Mathematical Tasks Without Words And Word Problems: Perceptions Of Reluctant Problem Solvers

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MATHEMATICAL TASKS WITHOUT WORDS AND WORD PROBLEMS:
PERCEPTIONS OF RELUCTANT PROBLEM SOLVERS

A Dissertation
presented in partial fulfillment of requirements
for the degree of Doctor of Education
in the School of Education
The University of Mississippi

by

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This qualitative research study used a multiple, holistic case study approach (Yin, 2009) to explore the perceptions of reluctant problem solvers related to mathematical tasks without words and word problems. Participants were given a choice of working a mathematical task without words or a word problem during four problem-solving sessions. Data were gathered from problem-solving sessions, in the form of session transcripts, written reflections, and interviews to determine how the reluctant problem solvers perceived the problems presented in each session. Participants' views of the problems before and after working were recorded and thick descriptions of the sessions including quotes from the participants are provided.

Findings indicated that the reluctant problem solvers typically chose to work tasks that appeared to be easier, indicating their desire to have high self-efficacy before working tasks. Findings also showed that participants did not expect to struggle, a natural occurrence during problem solving, making them less likely to engage in and persevere with challenging tasks. Participants demonstrated strategies that helped them to avoid struggling when working word problems, however, they did not demonstrate similar strategies when solving mathematical tasks without words. Therefore, mathematical tasks without words hold potential for engaging students in problem solving and possibly encouraging them to persevere when problem solving.
DEDICATION

This work is dedicated to my parents, John and Dianne, who lead by example, showing me that with hard work, achieving dreams is possible. They give sacrificially for me, believe in me, support me, and encourage me. Most importantly, they love me, and they taught me to love the Lord. To my mom and dad, the single most influential people in my life, I love you endlessly, and I cannot thank you enough for all that you do for me!

This work is also dedicated to my sisters, Joanna and Elizabeth, who provide me with the greatest source of joy and happiness in my life. They are my very best friends and show me daily that there is no sweeter thing than the relationship between sisters. To my sisters, Joanna and Elizabeth, you are sweet gifts from God, and I love you!

Finally, this work is dedicated to my grandparents, Bob and Joanne, who have prayed for me, supported me, and loved me as long as I can remember. They fostered in my sisters and me a wonderful imagination filled with creativity and excitement. To my grandparents, my love for you is “deep and wide!”
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CHAPTER I: INTRODUCTION

Introduction

International comparisons including the Program for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS) have found that students in the United States underachieve in mathematics when compared to other countries (NCES, 2009). The National Center for Educational Statistics reports that fifteen-year-old students in twenty-three of the twenty-nine other countries in the Organization for Economic Cooperation and Development (OECD) outperformed students in the United States as shown on the PISA (NCES, 2009). Additionally, on the TIMSS, the United States’ average scores for fourth-grade and eighth-grade students fell below that of eight other countries (NCES, 2009).

Consequently, the National Governor’s Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO) collaboratively led an effort to develop the Common Core State Standards for Mathematics (CCSSM) (CCSSI, 2010). The CCSSM is designed so that the expectations for students in the United States are equally as rigorous as the expectations for students in other countries (CCSSI, 2012a). According to the NGA Center and CCSSO, the CCSSM was created by examining models from high-performing states within the United States as well as high-performing countries across the world and utilizing that information in developing a new set of standards with the goal of college and workforce preparation for students. In response to the development of the CCSSM, forty-five states and three territories within the United States have adopted the curriculum (CCSSI, 2012b).
One major difference between the CCSSM and previous individual state curriculums is that the CCSSM introduces the Standards for Mathematical Practice, which include eight practices that “describe varieties of expertise that mathematics educators at all levels should seek to develop in their students” (CCSSI, 2010, p. 6). These Standards for Mathematical Practice are based on two primary documents. First, the standards were informed by the National Council of Teachers of Mathematics’ (NCTM) Process Standards, which include Problem Solving, Representations, Reasoning and Proof, Connections, and Communication (NCTM, 2000). Second, the standards were based on the National Research Council’s (NRC) strands associated with a goal of mathematical proficiency, which include Strategic Competence, Conceptual Understanding, Procedural Fluency, Adaptive Reasoning, and Productive Disposition (NRC, 2001). To understand the expectations for students found in the Standards for Mathematical Practice, a closer examination of these two foundational documents is provided.

The Process Standards

According to NCTM, “The Process Standards – Problem Solving, Reasoning and Proof, Communication, Connections, and Representation – highlight ways of acquiring and using content knowledge” (NCTM, 2000, p. 29). The National Council of Teachers of Mathematics recommends that teachers engage students in all five of the process standards in their learning of mathematics every day. A classroom in which students use or engage in the process standards is referred to as a “standards-based” classroom or as using “standards-based” instruction. To better understand standards-based instruction, each of the Process Standards will be described.

The first standard listed, Problem Solving, is an essential part of learning mathematics (NCTM, 2000).
Problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics but also a major means of doing so. (NCTM, 2000, p. 52)

Effective problem solvers constantly evaluate and redirect their work if needed (NCTM, 2000).

The Reasoning and Proof standard requires that students “make and investigate mathematical conjectures” (NCTM, 2000, p. 57). Additionally, students must explain their reasoning and evaluate their own work as well as the work of others. To do so, students must be engaged in the Communication standard. Students must “communicate their mathematical thinking coherently and clearly to peers, teachers, and others” (NCTM, 2000, p. 60). The Communication standard involves not only talking about the mathematics but also writing about it. This might include writing reflections detailing their thoughts about the mathematics.

For students to solve a problem for which they do not immediately have a strategy to solve, the problem likely will provide a context, which gives the students an entry point to understanding the problem. Problems also might provide connections among mathematics concepts previously learned, from which students might gain an idea for developing a strategy to solve. The Connections standard dictates that students “recognize and use connections among mathematical ideas, understand how mathematical ideas interconnect and build on one another to produce a coherent whole, and recognize and apply mathematics in contexts outside of mathematics” (NCTM, 2000, p. 64).

Finally, the Representation standard requires students to “create and use representations to organize, record, and communicate mathematical ideas” (NCTM, 2000, p. 67). Using pictures,
charts, tables, symbols, or manipulatives to model a problem would be an example of engaging in the Representation standard.

To compete with other countries in the job market, the United States must help students become willing and able to engage in mathematics in a way that allows them to make connections, think critically about the mathematics, and see relationships between concepts (NGA Center et al., 2012). Students’ engagement in the Process Standards promotes this type of activity in the classroom (NCTM, 2000, 2009).

Mathematically Proficient Students

In their book, *Adding It Up: Helping Children Learn Mathematics*, the National Research Council (2001) described their idea of mathematical proficiency as follows:

Recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics, [they] have chosen *mathematical proficiency* to capture what [they] believe is necessary for anyone to learn mathematics successfully . . .

As they go from pre-kindergarten to eighth grade, all students should become increasingly proficient in mathematics. That proficiency should enable them to cope with the mathematical challenges of daily life and enable them to continue their study of mathematics in high school and beyond. (p. 116)

There are five strands or components of mathematical proficiency that are equally important for students to learn mathematics. These include: Conceptual Understanding, Procedural Fluency, Strategic Competence, Adaptive Reasoning, and Productive Disposition. The Strands of Mathematical Proficiency, similar to the Process Standards, are not independent of each other; instead, strands are most effective working as a whole rather than individually (NRC, 2001). Students should learn mathematics with a goal of attaining mathematical
proficiency. Engagement in the Process Standards allows students to put into practice the behaviors that are required to build mathematical proficiency. Essentially, the Process Standards serve as a means for meeting the Goal of Mathematical Proficiency. For further clarity, each Strand of Mathematical Proficiency will be described, as well as how the Process Standards facilitate the acquisition of those strands.

