAC values calculated using mode shape magnitudes. Each column represent how well correlated each damaged case's mode is to the corresponding mode.
for the previous damage case. For example, when comparing DC 2 and 3, DC 2 is considered the baseline structure and DC 3 is the damaged structure. The value contained in the mode 1 column is a measure of how well correlated mode 1 within DC 2 is with mode 1 in DC 3. This correlation attempts quantify changes in mode shapes when cross bracing members are removed.

Table 3-1: MAC Results.

<table>
<thead>
<tr>
<th>Sequential Damage Scenario</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Mode 4</th>
<th>Mode 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC 0 vs. DC 1</td>
<td>0.4983</td>
<td>0.7980</td>
<td>0.5657</td>
<td>0.7331</td>
<td>0.6442</td>
</tr>
<tr>
<td>DC 1 vs. DC 2</td>
<td>0.7039</td>
<td>0.8427</td>
<td>0.8353</td>
<td>0.8153</td>
<td>0.8302</td>
</tr>
<tr>
<td>DC 2 vs. DC 3</td>
<td>0.7312</td>
<td>0.8667</td>
<td>0.7822</td>
<td>0.7925</td>
<td>0.5429</td>
</tr>
<tr>
<td>DC 3 vs. DC 5</td>
<td>0.7659</td>
<td>0.7834</td>
<td>0.7364</td>
<td>0.7447</td>
<td>0.6205</td>
</tr>
<tr>
<td>DC 5 vs. DC 6</td>
<td>0.7305</td>
<td>0.7809</td>
<td>0.5025</td>
<td>0.8080</td>
<td>0.7396</td>
</tr>
<tr>
<td>DC 6 vs. DC 7</td>
<td>0.6119</td>
<td>0.5946</td>
<td>0.7746</td>
<td>0.7850</td>
<td>0.5847</td>
</tr>
<tr>
<td>DC 7 vs. DC 8</td>
<td>0.8668</td>
<td>0.6284</td>
<td>0.8179</td>
<td>0.7749</td>
<td>0.6390</td>
</tr>
<tr>
<td>DC 8 vs. DC 9</td>
<td>0.7990</td>
<td>0.8102</td>
<td>0.4880</td>
<td>0.6069</td>
<td>0.7252</td>
</tr>
</tbody>
</table>

3.1.2.1 DC 0 vs. DC 1

The first row of Table 3-1 contains information about DC 0 vs. DC 1. In this particular case, the lowest MAC value of 0.4983 belongs to mode 1 and indicates that the first mode’s shape changes the most. This makes sense because member 8-15, which braces the structure solely in the X direction, was removed when transitioning from DC 0 to DC 1. Ideally, all Y direction modes (modes 2 and 5) should have MAC values that are relatively high since no braces in the Y direction were removed from the structure. Actually, mode 2 possesses the highest MAC value (0.7980) which matches with what one would expect in this situation. Additionally, the torsion mode (mode 3) exhibits typical behavior; one would expect the torsion mode to be affected because DC 0 is a symmetric structure while DC 1 is asymmetric.
This hypothesis is supported by the relatively low MAC value obtained for mode 3; the MAC value of 0.5657 denotes a 56.57% correlation between mode 3 of DC 0 and mode 3 of DC 1.

3.1.2.2 DC 1 vs. DC 2

The second row of Table 3-1 contains information about DC 1 vs. DC 2. For this sequential damage scenario, member 6-13 was removed when moving from DC 1 to DC 2. Because the member removed supports the structure in the X direction, one would expect to see a relatively low MAC value for mode 1. Calculated MAC values seem to give an expected result; mode 1 has the lowest MAC (0.7039) which means that mode 1 in DC 1 is 70.39% correlated to mode 1 in DC 2. While mode 1 is drastically affected, modes 2-5 seem to show relatively no change in mode shapes with MAC values higher than 0.8000. These high MAC values indicate that modes 2-5 do not change much when transitioning from DC 1 to DC 2. Note that mode 3 was not affected as much as expected; since DC 1 is an asymmetric structure and DC 0 a symmetric structure, the expected MAC value for mode 3 is lower than the actual MAC (0.8353).

3.1.2.3 DC 2 vs. DC 3

The third row of contains information about DC 2 vs. DC 3. Modifications were made solely in the X direction with the removal of member 7-16 was the replacement of member 6-13. This can be seen in the MAC results for mode 1 and mode 2. Mode 1’s MAC value, 0.7039, is almost 15% lower than mode 2’s MAC value (0.8667). This means that mode 1’s shape is more affected than mode 2’s shape, which matches the expected result. Conversely, the torsion mode, mode 3, possesses a MAC value much high than anticipated. Since DC 2 is a symmetric damage
case and DC 3 is an asymmetric case, there should be a significant amount of change to the third mode shape. However, mode 3 has a MAC of 0.7822 which is considered to be relatively high and consequently denotes that the torsion mode is not affected much when transition from DC 2 to DC 3. The damaged structure in this particular case, DC 3, is unique in the sense that it is the only damage case that removes and replaces a member. Perhaps the effect of replacing member 6-13 cannot be directly quantified by MAC values.

3.1.2.4 DC 3 vs. DC 5

The fourth row of Table 3-1 represents MAC values obtained when comparing DC 3 and DC 5. For this scenario, DC 3 is considered to be the baseline structure and DC 5 the damaged structure. The entire first story in DC 5 in not braced and because of this one would expect to see significant changes in the second order bending modes and probably not as much change in the first order bending modes. In reality, this is exactly the case; modes 4 and 5 exhibit MAC values that are all less 0.75, indicating that modes 4 and 5 of DC 3 are only approximately 75% similar to the respective modes in DC 5. The first order bending modes, modes 1 and 2 are not as affected and thus have relatively high MAC values. The torsion mode, mode 3, possesses a relatively low MAC value of 0.7364, indicating a 73.64% correlation between mode 3 in DC 3 and mode 3 in DC 5. This makes sense since DC 3 is an asymmetric structure while DC 5 is symmetric in geometry.

3.1.2.5 DC 5 vs. DC 6

The fifth row in Table 3-1 denotes data for DC 5 vs. DC 6. When moving from DC 5 to DC 6, member 15-24 is removed. Since the removed member spans the X direction, one would
expect to see significant changes in the X bending modes, especially the first bending mode in
the X direction, mode 1. Additionally, the torsion mode 3 should be affected due to the fact that
DC 5 is a symmetric structure while DC 6 is asymmetric. Modes 1 and 3 possess the lowest
MAC values, indicating that the shapes at these modes were significantly altered when
transitioning from DC 5 to DC 6. Mode 1 in DC 5 is approximately 73% similar to mode 1 in
DC 6, which is considered to be a relatively weak correlation.

3.1.2.6 DC 6 vs. DC 7

The sixth row of Table 3-1 represents calculated MAC values for DC 6 vs. DC 7. When
moving from DC 6 to DC 7, one brace is removed within the second story of the structure in the
X direction. Thus, one would expect to see mode 1’s shape to correlate poorly between DC 6
and 7. This is validated by the MAC value obtained for mode 1; there is a 61% correlation
between mode 1 in DC 6 and DC 7. This relatively low correlation denotes that mode 1 was
significantly affected by the removal of the cross brace that spans points 14 and 21. Although no
members were removed in the Y direction, modes 2 and 5, the first and second bending modes in
the Y direction respectively, have relatively low MAC values that could indicate significant
changes in the each mode shape. This does make sense since the MAC values should be
relatively high if no members were removed in the Y direction. Additionally, mode 3’s MAC
value is quite higher than expected. Since DC 6 is asymmetric in geometry and DC 7 a
symmetric structure, ideally the torsion mode should undergo significant changes when
comparing DC 6 and 7. The desired low MAC value for the torsion mode, mode 3, was not
obtained.
3.1.2.7 DC 7 vs. DC 8

The seventh row of Table 3-1 contains calculated MAC values for DC 7 vs. DC 8. For this particular case, DC 7 is considered the baseline structure while DC 8 is the damaged structure. When transitioning from DC 7 to DC 8, the members 13-22 and 16-23 were removed, leaving side 1 and 3 with no cross bracing between the second and third floors. Due to the fact that modifications were made solely in the X direction, one would expect mode 1’s MAC value to be relatively low. However, this is not the case; mode 1’s calculated MAC value is 0.8668 which indicates a fairly strong correlation between mode 1 in DC 7 and mode 1 in DC 8. Due to the removal braces solely spanning the X direction, the MAC value for mode 2 (first bending mode in Y direction) should be relatively high. Lastly, mode 3 possesses a MAC value of 0.7746, which matches the expected result for this damage scenario. The torsion mode, mode 3, should not be greatly affected since both DC 7 and DC 8 are symmetric structures.

3.1.2.8 DC 8 vs. DC 9

The last row of Table 3-1 represents the last sequential damage scenario, DC 8 vs. DC 9. The two members on side 2 between floors 2 and 3 were removed when moving from DC 8 to DC 9, resulting in only two cross bracing members remaining on the structure. Members 14-23 and 15-22, the removed cross braces, span the Y direction and therefore, one would expect to see a dramatic change in the mode shape for mode 2, the first bending mode in the Y direction. In reality, the calculated MAC value for mode 2 is the highest for this sequential damage scenario, indicating a strong correlation between mode 2 of DC 8 and mode 2 of DC 9. This is counter to the expected results of a relatively low MAC value for mode 2. Conversely, the torsion mode matches expected results; the MAC value for mode 3, 0.4880 is extremely low, denoting little
correlation between mode 3's shape in DC 8 and DC 9. This makes sense because DC 8 is a symmetric structure while DC 9 is asymmetric in geometry. When comparing DC 8 and DC 9, it overall appears that the second order modes (modes 4 and 5) are most affected, with MAC values less than 0.7300.

3.1.3 Cumulative Damage

In order to examine the detection of significant damage,

Figure 3-3 contains a plot of the calculated MAC values for the cumulative damage scenario, DC 0 vs. DC 9.

![Figure 3-3: MAC values calculated with mode shape magnitudes for the cumulative damage scenario, DC 0 vs. DC 9.](image)

Figure 3-3 shows that all MAC values for DC 0 vs. DC 9 are less than 0.8000. This is expected, since the entire structure was damaged when progressing from DC 0 to DC 9. All cross bracing was removed from the structure with the exception of members 13-24 and 16-21, which brace the Y direction within the second story of the model building. Mode 1's calculated MAC value
reflects massive changes within the structure; mode 1 in DC 0 is 28.73% similar to mode 1 in DC 9. Conversely, the first bending mode in the Y direction, mode 2, has the highest MAC value for this particular damage scenario. This indicates that there is a 76.06% correlation between mode 2 in DC 0 and mode 2 in DC 9. This could possibly be explained by the incomplete removal of all Y direction cross bracing members. The torsion mode, mode 3 exhibits typical behavior in this situation. Due to the fact that DC 0 is a symmetric structure and DC 9 an asymmetric structure, one would expect mode 3 to possess a relatively low MAC value. In fact, the calculated MAC value for mode 3 indicates that there is only a 40.75% correlation between mode 3 in DC 0 and DC 9. The fourth and fifth mode's MAC values are not as significant as the first order modes' results. The second bending mode in the X direction's MAC value denotes that there is a 60.29% correlation between mode 5 in DC 0 and DC 9. Finally, the fifth mode indicates that there is a 57.27% correlation between the second bending mode in the Y direction in DC 0 and DC 9. Overall, cumulative damage can be detected within the structure with mode 1's MAC value being the most influential and relevant damage indicator.

3.1.4 Conclusions on MAC

General conclusions can be made about utilizing MAC as a damage indicator for the test structure. First, the MAC values obtained from manual programming match well with those obtained from Star Modal; although the calculated MAC values are on average closer to 1 than Star Modal's values, their data trends are quite similar. Modes that showed damage within calculated MAC values also displayed damaged within the Star Modal values set. Second, single story damage (DC 0 vs. DC 1 through DC 3 vs. DC 5) is best indicated by high order modes. This means that modes 4 and 5 are most likely to report low MAC values for sequential damage
scenarios involving damage located solely between the first and second floors. Similarly, multiple story damage scenarios (DC 5 vs. DC 6 through DC 8 vs. DC 9) are best indicated by low order modes. Thus, modes 1, 2, and 3 are most likely to report low MAC values for sequential damage scenarios involving removal of braces solely in between the second and third floors. Third, the torsion mode is extremely sensitive when utilizing MAC as a damage indicator, especially when analyzing sequential damage. The third mode has low MAC values when the baseline structure is symmetric and damaged configuration is asymmetric (or vice-versa). Finally, MAC is sensitive enough to differentiate between sequential and cumulative damage; values obtained for cumulative damage are significantly lower than those obtained for sequential damage. For cumulative damage, the most crucial mode to analyze when examining MAC results is mode 1 as it contributes to the motion of the structure more than any other mode of vibration. Although MAC does not locate damage within the structure, it detects global damage within the structure and reports which modes are most affected by structural modifications.
3.2 COMAC

The next damage indicator utilized on the test structure is called coordinate modal assurance criterion (COMAC). Similar to MAC, coordinate modal assurance criterion calculates mode shape correlation, while incorporating multiple DOFs and identifying damage location. The undamaged and damaged structure’s $i^{th}$ modal displacement at coordinate $j$, $\phi_{ij}$ and $\phi_{ij}^*$, respectively, are the parameters used to calculate COMAC as

$$COMAC_j = \frac{\left( \sum_{i=1}^{m} |\phi_{ij} \cdot \phi_{ij}^*| \right)^2}{\sum_{i=1}^{m} \phi_{ij}^2 \cdot \sum_{i=1}^{m} \phi_{ij}^* \cdot \phi_{ij}^*}.$$  

(Eqn. 1-2)

For the three-story model building, the normalized mode shapes used to calculate MAC were also employed in the COMAC damage detection algorithm.

3.2.1 Initial Investigation of COMAC

A representative sequential damage case for DC 5 vs. DC 6 is examined herein. Figure 3-4 contains a plot of COMAC values calculated using different mode shape components as well as the values obtained from Star Modal. In this case, DC 5 is considered the baseline structure and DC 6 the damaged structure. For this particular example, every calculated COMAC value incorporated all five experimental modes.
In general, the COMAC values obtained from Star Modal are greater than the values calculated using both the real and imaginary portions of the mode shapes. The COMAC values exported from Star Modal are most similar to those values calculated using mode shape magnitudes; both follow a similar trend, rising and falling together with very similar values at each degree of freedom. The lowest values of each occur at 21X and 25X. Although these points may not actually contain damage, it is reassuring to have both COMAC plots match at these points.

The imaginary portions of the mode shapes lead to the greatest range of COMAC values, spanning from 18.9% correlated at 13X to 97.2% correlated at 18Y. Additionally, when comparing DC 5 and 6, COMAC calculated using imaginary portions of the mode shapes are, on average, 23.4% less than Star Modal's COMAC values. Due to these extreme values, the imaginary parts of mode shapes should not be used when attempting to detect mode shape

Figure 3-4: COMAC values calculated using various portions of the mode shapes compared to values obtained from Star Modal for DC 5 vs. DC 6. Real (blue), imaginary (red), magnitude (green), Star Modal (purple).
changes between damage cases at a specific degree of freedom. In addition, the real portions of
the mode shapes are not ideal when calculating MAC values since they produce values that are,
on average, 13.9% lower than values obtained from Star Modal. Utilizing mode shapes
magnitudes in calculations in the most efficient way to determine COMAC values. The values
calculated with mode shape magnitudes are, on average, only 0.7% greater than the values
obtained by Star Modal. Due to the similar trend behavior as Star Modal's values, COMAC
should be calculated using mode shape magnitudes.

Plots similar to Figure 3-4 can be generated for all damage scenarios. Moreover, trends
found in COMAC values for DC 5 vs. DC 6 are also observed in all damage scenarios.
Additionally, the real portions of mode shapes produce COMAC values that fluctuate often while
the imaginary parts of mode shapes lead to COMAC values that are quite extreme with a large
range. In all sequential damage scenarios, the COMAC values calculated utilizing mode shape
magnitudes best matched values obtained by Star Modal. Due to the similarity between Star
Modal's MAC values and trends, MAC values calculated for every damage scenario will
incorporate mode shape magnitudes.