Laying a foundation on which students can build mathematics concepts is important in developing their understanding of topics and relationships between them (Lambdin, 2003). The first Strand of Mathematical Proficiency, Conceptual Understanding, includes “comprehension of mathematical concepts, operations, and relations” (NRC, 2001, p. 5). To support the development of students’ conceptual understanding, they should be engaged in problem solving. Lambdin (2003) states “Understanding takes place in the students’ minds as they connect new information with previously developed ideas, and teaching through problem solving is a powerful way to promote this kind of thinking” (p. 11). The National Council of Teachers of Mathematics (2009) states that without building understanding of concepts through problem solving, “procedures may be forgotten as quickly as they are apparently learned” (p. 5).

After developing an understanding of a concept through problem solving, however, students should demonstrate Procedural Fluency, the second Strand of Mathematical Proficiency. Procedural fluency is a “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately” (NRC, 2001, p. 5). Students should learn mathematics by engaging in the Problem Solving standard, as opposed to learning procedures and algorithms from the beginning of a new topic (NCTM, 2000). Once students have learned the concept through problem solving, it becomes more efficient for them to transition to the use of procedures and algorithms to solve similar problems. Additionally, they are more likely to commit the concept to memory and
retrieve it when necessary (Lambdin, 2003). “The moral of the story is that, if students are to
develop both proficiency and understanding of skills, the most efficient instructional approach is
to build understanding into students’ experience from the beginning” (Hiebert, 2003, p. 18).

The third Strand of Mathematical Proficiency, Strategic Competence, describes the
“ability to formulate, represent, and solve mathematical problems” (NRC, 2001, p. 5). To
formulate or pose problems, students must determine what exactly the problem is and what
information is useful to solve the problem. “Problem posing can help students to see a standard
topic in a sharper light and enable them to acquire a deeper understanding of it as well. It can
also encourage the creation of new ideas derived from any given topic—whether a part of the
standard curriculum or otherwise” (Brown & Walter, 2005, p. 1). Additionally, the ability to
represent problems requires that students engage in the Representation standard, using materials
and manipulatives to model the problem (NCTM, 2000). Finally, to solve problems, students
must develop strategies, monitor problems that arise and reflect as they work as described by the
Problem Solving standard (NCTM, 2000). As described, engagement in the Representation and
Problem Solving standards fosters the building of strategic competence.

According to the National Research Council (2001), Adaptive Reasoning, the fourth
strand, is defined as “the glue that holds everything together, the lodestar that guides learning.
One uses it to navigate through the many facts, procedures, concepts, and solution methods and
to see that they all fit together in some way, that they make sense” (p. 129). It includes the
“capacity for logical thought, reflections, explanation, and justification” (NRC, 2001, p. 5). The
Communication standard is significant to Adaptive Reasoning as it is necessary for students to
communicate or share their thoughts, strategies and ideas related to the mathematics.
Additionally, engaging in the Reasoning and Proof standard allows students to explain and
justify their thoughts as they are working (NCTM, 2000). “A refocus on reasoning and sense making will increase understanding and foster meaning” (NCTM, 2009, p. 5).

Engaging in the Process Standards and practices described above allows students to take ownership of the mathematics, build confidence, and become autonomous learners (Lambdin, 2003). This confidence is described in the last Strand of Mathematical Proficiency, Productive Disposition, which refers to the “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (NRC, 2001, p. 5). Productive disposition is important as it allows students to see learning mathematics as worthwhile (NRC, 2001). The Connections standard is valuable for the fostering of productive disposition. Math-to-math connections support students in seeing mathematics as sensible, and math-to-real world connections support students in seeing mathematics as worthwhile and useful (NCTM, 2000). “Such interplay, in which mathematics illuminates a situation and the situation illuminates mathematics, is an important aspect of mathematical connections. Seeing the usefulness of mathematics contributes to students’ success in situations requiring mathematical solutions” (NCTM, 2000, p. 134).

Students must see mathematics as valuable, and they must see themselves as mathematicians for learning to occur (NRC, 2001). The National Research Council states that “helping students acquire mathematical proficiency calls for instructional programs that address all its strands” (NRC, 2001, p. 116). Developing mathematical proficiency is a goal for students, and the Process Standards provide students with a means for meeting that goal.

**Reluctant Problem Solvers**

Recognizing the role that the Process Standards play in reaching the Goal of Mathematical Proficiency, the focus turns to the most important Process Standard: Problem

Ideally, all students would be internally motivated to participate in problem solving or classified as “doers of mathematics” (Fillingim & Barlow, 2010, p. 82). This internal motivation is described by Cobb, Gresalfi, and Hodge (2009) as “personal identity,” for which students choose to participate in learning mathematics for themselves and not because it is expected of them by the teacher or others. Engaging in the Process Standards to learn mathematics just for the sake of engaging is not enough to become mathematically proficient; students must internalize the mathematical processes (Cobb et al., 2009; Fillingim & Barlow, 2010).

Furthermore, students’ engagement in Problem Solving is highlighted in the first Standard for Mathematical Practice.

Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. (CCSSI, 2010, p. 6)

Based on this excerpt, it is clear that meeting this standard requires students to engage in the problem-solving process, as described by Polya (1957). Unfortunately, many classrooms contain students who might be considered reluctant to engage in problem solving. “Typically, reluctant problem solvers [are] not as eager to engage, often relying on group members to do the work for them or asking the teacher to tell them what they should do” (Holbert & Barlow, 2012/2013, p. 312). The behavior of reluctant problem solvers does not support meeting the first Standard for Mathematical Practice as previously described.
Reluctant problem solvers, though hesitant to engage, must somehow be motivated to participate in the mathematics classroom. With the goal of all students achieving mathematical proficiency as defined by the National Research Council (NRC, 2001), as well as all students achieving the expectations of the Standards for Mathematical Practice in the CCSSM (CCSSI, 2010), it is imperative that mathematics educators identify learning opportunities that support reluctant problem solvers in engaging in the problem-solving process and help them reduce or defeat their reluctance in the mathematics classroom. A portrait is provided to aid the reader in understanding how a reluctant problem solver behaves in the classroom.

**A Portrait of a Reluctant Problem Solver**

To better understand how a reluctant problem solver might interact in a classroom, a scenario is provided based on my field notes from a pilot study. The following scenario is a compilation of my field notes on different students displaying behaviors of reluctant problem solvers combined into one fictitious student.

Imagine a fifth grade classroom comprised of approximately twenty-four students. Six groups of four students are arranged in the classroom. The teacher presents the class with a task or problem for which they must problem solve. Students are expected to understand the problem, develop strategies for solving the problem, solve the problem, and examine the reasonableness of the solution. Finally, students will present their work to their classmates and discuss their solution strategies and solutions. Students will ask questions of the peers and comment on their work. All discussion will be completed in a respectful manner.

On this day, the teacher presents the students with a task. Most students are excited about solving the task and eagerly begin to discuss the problem, the information provided, and their ideas for solving. A roar of noise circulates the room as different groups become more and more
excited about their ideas. With all of the excitement in the room, Luke sits quietly in one of the
groups. He leans over shuffling through papers under his desk. When his group members ask
what he thinks, he says little or nothing. Luke delays engaging in the task by sharpening his
pencil, searching through his backpack, or walking across the room to throw paper in the trash
can. He might ask to use the restroom or leave the room. The teacher has to prompt him several
times to begin working, as do his group members.

When Luke begins working on the problem, he asks his group members for help. His
group members discuss the problem with him. However, Luke starts coloring on his paper.
Finally, he stops and reads the problem and then he writes something on his paper. He turns his
paper into the teacher who states that he did not follow directions because he was not paying
attention when they were given.

Based on the repeated occurrences of these observations, including delaying engagement
in the mathematics task and being off task when the rest of the group is working, Luke is
classified as a reluctant problem solver. As previously stated, Luke was a fictional character
whose behaviors were taken from my field notes describing potential participants for a pilot
study.

**Mathematical Tasks without Words**

Mathematical tasks without words are problem-solving tasks in mathematics that do not
look like traditional word problems in that they require little to no reading (Holbert & Barlow,
2012/2013). Mathematical tasks without words might be considered games or puzzles since they
have few words. Holbert and Barlow (2012/2013) discussed the possibility of using these tasks
without words as a form of intervention for use with reluctant problem solvers. Students were
given the tasks to complete, and reluctant problem solvers often eagerly engaged in the
mathematical tasks without words. The recommendation of using mathematical tasks without words was offered, however, without data to support effectiveness. The effectiveness was limited to anecdotal notes.

Having taught mathematics to fourth grade students (2007-2008 and 2008-2009) and sixth grade students (2009-2010), I experienced the frustration that a teacher feels when one or more students display little to no desire to engage in mathematics. I experienced the feelings of wanting students to engage, but not knowing how to motivate them. In a pilot study, I examined the potential of using mathematical tasks without words with reluctant problem solvers. The experience triggered further questions that I wanted to explore, leading me to the current research study and informing my methods to be described in a later chapter.

**Statement of the Problem**

While it would be ideal if all students eagerly engaged in problem solving during mathematics class, realistically there are often students in classrooms who might be classified by their teachers as reluctant to engage. With standards and accountability currently prevalent in mathematics education (CCSSI, 2010; Lamdbin & Walcott, 2007; NCTM 1991, 2000; NRC, 2001), action must be taken to support these reluctant students in becoming eager problem solvers. Due to a lack of research on the topics of reluctant problem solvers related to mathematical tasks without words and/or word problems, research needs to be conducted to expose further information regarding the topics and any possible relationship among them to inform policy and practice.

**Purpose**

The purpose of this qualitative study was to examine reluctant problem solvers and their perceptions of mathematical tasks without words and word problems. Building on the
observations of students engaged in mathematical tasks without words in a pilot study completed during the Spring 2012, and adding an exploratory component related to word problems, this study aimed to answer one central research question: How do reluctant problem solvers perceive mathematical tasks without words and word problems?

**Significance of the Study**

With the requirement of students in the first Standard for Mathematical Practice that they “make sense of problems and persevere in solving them” (CCSSI, 2010, p. 6), it is necessary for teachers to provide opportunities for students to become engaged in the mathematics. To do so, teachers must find tasks that motivate students to willingly engage, and it is possible that mathematical tasks without words and/or word problems provide the motivation that some students need. “Well-chosen tasks can pique students’ curiosity and draw them into mathematics” (NCTM, 2000, p.19). This study aimed to provide a glimpse into the thought processes of reluctant problem solvers and their perceptions of mathematical tasks without words and word problems.

**Definitions**

Definitions for terms used frequently in the study are provided here for the reader’s reference. An understanding of these terms will help the reader to better grasp their meaning and significance in the study.

**Mathematical Proficiency**

Students who are able to problem solve, make connections between mathematical concepts, reason, reflect and explain are considered to be mathematically proficient (NRC, 2001).
Mathematical Tasks without Words

Mathematical tasks without words are non-routine mathematical tasks that engage students in problem solving and involve little to no words, often appearing as games or puzzles (Holbert & Barlow, 2012/2013). Mathematical tasks without words are abbreviated as “MTW” throughout the text.

Word Problems

For the purpose of this study, word problems are defined as non-routine, mathematical problems that engage students in problem solving and contain 50 or more words. Word problems are abbreviated as “WP” throughout the text.

Reluctant Problem Solvers

Reluctant problem solvers are slow to begin a mathematical task or problem. They often wait for others to motivate or help them (Holbert & Barlow, 2012/2013).

Standards-based Instruction

Standards-based instruction engages students in the Process Standards including: Problem Solving, Reasoning and Proof, Representations, Communication, and Connections (NCTM, 2000). Teachers in these classrooms serve as the facilitator of students’ thinking rather than that giver of information.

Traditional Instruction

Traditional instruction describes teachers instructing students on how to use algorithms to solve problems during a whole class setting, then having students practice problems independently, and finally reviewing answers to practice problems. The teacher explicitly makes connections between the mathematics for the students (Lambdin & Walcott, 2007).
Summary

With the adoption of the Common Core State Standards (CCSSI, 2010), it is crucial for teachers to implement standards-based instruction into mathematics classrooms for the purpose of all students becoming mathematically proficient (NRC, 2001) and meeting the expectations of the Standards for Mathematical Practice (CCSSI, 2010). The standards require all students to “make sense of problems and persevere in solving them” (CCSSI, 2010, p. 6), which means that all students are held accountable for the work, and not just those students who are willing to engage in mathematics.

Unfortunately, reluctant problem solvers exist in many classrooms. It is the responsibility of teachers to provide students with tasks that encourage them to engage in the mathematics. “When students work hard to solve a difficult problem or to understand a complex idea, they experience a very special feeling of accomplishment, which in turn leads to a willingness to continue and extend their engagement with mathematics” (NCTM, 2000, p. 21). Mathematical tasks without words could be a source of motivation for reluctant problem solvers. This study sought to determine if there is a relationship between reluctant problem solvers and mathematical tasks without words. The next chapter will describe the research relevant to this study including the theoretical framework.
CHAPTER II: REVIEW OF LITERATURE

Introduction

Reluctant problem solvers (Holbert & Barlow, 2012/2013) and their perceptions of and experiences in problem solving are the central topic in this study. The literature lacks research specifically examining students considered to be reluctant problem solvers in the mathematics classroom or their perceptions of mathematical tasks without words and/or word problems. Though the term reluctant problem solvers was not used in the literature, many links exist between reluctant problem solvers and the literature. The foundation for this study and a focus for this chapter is the self-determination theory (Deci & Ryan, 2000; Ryan & Deci, 2000b). Also relevant in this research and included in this chapter are ideas of self-efficacy (Akin & Kurbanoglu, 2011; Ashcraft & Moore, 2009; Hoffman, 2010; Lee, 2009) and mathematics anxiety (Akin & Kurbanoglu, 2011; Cates & Rhymer, 2003; Chinn, 2009; Hoffman, 2010; Kesici & Erdogan, 2010; Lee, 2009; Thomas & Higbee, 1999).

This chapter begins with a discussion of the self-determination theory, which states that all humans have three needs including the needs for autonomy, relatedness, and competence (Deci & Ryan, 2000; Jones, Uribe-Florez, & Wilkins, 2011; Ross & Bergin, 2011; Ryan & Deci, 2000a; Ryan & Deci, 2000b). Within this discussion, this chapter presents links from the literature to each of the three needs. Next, connections are drawn to reluctant problem solvers. A summary concludes the chapter.
Theoretical Framework: Self-Determination Theory

Recognizing that not all students are eager or even willing to engage in mathematics, researchers have explored why students might not want to engage and how to motivate students to engage (Deci & Ryan, 2000; Jones et al., 2011; Reeve & Jang, 2006; Ross & Bergin, 2011; Ryan & Deci, 2000a). To understand students’ behaviors in the classroom, human nature must be examined. Ryan and Deci (2000b) theorized that people have an innate curiosity that motivates them to learn.