The number of modes included in the COMAC analysis is an important choice: too few
causes a lack of spatial results, but too many may generate false positives. Figure 3-5
graphically compares COMAC values using various combinations of modes and values obtained
from Star Modal. COMAC was calculated utilizing modes 1-5, modes 1-3, and modes 1 and 2.
For this particular example, all calculated COMAC values were derived from mode shape
magnitudes.
Figure 3-5: COMAC values calculated using various combinations of modes compared to values from Star Modal for DC 5 vs. DC 6. All modes (blue), modes 1-3 (red), modes 1 and 2 (green), Star Modal (purple).

Overall, COMAC values calculated using all five modes match Star Modal values the best. COMAC values incorporating only modes 1 and 2 are extremely low and are, on average, 47% less than values obtained from Star Modal. The COMAC values calculated using modes 1 and 2 are too low to be considered a viable measure of mode shape correlation at each degree of freedom. Degrees of freedom that are not damaged should have relatively high COMAC values, and this is not observed in the values calculated using modes 1 and 2. Conversely, modes 1, 2, and 3 produce COMAC values that are fairly similar to those exported from Star Modal. However, only using modes 1, 2, and 3 is an incomplete analysis of the system. Many of the degrees of freedom in the X direction exhibit COMAC values that are completely different from those calculated using all modes. Overall, there is only a 7.7% difference between COMAC values calculated using all five experimental modes and incorporating only modes 1, 2, and 3.
A similar comparison can be made with all damage scenarios, and COMAC values calculated using all modes and mode shape magnitudes are generally most similar to those obtained from Star Modal. DC 7 vs. DC 8 is a singular exception; Figure 3-6 contains a graphical comparison for this case.

Figure 3-6: Calculated COMAC values (dark grey) compared to COMAC obtained from Star Modal (light grey) for DC 7 vs. DC 8.

Some of DC 7 vs. DC 8's calculated COMAC values align well with Star Modal values but some degrees of freedom conflict. In particular, 7X, 12X, 18X, 24X, 7Y, and 10Y's calculated COMAC values and Star Modal COMAC values greatly differ. At these particular degrees of freedom, the calculated COMAC "peaks" while Star Modal's COMAC negative "peaks," and vice-versa. Specifically, at 12X there is a 22.2% difference between Star Modal's COMAC and the calculated COMAC value. One of the most dramatic discrepancies occurs at 10Y. The calculated value of 0.92 indicates that there is a strong correlation between the mode shape coordinates at 10Y; however, Star Modal's COMAC value of 0.56 suggests that the mode shape...
shapes are poorly correlated at point 10 in the Y direction. Fortunately, this repeating pattern of opposing peaks is not observed in any other damage scenario. Moreover, calculated COMAC values herein will incorporate all five mode shapes due to their similarities with COMAC values obtained from Star Modal.

3.2.2 Sequential Damage

Calculated COMAC values were used to predict damage location within the structure for each sequential damage scenario. COMAC values less than 0.65 are considered to occur at locations that have poor correlations between all five experimental mode shapes' coordinates of the baseline and damaged structure. These particular degrees of freedom are probable locations of damage. Similarly, locations that may be damaged have COMAC values that are between 0.65 and 0.75. At these degrees of freedom the COMAC values indicate a medium correlation between the undamaged and damaged structure's mode shape coordinates. Table 3-2 summarizes the results obtained from carrying out the COMAC algorithm on all sequential damage scenarios. The column labeled "Damaged DOFs" contains the attachment points and directions of removed members.
### Table 3-2: Sequential Damage Prediction with COMAC.

<table>
<thead>
<tr>
<th>Sequential Damage Scenario</th>
<th>Damaged DOFs</th>
<th>Poor Correlation (COMAC&lt;0.65)</th>
<th>Some Correlation (0.65&lt;COMAC&lt;0.75)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DC 0 vs. DC 1</strong></td>
<td>8X, 15X</td>
<td>19X, 20X, 24X, 26X, 7Y</td>
<td>5X, 7X, 9X, 11X, 15X, 22X, 25X, 28X, 6Y, 9Y</td>
</tr>
<tr>
<td><strong>DC 1 vs. DC 2</strong></td>
<td>6X, 13X</td>
<td>20X</td>
<td>13X, 16X, 7Y</td>
</tr>
<tr>
<td><strong>DC 2 vs. DC 3</strong></td>
<td>7X, 16X (add: 6X, 13X)</td>
<td></td>
<td>12X, 16X, 5Y, 6Y, 7Y</td>
</tr>
<tr>
<td><strong>DC 5 vs. DC 6</strong></td>
<td>15X, 24X</td>
<td>21X, 25X</td>
<td>5X, 14X, 20X, 10Y</td>
</tr>
<tr>
<td><strong>DC 7 vs. DC 8</strong></td>
<td>13X, 16X, 22X, 23X</td>
<td>8X, 16X, 8Y, 12Y, 16Y</td>
<td>20X, 25Y</td>
</tr>
<tr>
<td><strong>DC 8 vs. DC 9</strong></td>
<td>14Y, 15Y, 22Y, 23Y</td>
<td>16X, 17X, 18X, 20X</td>
<td>5X, 7X, 8X, 9X, 12X, 22X, 24X, 26X, 5Y, 10Y, 14Y</td>
</tr>
</tbody>
</table>

#### 3.2.2.1 DC 0 vs. DC 1

The first row of Table 3-2 contains COMAC damage detection data for DC 0 vs. DC 1. Only 2 DOFs, 8X and 15X, are expected to exhibit damaged with the removal of member 8-15. 15X is detected as a possible damage location since its COMAC value falls between 0.65 and 0.75. Points 7 and 9 show possible damage in both the X and Y directions. Points 7 and 9 are both located within the first story and could be directly affected by changes at points 8 and 15. The algorithm identified several false positives, including 25X, 26X, and 28X, which are situated at the very top of the structure; these are not near actual structural damage but are close to the measurement point. Thus, signal noise is considered the error source. It is also interesting to
note that most of the identified possible damage locations are DOFs in the X direction. This makes sense since modifications were in the X direction when moving from DC 0 to DC 1 with the removal of member 8-15. Figure 3-7 pictorially represents the results tabulated in Table 3-2 for DC 0 vs. DC 1: the arrows in the drawing represent identified damaged locations. These vectors are either pointing in the X or Y direction, and damage extent is shown as either red (COMAC<0.65) or orange (0.65<COMAC<0.75). The actual structurally damaged locations are denoted by blue stars. The degrees of freedom that are affected in the system are labeled in black. The corresponding COMAC values for DC 0 vs. DC 1 are presented in Figure 3-8.

Figure 3-7: Schematic of possible damage locations for DC 0 vs. DC 1 using COMAC.
The lowest COMAC value for this particular case occurs at 20X which suggests that 20X is the most damaged DOF in when comparing DC 0 and DC 1. At point 20 in the X direction, there is a 48.6% correlation between modal coordinates of DC 0 and DC 1. The highest COMAC value of 0.972 occurs at 12Y, which indicates that point 12 in the Y direction is the least damaged DOF. In general, there is more variability in COMAC for the X direction DOFs than for the Y direction DOFs. This could be explained by most of the damage occurring in the X direction.

3.2.2.2 DC 1 vs. DC 2

The second row in Table 3-2 contains COMAC damage detection data for DC 1 vs. DC 2. Since member 6-13 was removed while transitioning from DC 1 to DC 2, the actual structurally damaged DOFs in the case are 6X and 13X. 13X is detected as a possible damage location since its COMAC value falls between 0.65 and 0.75. 5X, 9X, 11X, 13X, 16X, and 7Y
are all located within the first story and may be possible damage locations. This makes sense because point 6 and 13 are also within the first story and any bracing and columns around member 6-13 could be affected by its removal. An unusually high amount of damage is predicted to occur at point 20 in the X direction. Although point 20 is on the opposite side of the structure than member 6-13, it is identified as a likely damaged DOF. Nonetheless, since structural modifications were made in the X direction, point 20 in the X direction could have been affected by the removal of member 6-13, albeit indirectly. Figure 3-9 pictorially represents the tabulated for DC 1 vs. DC 2; the same representation scheme is used as in Figure 3-7.

Figure 3-9: Schematic of possible damage locations for DC 0 vs. DC 1 using COMAC.
The corresponding plot of calculated COMAC values for DC 1 vs. DC 2 is displayed in Figure 3-10.

The lowest COMAC value for this particular case, 0.607, occurs at 20X which suggests that 20X is the most damaged DOF when comparing DC 1 and DC 2. At point 20 in the X direction, there is a 60.7% correlation between modal coordinates of DC 0 and DC 1. The highest COMAC value of 0.982 occurs at 14Y, which indicates that point 14 in the Y direction is the least damaged DOF. In general, there is more variability in COMAC for the X direction DOFs than for the Y direction DOFs. This could be explained by most of the damage occurring in the X direction.

3.2.2.3 DC 2 vs. DC 3

The third row of Table 3-2 contains COMAC damage detection data for DC 2 vs. DC 3. For this damage scenario one brace (member 7-16) was removed and member 6-13 was replaced.
on the structure. Thus, differences are expected to occur at points 6, 7, 13, and 16. The COMAC algorithm predicted that there was damage at points 13 and 16 in the X direction. These are the exact DOFs directly affected with the removal of one member and the replacement of another. Point 7 in the Y direction is also detected as a possible damage location with a COMAC between 0.65 and 0.75. This makes sense since point 7 is directly affected with the removal of member 6-7 and could also be affected by the addition of brace 6-13. Most of the identified possible damage locations are DOFs in the X direction. This makes sense since modifications were in the X direction when moving from DC 2 to DC 3 with the removal of member 8-15. Figure 3-11 pictorially represents the tabulated results for DC 2 vs. DC 3; the same representation scheme is used as in Figure 3-7. The corresponding plot of the calculated COMAC values for DC 2 vs. DC 3 is displayed within Figure 3-12.
Figure 3-11: Schematic of possible damage locations for DC 2 vs. DC 3 using COMAC.
The lowest COMAC value for this particular case, 0.653, occurs at 6Y which suggests that 6Y is the most damaged DOF in when comparing DC 2 and DC 3. At point 6 in the Y direction, there is a 65.3% correlation between modal coordinates of DC 2 and DC 3. The highest COMAC value of 0.990 occurs at 25X, which indicates that point 25 in the X direction is the least damaged DOF. In general, there is more variability in COMAC for the X direction DOFs than for the Y direction DOFs. This may be explained by most of the damage occurring in the X direction.

3.2.2.4 DC 3 vs. DC 5

The fourth row of Table 3-2 contains COMAC damage detection data for DC 3 vs. DC 5. Since members 5-14, 5-16, 6-13, 6-15, 7-16, 7-14, and 8-13 were removed when moving from DC 3 to DC 5, one would expect all DOFs between the first and second floor to be directly
affected. COMAC results should indicate possible damage locations at DOFs contained between the first and second floors. Eleven out of the eighteen identified possible damaged DOFs are located between the first and second floors. Overall, the algorithm identified the general area of most of the damage to the structure. However, the COMAC damage indicator also identified several false positives, including 21 and 25 in both the X and Y directions as well as 24X and 28X. All these DOFs are located at the top of the structure, far from any structural modification but near to the measurement point. Figure 3-13 pictorially represents the tabulated results for DC 3 vs. DC 5.

Figure 3-13: Schematic of possible damage locations for DC 3 vs. DC 5 using COMAC.
The corresponding plot of calculated COMAC values for DC 3 vs. DC 5 is displayed in Figure 3-14.

The lowest COMAC value of 0.331 occurs at 21X which suggests that 21X is the most damaged DOF in when comparing DC 3 and DC 5. At point 21 in the X direction, there is a 33.1% correlation between modal coordinates of DC 3 and DC 5. The highest COMAC value of 0.967 occurs at 5Y, which indicates that point 5 in the Y direction is the least damaged DOF. This does not agree with the expect result of 5Y being one of the most damaged DOFs. Since member 5-16 was removed when transitioning from DC 3 to DC 5, 5Y should be directly affected and identified as a possible damage location. There is variability in COMAC for both directions due to the structure being damaged in both the X and Y directions.
3.2.2.5 DC 5 vs. DC 6

The fifth row of Table 3-2 contains COMAC damage detection data for DC 5 vs. DC 6. Since member 15-24 was removed when moving from DC 5 to DC 6, one would expect the X direction DOFs between the second and third floors to be directly affected. The most severe damage is located at points 21 and 25 in the X direction. These COMAC values indicate that the mode shapes in the X direction, near the top of the structure are poorly correlated. In addition to 21X and 25X, the algorithm also identified moderate damage at points 14 and 20 in the X direction. These DOFs could be affected by the removal of member 15-24 since they are adjacent to points 15 and 24, respectively. Although the general area of actual damage within the structure is effectively located, there are some locations that are identified as damaged DOFs that do not seem logical. More specifically, 5X and 10Y may be false positive damage locations. Figure 3-15 pictorially represents the tabulated the results for DC 5 vs. DC 6.
Figure 3-15: Schematic of possible damage locations for DC 5 vs. DC 6 using COMAC.
A plot of calculated COMAC values for DC 5 vs. DC 6 is displayed in Figure 3-16.

The lowest COMAC value of 0.540 occurs at 21X which suggests that 21X is the most damaged DOF in when comparing DC 5 and DC 6. At point 21 in the X direction, there is a 54% correlation between modal coordinates of DC 5 and DC 6. The highest COMAC value of 0.987 occurs at 26Y, which indicates that point 27 in the Y direction is the least damaged DOF. The top of the structure is expected to be most affected by the removal of member 15-24 which is contrary to the experimental results. 26Y may be affected by both brace removal on the same face as well as measurement noise. In general, there is more variability in COMAC for the X direction DOFs than for the Y direction DOFs. This may explained by the actual damage occurring in the X direction.
3.2.2.6 DC 6 vs. DC 7

The sixth row of Table 3-2 contains COMAC damage detection data for DC 6 vs. DC 7. Since member 14-21 was removed when moving from DC 6 to DC 7, one would expect DOFs in the X direction between the second and third floor to be directly affected. A majority of the identified damage locations are contained between the first and second floors, contrary to the expected results. However, some of the most severe damage occurs in the X directions at points 5 and 8. These DOFs could be indirectly affected by the removal of cross bracing member supporting the structure in the X direction. The algorithm correctly identifies points 14 (X direction) and 21 (X and Y directions) as possible damage locations. Although the removed member location can be identified, there are several DOFs that provide false positives. Most of the DOFs between the first and second floors, especially those in the Y direction are not expected damage locations. Figure 3-17 pictorially represents the tabulated results for DC 6 vs. DC 7.
Figure 3-17: Schematic of possible damage locations for DC 6 vs. DC 7 using COMAC.
The lowest COMAC value of 0.562 occurs at 25Y which suggests that 25Y is the most damaged DOF in when comparing DC 6 and DC 7. At point 25 in the Y direction, there is a 56.2% correlation between modal coordinates of DC 6 and DC 7. Point 25 is adjacent to point 21, thus it is reasonable to have significant changes in the mode shape coordinates. The highest COMAC value of 0.988 indicates that point 27 in the Y direction is the least damaged DOF.

3.2.2.7 DC 7 vs. DC 8

The seventh row of Table 3-2 contains COMAC damage detection data for DC 7 vs. DC 8. Since members 13-22 and 16-23 were removed when transitioning from DC 7 to DC 8, one would expect DOFs between the second and third floor to be directly affected. Point 16 in both the X and Y directions is identified as possible damage locations and have COMAC values less than 0.65. This matches the expected result of relatively low COMAC values for the points

Figure 3-18: Calculated COMAC values for DC 6 vs. DC 7.
directly damaged by cross bracing removal. The algorithm also identified 8X, 8Y, and 12Y as points with severe damage. Although these points were not directly affected by brace removals, they reside on the same column as point 16 and, therefore, could be indirectly affected. Additionally, 20X and 25Y were identified as locations with moderate damage. These COMAC values make sense since these DOFs are near the top of the structure and close to the actual damage. Figure 3-19 pictorially represents the tabulated results for DC 7 vs. DC 8.

Figure 3-19: Schematic of possible damage locations for DC 7 vs. DC 8 using COMAC.
A plot of calculated COMAC values for DC 7 vs. DC 8 is displayed in Figure 3-20.

Figure 3-20: Calculated COMAC values for DC 7 vs. DC 8.