The fullest representations of humanity show people to be curious, vital, and self-motivated. At their best, they are agentic and inspired, striving to learn; extend themselves; master new skills; and apply their talents responsibly. That most people show considerable effort, agency, and commitment in their lives appears, in fact, to be more normative than exceptional, suggesting some very positive and persistent features of human nature. (Ryan & Deci, 2000b, p. 68)

If people are naturally curious and strive to learn (Ryan & Deci, 2000b), why are some students reluctant to engage in learning mathematics? Ryan and Deci discussed three levels of motivation exhibited by people: amotivation, extrinsic motivation, and intrinsic motivation. Reluctant problem solvers avoid engaging in mathematics (Holbert & Barlow, 2012/2013), implying a lack of motivation to engage. The Common Core State Standards for Mathematics seeks to develop in students the ability to “make sense of problems and persevere in solving them” (CCSSI, 2010, p. 6). Students likely need intrinsic motivation to solve problems if they are expected to persevere when doing so. Without intrinsic motivation, reluctant problem solvers might not display perseverance to solve problems. People are motivated for different reasons and in various amounts; therefore, not all students are motivated to participate in the classroom. With
the goal of developing mathematically proficient students (NRC, 2001), however, it is necessary to find approaches for motivating the reluctant students in the mathematics classroom.

The self-determination theory states that human beings have basic psychological needs in place at birth that need to be continually met throughout development (Jones et al., 2001). These needs include autonomy, relatedness, and competence (Jones et al., 2011; Ross & Bergin, 2011; Ryan & Deci, 2000b). None of these needs is greater than the other two. Instead, all three needs are important for the occurrence of intrinsic motivation. “In short, psychological health requires satisfaction of all three needs; one or two are not enough” (Deci & Ryan, 2000, p. 229).

Self-determination theory explores motivation and personality that makes humans act in a certain manner due to experiences that guide the development of personality and regulation of behavior (Ryan, Kuhl, & Deci, 1997). The fulfillment of the needs for autonomy, relatedness, and competence guide the development of personality, self-motivation, and behavior. Additionally, overall development of social skills and well-being is affected by the level of fulfillment of these needs (Ryan & Deci, 2000b).

The foundations for self-determination theory are the three psychological needs: autonomy, relatedness, and competence. It can be difficult for some teachers to meet these needs in the classroom, though the degree to which these needs are met influences the health of individuals (Ross & Bergin, 2011).

Each person’s psychological well-being and growth depend on how fully these needs are satisfied. People are naturally inclined to pursue fulfilling these needs. When needs are not satisfied, people tend to experience depressed motivation, psychological problems, and ill-being. (Ross & Bergin, 2011, p. 57)
The self-determination theory, a recognized theory of motivation, has been examined regarding students in classrooms (Jones et al., 2011; Reeve & Jang, 2006; Ross & Bergin, 2011). Further exploration of self-determination theory in relation to mathematics classrooms, however, is needed (Ross & Bergin, 2011). In the following sections, each of the three needs will be discussed and related to the literature relevant to the study.

**Autonomy**

The first psychological need, autonomy, is described as the “inner approval of one’s actions and the sense that one’s actions emanate from oneself” (Ross & Bergin, 2011, p. 57). Students need to feel that they are doing work because they choose to do the work. Students do not need to work on tasks separate from others in the classroom, but they need to feel that they had the option to choose between tasks to work. Choosing which assignments to work as well as self-guiding through the process supports autonomy (Deci & Ryan, 1985; Ross & Bergin, 2011). Students who are given opportunities to build autonomy in the classroom prosper, because they tend to display greater motivation and yearning to learn and be challenged (Deci, Nezlek, & Sheinman, 1981; Flink, Boggiano, & Barrett, 1990; Ryan & Grolnick, 1986).

Due to enhanced intrinsic motivation and even academic performance when autonomy is supported by teachers (Deci, Nezlek, & Sheinman, 1981; Deci & Ryan, 1985; Deci & Ryan, 2000; Flink, et al., 1990; Reeve & Jang, 2006; Ross & Bergin, 2011; Ryan & Grolnick, 1986), researchers have suggested that students be given opportunities to build autonomy in the classroom. There are many ways to support autonomy in the classroom and many positive outcomes when students feel autonomous (Ross & Bergin, 2011; Ryan & Deci, 2000). Students who have been given opportunities to feel that their behavior is autonomous display more intrinsic motivation (Ross & Bergin, 2011). According to Ross and Bergin (2011), autonomy-
supporting environments are typically associated with flexibility to be creative. Additionally, these environments inspire excitement, independence, self-esteem, trust, persistence, high academic achievement, and increased physical and emotional well-being. This type of environment is similar to the classroom described by the National Council of Teachers of Mathematics (NCTM, 2000) in which students are encouraged to be creative when problem solving.

**Problem solving.** Students are engaged in problem solving when they begin working a non-routine problem, one for which they do not immediately know how to begin solving (NCTM, 2000). “In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings” (p. 52). The National Council of Teachers of Mathematics has a long tradition of promoting problem solving in the mathematics classroom (NCTM, 1989, 1991, 2000, 2010). “The term ‘problem solving’ refers to mathematical tasks that have the potential to provide intellectual challenges for enhancing students’ mathematical understanding and development” (NCTM, 2010, p.1). Problem solving could potentially occur in all mathematical challenges including word problems and mathematical tasks without words. However, not every problem elicits problem solving. Story or word problems often come to mind in a discussion about problem solving. However, this conception of problem solving is limited. Some ‘story problems’ are not problematic enough for students and hence should only be considered as exercises for students to perform. For example, students may be asked to find the perimeter of a polygon, given the length of each side. They can mindlessly add these numbers and get the answer without understanding the concept of perimeter and the problem situation.
However, some nonstory problems can be true problems, such as those found, for example, while playing mathematical games. (NCTM, 2010, p.1)

From this example, the perimeter problem did not engage students in problem solving because they simply added the numbers around the polygon, applying a previously learned procedure to solve the problem. The students were not challenged to problem solve since they immediately knew how to find a solution. Students should be engaged in problem solving by working non-routine problems and strategizing methods for solving the problem.

Engaging students in problem solving as described here (NCTM, 2000) supports the need for autonomy in many ways. Through the problem-solving process, autonomy is supported as students are provided opportunities to be independent, to strategize, and to be creative. The support that problem solving affords the development of autonomy is described in the following sections.

*Independence.* Engaging students in problem solving provides them with opportunities to be independent in the classroom, which supports their need for autonomy (Ross & Bergin, 2011). Teachers may allow students to decide which manipulatives or materials to use when solving problems, and students are often responsible for retrieving supplies from their designated home and returning them when finished. Also, it is “important that students learn to use the tools to construct and make sense of concepts for themselves rather than just see teachers using the tools to demonstrate mathematical ideas” (Koestler, Felton, Bieda, & Otten, 2013, p. 60).

Students are expected to work alone or together in groups to solve problems, however, each student is held accountable for contributing in some way (NCTM, 2000). The group is expected to be independent, not relying on the teacher or other groups for help. Allowing
students to feel independent in the classroom supports their need for autonomy (Ross & Bergin, 2011).

**Strategies.** During the problem-solving process, students are expected to analyze the information given in the problem, determine what the problem is asking them to do, and develop strategies for solving the problem (NCTM, 2000). As students problem solve, they must monitor their progress (NCTM, 2000) to determine if the strategy they are using will lead to a reasonable solution. If the strategy appears to be leading to an incorrect solution, students must adjust their strategy or develop a new one (NCTM, 2000). Worthwhile tasks “can be approached in more than one way, such as using an arithmetic counting approach, drawing a geometric diagram and enumerating possibilities, or using algebraic equations, which makes the tasks accessible to students with varied prior knowledge and experience” (NCTM, 2000, p. 19). Students have the power to determine what solution paths to use when problem solving, and this self-guidance supports their need for autonomy in the classroom (Deci & Ryan, 1985; Ross & Bergin, 2011).

**Creativity.** As students are problem solving, they have the freedom to be creative not only when choosing solution paths, but also when choosing how to represent their work. Students may choose from many different methods to help them solve the problem such as drawing pictures, drawing diagrams, using manipulatives, acting out the problem, organizing information in lists and tables, and using any other method they find (NCTM, 2000). Students are only limited by their imagination. “Teaching mathematics well involves creating, enriching, maintaining, and adapting instruction to move toward mathematical goals, capture and sustain interest, and engage students in building mathematical understanding” (NCTM, 2000, p. 18). Allowing students the opportunity to be creative when problem solving supports their need for autonomy (Ross & Bergin, 2011).
**Obstacles.** As described, students’ need for autonomy is supported through problem solving, however, there are many obstacles that hinder the meeting of the need. The need is not met if students are not engaged in authentic problem solving as described by the National Council of Teachers of Mathematics (NCTM, 2000). For example, the students that solved the previously discussed perimeter problem simply identified the numbers labeling the sides of the polygon, added them together, and found a solution. Though they found a correct solution, the students were not engaged in problem solving (NCTM, 2000), because they used a previously learned procedure to work the problem. This type of work does not support autonomy, because students are not given the independence and creativity to develop their own solution paths.

In other instances, students neither use algorithms nor engage in problem solving to arrive at solutions. Some students take numbers from within word problems and, in essence, guess which mathematical operation to perform to reach a solution, as described by Xin, Wiles, and Lin (2008). In their study, Xin and colleagues noted, “It was like gambling. That is, if an answer resulting from one operation did not work (e.g., did not result in a ‘good-sized’ number or a ‘good-looking’ integer), the students would try different operations” (p. 175). These participants did not problem solve as defined and encouraged by the National Council of Teachers of Mathematics (2000). Instead, the participants took the numbers from the problem and utilized key words in the problem to determine which operation to use. This strategy is harmful to students because they use it instead of engaging in problem solving (Clement & Bernhard, 2005). In some instances, this key word approach can lead to a correct solution, but in many instances, it fails the students and leads to a false sense of success (Clement & Bernhard, 2005). The key word approach does not provoke engagement in problem solving and therefore, does not support the need for autonomy.
**Summary.** A student’s need for autonomy is supported when he or she is engaged in problem solving as described by the National Council of Teachers of Mathematics (2000). Problem solving allows students to be independent (Ross & Bergin, 2011), to develop their own strategies and solution paths (Deci & Ryan, 1985; Ross & Bergin, 2011), and to be creative (Ross & Bergin, 2011). Students who are engaged in problem solving have the freedom and power to plan and develop their work as they see fit. Also, they have the flexibility to adjust their work as needed (NCTM, 2000). Through problem solving, students are engaged in all of the previously mentioned activities and processes, which, in turn, support autonomy. Therefore, students should be engaged in problem solving to support their need for autonomy.

**Relatedness**

Relatedness is the second psychological need that all humans have. It is an important aspect of intrinsic motivation and self-determination. Relatedness is the need to feel connected to other people (Jones et al., 2011; Ross & Bergin, 2011). A key reason for people displaying behaviors of motivation is that they feel connected to significant other people or groups of people to whom they would like to feel connected (Ryan & Deci, 2000a).

In its document, *the Principles and Standards for School Mathematics*, the National Council of Teachers of Mathematics (2000) provided a vision for the classroom. Ideally, students should engage in appropriate mathematical tasks, exploring many different entry points and solution paths while monitoring their progress and adjusting their strategies to reach solutions (NCTM, 2000). Students should represent their work using various materials, present and justify their methods and solutions, and critique their classmates’ work (CCSSI, 2010). Additionally, students should make and explore conjectures based on their findings (NCTM, 2000). Students
should work cooperatively to explore mathematics, problem solve, and communicate ideas (NCTM, 2000). Working collaboratively with peers supports students’ need for relatedness.

Relationships among students and relationships between the student and the teacher are valuable in meeting the need for relatedness. Students must engage in the Process Standards (NCTM, 2000) and the Standards for Mathematical Practice (CCSSI, 2010) to fulfill the vision for the classroom (NCTM, 2000). Each set of standards requires that students work together, communicate and critique ideas. For students to communicate effectively, they must feel that the environment is safe and that they will not be mocked. To this end, positive relationships must exist in the classroom. Students must have working relationships with each other to allow them to confidently engage in the standards (CCSSI, 2010; NCTM, 2000) as expected of them. Additionally, students’ relationship with their teacher is of great importance. Research shows that students have more confidence in their ability when they perceive their mathematics teacher as caring and supportive (Fast et al., 2010). The teacher must serve as a facilitator of learning and encourage students in their problem-solving pursuits. The relationships in the classroom promoted by the National Council of Teachers of Mathematics (2000) support the need for relatedness.

In summary, it is necessary for students to develop and foster relationships in the mathematics classroom to support their need for relatedness. “The relatedness need not occur in the moment of instruction but should occur in a general base of relatedness toward teacher and peers” (Ross & Bergin, 2011, p. 63). Relatedness should occur naturally in the classroom if the teacher has created an environment in which students communicate regularly and respectfully with each other and with the teacher (Ross & Bergin, 2011). “Students feeling respected and cared for by the teacher is essential for their willingness to accept [preferred] classroom values
(Ryan & Deci, 2000a, p. 64). Simply having students work together, however, does not foster relatedness; meeting the relatedness need requires that students have relationships that are fulfilling and safe (Jones et al., 2011), the same type of relationships that are promoted in the classroom by the National Council of Teachers of Mathematics through problem solving (NCTM, 2000). Problem solving to learn and understand is “further enhanced by classroom interactions” which require students to “propose mathematical ideas and conjectures” and “learn to evaluate their own thinking and that of others” (NCTM, 2000, p. 21). Additionally, “Classroom discourse and social interaction can be used to promote the reorganization of knowledge.” (NCTM, 2000, p. 21). Therefore, problem solving (NCTM, 2000) holds the potential for meeting the need for relatedness.

**Competence**

Competence, the third psychological need that humans have, is the degree to which students feel that they can be successful when solving a problem (Ross & Bergin, 2011). People frequently participate in activities purely to experience success and meet their need for competence (Deci & Ryan, 2000). Challenges within an individual’s capability elicit feelings of competence.

**Supporting competence.** There are many ways to support the need for competence (Deci & Ryan, 2000; Jones et al., 2011; Ross & Bergin, 2011). Allowing students to share ideas in a group setting before sharing with the whole class is one way to encourage competence. In this way, students have the opportunity to receive feedback on their ideas from a small number of peers before discussing with the entire class (Jones et al., 2011). This allows students to practice sharing their thoughts and adjust their thinking, if needed, based on peer comments before presenting their ideas to the whole class. Positive feedback also supports competence, though
students must feel that their performance was a success for the positive feedback to be meaningful (Deci & Ryan, 2000).

Additionally, students’ use of manipulatives in the classroom can support their need for competence (Jones, et al., 2011). For example, students can feel that manipulating objects is easier than manipulating abstract numbers and letters in mathematics so they might feel more capable of completing a task (Jones, et al., 2011). Students should become familiar with manipulatives and tools available for their use in the classroom (CCSSI, 2010; Jones et al., 2011), and should be able to choose which manipulatives are appropriate to complete a given task and use the tools as they see fit (CCSSI, 2010; Koestler et al., 2013). Allowing students to choose their own manipulative supports their need for competence, because they can choose a manipulative that they feel will help them be successful in completing the task.

When students have control of choosing manipulatives and using them however they decide to solve a task, they attribute their success in problem solving to their own competence (Jones et al., 2011). For students to complete a task, feel successful, and have their need for competence met, they would first have to feel capable enough to attempt solving the task. Essentially, they would need to have a high self-efficacy. The following sections describe factors that support the need for competence by promoting the development of self-efficacy.