The lowest COMAC value of 0.293 occurs at 16Y which suggests that 16Y is the most damaged DOF in when comparing DC 7 and DC 8. At point 16 in the Y direction, there is a 29.3% correlation between modal coordinates of DC 7 and DC 8. This agrees with expected results since point 16 is directly affected by the removal of member 16-23. The highest COMAC value of 0.995 occurs at 7Y, which indicates that point 7 in the Y direction is the least damaged DOF. This result is expected since the actual damage is nowhere near point 7. There is variability in COMAC for the both the X and Y direction DOFs indicating that the structure was affected in both directions while actual damage was implemented solely in the X direction.

3.2.2.8 DC 8 vs. DC 9

The eighth row of Table 3-2 contains COMAC damage detection data for DC 8 vs. DC 9. Since members 14-23, and 15-22 were removed when moving from DC 8 to DC 9, one would
expect possible damage locations to occur at DOFs between the second and third floors of the structure. COMAC values for DOFs contained within the top half of the building should be relatively low. The COMAC algorithm identified points 16, 17, 18, and 20 in the X direction as severely damaged locations. Additionally, moderate damage is indicated at point 22, 24, and 26 in the X direction and at point 14 in the Y direction. All DOFs between the second and third floor identified as damaged locations are logical areas that could be affected by the removal of two cross bracing members. Although one would expect the Y direction DOFs to be most affected, more X direction DOFs are identified as damaged compared to Y direction DOFs. It is also interesting to note the amount of moderately damaged DOFs that were detected between the first and second floors; more specifically, 5X, 5Y, 7X, 8X, 9X, 10Y, and 12Y were recognized as DOFs with moderate damage. These DOFs could be affected due to the increase in torsion when moving from a symmetric (DC 8) to an asymmetric (DC 9) structure. Figure 3-21 pictorially represents the tabulated results for DC 8 vs. DC 9.
Figure 3-21: Schematic of possible damage locations for DC 8 vs. DC 9 using COMAC.
A plot of calculated COMAC values for DC 8 vs. DC 9 is displayed in Figure 3-22.

Figure 3-22: Calculated COMAC values for DC 8 vs. DC 9.

The lowest COMAC value of 0.087 occurs at 16X which suggests that 16X is the most damaged DOF in when comparing DC 8 and DC 9. At point 16 in the X direction, there is an 8.7% correlation between modal coordinates of DC 8 and DC 9. This agrees with the expected result of low COMAC values for DOFs between the second and third floors. Conversely, the highest COMAC value of 0.975 occurs at 23X, which indicates that point 23 in the X direction is the least damaged DOF. This does not agree with the expect result of 23X being one of the most damaged DOFs. Since member 14-23 was removed when transitioning from DC 8 to DC 9, 23X should be directly affected and identified as a damage location. There is more variability in the COMAC values for X direction DOFs than Y direction DOFs. This contradicts with what one may expect; although cross bracing members that support the Y direction of the structure were removed, most damage was identified in X direction DOFs. This again indicates that torsion is a significant mode this sequential damage scenario.
3.2.3 Cumulative Damage

Table 3-3 summarizes the degrees of freedom that were identified by the COMAC damage detection algorithm for the cumulative damage scenario, DC 0 vs. DC 9.

<table>
<thead>
<tr>
<th>Cumulative Damage Scenario</th>
<th>Damaged DOFs</th>
<th>Poor Correlation (COMAC&lt;0.65)</th>
<th>Some Correlation (0.65&lt;COMAC&lt;0.75)</th>
</tr>
</thead>
</table>

Table 3-3 contains COMAC damage detection data for DC 0 vs. DC 9. Since all cross bracing members except 13-24 and 16-21 were removed when transitioning from DC 0 to DC 9, one would expect possible damage locations to occur at all degrees of freedom. The COMAC algorithm identified most points on side 2 in the Y direction of the model building as severe damaged locations. This makes sense due to the fact that some cross bracing remains on the building on side 4, directly opposite of side 2. The remaining cross bracing members on side 4 creates more torsion movement in the building because DC 0 is considered to be a symmetric structure and DC 9 an asymmetric structure. This torsion movement is evident by the abundance of Y direction severe damage locations. Additionally, severe and moderate damage is indicated at points along side 4, especially in the X direction. Since cross braces remain on side 4 between the second and third floors, it is expected that X direction DOFs along side 4 will be identified as damaged. Therefore, the experimental results agree well with the expected damaged locations. Figure 3-23 pictorially represents the results tabulated in Table 3-3 for DC 0 vs. DC 9.
The corresponding plot of the calculated COMAC values for DC 0 vs. DC 9 is displayed within Figure 3-24.

Figure 3-23: Schematic of possible damage locations for DC 0 vs. DC 9 using COMAC.
Figure 3-24: Calculated COMAC values for DC 0 vs. DC 9.

The lowest COMAC value of 0.239 occurs at 10Y which suggests that 10Y is the most damaged DOF when comparing DC 0 and DC 9. At point 10 in the Y direction, there is a 23.9% correlation between modal coordinates of DC 0 and DC 9. This agrees with the expected result of low COMAC values for Y direction DOFs on side 2. Conversely, the highest COMAC value of 0.961 occurs at 17Y, which indicates that point 17 in the Y direction is the least damaged DOF. This matches the expected result of no identified damage between the second and third floor on side 4 due to the fact that cross braces remain on the structure and support the building in Y direction. There is variability in the COMAC values for both X and Y direction DOFs, indicating that damage is predicted at multiple locations throughout the structure in both the X and Y directions.
3.2.4 Conclusions on COMAC

Several general conclusions can be made about COMAC as a damage indicator. The calculated COMAC values correlate well with COMAC data exported from Star Modal, verifying that the manual programming used to calculate COMAC is accurate. In addition, it is evident that this particular algorithm can successfully locate and quantify damage within the test structure. This method usually identifies general areas of damage but has difficulties pinpointing the exact location of damage, especially with incremental damage. Although general areas and directions of damage can be predicted, the algorithm may produce a great amount of false positives which further decreases the precision of this method. The final observation and perhaps the most significant is that COMAC identified a greater amount of damaged locations for cumulative damage than any sequential scenario. Thus, COMAC can differentiate between incremental and cumulative damage.

3.3 Modal Curvatures

Modal curvature is mathematically defined as the second derivative of the modal displacement function and is thus quite sensitive to damage. The occurrence of damage reduces stiffness and ultimately increases the curvature. The change in mode shape curvature between two damage states of a structure may effectively estimate the level of damage present. The curvature of the $i^{th}$ mode shape at the $j^{th}$ measurement coordinate of an undamaged structure, $\kappa_{ij}$, is approximated with the central difference method:

$$\kappa_{ij} = \frac{\phi_i(j+1) - 2\phi_{ij} + \phi_i(j-1)}{h^2},$$

(Eqn. 1-8)

where $h$ is the distance between the coordinates of the measurements and $\phi_{ij}$ is the modal displacement of coordinate $j$ at mode $i$. Eqn. 1-8 can be similarly applied to the damaged
structure in order to determine the mode shape curvature, $\kappa_{ij}$, of the $i^{th}$ mode shape at the $j^{th}$ measurement coordinate of a damaged structure. The mode shapes used in this term are those obtained through experimental measurement of the damaged state. All mode shapes utilized in the calculation of the modal curvatures are ones that have been normalized to a magnitude of 1 by dividing the all modal coordinates by the maximum displacement within that particular mode shape column matrix.

3.3.1 COMAC of Curvatures

In order to compare the curvatures of the undamaged structure to the damaged structure, a correlation coefficient is calculated. This correlation measure is considered to be the COMAC of curvatures and is mathematically defined by

$$COMAC of \kappa(j) = \frac{|\sum_{j=1}^{m} \kappa_{ij} \cdot \kappa_{ij}^*|^2}{\sum_{j=1}^{m} \kappa_{ij}^2 \cdot \sum_{j=1}^{m} \kappa_{ij}^*^2}.$$  (Eqn. 1-11)

The total number of compared modes is defined by the variable $m$. For this experiment, there are five total experimental modes. The result from this damage detection algorithm are values from 0 (no correlation) to 1 (complete correlation) representing a measure of similarity between two sets of mode shape curvatures. Excluding points 5 and 28, each degree of freedom has a calculated COMAC value that could indicate possible damage at that particular location on the structure. Points 5 and 28 are not included in these calculations due to the nature of the curvature algorithm; as seen in Eqn. 1-8, the modal curvature at a particular point is numerically estimated as a function of the locations of the original point and two neighboring points. Since points 1, 2, 3, and 4 are situated on the structure's foundation, they are considered fixed boundary
points. Thus, points 6 and 27 are the first and last degree of freedoms that can truly be examined by this damage detection technique.

3.3.1.1 Example of Sequential Damage: DC 5 vs. DC 6

Figure 3-25 displays the plot of COMAC values calculated using modal curvatures for DC 5 vs. DC 6. In this case, DC 5 is considered to be the baseline case, while DC 6 is the damaged structure.

![Figure 3-25: COMAC values calculated using the mode shape curvatures for DC 5 vs. DC 6.](image)

Figure 3-25: COMAC values calculated using the mode shape curvatures for DC 5 vs. DC 6.

The highest value of COMAC within Figure 3-25 is 0.99 at point 24. This suggests that the modal curvatures at point 24 for DC 5 are 99% similar to those at point 24 for DC 6. This result does not agree with the expected result of point 24 having the most damage since member 15-24 was removed from the structure when transitioning from DC 5 to DC 6. The lowest COMAC value of 0.57 at point 7 indicates that there is only a 57% correlation between the mode
shape curvatures at point 7 for DC 5 and DC 6. This lack of correlation could be explained by point 7 location along the same column as point 15. Since point 15 was directly affected by the removal of member 15-24, it makes sense that point 7 could experience some residual effects. Thus, this high curvature COMAC values appear more suspect than lower values.

Figure 3-26 is a pictorial representation of the damage detection algorithm's results when comparing DC 5 and DC 6. Blue stars represent the endpoints of the removed member while the orange and red dots indicate moderate and severe damage predictions, respectively.

![Figure 3-26: Predicted damage locations using the COMAC of Curvatures for DC 5 vs. DC 6.](image)

Damage is predicted to have occurred only on two columns of the structure. Points 7 and 11 make logical sense as damaged locations since they are collinear with point 15. Point 21 is
adjacent to the directly affected point 24, indicating a general area of damage. However, points 13 and 17 may be indirectly affected, especially within the first mode or the X direction motion since the removed brace supported the structure in the X direction.

3.3.1.2 Sequential Damage

Table 3-4 contains the results for all sequential damage scenarios when using the COMAC of curvatures as a damage indicator. DOFs with COMAC values less than 0.65 are considered to be locations with severe damage. Similarly, COMAC values between 0.65 and 0.75 indicate moderate damage. These thresholds were used in order to be able to make direct comparisons to the previously calculated COMAC (of direct mode shapes) values. If slightly higher thresholds are used, the COMAC of the curvatures algorithm would successfully identify at least one possible damage location for every sequential damage scenario.

<table>
<thead>
<tr>
<th>Sequential Damage</th>
<th>Damaged DOFs</th>
<th>Severe Damage (COMAC&lt;0.65)</th>
<th>Moderate Damage (0.65&lt;COMAC&lt;0.75)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC 0 vs. DC 1</td>
<td>8X, 15X</td>
<td>18</td>
<td>8, 12, 14, 16, 20</td>
</tr>
<tr>
<td>DC 1 vs. DC 2</td>
<td>6X, 13X</td>
<td>14</td>
<td>--</td>
</tr>
<tr>
<td>DC 2 vs. DC 3</td>
<td>7X, 16X (add: 6X, 13X)</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>DC 3 vs. DC 5</td>
<td>5X, 5Y, 6X, 6Y, 7X, 7Y, 8Y, 13X, 13Y, 14X, 14Y, 15Y, 16Y</td>
<td>22</td>
<td>26</td>
</tr>
<tr>
<td>DC 5 vs. DC 6</td>
<td>15X, 24X</td>
<td>7, 17</td>
<td>11, 13, 21</td>
</tr>
<tr>
<td>DC 6 vs. DC 7</td>
<td>14X, 21X</td>
<td>7</td>
<td>9, 17</td>
</tr>
<tr>
<td>DC 7 vs. DC 8</td>
<td>13X, 16X, 22X, 23X</td>
<td>7, 9, 13</td>
<td>17</td>
</tr>
<tr>
<td>DC 8 vs. DC 9</td>
<td>14Y, 15Y, 22Y, 23Y</td>
<td>9, 13, 18, 22</td>
<td>19, 21, 26</td>
</tr>
</tbody>
</table>
The resulting COMAC values for every sequential damage case are graphically displayed within Figure 3-27. The orange and red lines on the graph represent the threshold values for moderate and severe damage, respectively.

![Figure 3-27: COMAC of Curvatures for all sequential damage scenarios. Darker intensity represents data for DC 0 vs. DC 1 while lightest intensity represents data for DC 8 vs. DC 9.](image)

Examining both Table 3-4 and Figure 3-27 together leads to several observations. DC 2 vs. DC 3 has the smallest range of COMAC values with the maximum of 0.981 occurring at point 26 and a minimum of 0.781 at point 7. The small range indicates that there is very little change in mode shape curvatures when moving from DC 2 to DC 3. One must note that one cross brace was removed and one replaced when transitioning from DC 2 to DC 3 and could explain why DC 2 and DC 3 have such similar mode shape curvatures. Conversely, DC 7 vs. DC 8 has the largest range of COMAC values; the minimum value of 0.391 is located at point 9 while the maximum COMAC value of 0.994 is achieved at point 15. For this sequential damage scenario, two members were removed when transitioning from DC 7 to DC 8. In general, when
comparing two damage cases the higher the number of braces removed between them, the larger the COMAC range becomes. This could be due to more severe damage occurring when multiple braces are removed than just when one is removed. Additionally, most of the severe damage occurs in sequential damage scenarios DC 7 vs. DC 8 and DC 8 vs. DC 9. More specifically, points 9 and 13 are identified as severe damaged locations in both sequential damage scenarios. This could be due to the direct effect of removing cross bracing connecting floor 2 and floor 3. Point 9 is located along the same column as point 13 and could be indirectly affected by any damage inflicted between the second and third floors of the structure.

Overall, trends in the data can be compared and contrasted between various damage scenarios, but the actual damaged locations predicted do not, in general, line up with expected results. However, a general area of damage, although more imprecise than regular COMAC, can successfully be predicted.
3.3.1.3 Cumulative Damage

Figure 3-28 displays the plot of COMAC values calculated using modal curvatures for DC 0 vs. DC 9. In this case, DC 0 is considered to be the baseline case while DC 9 is the damaged structure.

![Figure 3-28: COMAC of Curvatures for DC 0 vs. DC 9.](image)

The highest value of COMAC within Figure 3-28 is 0.919 at point 10. This suggests that the modal curvatures at point 10 for DC 0 are 92% similar to those at point 24 for DC 9. This result does not agree with the expected result of the majority of the DOFs showing some sort of damage due to the fact that 14 cross bracing members, all over the structure, were removed when moving from DC 0 to DC 9. Conversely, the lowest COMAC value of 0.417 at point 22 indicates that there is only a 42% correlation between the mode shape curvatures at point 22 for DC 0 and DC 9. This lack of correlation could be explained by point 22's decrease in stiffness due to the removal of brace 13-22.
In addition to the direct stiffness change, point 22 could be experiencing the most motion within the structure since it is located at the top of the model, adding to the flexibility of the DOF and therefore, changing its curvature significantly.

Perhaps the most important piece of information extracted from this analysis is the number of identified damage locations in the cumulative damage scenario. Twelve separate DOFs were identified as possible damaged locations within the cumulative damage scenario. This is far greater than the number of predicted damage locations obtained for any sequential damage scenario. Since the total damage via removed members occurred in the cumulative scenario is much greater than the amount damage induced on any sequential scenario, the damage detection algorithm is expected to identify more possible damaged locations within the cumulative scenario than any sequential scenario.

Figure 3-29 is a pictorial representation of the damage detection algorithm's results when comparing DC 0 and DC 9. Blue stars represent the endpoints of removed members while the orange and red dots indicate moderate and severe damage predictions, respectively.
Figure 3-29: Predicted damage locations using the COMAC of Curvatures for DC 0 vs. DC 9.