**High self-efficacy.** As previously described, competence is a psychological need that humans need to have met. For this to occur, however, the need for competence suggests that students might also need to have high self-efficacy. Self-efficacy is one’s belief about his or her potential and capability to complete successfully a given task and produce specific, desired results (Bandura, 1997). A lack of self-efficacy, meaning that students do not believe they have the ability to complete a task, could hinder the meeting of their need for competence. Therefore,
it is important for teachers to support the development of high-self efficacy in students. The following sections discuss ways that teachers might support self-efficacy.

**Supporting high self-efficacy.** Self-efficacy is influenced by factors in the classroom (Fast et al., 2010), and it is supported when students feel that their classroom environment is caring and challenging (Fast et al., 2010). When students are successful at completing a challenging task, self-efficacy is increased, and in turn, the need for competence is met (Ross & Bergin, 2011). The task, however, must be appropriately challenging. If the task is too easy, the success will not be as meaningful. Alternatively, a task that is too difficult might make students feel that they cannot succeed (Ross & Bergin, 2011), decreasing their self-efficacy. Tasks should be worthwhile, interesting to students, and challenging in a way that students feel they can meet with hard work (NCTM, 2000), causing increased self-efficacy (Fast et al., 2010).

Students have higher self-efficacy levels when they perceive their mathematics classroom to be caring, challenging, and mastery-oriented (Fast et al., 2010). Consequently, students’ feelings pertaining to their classroom environment, including feelings that their teachers and even their peers care about them, affect their levels of self-efficacy toward mathematics tasks. Also, students’ self-efficacy is improved when they see that their performance is better than that of their peers (Arslan, 2012). Other components in the classroom affect students’ self-efficacy as well, as the following paragraphs describe.

**Appropriate tasks.** “Providing an opportunity to learn means setting up the conditions for learning that take into account students’ entry knowledge, the nature and purpose of the tasks and activities, the kind of engagement required, and so on” (Hiebert, 2003, p. 10). Students should be given tasks that are within their ability level and related to their real world (NCTM, 2000). Completing these appropriately challenging mathematics tasks can help students develop
confidence in their ability to problem solve (NCTM, 2000). This confidence occurs because students believe that they can be successful at solving the problem with some effort on their part (Ross & Bergin, 2011). Consequently, their self-efficacy is increased. Self-efficacy directly affects students’ engagement in mathematics tasks, persistence in working mathematics tasks, attitudes toward mathematics in general (Akin & Kurbanoglu, 2011; Hoffman, 2010), and performance in mathematics (Bandura & Schunk, 1981; Lee, 2009). Consequently, self-efficacy should be supported for students in the classroom by having opportunities to solve appropriate mathematics tasks.

*Probing questions.* Alter, Brown, and Pyle (2011) discovered that students were better able to successfully solve word problems when they were asked probing questions by the researcher. “The use of the cuing format that asked probing questions to further students’ conceptual understanding of problem strategies also played a critical role in improving student results in both percentage of problems solved correctly and time on-task” (Alter et al., 2011). Given the in-depth thinking and processing needed to solve problems, it is crucial for students to develop habits of mind (NRC, 2001) and persevere in solving problems (CCSSI, 2010) to be able to reach correct solutions. “Children who doubt their capabilities quit sooner than those who believe they can eventually master the task should they persevere” (Bandura & Schrunk, 1981, p. 594). Asking questions of students as they are working mathematics problems, without giving hints as to how to work the problems, could prompt students’ thinking about the mathematics, help them improve problem-solving skills (Alter et al., 2011), and potentially increase perseverance in problem solving. This improvement of problem-solving skills, resulting in the feeling of success in problem solving, fosters higher self-efficacy (Hoffman, 2010).
Strategies for success. When students feel that they successfully completed a task, their self-efficacy increases (Hoffman, 2010). Consequently, it would be beneficial to help students develop strategies to become successful at completing tasks. Research suggests that while problem solving to find solutions for tasks, students are more successful when they record and organize the given information (Pape, 2004). They are better able to explain their methods for solving, justify their work, apply contextual information, and use the problem context to reinforce their solutions by checking for reasonability. Additionally, they can solve more problems while committing fewer errors (Pape, 2004). Students are more successful at problem solving when they create a drawing or picture to represent the problem (Abdullah, Zakaria, & Halim, 2012). Pictures provide visual representations that students can manipulate to help find strategies for solving problems. “When the teaching approaches encourage students to apply thinking strategies through using visual representation, students are able to gain a better conceptual understanding and eventually improve their mathematics achievement” (Abdulla et al., 2012, p. 35). Encouraging students to use some of these strategies when problem solving could help them become more successful, therefore increasing their self-efficacy, and, in turn, meeting their need for competence.

Goals, persistence, and efficiency. Students who set short-term, motivational goals master skills more rapidly than students with long-term goals or students with no goals (Bandura & Schunk, 1981). Bandura and Schunk found that students who set small, daily goals “progressed rapidly in self-directed learning, achieved substantial mastery of mathematical operations, and heightened their perceived self-efficacy and interest in activities that initially held little attraction for them” (Bandura & Schunk, 1981, p. 595). Essentially, students are more likely to be successful at mathematics tasks when they set small, achievable goals that provide almost instant
gratification. Students who are confident in their abilities, or have high levels of self-efficacy, persist in solving problems longer than students who are less confident. Additionally, students are more likely to develop an interest in a subject when they are exposed to the subject and experience success in the subject (Bandura & Schunk, 1981).

Hoffman (2010) found that self-efficacy was positively related to problem-solving accuracy and efficiency, which takes into consideration the amount of time spent on solving problems as well as correctness of the solution. In essence, when students’ self-efficacy was high, their problem-solving performance and efficiency was also high. Reaffirming Hoffman’s (2010) findings, Arslan (2012) and Fast et al. (2010) also found that higher levels of self-efficacy had positive affects on students’ performance in mathematics. Students with high levels of self-efficacy are better able to successfully complete mathematics problems as opposed to their peers with lower self-efficacy toward the same problems (Pajares & Miller, 1994). Due to high levels of self-efficacy, which is a predictor for mathematics performance (Hoffman, 2010), students will persevere when solving problems until they are able to reach the desired outcome, thus meeting their need for competence.

**Hindering competence.** When students have low self-efficacy, they do not feel that they have the capability to complete a task and achieve desired results (Hoffman, 2010). This low self-efficacy inhibits students from meeting their need for competence and contributes to high levels of mathematics anxiety.

**Mathematics anxiety.** The percentage of students in the United States with an actual mathematical disability is only around 6% (Badian, 1999; Siegler, 2003). Many students who do not have a mathematical disability, however, experience anxiety when facing mathematical challenges (Hoffman, 2010; Kesici & Erdogan, 2010; Waddlington & Waddlington, 2008; Witt,
In the classroom, even children in younger grades know which of their peers is proficient or advanced in mathematics and which of their peers is not proficient in mathematics. In many instances, students with low self-efficacy begin to avoid mathematics and develop what is referred to as mathematics anxiety, which inhibits students’ ability to complete mathematical tasks.

A definition, provided by Hoffman (2010), explained that mathematics anxiety is “the state of nervousness and discomfort brought upon by the presentation of mathematical problems” (p. 276). When faced with mathematical challenges, students with mathematics anxiety may be overcome with feelings of agitation and vulnerability. Students may even feel threatened or intimidated, causing them to acquire and maintain negative feelings toward mathematics (Akin & Kurbanoglu, 2011; Hoffman, 2010). Anxiety is activated when circumstances seem threatening (Hoffman, 2010). In the mathematics classroom, threatening circumstances might include problems that seem unfamiliar, foreign, and/or difficult. Many factors affect mathematics anxiety, and Kesici and Erdogan (2010) maintain that students’ achievement motivation, in particular, plays an important role in their mathematics anxiety. Students with mathematics anxiety learn to avoid mathematics and might even feel that mathematics is useless (Akin & Kurbanoglu, 2011).