Damage is predicted to have occurred all over the structure. Severe damage is detected at DOFs above the second floor, indicating that the top of the structure is more damaged than the bottom. Points 8, 15, 16, and 22 were accurately identified as damaged locations. Members were removed from these four points and consequently, they are expected to show damage. Although the COMAC of curvatures correctly identified four directly affected DOFs, it failed to recognize seven other locations where cross bracing was removed.
3.3.2 Modal Curvature Division

Although COMAC is a good measure of correlation, there are several other ways to algebraically manipulate mode shape curvatures to form a damage index. One method is to divide the mode shape curvature matrix of the damaged structure, $\kappa_{ij}^*$, by the mode shape curvature matrix of the baseline structure, $\kappa_{ij}$, to form a damage indicator that predicts damage at varying degrees of freedoms and mode. The mode shape curvature matrices are calculated using Eqn. 1-8 and the actual damage indicator, $\Delta\kappa_{ij}$, calculated using the following equation:

$$\Delta\kappa_{ij} = \frac{\kappa_{ij}^*}{\kappa_{ij}}.$$

(Eqn. 3-1)

After calculating the damage indicator matrix, $\Delta\kappa_{ij}$, the entire matrix is normalized with respect to the mean and standard deviation of all the values within the matrix. This converts the data to the standard normal distribution with a mean of 0 and a standard deviation of 1. Next, the absolute values of all normalized values were calculated: these positive standard normalized values are considered to be a damage indicator.

3.3.2.1 Example of Sequential Damage: DC 5 vs. DC 6

Figure 3-30a contains a plot representing the results of dividing the modal curvatures matrices to form a damage indicator for DC 5 vs. DC 6. The thick black line represents all damage indicator values for mode 3. The corresponding plot in Figure 3-30b is a 2-D plot of mode three's values.
Figure 3-30: For DC 5 vs. DC 6, (a) 3-D plot of damage indicator DI via dividing modal curvatures and (b) 2-D plot outlining DI values at Mode 3.
The highest value within Figure 3-30a is 10.11 at point 13 in mode 5. This means that the modal curvature at point 13 in the fifth mode for DC 6 is extremely different than the modal curvature at point 13 in the fifth mode for DC 5. The damage indicator at point 13 in mode 5 is approximately 10 standard deviations from the mean damage indicator value, meaning that this value is extreme compared to the rest of the results for DC 5 vs. DC 6. Point 13 in mode 5 is considered to be the most damaged DOF within the structure when transitioning from DC 5 to DC 6. Conversely, the lowest value within Figure 3-30a of 0.005 occurs at point 21 in the fifth mode. This indicates that modal curvatures at point 21 in the fifth mode for DC 5 and DC 6 are quite similar and that there is no damage identified there.

Figure 3-31 is a pictorial representation of the damage detection algorithm's results when comparing DC 5 and DC 6. Blue stars represent the endpoints of the removed member while locations identified by the algorithm as damaged are indicated by a series of arrows and colors corresponding to Table 3-5. Severe damage occurs when the value of the damage indicator exceeds 3, while moderate damage happens when the damage indicator is between 1.5 and 3. Any value lower than 1.5 is considered undamaged. These thresholds were picked keeping the standard normal distribution in mind.
Figure 3-31: Predicted damage locations by dividing modal curvatures for DC 5 vs. DC 6.

Table 3-5: Legend for health algorithms that predict damaged modes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Severe Damage</th>
<th>Moderate Damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>![Red Arrow]</td>
<td>![Orange Arrow]</td>
</tr>
<tr>
<td>2</td>
<td>![Red Arrow]</td>
<td>![Orange Arrow]</td>
</tr>
<tr>
<td>3</td>
<td>![Red Arrow]</td>
<td>![Orange Arrow]</td>
</tr>
<tr>
<td>4</td>
<td>![Red Arrow]</td>
<td>![Orange Arrow]</td>
</tr>
<tr>
<td>5</td>
<td>![Red Arrow]</td>
<td>![Orange Arrow]</td>
</tr>
</tbody>
</table>
Damage is predicted to have occurred all over the structure, with the highest concentration of identified locations between the second and third floors. Note that multiple DOFs between the second and third floors are identified to be damaged in the third mode. This makes sense since DC 5 is a symmetric configuration while DC 6 is an asymmetric structure and the torsion mode should be greatly affected in this sequential damage case. Points 7 and 8 are affected in the X direction bending modes. Since member 15-24 supported the structure in the X direction, it makes sense that X direction DOFs would be greatly affected by the member's removal.

3.3.2.2 Sequential Damage

Table 3-6 contains the results for all sequential damage scenarios when dividing modal curvatures and using it as a damage indicator. DOFs with values greater than 3 are considered to be locations with severe damage. Similarly, values between 1.5 and 3 indicate moderate damage. In theory, a damage indicator value greater than 3 has a 0.27% chance of occurring. Damage indicator values greater than 3 indicate an extreme deviation in curvatures from one damage case to another. The third and fourth columns indicate the affected degree of freedom and corresponding mode. Each mode is labeled with an "M" preceding the numerical mode number.
Table 3-6: Results of curvature division damage detection method for all sequential damage scenarios.

<table>
<thead>
<tr>
<th>Sequential Damage Scenarios</th>
<th>Damaged DOFs</th>
<th>Severe Damage (Z&gt;3)</th>
<th>Moderate Damage (1.5&lt;Z&lt;3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC 0 vs. DC 1</td>
<td>8X, 15X</td>
<td>8 (M 1), 16 (M 2), 17 (M 3), 14 (M 4), 24 (M 5)</td>
<td>20 (M 1), 19 (M 2)</td>
</tr>
<tr>
<td>DC 1 vs. DC 2</td>
<td>6X, 13X</td>
<td>26 (M 5)</td>
<td>--</td>
</tr>
<tr>
<td>DC 2 vs. DC 3</td>
<td>7X, 16X</td>
<td>10 (M 1), 15 (M 1), 24 (M 3), 9 (M 4)</td>
<td>11 (M 1), 21 (M 2), 25 (M 2)</td>
</tr>
<tr>
<td>DC 3 vs. DC 5</td>
<td>5X, 5Y, 6X, 6Y, 7X, 7Y, 8Y, 13X, 13Y, 14X, 14Y, 15Y, 16Y</td>
<td>11 (M 2), 12 (M 2), 17 (M 3), 18 (M 5)</td>
<td>--</td>
</tr>
<tr>
<td>DC 5 vs. DC 6</td>
<td>15X, 24X</td>
<td>7 (M 1), 13 (M 1), 8 (M 4), 13 (M 5), 26 (M 5)</td>
<td>19 (M 2), 26 (M 2), 13 (M 3), 18 (M 3), 22 (M 3), 22 (M 5)</td>
</tr>
<tr>
<td>DC 6 vs. DC 7</td>
<td>14X, 21X</td>
<td>27 (M 1), 6 (M 2)</td>
<td>--</td>
</tr>
<tr>
<td>DC 7 vs. DC 8</td>
<td>13X, 16X, 22X, 23X</td>
<td>11 (M 1), 26 (M 1), 23 (M 2)</td>
<td>8 (M 3), 22 (M 3), 11 (M 4), 18 (M 4)</td>
</tr>
<tr>
<td>DC 8 vs. DC 9</td>
<td>14Y, 15Y, 22Y, 23Y</td>
<td>22 (M 1), 11 (M 3), 22 (M 4), 18 (M 5)</td>
<td>--</td>
</tr>
</tbody>
</table>

Overall, for sequential damage scenarios, dividing the curvatures in order to detect damage is somewhat successful. Perhaps the correct location on the structure is not identified, but in general, the direction of damage is recognized correctly. For example, when cross bracing supporting X direction motion is removed, the damage indicator results show modes 1 and 4 to be affected the most. Additionally, second order modes seem to indicate damage more often than first order modes. This suggests that modes with higher frequencies are more sensitive to damage than lower frequency modes.
3.3.2.3 Cumulative Damage

Figure 3-32 contains a plot representing the results of dividing the modal curvatures matrices to form a damage indicator for DC 0 vs. DC 9.

The highest value within Figure 3-32 is 8.08 at point 19 in mode 2. This means that the modal curvature at point 19 in the second mode for DC 9 is extremely different from the modal curvature at point 19 in the second mode for DC 0. The damage indicator at point 19 in mode 2 is approximately 8 standard deviations from the mean damage indicator value, meaning that this value is extreme compared to the rest of the results for DC 0 vs. DC 9. Point 19 in mode 2 is considered to be the most damaged DOF within the structure when transitioning from DC 0 to DC 9. Conversely, the lowest value within Figure 3-32, 0.0157, occurs at point 23 in the fifth
mode. This indicates that modal curvatures at point 23 in the fifth mode for DC 0 and DC 9 are quite similar and that there is no damage identified at point 23 in the fifth mode.

Figure 3-33 is a pictorial representation of the damage detection algorithm's results when comparing DC 0 and DC 9. Blue stars represent the endpoints of the removed members while locations identified by the algorithm as damaged are indicated by a series of arrows and colors corresponding to Table 3-5.

![Figure 3-33: Pictorial representation of predicted damage using the modal curvature division method for DC 0 vs. DC 9.](image)

As Figure 3-33 shows, all of the identified possible damaged locations are contained between the second and third floors of the structure. Since the structure moves most at the top, it
makes sense that the modal curvatures were most affected towards the top of the structure when moving from DC 0 to DC 9. Since the amount of damage between DC 0 and DC 9 is much greater than any sequential damage scenario, it is expected that the algorithm identifies a large quantify of damaged locations. However, this is not the case; in fact, this damage detection method only identified 6 possible damaged locations. This discrepancy in expected and actual results is most likely generated by the normalization scheme, which causes a loss of sensitivity for lesser damage.

3.3.3 Alternative Curvature Methods

In addition to division and COMAC, the modal curvatures can be algebraically manipulated to form several other damage indicators. For example, modal curvature matrices can be subtracted and the differences normalized and used as a damage index. The result of this type of algorithm is shown in Figure 3-34 for DC 0 vs. DC 9.
The results for the modal curvature subtraction method are directly comparable to the results obtained for the modal curvature division algorithm; several observations can be made when comparing Figure 3-32 and Figure 3-34. For the cumulative damage scenario the subtraction damage indicator is more sensitive in the lower degrees of freedom than the division method. This is evidenced by several peaks occurring in Figure 3-34 at DOFs lower than point 15. Additionally, there are far too many false positives for the subtraction method to be seriously considered a dependable damage indicator for the test structure.

Furthermore, the modal curvatures can be summed up over all five modes and then divided and normalized. The results of this particular method are displayed in Figure 3-35 for DC 0 vs. DC 9.
This particular method attempts to sum over all modes and produces a two-dimensional plot that can be directly compared to the results for mode 3 in the modal curvature division algorithm (Figure 3-30). Overall, the peaks contained in Figure 3-35 are not consistent with the peaks indicated by mode 3 in Figure 3-30. This may indicate that this particular method does not capture all the information needed to make accurate damage predictions; more specifically, a significant portion of the modal information is lost when the damage indicator is summed over all five experimental modes. Since the results for the summed division method do not align well with previously discussed damage indicators or indicate damage where actual structural damage occurs, this method is not a reliable or accurate damage detection algorithm.
3.3.4 Conclusions on Modal Curvatures Methods

The COMAC of modal curvatures serves as a damage indicator that can locate the general area of damage. It does not identify a direction of damage and even excludes points 5 and 28 from the analysis. It also is not as precise as the regular COMAC measure (using mode shapes) and often produces many false positives. However, the method does accurately differentiate between incremental and cumulative damage; more possible damaged locations are identified in the cumulative scenario than any other sequential damage scenario.

The modal curvature division method cannot accurately predict damage location; instead, it is successful in determining the correct direction of damage within the structure. For example, when cross bracing supporting X direction motion is removed, the damage indicator results show modes 1 and 4 to be most affected. Additionally, second order modes indicate damage more often than first order modes. This suggests that modes with higher frequencies are more sensitive to damage than lower frequency modes with this particular damage indicator.
3.4 Flexibility Based Damage Indicators

In order to formulate a damage index that incorporates both natural frequencies and mode shapes, an estimated flexibility matrix may be constructed based upon experimental data. Huth et al. [14] proposed a way to calculate the flexibility matrix as

\[
F_{ij} = \sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_i \phi_i^T
\]  

(Eqn. 1-13)

where \( \omega_i \) is the natural frequency corresponding the \( i^{th} \) mode and \( \phi_i \) is mode shape vector for the \( i^{th} \) mode. The flexibility matrix estimated by Eqn. 1-13 is calculated by summing over all \( m \) modes. Although several researchers have utilized this method, it should be noted that for this particular data, this is not an estimation of the true flexibility matrix of the structure. Instead, it is more similar to a relative flexibility matrix that gives a measure of displacement or deflection within the structure. For example, values in \( F_{ij} \) that are relatively high indicate areas on the structure that extremely flexible.

3.4.1 Flexibility Division

Several algebraic operations may be performed on the relative flexibility matrix, much like the modal curvature matrix. For example, the flexibility matrix of a damaged structure can be divided by the flexibility matrix of the baseline case. All elements in this new matrix can then be normalized with respect to their mean and standard deviation in order to form a damage index that compares two state's dynamic properties. For clarity, the absolute value of the index is taken to be the actual damage indicator in this case. Essentially, this damage indicator tracks changes in flexibility (and stiffness) within the structure when moving from one damage case to another.
3.4.1.1 Example of Sequential Damage: DC 5 vs. DC 6

The sequential damage scenario DC 5 vs. DC 6 is analyzed first in order to evaluate this particular damage detection algorithm. Figure 3-36 displays a three-dimensional plot of the normalized damage indices obtained by dividing flexibilities for DC 5 vs. DC 6.

Figure 3-36: 3-D plot results for flexibility matrix division damage algorithm for DC 5 vs. DC 6.

The highest value within Figure 3-36 is 9.93 at point 25 in mode 3. This implies that point 25 in the third mode is the most damaged DOF and that the relative flexibility at point 25 in the mode 3 for DC 6 is dramatically different from that for DC 5. The damage indicator at point 25 in mode 3 is approximately 10 standard deviations from the mean damage indicator value, meaning that this value is extreme compared to the rest of the results for DC 5 vs. DC 6.
Additionally, the relatively high value obtained from applying the flexibility matrix division damage algorithm confirms the data and calculated results are statistically significant.

Conversely, the lowest value within Figure 3-36 of 0.0193 occurs at point 6 within the fourth mode. This relatively low value indicates that the modal flexibility matrices for DC 5 and DC 6 are numerically similar at point 6 in the fourth mode. According to this particular damage detection technique, point 6 in the fourth mode is the least damaged DOF when comparing DC 5 and DC 6.

Figure 3-37 contains a pictorial representation of the algorithm's results when comparing DC 5 and DC 6. Blue stars represent the endpoints of the removed member while locations identified by the algorithm as damaged are indicated by a series of arrows and colors corresponding to Table 3-5. Severe damage occurs when the value of the damage indicator exceeds 3, while the location of moderate damage is identified when the damage indicator is between 1.5 and 3. Any DOF with a value lower than 1.5 is considered undamaged.
As depicted in Figure 3-37, damage is predicted to have occurred at points 8, 17, 21, and 25. Severe damage is indicated at the top of the structure at points 21 and 25 within the third mode. These identified locations make sense because they are situated on the opposite face of structure of the points that were directly affected by the removal of member 15-24. In addition, point 8 is predicted to have moderate damage within the first mode. Since the first mode controls motion within the X direction, point 8 could have easily been affected by the removal of a cross bracing member that supports the structure along the X direction. Overall, the DOFs identified as possible damaged locations correspond well with expected results.
3.4.1.2 Sequential Damage

Table 3-7 displays the results of this damage detection algorithm for all sequential damage scenarios. The second column, labeled "Damaged DOFs," contains the DOFs that were directly affected from the removal of members. The third column, labeled "Severe Damage," lists locations and modes (contained within parentheses with an "M" preceding the numerical mode) at which damage is most likely to occur. DOFs fall into this particular category when their flexibility damage index is greater than 3. DOFs with values between 1.5 and 3 are considered to be locations that may contain damage and are tabulated in the fourth column of Table 3-7.

The algorithm identified a large amount of damaged locations for DC 6 vs. DC 7, which implies that there are several false positives for this particular damage scenario. This could be explained by inconsistency in the data, particularly the natural frequencies for DC 7. The natural frequency of mode 1 is reduced by 38% when moving from DC 6 to DC 7. This dramatic change could also contribute to the immense amount of false positives provided by the damage indicator. However, in general, the flexibility matrix division damage detection algorithm successfully locates the general area of damage as well as modes of vibration that may be directly affected for most sequential damage scenarios with some false positives. Overall, this algorithm effectively locates incremental damage within the test structure.
Table 3-7: Results of flexibility matrix division damage indication for sequential damage.

<table>
<thead>
<tr>
<th>Sequential Damage Scenario</th>
<th>Damaged DOFs</th>
<th>Severe Damage ($Z&gt;3$)</th>
<th>Moderate Damage ($1.5&lt;Z&lt;3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC 0 vs. DC 1</td>
<td>8X, 15X</td>
<td>17 (M 1), 21 (M 1), 25 (M 1)</td>
<td>26 (M 1)</td>
</tr>
<tr>
<td>DC 1 vs. DC 2</td>
<td>6X, 13X</td>
<td>8 (M 2), 13 (M 2), 13 (M 3), 24 (M 3), 28 (M 3), 13 (M 4), 19 (M 4)</td>
<td>8 (M 1), 22 (M 1), 9 (M 3), 14 (M 3), 14 (M 4)</td>
</tr>
<tr>
<td>DC 2 vs. DC 3</td>
<td>7X, 16X (add: 6X, 13X)</td>
<td>7 (M 5), 11 (M 5)</td>
<td>--</td>
</tr>
<tr>
<td>DC 3 vs. DC 5</td>
<td>5X, 5Y, 6X, 6Y, 7X, 7Y, 8Y, 13X, 13Y, 14X, 14Y, 15Y, 16Y</td>
<td>21 (M 1), 5 (M 2), 6 (M 2), 21 (M 2)</td>
<td>25 (M 1), 10 (M 2), 25 (M 2)</td>
</tr>
<tr>
<td>DC 5 vs. DC 6</td>
<td>15X, 24X</td>
<td>21 (M 3), 25 (M 3)</td>
<td>8 (M 1), 17 (M 3)</td>
</tr>
<tr>
<td>DC 6 vs. DC 7</td>
<td>14X, 21X</td>
<td>10 (M 1), 14 (M 1), 18 (M 1), 23 (M 1), 27 (M 1), 5 (M 2), 8 (M 3), 14 (M 3), 14 (M 4), 16 (M 5)</td>
<td>5 (M 1), 6 (M 1), 7 (M 1), 8 (M 1), 9 (M 1), 12 (M 1), 16 (M 1), 19 (M 1), 21 (M 1), 22 (M 1), 26 (M 1), 28 (M 1), 6 (M 2), 14 (M 2), 17 (M 2), 18 (M 2), 21 (M 2), 22 (M 2), 23 (M 2), 25 (M 2), 26 (M 2), 27 (M 2), 6 (M 3), 18 (M 3), 6 (M 4), 19 (M 4), 6 (M 5), 7 (M 5), 8 (M 5), 10 (M 5), 13 (M 5), 14 (M 5), 17 (M 5), 19 (M 5), 21 (M 5), 25 (M 5)</td>
</tr>
<tr>
<td>DC 7 vs. DC 8</td>
<td>13X, 16X, 22X, 23X</td>
<td>6 (M 1)</td>
<td>6 (M 3)</td>
</tr>
<tr>
<td>DC 8 vs. DC 9</td>
<td>14Y, 15Y, 22Y, 23Y</td>
<td>12 (M 1), 16 (M 2), 17 (M 4)</td>
<td>16 (M 1), 12 (M 4)</td>
</tr>
</tbody>
</table>
3.4.1.3 Cumulative Damage

Figure 3-38 displays a three-dimensional plot of the normalized damage indices obtained by dividing flexibilities for DC 0 vs. DC 9.

The highest value within Figure 3-38 is 9.43 at point 21 in mode 1. This implies that point 21 in the first mode is the most damaged DOF and that the relative flexibility at point 21 in the first mode for DC 0 is dramatically different from the relative flexibility at point 21 in the first mode for DC 9. The damage indicator at point 21 in mode 1 is approximately 9.4 standard deviations from the mean damage indicator value, meaning that this value is extreme.

Conversely, the lowest value within Figure 3-38 of 0.0108 occurs at point 20 within the first mode. This relatively low value indicates that the modal flexibility matrices for DC 0 and
DC 9 are numerically similar at point 20 in the first mode. According to this particular damage detection technique, point 20 in the fourth mode is the least damaged DOF when comparing DC 0 and DC 9. Essentially, there is no damage identified at point 6 in the fourth mode of vibration.

Figure 3-39 contains a pictorial representation of the damage detection algorithm's results when comparing DC 0 and DC 9. Blue stars represent the endpoints of the removed members while damage locations identified by the algorithm are indicated by a series of arrows and colors corresponding to Table 3-5. Severe damage occurs when the value of the damage indicator exceeds 3, while the location of moderate damage is identified when the damage indicator is between 1.5 and 3. Any DOF with a value lower than 1.5 is considered undamaged.

![Figure 3-39: Pictorial representation of results for flexibility matrix division damage algorithm for DC 0 vs. DC 9.](image-url)
Figure 3-39 shows that all the damage is concentrated towards the top of the structure in the X direction (mode 1). The sides of the structure that align with the X axis are free of all bracing in DC 9, and therefore it is probable that damage has occurred in the first mode, especially around the top which is the part of the structure with the most displacement. However, damage should be identified all over the structure, especially at the points labeled with blue stars. The small amount of identified damaged locations could be explained by the normalization of the damage indicator. Since most of the structure is actually damaged, it is not sufficient to only select extreme damage indicator values. Overall, the flexibility matrix division method is not sensitive enough to detect cumulative damage within the test structure.

3.4.2 Percent Difference of Flexibilities

In addition to dividing the relative flexibility matrices, one may calculate the percent difference between a pair of flexibility matrices. In theory, this method should be more robust because it incorporates subtraction as well as division. The equation utilized to calculate the damage indicator used in this section is

\[
\Delta F_{ij} = \frac{|F_{ij}^* - F_{ij}|}{F_{ij}}
\]  

(Eqn. 3-2)

where \(F_{ij}\) and \(F_{ij}^*\) are the relative flexibility matrices for the baseline and damaged structures, respectively. These relative flexibility matrices were calculated using Eqn. 1-13. Similar to the flexibility matrix division method, the \(\Delta F_{ij}\) matrix is normalized with respect to the elements mean and standard deviation and in order to form a damage indicator. The absolute value of the normalized \(\Delta F_{ij}\) is the selected damage indicator for this method.
Figure 3-40 displays a 3-D plot of the results obtained from implementing the percent difference algorithm on the relative flexibility matrices for the sequential case DC 5 vs. DC 6 (Figure 3-40a) and the cumulative scenario, DC 0 vs. DC 9 (Figure 3-40b).

Figure 3-40: 3-D results for flexibility percent difference algorithm for (a) DC 5 vs. DC 6 and (b) DC 0 vs. DC 9.

Both plots contained within Figure 3-40 are quite similar to those in Figure 3-36 and Figure 3-38. For this particular test structure, the flexibility division method and the flexibility percent difference method produce the same results and identify the same damaged DOFs. Although the percent difference method may not be optimal for the three-story test structure, it should still be tested on other structures before complete dismissal.

3.4.3 Flexibility Subtraction

The last explored algebraic manipulation of the relative flexibility matrix involves simply subtracting the undamaged matrix from the damaged matrix. The normalization and absolute value operations are again implemented on the "net" relative flexibility matrix in order to form a unique damage indicator.
Figure 3-41 displays a 3-D plot of the results obtained from implementing the subtraction algorithm on the relative flexibility matrices for the sequential case DC 5 vs. DC 6 (Figure 3-41a) and the cumulative scenario, DC 0 vs. DC 9 (Figure 3-41b). Both parts of Figure 3-41 indicate significant damage within the first and second modes. This is a false result: the lower order modes' relative flexibility values are magnified due to the nature of flexibility calculation. One cannot simply subtract flexibility matrices with intentions of being able to detect damage within a structure. Since the equation for the flexibility matrix (Eqn. 1-13) involves dividing by the square of each natural frequency, the modes with smaller natural frequencies will have significantly higher values than modes with relatively high natural frequencies. Due to this low frequency bias, subtracting relative flexibility matrices is not an effective damage indicator.
3.4.4 Conclusions on Flexibility Based Methods

Overall, the flexibility matrix division algorithm successfully locates the general area of damage as well as modes of vibration that may be directly affected for most sequential damage scenarios with a small amount of false positives. However, cumulative damage cannot be successfully identified by the flexibility division method, as there was only a small amount of damage predicted within the cumulative case. The small amount of identified damaged locations could be explained by the normalization of the damage indicator; the flexibility matrix division method is not sensitive enough to detect cumulative damage within the test structure.

Additionally, the flexibility method that incorporates a percent difference has similar results to the division method and for the test structure they are essentially interchangeable. Conversely, the damage detection method that includes subtraction of flexibility matrices is not a reliable method to use on the test structure. This method has an extreme bias toward low frequency modes; the first and second modes have larger magnitude damage indices than those calculated for modes 3, 4, and 5. Due to this low frequency bias, subtracting relative flexibility matrices is not an effective damage indicator.

3.5 Story Stiffness Method

The next stiffness damage indicator examined herein is intended for use on multi-story buildings and thus directly applicable to the model three-story structure discussed within this work. Wang et al. [32] described a damage detection technique that quantifies damage in every story of a building. The story stiffness damage index of the $k^{th}$ story, $SSDI_k$ is expressed as:
where * denotes parameters from the damaged state. $\phi_{ik}$ represents the value of the $i^{th}$ mode shape at the $k^{th}$ story, $\omega_i$ describes the natural frequency corresponding to the $i^{th}$ mode, and $L$ is the total number of floors. The change in mode shape, $\Delta \phi_{ik}$, is defined as

$$\Delta \phi_{ik} = \begin{cases} 
\phi_{ik} - \phi_{i(k-1)}, & \text{for } k = 2,3,...,L \\
\phi_{ik}, & \text{for } k = 1 
\end{cases}$$

(Eqn. 1.22)

For the three-story test structure, $L$ is equal to 3 and the mode shape at each floor was calculated by averaging the modal coordinates at every corner of the floor. For example, in order to calculate the mode shape for floor 1, the modal coordinates at points 5, 6, 7, and 8 would be averaged. Once all story stiffness indices were calculated, they were then normalized with respect to their mean and standard deviation in order to be directly comparable to other damage detection techniques outlined in this work.

3.5.1 Sequential Damage Example: DC 5 vs. DC 6

The story stiffness damage indicator was calculated for each floor at each mode for every damage scenario. This method uses a total of fifteen DOFs (3 floors x 5 modes) in its analysis of the three-story spatial frame. The results of this algorithm for DC 5 vs. DC 6 are graphically displayed in Figure 3-42.
The highest value within Figure 3-42 is 3.41 at the third floor in mode 3. This implies the third floor in mode 3 is the most damaged DOF and that the relative stiffness of the third floor for DC 6 is dramatically different than the stiffness of the third floor in DC 5 (in the third mode). The damage indicator at floor 3 in the third mode is approximately 3.5 standard deviations from the mean damage indicator value, meaning that this value is extreme when compared to the rest of these results. Additionally, the relatively high value obtained from applying the story stiffness damage algorithm confirms the experimental data used as well as the calculated results are statistically significant.
Conversely, the lowest value of 0.245 within Figure 3-42 occurs at the first floor within the fourth mode. This relatively low value indicates that the stiffness of the first floor for DC 5 and DC 6 are numerically similar in the fourth mode. According to this particular damage detection technique, the first floor in the fourth mode of vibration is the least damaged DOF when comparing DC 5 and DC 6. Essentially, there is no damage identified at the first floor in the fourth mode. This is agrees with the expected result; since there were no members removed between the first and second floors, there should not be any damage indicated within the first floor.

Figure 3-43 contains a pictorial representation of the damage detection algorithm's results when comparing DC 5 and DC 6. Blue stars represent the endpoints of the removed member while floors identified by the algorithm as damaged are highlighted in grey and incorporate a series of arrows and colors corresponding to Table 3-5. Severe damage occurs when the standardized value of the damage indicator exceeds 3, while the location of moderate damage is identified when the damage indicator is between 1.5 and 3. Any DOFs with a value lower than 1.5 is considered undamaged.
As depicted in Figure 3-43, most damage occurs within the third floor, and its first and third modes are identified as severely damaged. Since the removed cross brace, member 15-24, spanned and supported the structure in the X direction, it is expected that the first mode would be affected, especially near the actual location of damage (points 8 and 15). Additionally, the torsion mode should be affected since DC 5 is a symmetric configuration and DC 6 is an asymmetric structure. Moderate damage is predicted to have occurred at the first floor within the second mode of vibration. Since the torsion mode at the top of the structure (floor 3) was dramatically changed when moving from DC 5 to DC 6, the second mode on a lower portion of
the structure could have been affected. However, it is more likely that the moderate damage indicated on the first floor is a false positive. In general, the damage is well identified and generally located using the story stiffness method.

3.5.2 Sequential Damage

Table 3-8 displays the results of this damage detection algorithm for all sequential damage scenarios. The second column, labeled "Damaged DOFs", contains the DOFs that were directly affected from the removal of members. The third column, labeled "Severe Damage", lists floors (F) and modes (M) at which damage is most likely to occur. DOFs fall into this particular category when their flexibility damage index is greater than 3. DOFs with values between 1.5 and 3 are considered to be locations that may contain damage and are tabulated in the fourth column of Table 3-8.

Table 3-8: Results of story stiffness damage indication for sequential damage.

<table>
<thead>
<tr>
<th>Sequential Damage Scenario</th>
<th>Damaged DOFs</th>
<th>Severe Damage (Z&gt;3)</th>
<th>Moderate Damage (1.5&lt;Z&lt;3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC 0 vs. DC 1</td>
<td>8X, 15X</td>
<td>F 3 (M 4)</td>
<td>F 3 (M 1), F 2 (M 2), F 3 (M 3)</td>
</tr>
<tr>
<td>DC 1 vs. DC 2</td>
<td>6X, 13X</td>
<td>F 2 (M 1), F 3 (M 5)</td>
<td>F 3 (M 3)</td>
</tr>
<tr>
<td>DC 2 vs. DC 3</td>
<td>7X, 16X (add: 6X, 13X)</td>
<td>F 3 (M 1)</td>
<td></td>
</tr>
<tr>
<td>DC 3 vs. DC 5</td>
<td>5X, 5Y, 6X, 6Y, 7X, 7Y, 8Y, 13X, 13Y, 14X, 14Y, 15Y, 16Y</td>
<td>F 3 (M 2)</td>
<td>F 2 (M 4), F 3 (M 4), F 3 (M 5)</td>
</tr>
<tr>
<td>DC 5 vs. DC 6</td>
<td>15X, 24X</td>
<td>F 3 (M 1), F 3 (M 3)</td>
<td>F 1 (M 2), F 3 (M 2), F 3 (M 4)</td>
</tr>
<tr>
<td>DC 6 vs. DC 7</td>
<td>14X, 21X</td>
<td>F 3 (M 5)</td>
<td>F 3 (M 2), F 2 (M 4)</td>
</tr>
<tr>
<td>DC 7 vs. DC 8</td>
<td>13X, 16X, 22X, 23X</td>
<td>F 2 (M 1)</td>
<td>F 3 (M 2), F 2 (M 4), F 3 (M 4)</td>
</tr>
<tr>
<td>DC 8 vs. DC 9</td>
<td>14Y, 15Y, 22Y, 23Y</td>
<td>F 3 (M 3), F 2 (M 4)</td>
<td>F 1 (M 1), F 3 (M 1), F 3 (M 4)</td>
</tr>
</tbody>
</table>
In general, the story stiffness damage detection method successfully locates floors that may be damaged as well as modes of vibration that may be directly affected by the removal of cross bracing members for most sequential damage scenarios. Damage scenarios involving the detachment of cross bracing members between the second and third floors yielded the most accurate damage predictions. Sequential damage scenarios that required the removal of cross braces between the first and second floors produced more false positives when utilizing the story stiffness algorithm. However, overall, this algorithm effectively locates incremental damage within the test structure, especially for damage occurring between the second and third floors.

3.5.3 Cumulative Damage

Figure 3-44 displays a three-dimensional plot of the normalized damage indices obtained utilizing the story stiffness method for DC 0 vs. DC 9.
The highest value within Figure 3-44, 6.119, occurs at the third floor in mode 4. This implies the third floor in mode 4 is the most damaged DOF and that the relative stiffness of the third floor in DC 6 is dramatically different than the stiffness of the third floor in DC 5 (in the third mode).