Lee (2009) stated that anxiety “can be manifested as one’s physio-emotional reactions” (p. 355) even from simply thinking about a task or from actually working a task. For students, mathematics anxiety occurs when they become anxious about completing a mathematics task or problem in the classroom or at home. Students who tend to fail in mathematics may develop anxiety toward the subject. Conversely, mathematics anxiety leads to more failure. The cycle
leads to more failure in mathematics and more anxiety toward mathematics (Waddlington & Waddlington, 2008).

Specific mathematics anxiety occurs when an individual becomes anxious about certain mathematical situations. For example, the individual might be anxious about completing a specific task given by the teacher during mathematics class. Individuals with specific mathematics anxiety do not typically fear mathematics overall, because they generally feel that they can learn the content when provided instruction (Waddlington & Waddlington, 2008). Global mathematics anxiety occurs when an individual feels pressure in all mathematical situations. These individuals typically loathe mathematics and “spend time and energy in avoiding mathematics” (Wadlington & Wadlington, 2008, p. 3). Students with global mathematics anxiety might try to find ways to distract themselves when given a new mathematics task to complete. Students with global mathematics anxiety could potentially be referred to as reluctant problem solvers, as defined by Holbert and Barlow (2012/2013), due to the fact that reluctant problem solvers also frequently avoid engaging in mathematics.

Math anxiety is directly related to working memory (Ashcraft & Krause, 2007; Hoffman, 2010; Krinzinger, Kaufmann, & Willmes, 2009; Witt, 2012). Witt (2012) found that students who had high levels of mathematics anxiety experienced a decrease in working memory. The increase in mathematics anxiety occupied working memory, allowing less working memory to be used to complete the mathematics task. Interestingly, Witt (2012) also found that students have high levels of mathematics anxiety and disruptions to working memory “in situations where they are dealing with digits, even if there is no explicit mathematical processing required” (p. 271). Even small disruptions to working memory could considerably lessen mathematical performance (Witt, 2012). His findings coincided with those of Ashcraft and Kirk (2001) who found that even
counting generated anxiety in some individuals. “The mere presence of digits as to-be-remembered stimuli can trigger an anxious response that inhibits central executive functioning” (Witt, 2012, p. 271). Findings from Ashcraft and Kirk (2001) and Witt (2012) could potentially be relevant to reluctant problem solvers, given that they avoid mathematics, possibly, for the reasons identified by the researchers.

**Mathematics anxiety and self-efficacy.** There is a negative relationship between mathematics anxiety and mathematics performance (Akin & Kurbanoglu, 2011; Cates & Rhymer, 2003; Hoffman, 2010; Kesici, & Erdogan, 2010; Thomas & Higbee, 1999). Kesici and Erdogan (2010) described mathematics anxiety as “one of the most significant reasons preventing mathematics achievement” (p. 54). Further, students’ negative experiences related to mathematics in younger grades produce mathematics anxiety for those students in upper grades in school. The relationship between mathematics anxiety and performance in mathematics is strong and negative (Hoffman, 2010). When students are faced with difficult problems that are viewed as nearly impossible to solve, anxiety and worry take over and leave little working memory to actually problem solve (Ashcraft & Krause, 2007; Hoffman, 2010; Krinzinger, Kaufmann, & Willmes, 2009).

Ashcraft and Moore (2009) expressed that mathematics anxiety affects self-efficacy. Students who maintain high levels of mathematics anxiety view themselves as less capable of completing mathematics tasks than their classmates who have less mathematics anxiety, indicating lower levels of self-efficacy. Students with very low self-efficacy may have global mathematics anxiety if they feel that they cannot complete mathematics tasks because they are not good at mathematics in general. Other students may have low self-efficacy and have only specific mathematics anxiety because they only feel incapable of completing specific

Alleviating mathematics anxiety. “Often before students can be successful in mathematics, they must start to overcome mathematics anxiety. Conversely, becoming successful in mathematics will help students to overcome [mathematics] anxiety” (Waddlington & Waddlington, 2008, p. 4). The situation given is somewhat of a circular cause and consequence. Research indicates that increasing self-efficacy lessens mathematics anxiety, therefore increasing mathematics achievement (Arslan, 2012; Bandura & Schunk, 1981; Fast et al., 2010; Hoffman, 2010). Students with low self-efficacy and high mathematics anxiety need to be given opportunities that allow them to be successful in mathematics to raise their levels of self-efficacy and lessen their mathematics anxiety, in turn, enhancing their mathematics performance.

Self-efficacy regulates the level of anxiety a person has toward mathematics, as self-efficacy is negatively related to mathematics anxiety (Hoffman, 2010). Students who have opportunities to feel successful in problem solving experience higher levels of self-efficacy, which decreases mathematics anxiety (Hoffman, 2010). Students with lower mathematics anxiety and higher self-efficacy when faced with mathematics tasks would have more working memory with which to complete the tasks. “Intervention of anxiety before it is cultivated may avoid negative self-evaluation to overcome task avoidance and preempt the ‘I think I can, but I’m afraid to try’ approach of many mathematics anxious individuals” (Hoffman, 2010, p. 281). Students need to engage in appropriately challenging mathematical tasks so that they may have
opportunities to be successful, increase their self-efficacy and decrease their anxiety. Increased self-efficacy caused by success in problem solving allows students to have their need for competence met.

**Struggling in reading comprehension.** Word problems are challenging for students who struggle with reading comprehension and could lead to lower self-efficacy levels in these students. Beyond simply reading problems, students are influenced by their own backgrounds and may interpret problems differently than other students. “Guided by expectations, previous experience, and knowledge, readers mediate their transactions with text by selecting specific elements to which they attend” (Pape, 2004, p. 190). As students read the problem, their interpretation is altered each time they read and interpret the problem. After initially reading a word problem, students generate a mental idea of the problem, including the information given and the question asked. If students struggle with reading comprehension, the result may be an inaccurate portrayal of the problem, which leads to an incorrect solution (Pape, 2004). Errors in solving problems are frequently due to miscomprehension of the word problem, which might occur due to language deficiencies, difficulties in reading comprehension, and insufficient mathematics knowledge (Pape, 2004). Errors and incorrect solutions lead to lower self-efficacy and even avoidance of mathematical tasks (Hoffman, 2010). Since students with low self-efficacy do not feel capable of completing tasks, they will not experience the feeling of success, which in turn fails to meet the need for competence (Ross & Bergin, 2011). Therefore, students with low self-efficacy do not have their need for competence met.

Mathematical tasks without words (Holbert & Barlow, 2012/2013) are non-routine problems that have few to no words and require students to engage in problem solving (NCTM, 2000) to reach a solution. The fact that these tasks have few to no words could potentially be
appealing to students who struggle with reading comprehension, as Holbert and Barlow (2012/2013) hypothesized.

The tasks appeal to reluctant problem solvers for two reasons: (1) The reading requirement is removed, which may interest a student who considers himself or herself to be a struggling reader; and (2) the lack of words gives the task more of a puzzle-like nature, thus appealing to a student’s curiosity. (p. 315)

Students are allowed to focus on the problem-solving aspect of the task since the challenge of reading has been eliminated. Consequently, mathematical tasks without words could potentially provide opportunities for struggling readers to be successful in mathematics, therefore increasing their levels of self-efficacy. The increased self-efficacy could lead to success when problem solving, which meets the need for competence (Ross & Bergin, 2011).