The damage indicator at floor 3 in the fourth mode is approximately 6 standard deviations from the mean damage indicator value, meaning that this value is extreme compared to the rest of the results for DC 0 vs. DC 9.

Conversely, the lowest value of 0.003 within Figure 3-44 occurs at the third floor within the first mode. This relatively low value indicates that the stiffness of the third floor for DC 0 and DC 9 are numerically similar in the first mode. According to this particular damage
detection technique, the third floor in the first mode of vibration is the least damaged DOF when comparing DC 0 and DC 9. According the story stiffness damage detection method, the third floor within the first mode is the least damaged DOF.

Figure 3-45 contains a pictorial representation of the damage detection algorithm's results when comparing DC 0 and DC 9. Blue stars represent the endpoints of the removed member while floors identified by the algorithm as damaged are highlighted in grey and incorporate a series of arrows and colors corresponding to Table 3-5. Severe damage occurs when the value of the damage indicator exceeds 3, while the location of moderate damage is identified when the damage indicator is between 1.5 and 3. Any DOFs with a value lower than 1.5 are considered undamaged.

Figure 3-45: Pictorial representation of results for story stiffness damage detection algorithm for DC 0 vs. DC 9.
As depicted in Figure 3-45, damage is identified on all three floors, with the most severe damage occurring on floors 2 and 3. This agrees with the expected result of the entire structure accruing damage, since the simulated damage was significant and cumulative. Moderate damage is identified in the first and second floors with severe damage concentrated toward the top of the structure. This gradation could be due to the sensitivity of the signals being increased when analyzing DOFs in close proximity to the accelerometer at point 28. Moreover, increased damage severity on the top half of the structure could be attributed to the fact that the points on the structure further away from the foundation move with the most displacement and their modal properties are most affected by structural modifications.

In addition to location of damage, it is interesting to note which modes are most affected in the cumulative damage scenario. Only the second order modes, modes 4 and 5, are identified as damaged by the story stiffness damage indicator for DC 0 vs. DC 9. This makes sense because second order modes are higher frequency and by nature are more sensitive than lower frequency modes. Overall, the story stiffness damage detection method effectively locates and quantifies the damage in each floor of the three-story test structure.

3.5.4 Conclusions on Story Stiffness

In general, the story stiffness damage detection method successfully locates floors that may be damaged as well as modes of vibration that may be directly affected by the removal of cross bracing members. Damage scenarios involving cross brace removal between the second and third floors yielded the most accurate damage predictions; however, scenarios involving cross brace removal between the first and second floors produced more false positives when
utilizing the story stiffness algorithm. This algorithm effectively locates incremental damage within the test structure, especially for damage occurring between the second and third floors.

Cumulative damage is also accurately identified on every floor of the structure using the story stiffness method. Only the second order modes, modes 4 and 5, are identified as damaged for DC 0 vs. DC 9. This makes sense because second order modes are higher frequency and by nature are more sensitive than lower frequency modes. Overall, the story stiffness damage detection method effectively locates and quantifies the damage in each floor of the three-story test structure.
3.6 Direct FRF Comparison

Requiring no modal processing, the final damage detection algorithm detailed herein directly employs the frequency response function (FRF) signals each impact/response. As variations of the Damage Location Vector (DLV), the first damage index involves subtracting FRFs while the second indicator utilizes division. Both of these methods identify possible damaged locations by assessing the change in FRF magnitudes for a specific impact/response combination. For this study, this means that there are a total of fifty-two DOFs considered due to the twenty-six impact locations and two acceleration directions.

3.6.1 FRF Subtraction

The DLV is a damaged indicator that allows for direct comparison of two damage states' FRFs. The formula for the DLV at any \( j \) DOF is given by the following equation:

\[
DLV_j = K \sum_{s=1}^{S} (FRF_{sj}^* - FRF_{sj}) .
\]

(Eqn. 1-26)

The \( K \) matrix represents the undamaged structure's stiffness matrix. Since the three-story spatial frame's \( K \) matrix cannot be accurately calculated with the available data, it is dropped from the equation. Instead, the following relation defines the FRF subtraction damage indicator for each DOF, \( FRF_s \):

\[
FRF_s = \sum_{s=1}^{S} (FRF_{sj}^* - FRF_{sj}) ,
\]

(Eqn. 3-3)

where \( FRF_{sj} \) and \( FRF_{sj}^* \) are the values of frequency response functions corresponding the \( s^{th} \) frequency for the undamaged and damaged structure, respectively. By choice, the maximum value of \( s \), denoted as \( S \) (capital), is equal to 250 Hz and divided into 0.061035 Hz frequency
steps. The summation term symbolizes the difference between FRFs for any \( j \) degree of freedom totaled over all frequencies. All FRF data utilized in this damage indicator were normalized by dividing each FRF matrix column by the maximum value contained in that particular FRF. After damage indices were calculated for each impact/response combination, they were normalized utilizing the mean and standard deviation of all \( FRF_s \) values for a particular damage scenario.

3.6.1.1 Example of Sequential Damage: DC 5 vs. DC 6

In order to illustrate the effectiveness of the FRF subtraction method, a pictorial representation of the damage algorithms results for a sequential damage scenario, DC 5 vs. DC 6, is created and presented in Figure 3-46. Severe damage occurs when the damage index exceeds 1.5 and is represented by large red arrows pointing in either the X or Y direction. Moderate damage is considered to happen when the damage indicator value is in between 1 and 1.5 and is represented by smaller orange arrows pointing in either the X or Y direction. These particular thresholds are chosen because the FRF indicator typically has lower values than other indices already discussed. Cross-correlated DOFs (i.e. impact in the X direction and response in the Y direction) that are identified as damaged locations are denoted by a large cross or "\( \times \)". Orange crosses represent moderate damage while red crosses indicate severe damage. As in previous sections, the blue stars mark the points of the removed member when transitioning from DC 5 to DC 6.
The highest damage indicator value of 3.58 occurs at 16yy (impact in the Y direction at point 16 with response at point 28 in the Y direction). This implies that point 16 in the Y direction is the most damaged DOF and that the response in the Y direction from impact at point 16 in the Y direction for DC 5 is dramatically different from the same impact's response in DC 6. Conversely, the lowest damage indicator value of 0.0017 occurs at 21xx (impact in the X direction at point 21 with response at point 28 in the X direction). This low value indicates that the responses in the X direction from impact at point 21 in the X direction are similar when comparing DC 5 and DC 6. According to this particular damage detection technique, 21xx is the least damaged DOF when comparing DC 5 and DC 6.
As depicted in Figure 3-46, the highest concentration of possible damage occurs above the second floor of the model building. The most severe damage is predicted to occur at point 28 on the structure, which is also the location of the tri-axial accelerometer. In order to collect data for point 28, the spatial frame had to be excited at that point 28; this proved difficult due to the location of the sensor with respect to where the impact must occur. Due to the close proximity of the impact to the accelerometer at point 28, the FRFs calculated for impact at point 28 were extremely sensitive, and the damage indicator values produced for point 28 were falsely magnified.

Besides point 28, most of the damage occurs between the second and third floors. In particular, point 16 is identified as damaged in the X, Y, and cross-correlated terms. This is plausible since point 16 is located on the same face as the removed member and is directly adjacent to the two points that were actually damaged, points 8 and 15. All other DOFs identified as possible damaged locations between the second and third floors are affected in their cross-correlated terms. This makes sense since DC 5 is a symmetric structure while DC 6 is asymmetric, so the twisting motion of the structure is expected to be affected. The cross-correlated FRFs are the most sensitive to changes in the structure's torsion mode, and therefore the results obtained from the FRF subtraction method match expected results. There are some false positives produced by this damage detection technique, such as the overly sensitive point 28 and the severely damaged point 5. Nevertheless, the subtraction FRF method effectively locates and quantifies damage within the three-story frame when comparing DC 5 and DC 6.
3.6.1.2 Sequential Damage

Table 3-9 displays the results of the FRF subtraction damage identification method for all sequential damage scenarios. The second column, labeled "Damaged DOFs," contains the DOFs that were directly affected from the removal of members. The third column, labeled "Severe Damage," lists the DOFs at which damage is most likely to occur. DOFs fall into this particular category when their $F_{FRF}^5$ value is greater than 1.5. DOFs with values between 1 and 1.5 are considered to be locations that may contain damage and are tabulated in the fourth column of Table 3-9. Note the naming scheme for each DOF within Table 3-9; for example, 14xx refers to impact at point 14 in the X direction with acceleration response at point 28 in the X direction, 14yy represents impact at point 14 in the Y direction with response measured in the Y direction at point 28. The cross-correlated terms, 14xy and 14yx, indicate that the structure was hit at point 14 in the X (or Y) direction and the response was collected in the Y (or X) direction at point 28.

Overall, the FRF subtraction method successfully locates points and directions of motion that may be damaged by the removal of cross bracing members for most sequential damage scenarios. This particular damage detection technique is relatively accurate (when compared to other damage indicators as discussed). For each sequential damage scenario, the algorithm picks out at least one of the DOFs contained in the column labeled "Damaged DOFs" within Table 3-9. Along with this precision, the algorithm also identifies several DOFs surrounding the expected areas of damage. A weakness of this method includes magnified FRF signals when impacting around or near the sensor location, point 28. However, since the location of the accelerometer is known, point 28 can simply be ignored.
In general, the FRF subtraction damage indicator accurately locates and quantifies (moderate or severe) damage within the test structure with few false positives and does so without any specialized modal decomposition software.

<table>
<thead>
<tr>
<th>Sequential Damage Scenario</th>
<th>Damaged DOFs</th>
<th>Severe Damage (Z&gt;1.5)</th>
<th>Moderate Damage (1&lt;Z&lt;1.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DC 0 vs. DC 1</strong></td>
<td>8X, 15X</td>
<td>14xx, 22xx, 22xy, 23xx, 24xx, 28yx</td>
<td>5yx, 5yy, 14xy, 15xy, 15yx, 22yx, 23xy, 23yy</td>
</tr>
<tr>
<td><strong>DC 1 vs. DC 2</strong></td>
<td>6X, 13X</td>
<td>6xx, 7yy, 8xx, 14xx, 22xx, 22yx, 28xx, 28xy</td>
<td>6yx, 7xy, 13xx, 14xy, 14yx, 15yx, 15xy, 22yy, 23yy</td>
</tr>
<tr>
<td><strong>DC 2 vs. DC 3</strong></td>
<td>7X, 16X (add: 6X, 13X)</td>
<td>6yy, 8xx, 14xx, 21yy, 24xx, 28xy</td>
<td>6yx, 7yy, 8yx, 8yy, 14xy, 15xx, 16yx, 16yy, 21yx, 22xx, 23yy, 24yx, 28xx</td>
</tr>
<tr>
<td><strong>DC 3 vs. DC 5</strong></td>
<td>5X, 5Y, 6X, 6Y, 7X, 7Y, 8Y, 13X, 13Y, 14X, 14Y, 15Y, 16Y</td>
<td>6yy, 8xx, 8yx, 21xx, 24yx</td>
<td>6xx, 13xy, 13yy, 15yy, 16xy, 16yx, 21yy, 22xx, 22yy, 23xx, 23yy, 24xy, 24yx, 28xx, 28xy, 28yx</td>
</tr>
<tr>
<td><strong>DC 5 vs. DC 6</strong></td>
<td>15X, 24X</td>
<td>5xx, 16yy, 22xy, 28xx, 28yx, 28yy</td>
<td>14yx, 16xx, 16yx, 21yx, 23xy, 24yx</td>
</tr>
<tr>
<td><strong>DC 6 vs. DC 7</strong></td>
<td>14X, 21X</td>
<td>16yx, 16yy, 21xy, 22xy, 28xy</td>
<td>6yx, 7xy, 8xy, 8yx, 14xx, 15xx, 15xy, 22yx, 23xx, 23xy, 23yy</td>
</tr>
<tr>
<td><strong>DC 7 vs. DC 8</strong></td>
<td>13X, 16X, 22X, 23X</td>
<td>6yx, 6yx, 13yy, 14xx, 14yx, 21yy, 28xy</td>
<td>8yx, 13xx, 13yx, 14yy, 16xy, 16yx, 22yx, 23xx, 28yx, 28yy</td>
</tr>
<tr>
<td><strong>DC 8 vs. DC 9</strong></td>
<td>14Y, 15Y, 22Y, 23Y</td>
<td>6yx, 7xy, 14xy, 16xy, 16yx, 23yx, 23yy</td>
<td>7yy, 8xy, 8yx, 15xy, 22yy, 24yx, 28yy</td>
</tr>
</tbody>
</table>

### 3.6.1.3 Cumulative Damage

In order to assess the effectiveness of the FRF subtraction method on cumulative damage, a pictorial representation of the damage algorithm results for DC 0 vs. DC 9 is created and displayed in Figure 3-47. Severe damage occurs when the damage index exceeds 1.5 and is
represented by large red arrows pointing in either the X or Y direction in Figure 3-47. Moderate damage is considered to happen when the damage indicator value is in between 1 and 1.5 and is represented by smaller orange arrows pointing in either the X or Y direction. Cross-correlated DOFs (i.e. impact in the X direction and response in the Y direction) that are identified as damaged locations are denoted by a large cross or "\(\bigotimes\)". Orange crosses represent moderate damage while red crosses indicate severe damage. As in previous sections, the blue stars mark the points of the removed members when transitioning from DC 0 to DC 9.

Figure 3-47: Pictorial representation of results for FRF subtraction damage detection algorithm for DC 0 vs. DC 9.
The highest damage indicator value of 2.06 occurs at 23yy (impact in the Y direction at point 23 with response at point 28 in the Y direction). This implies that point 23 in the Y direction is the most damaged DOF: the Y response from a Y impact at point 23 for DC 0 is dramatically different from the matching response in DC 9. Conversely, the lowest damage indicator value of 0.0219 occurs at 8xy (impact in the X direction at point 8 with response at point 28 in the Y direction). This relatively low value indicates that the responses in the Y direction from impact at point 8 in the X direction are similar when comparing DC 0 and DC 9. According to this particular damage detection technique, 8xy is the least damaged DOF when comparing DC 0 and DC 9.

As depicted in Figure 3-47, damage has been predicted to have occurred all over the structure. However, most of the severe damaged locations are situated above the second floor of the test structure. This is likely due to the fact that the closer one moves toward the top of the structure, the more the structure will displace. Since points 5-8 are located close to the extremely stiff foundation, they will not displace much. Due to the relative stiffness of the foundation braces compared to the rest of the structure, it is difficult to cause large changes in structure's response near the base by simple removing exterior cross bracing members. Thus, it is reasonable to have more damaged DOFs toward the top of the structure than near the foundation.

Every point labeled with a blue star on the structure indicates some form of damage. This agrees with the expected results of damage occurring everywhere on the building. As with sequential damage, data for point 28 is extremely sensitive and will always show damage. Other than point 28, there is general agreement with expected results and obtained results from the FRF subtraction damage identification technique.
3.6.2 FRF Division

An additional FRF damage indicator analyzed involves the division of two damage cases' FRF magnitudes for each impact/acceleration DOF. The FRF division damage indicator, $FRF_d$, is defined by the following relation

$$FRF_d = \sum_{s=1}^{S} \left( \frac{FRF_{sj}^*}{FRF_{sj}} \right),$$

(Eqn. 3-4)

where $FRF_{sj}$ and $FRF_{sj}^*$ are the values of frequency response functions corresponding the $s^{th}$ frequency for the undamaged and damaged structure, respectively. By choice, the maximum value of $s$, denoted as $S$ (capital), is equal to 250 Hz and divided into 0.061035 Hz frequency steps. The summation term symbolizes the difference between FRFs for any $j$ degree of freedom totaled over all frequencies. All FRF data utilized in this damage indicator were normalized by dividing each FRF matrix column by the maximum value contained in that particular FRF. After damage indices were calculated for each impact/response combination, they were normalized utilizing the mean and standard deviation of all $FRF_d$ values for a particular damage scenario.

3.6.2.1 Example of Sequential Damage: DC 5 vs. DC 66

In order to determine the effectiveness of the FRF division method, a pictorial representation of the damage algorithms results for a sequential damage scenario, DC 5 vs. DC 6, is created and presented in Figure 3-48. Severe damage occurs when the damage index exceeds 1.5 and is represented by large red arrows pointing in either the X or Y direction in Figure 3-46. Moderate damage is considered to happen when the damage indicator value is in between 1 and 1.5 and is represented by smaller orange arrows pointing in either the X or Y direction. Cross-correlated DOFs (i.e. impact in the X direction and response in the Y direction) that are
identified as damaged locations are denoted by a large cross or "\(\times\)". Orange crosses represent moderate damage while red crosses indicate severe damage. As in previous sections, the blue stars mark the points of the removed member when transitioning from DC 5 to DC 6.

The highest damage indicator value of 4.78 occurs at 28yy (impact in the Y direction at point 28 with response at point 28 in the Y direction). This implies that point 28 in the Y direction is the most damaged DOF: the Y response from a Y impact at point 28 for DC 5 is dramatically different from the matching response in DC 6. Conversely, the lowest damage indicator value of 0.0530 occurs at 24yy (impact in the Y direction at point 24 with response at
point 28 in the Y direction). This relatively low value indicates that the responses in the Y direction from impact at point 24 in the Y direction are similar when comparing DC 5 and DC 6. According to this particular damage detection technique, 24yy is the least damaged DOF when comparing DC 5 and DC 6. However, this particular result does not agree with expected results for point 24. Since member 15-24 was removed when moving from DC 5 to DC 6, it is expected that a large amount of damage has occurred in point 24.

As depicted in Figure 3-46, the highest concentration of possible damage occurs above the second floor of the model building. The most severe damage is predicted to occur at point 28 on the structure, which is also the location of the tri-axial accelerometer. As mentioned in previous FRF damage detection method sections, the readings at point 28 are unusually high and can be disregarded because of the location of the accelerometer. Aside from point 28, all of the damage occurs between the second and third floors. In particular, point 16 is identified as severely damaged in the Y direction. This is plausible since point 16 is located on the same face of the structure as the removed member and is directly adjacent to the two points that were actually damaged, points 8 and 15. All other DOFs identified as possible damaged location in between the second and third floors are affected in their cross-correlated terms, which again makes sense due to the symmetry/asymmetry and its connection to the torsion mode. Overall, the FRF division method can locate a general area of damage, but has difficulty in accurately pinpointing the original damaged DOFs when comparing DC 5 and DC 6.
3.6.2.2 Sequential Damage

Table 3-10 displays the results of the FRF division damage identification method for all sequential damage scenarios. The second column, labeled "Damaged DOFs," contains the DOFs that were directly affected from the removal of members. The third column, labeled "Severe Damage," lists the DOFs at which damage is most likely to occur. DOFs fall into this particular category when their $FRF_d$ value is greater than 1.5. DOFs with values between 1 and 1.5 are considered to be locations that may contain damage and are tabulated in the fourth column of Table 3-10.

Table 3-10: Results of dividing FRFs for all sequential damage scenarios.

<table>
<thead>
<tr>
<th>Sequential Damage Scenario</th>
<th>Damaged DOFs</th>
<th>Severe Damage ($Z&gt;1.5$)</th>
<th>Moderate Damage ($1&lt;Z&lt;1.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC 0 vs. DC 1</td>
<td>8X, 15X</td>
<td>22xx, 22xy</td>
<td>23xx</td>
</tr>
<tr>
<td>DC 1 vs. DC 2</td>
<td>6X, 13X</td>
<td>13xy, 14xx, 28xy</td>
<td>5yx, 6xx, 7yy, 8xx, 13xx, 15yx, 22xy, 28xx, 28yx</td>
</tr>
<tr>
<td>DC 2 vs. DC 3</td>
<td>7X, 16X (add: 6X, 13X)</td>
<td>6yy, 7yy, 8xx, 15xx, 24xx</td>
<td>7yx, 8yy, 14xx, 16yy, 21yy, 28xx, 28xy</td>
</tr>
<tr>
<td>DC 3 vs. DC 5</td>
<td>5X, 5Y, 6X, 6Y, 7X, 7Y, 8Y, 13X, 13Y, 14X, 14Y, 15Y, 16Y</td>
<td>6xx, 8yx, 14xx, 24xy, 28xy</td>
<td>6yy, 7xx, 7yx, 8xx, 13xy, 15yy, 16yx, 21xx, 21yy, 22xx, 22yy, 24yx, 28xx, 28xy, 28yx</td>
</tr>
<tr>
<td>DC 5 vs. DC 6</td>
<td>15X, 24X</td>
<td>16yy, 28yx, 28yy</td>
<td>22xy, 23xy, 23yx</td>
</tr>
<tr>
<td>DC 6 vs. DC 7</td>
<td>14X, 21X</td>
<td>7xx, 7xy, 8yx, 15xx, 15xy, 28xy</td>
<td>6xy, 16yx, 16yy, 23yy</td>
</tr>
<tr>
<td>DC 7 vs. DC 8</td>
<td>13X, 16X, 22X, 23X</td>
<td>6xy, 13yx, 13yy, 22yx</td>
<td>6yx, 8xy, 14xx, 23xx, 24yx</td>
</tr>
<tr>
<td>DC 8 vs. DC 9</td>
<td>14Y, 15Y, 22Y, 23Y</td>
<td>6yx, 6yy, 8xy, 16xy, 23yx, 23yy</td>
<td>--</td>
</tr>
</tbody>
</table>

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Overall, the FRF subtraction method locates the general area of the damage caused by the removal of cross bracing members for most sequential damage scenarios. This particular damage detection technique is not as accurate as the FRF subtraction method for sequential damage.

3.6.2.3 Cumulative Damage

In order to assess the effectiveness of the FRF division method on cumulative damage, a pictorial representation of the damage algorithms results for DC 0 vs. DC 9 is created and displayed in Figure 3-49. Severe damage occurs when the damage index exceeds 1.5 and is represented by large red arrows pointing in either the X or Y direction in Figure 3-49. Moderate damage is considered to happen when the damage indicator value is in between 1 and 1.5 and is represented by smaller orange arrows pointing in either the X or Y direction. Cross-correlated DOFs (i.e. impact in the X direction and response in the Y direction) that are identified as damaged locations are denoted by a large cross or "✗". Orange crosses represent moderate damage while red crosses indicate severe damage. As in previous sections, the blue stars mark the points of the removed members when transitioning from DC 0 to DC 9.
The highest damage indicator value of 2.83 occurs at 22xy (impact in the X direction at point 22 with response at point 28 in the Y direction). This implies that point 22 is the most damaged DOF: the X response from a Y impact at point 22 for DC 0 is dramatically different than the matching response in DC 9. This implies that point 23 is the most damaged DOF and that the response in the Y direction from impact at point 23 in the X direction for DC 0 is dramatically different from the same impact's response in DC 9. Conversely, the lowest damage indicator value of 0.0265 occurs at 22yy (impact in the Y direction at point 22 with response at point 28 in the Y direction). This relatively low value indicates that the responses in the Y
direction from impact at point 22 in the Y direction are similar when comparing DC 0 and DC 9. According to this particular damage detection technique, 22yy is the least damaged DOF when comparing DC 0 and DC 9; however, point 22 does show severe damage in the cross correlated term of 22xy.

As depicted in Figure 3-49, damage has been predicted to have occurred all over the structure. Every point labeled with a removed member or blue star on the structure should indicate some level of indicated damage; however, points 15, 21, and 24 are not identified by the algorithm. As with sequential damage, data for point 28 is extremely sensitive and will always show damage. Other than point 28, there is general agreement with expected results and obtained results from the FRF division damage identification technique for the cumulative damage scenario, DC 0 vs. DC 9.

3.6.3 Conclusions on FRF Damage Indicators

Overall, the FRF subtraction and division methods locate points and directions of motion that may be damaged by the removal of cross bracing members for most sequential damage scenarios. However, the algorithm incorporating subtracting is more precise than the one involving division. The FRF subtraction method is more accurate than any other damage indicator discussed in this chapter and for each sequential damage scenario, the algorithm picks out at least one of the DOFs contained in the column labeled "Damaged DOFs" within Table 3-9. Along with this precision, the algorithm also identifies several DOFs surrounding the expected areas of damage. Additionally, the cross-correlated terms are the most significant form of damage indication with this method; the torsion mode is most affected by the change from a symmetric to an asymmetric structure, and the cross correlated terms accurately depict this
phenomenon. A weakness of this method includes magnified FRF signals when impacting around or near the sensor location, point 28. However, since the location of the accelerometer is known, point 28 can simply be ignored. Similar results are produced for cumulative damage. In both sequential and cumulative scenarios, the most damage is predicted to have occurred at and above the second floor of the structure. Due to the relative stiffness of the foundation braces compared to the rest of the structure, it is difficult to cause large changes in structure's response near the base by simple removing exterior cross bracing members. Thus, it is reasonable to have more damaged DOFs toward the top of the structure than near the foundation.

3.7 Overall Comparisons and Discussion

Twelve different algorithms involving six major damage indices have been applied to the same experimental data. In order to compare and contrast methods herein, a conglomerate with each damage indicator's pictorial results is presented in Figure 3-50 for the example sequential damage scenario, DC 5 vs. DC 6. A similar figure is displayed in Figure 3-51 for cumulative damage.

Twelve separate damage detection algorithms have been applied to the example sequential scenario (DC 5 vs. DC 6). Analyzing Figure 3-50 leads to several observations about damage prediction for sequential cases. Multiple methods are most sensitive towards the top of the structure; COMAC, flexibility division, story stiffness, FRF subtraction, and FRF division identified most damage as occurring above the second floor. In addition, there are several algorithms that indicate a significant amount of damage within the torsion mode. These methods include curvature division, flexibility division, story stiffness, FRF subtraction, and FRF division. Finally, the COMAC of curvatures index is not adequate for use on sequential damage.
scenarios because it does not accurately locate damage. Note that MAC results are not included in Figure 3-50 because it cannot predict damage location. However, for DC 5 vs. DC 6, MAC correctly identifies the torsion mode as most affected.

As portrayed in Figure 3-51, the cumulatively damaged case of DC 0 vs. DC 9 is examined in order to find similarities between detection methods. Minimal damage is predicted by COMAC of curvatures, curvature division, and flexibility division. These methods do not accurately depict true damage within the structure and are deemed inadequate. Conversely, several algorithms tend to accurately identify damage. COMAC, story stiffness, FRF subtraction, and FRF division methods are among those that indicate a significant amount of damage throughout the test structure. Note that false positives cannot be correctly identified as damage should be indicated throughout the entire model building. Again, MAC is not included in Figure 3-51, but it does correctly indicate that all modes are significantly affected for the cumulative scenario.
Figure 3-50: Direct comparison of damage detection results for DC 5 vs. DC 6.

Figure 3-51: Direct comparison of damage detection results for DC 0 vs. DC 9.
There are a few sources of error that may contribute to the inaccuracy of the applied damage detection techniques. First, a uniform amount of torque was not applied to each screw when tightening the bolts, which may have resulted in varying strengths of each connection. The torque was difficult to measure for such small and low strength screws, but slight variations in connections could have caused rattling that contributed to extra noise within the captured acceleration signals. Additionally, the extreme sensitivity towards the top of the structure needs to be carefully scrutinized. The response at the top of the model may be amplified due to the location of the sensor at the top of the structure. However, the overly sensitivity top half of the structure could also be attributed to the fact that the most motion occurs furthers away from the foundation, towards the top of the structure. With increased motion, the dynamic parameters are more likely to be affected by damage at the top of the structure. In order to investigate this, sensors should be placed at other points on the structure in order to verify the results obtained in this work. A single-input, multiple-output dynamic test's damage indication results could be compared to those obtained in this work from a multiple-input, single-output instrumentation system.
4. CONCLUSION

4.1 Summary

In order to evaluate potential damage indices, a three-story metal frame building was constructed. Using Star Modal software, dynamic structural properties were obtained from modal decomposition on experimental tap test responses. The natural frequencies and mode shapes of the structure established an as-built baseline for comparison to ten other scenarios with removed bracing. Once modal properties for each case were determined, six unique damage indicators were applied to identical experimental data via twelve algorithms. The effectiveness of each damage detection technique was assessed, and final recommendations for the three-story model building were made.

Significant observations can be made about each mode shape based damage detection algorithm. MAC is adequate for detecting incremental damage that affects the twisting motion of the structure. Modes 3 and 1 are the most sensitive for sequential and cumulative damage, respectively. Although COMAC can predict general areas of damage, it has difficulty identifying exact damaged locations. However, it does effectively predict damage for the cumulative scenario. COMAC of modal curvatures is not suitable for sequential cases due to inaccurate damage prediction and the abundance of false positives. It also is not a reliable detection method for the cumulative case. The modal curvature division algorithm successfully determines direction of damage and works best for incremental scenarios with brace removal in one direction. It also has a high frequency bias with second order modes being the most sensitive to damage. Next, the modal curvature subtraction method produces too many false
positives and is overly sensitive near the foundation. Similarly, the summed modal curvature division algorithm is not adequate as it does not accurately identify damaged locations. Thus, curvature subtraction and summation are not suitable for the model building.

Conclusions have also been reached regarding the applicability of flexibility, story stiffness, and direct FRF indicators to the test structure. The flexibility division and percent difference methods are quite accurate for sequential damage with few false positives. However, these algorithms are not sensitive enough to detect cumulative damage; this could be due to the utilized normalization scheme. Flexibility subtraction is not suitable for either sequential or cumulative damage within the test structure due to a low frequency bias that incorrectly magnifies first order mode results. The story stiffness algorithm yields accurate damage prediction for sequential cases with brace removal between the second and third floors; conversely, damage between the first and second floors produces many false positives. Additionally, second order modes are most sensitive to cumulative damage. A major shortcoming, the story stiffness method can predict which global floor but not local element is damaged. Finally, both FRF subtraction and FRF division work well, but subtraction is more accurate below the second floor. For direct FRF indicators, cross-correlated terms are the most significant due to the extreme sensitivity of the torsion mode. However, the top of the structure is overly sensitive to damage due to sensor location. Of all the implemented algorithms, FRF subtraction using the FRFs as a direct indicator is the most accurate damage detection scheme for the three-story test structure.
4.2 Recommendations and Contributions

Table 4-1: Overall damage prediction accuracy for test structure.

<table>
<thead>
<tr>
<th>Damage Indicator</th>
<th>Algorithm</th>
<th>Recommendations</th>
<th>Recommendations</th>
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</thead>
<tbody>
<tr>
<td>MAC</td>
<td>MAC</td>
<td>Adequate; mode 3 most sensitive</td>
<td>Adequate; mode 1 most sensitive</td>
</tr>
<tr>
<td>COMAC (Modal curvatures)</td>
<td>COMAC of modal curvatures</td>
<td>Not suitable</td>
<td>Not suitable</td>
</tr>
<tr>
<td></td>
<td>Modal curvature division</td>
<td>Works well when brace removal is in one direction</td>
<td>Not suitable</td>
</tr>
<tr>
<td></td>
<td>Modal curvature subtraction</td>
<td>Not suitable; many false positives</td>
<td>Not suitable</td>
</tr>
<tr>
<td></td>
<td>Summed modal curvature division</td>
<td>Not suitable</td>
<td>Not suitable</td>
</tr>
<tr>
<td>Flexibility</td>
<td>Flexibility division</td>
<td>Adequate; few false positives</td>
<td>Not sensitive enough</td>
</tr>
<tr>
<td></td>
<td>Flexibility percent difference</td>
<td>Adequate; few false positives</td>
<td>Not sensitive enough</td>
</tr>
<tr>
<td></td>
<td>Flexibility subtraction</td>
<td>Not suitable</td>
<td>Not suitable; low frequency bias</td>
</tr>
<tr>
<td>Story stiffness</td>
<td>Story stiffness</td>
<td>Suitable for brace removal between second and third floors</td>
<td>Adequate; second order modes most sensitive</td>
</tr>
<tr>
<td>Direct FRF comparison</td>
<td>FRF subtraction</td>
<td>Suitable; torsion most significant; <strong>most accurate</strong></td>
<td>Suitable; top of structure most sensitive; <strong>most accurate</strong></td>
</tr>
<tr>
<td></td>
<td>FRF division</td>
<td>Adequate; torsion most significant</td>
<td>Adequate; top of structure most sensitive</td>
</tr>
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</table>
4.3 Future Work

Short term as well as long term future work are presented in this section. Short term objectives could be implemented if more project time were available. Conversely, long term efforts include ideas that may be implemented by another researcher upon reading this work.

4.3.1 Short Term

In order to simulate a stiffer and thus more realistic building, the three-story structure was retrofitted with twelve 1/8" thick, 1" wide beams across the side of each floor. This configuration is presented in Figure 4-1.

Figure 4-1: Test structure with twelve reinforcing beams attached.

Three additional damage cases were considered: the reinforcing beams were removed from structure one side at a time until the model building returned to the baseline of DC 0. As preliminary analysis, tap tests were performed on four reinforced damage cases, and the resulting
modal peaks were generated in order to map the global modes. Figure 4-2 displays the modal map for all reinforced cases. Although further analysis needs to be completed, both frequency shifts and mode splitting are witnessed, much like with damage cases 0 through 10. However, the reinforced cases are much more complex in nature as compared to the unreinforced configurations. Due to increased stiffness, higher frequencies must be examined; both signal noise and mode uncertainty are introduced. A more in-depth and lengthy analysis of these reinforced cases is required before damage algorithms can even be applied.

4.3.2 Long Term

A limited amount of damage detection algorithms were discussed in this work. In order to improve upon results and perform a more complete analysis, it would be wise to investigate additional damage indication methods. Novel algorithms involving other dynamic parameters besides mode shapes and natural frequencies should be examined. It is also imperative to test a wide variety of damage indicators on laboratory-built structures of all scales and sizes. Including the most common reinforced concrete buildings, structures composed of non-metal materials are of particular interest. In addition to material and scale changes, studying the dynamic response and attempting damage detection within joints and connections of structures could lead to new discoveries. When collecting dynamic response data on future model structures, one should also explore diverse non-destructive testing methods with various input excitations. Different types of sensors besides accelerometers could measure shaker table or ambient vibration response parameters. Researchers could also utilize single-input, multiple-output methods in order to acquire response signals. Finally, several types of mechanical damage could be induced. The
damage within this work was in the form of brace removal; temporal deterioration, member cracking, and plastic deformation could be simulated.

Although laboratory experimentation is important to fundamental understanding, the true issue is the application of damage detection algorithms to real world structures. There are several quandaries that must be addressed when attempting to evaluate the health state of a real building or bridge. One single damage indicator has not been proven to be effective for all civil structures. At this time, damage may be best detected in multiple structures through different techniques. The vibration-based methods outlined in this work are quite common for steel and concrete structures, but a major topic that has not been addressed within this work is the frequency range of interest. The model building herein had a fundamental natural frequency of approximately 70 Hz; massive infrastructure has much lower natural frequencies with a first mode usually below 5 Hz. Extra care must be taken to ensure that required modes are properly excited. If local damage detection is desired, higher frequency modes are required but still usually below 250 Hz. Conversely, global damage indication is better estimated with low frequency modes, perhaps less than 50 Hz. In addition to different frequency ranges, a wide variety of sensor and excitation techniques should be explored and applied to real-world structures. Ambient vibration excitation paired with an output-only damage detection algorithm is currently the most promising method. Output-only methods allow engineers to use ambient measurements under normal loading conditions, but the time history signals typically need to be several minutes in length for researchers to effectively extract dynamic parameters. Overall, the excitation and sensors utilized in field structural health evaluation test will dictate the sensitivity of the damage indication results.
After considering varieties of both real and laboratory structures, a robust damage indicator (or a set) can be formulated. Ultimately, a computer program should be developed to aid first responders in damage assessment. This program would provide a simple interface that would require no structural dynamic expertise to operate. It would extract the appropriate dynamic parameters from real-time response data, calculate natural properties, apply an optimal damage indicator, and explicitly output damaged members. The computer program will also alert engineers to structural weaknesses and would represent structural integrity in a simplified pictorial manner.
Figure 4-2: Mode map for reinforced damage cases.

11.1: general sway in X; Pt 21 has a lot of displacement (not strong)

11.2: sway along diagonal 1-3

11.3: 1st torsion mode (strong)

11.4: 2nd bending in X; side 1 displaces more than other sides

11.5: 2nd bending in Y;Pts 8 and 24 moving slightly out of phase

12.1: 1st bending in X; Col 2-26 moving out of phase

12.2: 1st bending in Y; 2nd floor twisting a bit

12.3: 1st torsion; Pt 22 is twisting out of phase

12.4: 2nd bending in X; some twisting of 1st floor; side 1 has most displacement

12.5: 2nd bending in Y (strong)

13.1: 1st bending in X; Pts 7, 16, and 24 moving out of phase

13.2: 1st bending in Y; Pts 14 and 22 moving with a lot of displacement

13.3: 1st torsion mode (strong)

13.4: 2nd bending along the 2-4 diagonal

13.5: 2nd bending in Y (strong)

14.1: 1st bending in X (strong)

14.2: 1st bending in Y; Pts 7, 21, and 25 twisting out of phase (not strong)

14.3: 1st torsion mode; side 4 with most displacement

14.4: 2nd bending in X; Col 1-25 more displacement than other cols

14.5: 2nd bending in Y; side 4 has most displacement; pt. 14 moving out of phase

0.1: 1st bending mode in X; not much displacement; columns 3-17 and 4-18 clearly swaying along X

0.2: clear 1st bending in Y (strong)

0.3: very clear torsion (strong)

0.4: 2nd bending in X

0.5: 2nd bending in Y; Pt 14 out of phase

DC 0

DC 11

DC 12

DC 13

DC 14
LIST OF REFERENCES


LIST OF APPENDICES
APPENDIX A: Modal Peak Trends

Mode Map for Single Story Damage

DC 0

**One brace removed, one added**

DC 1

DC 2

DC 3

DC 4

DC 5
Mode Map for Multiple Story Damage

DC 0

DC 6

DC 7

DC 8

DC 9

DC 10
Mode Map for Symmetric Damage

DC 0

DC 2

DC 4

DC 5

DC 7

DC 8

DC 10
Mode Map for Asymmetric Damage

DC 0

DC 1

DC 3

DC 6

DC 9
APPENDIX B: Manual for Obtaining Mode Shapes and Natural Frequencies in Star Modal

Damage case 10 of the three story test structure is used herein as an example to demonstrate how to obtain experimental mode shapes using FRF data in Star Modal. The Star Modal operation manual was used to create this step-by-step procedure [11].

The Workspace

This is the main interface of Star Modal. After opening a file, the user will see this workspace screen. Labeled 1, the Workspace Tool is the entire white panel on the left side of the screen. This is a directory of all project files within the current workspace. A workspace file links several project files together in order to allow the user to work on several variations of one project, combining different measurements and structural modifications into one convenient interface. In order to toggle between different projects, one must right click on a project and select "Set as Master Project." The current Master Project is always indicated by bold typeface.

The black part of the screen, labeled 2, contains the model view which allows one to view mode shape deflection animations in a 3-D environment. The box at the bottom, labeled 3, contains useful buttons that control the model and graph views. Click the graph icon to toggle to the graph view, and click the cube icon in order to return to the model view.
Building the Model

Open the Model Table by expanding a project in the workspace tool and right clicking on "Model" then "Setup."
A window opens as shown below. The buttons along the top row of the Model Table are used to toggle between the Points, Lines, and Surfaces Tables. Coordinates of node points are directly entered into the table entitled Points Table. A measurement point label will automatically be assigned to each global node point. In this example, direction 1 corresponds to the X axis and direction 2 and 3 to the Y and Z axes, respectively.

For the columns, the Lines Table is displayed in the figure below. A line is defined by two measurement points - the "From (Pt)" to the "To (Pt)". When the "New" box is checked a new line will be drawn starting at "From (Pt)"; however, if unchecked, the line will be a continuation of the previous line. If the "Inclusive" is checked the line will be drawn between each point within the range from "From (Pt)" to "To (Pt)". Otherwise, if the "Inclusive" is left unchecked, a straight line will be drawn directly "From (Pt)" to "To (Pt)."
In order to draw the floors of the model, the Model Editor needs to be opened. Do this by clicking Setup>Model Editor from the top toolbar.

A panel on the right side of the screen will pop up that will allow the user to have point and click control over model construction. Any change or addition made to points, lines, or surfaces with the Model Editor will be automatically updated within the Model Tables. Since the points and
lines are already set up for this particular example, the Surfaces tab should be selected under
"Draw." Click the "Quad" bullet in order to enable surfaces to be defined by 4 measurement
points. Now, click on 4 points comprising a floor on the model; this will create 4-sided surfaces
that will appear in dark blue. The user can check to the Surfaces Table within in the Model
Table to be sure that the surface was in fact created. The Surfaces Table will only display
surfaces in terms of 3 points. For example, a quadrilateral surface will be split up into two
triangular surfaces within the Surfaces Table.
Importing Measurements

Once the model has been constructed, it is time to import frequency response measurements. Expand the project that needs measurement files and click on "Measurement Data" in the Workspace Tool. If there are no measurements currently linked to the project, a window will pop up asking the user to import measurement files. Click "Show Me" and a file dialogue will pop up prompting the user to select files to import.

Select all the measurement (.frf) files via shift-click and click "Open". Note that the measurement files are a result from data processing as described in Figure 2-6.
Star Modal then asks what types of measurements are being imported: fixed excitation, fixed response, or ODS. Since the experimental set up included a single accelerometer at a fixed location, "Fixed Response" has been selected for this example. More data can always be added later via the append check box.

Once the measurements have been imported, they will appear in the Measurement Table which can be accessed by clicking "Measurement Data" under a project in the Workspace Tool. Each row corresponds to one frequency response function (FRF). For example, the first line in the Measurement Table below refers to the FRF produced by impacting the structure at point 5 in the Operating Deflection Shapes (model updating).
X direction and capturing the response at point 28 in the X direction, which is from file 005x028x.frf.

Defining Constraints

Often times an experimental FRF is not available for every point in every direction. Constraints are needed in order to determine the motion of unmeasured points on a model. Star Modal defines the motion of unmeasured points by a linear combination of measured points. The user should be concerned with applying constraints to his/her model when 1) a geometrically complex model is needed for visual understanding, but only a small subset of measured points may be needed to describe the motion of the structure or 2) motion at a certain point or direction cannot be easily measured. In order to setup the constraints within a model, one should right click Constraints>Setup in the Workspace Tool. This action opens the Constraints Table. Each line of this table needs to be manually created or modified. The “Dependent” column contains point/direction combinations that are unmeasured while the Independent column contains point/direction combinations that have been measured.
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<th>Independent</th>
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Modal Analysis

On the bottom left hand portion of the workspace interface, click the graph icon to toggle to the graph view. A plot of an FRF should show up, assuming that the experimental data has been properly imported. Click Results>Mode Indicators on the top toolbar to bring up the Modal Peaks Calculator. The right side of the screen should display both the DOF Selector and Mode Indicators panels.

Next, select all the DOF measurements that need to be included in the Modal Peak calculations. In this example, all DOFs and references have been selected. Under the Mode Indicators panel, there are six different ways of calculating the modal peaks of a structure. The modal peaks graph is essentially the cumulative frequency response function incorporating all geometry and measurements. In this example, the modal peaks are calculated using Magnitude². Click "Calculate" once the correct parameters are selected.
In order to view the calculated modal peaks, select "Modal Peaks" in the graph view drop-down menu at the top of the screen.

The Modal Peaks plot is displayed below. Sometimes the user is only interested in a certain frequency range. The scales on the axes can be directly edited by double clicking on the last or first number on the independent or dependent axis. The following screenshot demonstrates changing the frequency scale from the 0 to 1000 Hz range to the 0 to 250 Hz range.
A correctly scaled plot of the modal peaks for the structure from 0 to 250 Hz is displayed below.
Currently the Y axis corresponds to the real values of the modal peaks; however, for this example, the magnitude ($\sqrt{\text{Real}^2 + \text{Imaginary}^2}$) values are needed. In order to switch the Y axis to magnitude, right click on the Y axis and select "Magnitude".

Various Y-axis units can be selected.
Now that the plot of magnitude of the modal peaks from 0 to 250 Hz is correctly formatted, the user should calculate Frequency and Damping values (F & D). This is done by clicking on the "Estimate FD" Tab under the Mode Indicators panel on the right side of the screen. In this particular example, the user is only interested in the modes that fall within the range of 0-250Hz. This is done by selecting the proper low and high frequency values. The Model Size refers to how many iterations the program will loop through to find global modes; thus, a high model size is preferred, although too high may crash the computer. In this example, the model size range is 1 to 75. Press "Calculate" when the all the settings are correct. In order to clarify the results and discard data points that do not meet a certain criteria (and therefore are not considered a mode), one must check the "% Critical Damping" and "Damping" boxes as well as enter in values for these parameters. For the 3 story test structure, values of either 3% or 5% were used.

After calculating the frequency and damping values, the stability diagram must be turned on in order for the user to visualize frequency and damping values. This is done by clicking on the Stability Diagram icon in the top toolbar and making sure it is depressed.
The stability diagram should look something like the following:

Now it is time for the experienced user to pick modes that look "right." Modes that look "right" will usually be indicated by a large peak in the modal peaks plot (peak frequency) as well as a large column of filled-in dots (stability). There are five major modes in the example provided. A circle within each column of dots should be selected, in order of lowest mode to highest mode. The selected modes are highlighted in red. These modes are later verified by shape animation and user experience.
Advanced Curve Fitting

While the raw results are complete, the next step is to curve fit FRFs at each degree of freedom (or measurement/node point). To begin, check the modal participation factors to see which reference point/direction to which the measurements should be fitted. On the top toolbar, click Setup>Modal Participation Factors to open the Modal Participation Factors Table.
Each row in this table corresponds to one mode selected in the stability diagram. The values contained within this table represent a measure of how sensitive each reference degree of freedom (DOF) is to each mode of vibration. The user should take note of which reference DOF has the highest average modal participation factor. In this example, the highest average modal participation factor belongs to 28X. This means that the selected modes' shapes should be calculated based on curve fitting FRFs at each DOF with respect to the reference DOF 28X.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Damping (Hz)</th>
<th>28X</th>
<th>28Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18.73</td>
<td>0.16</td>
<td>41.96</td>
<td>100.00</td>
</tr>
<tr>
<td>2</td>
<td>27.31</td>
<td>0.16</td>
<td>100.00</td>
<td>54.98</td>
</tr>
<tr>
<td>3</td>
<td>53.53</td>
<td>0.42</td>
<td>100.00</td>
<td>43.32</td>
</tr>
<tr>
<td>4</td>
<td>197.02</td>
<td>1.07</td>
<td>64.60</td>
<td>100.00</td>
</tr>
<tr>
<td>5</td>
<td>211.57</td>
<td>1.57</td>
<td>100.00</td>
<td>73.75</td>
</tr>
</tbody>
</table>

On the top toolbar, click Analysis>Curve Fitting to bring up the Curve Fitting panel on the right side of the screen.
Click on the "Poly-Reference" tab within the Curve Fitting panel. "Complex Shape" should be checked in order to ensure that both the real and imaginary portions of modes are calculated. "Flexibility Residues" as well as "Mass Residues" should be checked in order to capture coupled modes. The curve fitting process is calculated with respect to the reference DOF, which was identified by the Modal Participation Factors Table. Click "Fit All" to calculate mode shapes.

In order to visually inspect the mode shapes, switch to the Model view by clicking the Cube icon in the bottom left hand corner of the screen. Then in the drop down animation menu, under , select "Mode Shape (Mode)".
At this moment, motion is only assigned to the measured DOFs. The constraints need to be applied to the frequency results in order for the entire structure to move in the mode shape animation. Open the Constraints Table, and click Options>Add to Frequency Results.

Mode shape animations are now ready to be viewed. Toggle between each mode with the drop down menu in the top toolbar.
In the Workspace Tool, open the Frequency Results Table in order to obtain mode shape deflection values at each DOF.

Below is an example of the Frequency Results Table.
VITA

Originally from Basking Ridge, New Jersey, Samantha Sabatino graduated high school in 2005 and enrolled at Vanderbilt University for undergraduate studies. After receiving bachelor degrees in civil engineering and mathematics, she applied for graduate school. In 2009, she began her studies at the University of Mississippi in order to obtain a Master's degree in civil engineering. During Samantha's two years at the University of Mississippi, she was awarded the Outstanding Graduate Award by the School of Engineering and won third place in the Graduate School's annual poster symposium. Additionally, she intends to submit her thesis work to a reputable engineering journal for publishing. Finally, she completed her thesis entitled "Experimental Damage Diagnosis of a Three-Story Spatial Frame" in 2011 and looks forward to the next step in her education: obtaining a Ph.D. in structural engineering at Lehigh University.