**Summary.** When students experience success in mathematics, their self-efficacy is improved, meaning that they feel more confident in their ability to problem solve. Experiencing success in mathematics also meets the need for competence (Ross & Bergin, 2011). Essentially, high self-efficacy is required to meet the need for competence. Teachers can help students experience success by providing them with appropriately challenging mathematics tasks (Ross & Bergin, 2011), asking probing questions (Alter et al., 2011) to facilitate their thinking, and creating a caring and safe environment in the classroom (Fast et al., 2010). Low levels of self-efficacy lead to high levels of mathematics anxiety (Hoffman, 2010). Mathematics anxiety can potentially be diminished in struggling readers when they are given opportunities to be successful in solving mathematical tasks without words (Holbert & Barlow, 2012/2013).
Summary

The self-determination theory suggests that people have three psychological needs that must be met to feel self-determined: autonomy, relatedness, and competence (Deci & Ryan, 2000; Jones et al., 2011; Ross & Bergin, 2011; Ryan & Deci, 2000a; Ryan & Deci, 2000b). Engaging in problem solving as promoted by the National Council of Teachers of Mathematics (NCTM, 2000) provides students with opportunities to meet each of the three needs. Students face obstacles, however, that prohibit them from having their needs met. Those obstacles can be overcome when students are provided opportunities to help them experience success.

Reluctant Problem Solvers

Holbert and Barlow (2012/2013) described reluctant problem solvers as students in the classroom who frequently avoid engaging in mathematics. Research indicates that students who have high mathematics anxiety typically try to avoid mathematics tasks or challenges (Chinn, 2009). Potentially, these reluctant problem solvers (Holbert & Barlow, 2012/2013) may be reluctant because they are anxious about the mathematics and/or maintain a low self-efficacy related to mathematics. Research has not been conducted, however, to explore the levels of mathematics anxiety and self-efficacy of students classified as reluctant problem solvers. Therefore, no specific relationship can be determined between levels of mathematics anxiety, self-efficacy, and reluctant problem solvers without conducting research.

Low Levels of Self-Efficacy.

Bandura’s (1997) description of students with low self-efficacy shares commonalities with Holbert and Barlow’s (2012) description of reluctant problem solvers. Reluctant problem solvers tend to avoid engaging in the mathematics, distract themselves and others, and wait for the teacher or their peers to prompt them to engage. Providing reluctant problem solvers with
opportunities to be successful could increase their self-efficacy. Increasing self-efficacy in reluctant problem solvers could potentially help them become less reluctant. Bandura and Schunk (1981) stated that high levels of self-efficacy and success in particular activities increase students’ interests in those activities. Consequently, students choose to engage in activities in which they feel capable of completing and/or activities in which they have already experienced success. The research suggests that students can improve their self-efficacy and gain an interest in an activity when exposed to appropriately challenging tasks related to the activity and when small goals related to the activity are met (Bandura & Schrunk, 1981). Further, when students experience success, their need for competence is met (Ross & Bergin, 2011). The more success they experience, the more interested they will become in the activity (Bandura & Schrunk, 1981). Students are not born with the desire or interest in certain sports or activities, rather they must experience favorable involvement in the activity for it to become important to them (Bandura & Schrunk, 1981). For example, students are not born with a desire to play football. If they are exposed to the sport of football, practice football, and experience success in football, they could have an increase in self-efficacy toward football, fostering their interest in football (Bandura & Schrunk, 1981). In essence, students must have comprehensive experiences related to football to become interested in it. If mathematics was considered a sport, students should be exposed to mathematics, practice mathematics, and experience success in mathematics, thereby increasing their self-efficacy and developing an interest in mathematics and a desire to persevere in working mathematical tasks (Bandura & Schrunk, 1981).

**Lack of willingness to engage.** Self-efficacy affects students’ choice of activities in which to engage, amount of effort put into those activities, and persistence in completing the activities (Bandura & Schunk, 1981; Bandura, 1997). Reluctant problem solvers display, in
particular, a lack of willingness to engage in mathematics (Holbert & Barlow, 2012/2013), possibly indicating low self-efficacy.

People who have a low sense of efficacy in a given domain shy away from difficult tasks, which they perceive as personal threats. They have low aspirations and weak commitment to the goals they choose to pursue. They maintain a self-diagnostic focus rather than concentrate on how to perform successfully. When faced with difficult tasks, they dwell on their personal deficiencies, on the obstacles they will encounter, and on all kinds of adverse outcomes. They slacken their efforts and give up quickly in the face of difficulties. (Bandura, 1997, p. 144)

Potentially, reluctant problem solvers have low levels of self-efficacy, which negatively affect their desires to engage.

**Fewer opportunities to be successful.** Self-efficacy determines how confident a student feels in his or her ability to complete or solve a given mathematics task. Students with high levels of self-efficacy toward mathematics in general would accept more challenges (Hoffman, 2010), whereas students with low self-efficacy tend to avoid mathematics challenges or tasks. Additionally, self-efficacy influences persistence level when completing mathematics tasks (Akin & Kurbanoglu, 2011; Bandura & Schunk, 1981). Students with high self-efficacy will spend more time attempting to solve a challenge and exhausting many problem-solving strategies to find a solution. Alternatively, students with low self-efficacy are more likely to give up when a task appears too difficult.

Reluctant problem solvers often avoid engaging in mathematics (Holbert & Barlow, 2012/2013), indicating that they have low self-efficacy. Since they avoid engaging in mathematics, they have fewer opportunities to be successful, and experiencing success could
increase their levels of self-efficacy (Arslan, 2012). Alternatively, when students do not feel successful, they experience decreased levels of self-efficacy. If students, such as reluctant problem solvers, avoid engaging in the mathematics, they will not have opportunities to be successful, resulting in low self-efficacy.

**Need for competence not met.** As previously stated, reluctant problem solvers allow themselves fewer opportunities to experience success in problem solving. The feeling of success increases self-efficacy (Arslan, 2012) and also meets the need for competence (Ross & Bergin, 2011). Therefore, it is necessary for students to experience success in mathematics, resulting in higher levels of self-efficacy, to have their need for competence met. Reluctant problem solvers, however, do not often experience success or have high levels of self-efficacy. As a result, their need for competence is not met (Ross & Bergin, 2011). Without their need for competence met, reluctant problem solvers do not have all of their three psychological needs met, which is potentially, the reason that they are reluctant to engage.

**Summary**

This chapter discussed the literature relevant to the study, focusing on the self-determination theory (Deci & Ryan, 2000; Jones et al., 2011; Ross & Bergin, 2011; Ryan & Deci, 2000a, 2000b) which states that all humans have three psychological needs that are necessary to make them feel self-determined, including the needs for autonomy, relatedness, and competence. Students must be engaged in problem solving (NCTM, 2000), be connected to their classmates and teacher, and be provided opportunities to be successful in mathematics to have their needs met. The literature pertaining to self-efficacy (Akin & Kurbanoglu, 2011; Ashcraft & Moore, 2009; Hoffman, 2010; Lee, 2009) and mathematics anxiety (Akin & Kurbanoglu, 2011;
There is a lack of research related to reluctant problem solvers (Holbert & Barlow, 2012/2013); however, there is a link between reluctant problem solvers, self-efficacy, and competence in the mathematics classroom. Meeting reluctant problem solvers’ needs of the three components of the self-determination holds the potential for helping those students become less reluctant and possibly more confident in the mathematics classroom.

There is little to no literature pertaining to the perceptions of reluctant problem solvers related to mathematical tasks without words or word problems, leaving a gap in the literature. The current study sought to fill that gap by gaining insight into the perceptions of reluctant problem solvers related to mathematical tasks without words and word problems. Further, reluctant problem solvers’ perceptions could reveal the potential that these tasks hold for meeting their need for competence. Chapter III provides information regarding the methods used in the study.
CHAPTER III: METHODOLOGY

Introduction

It is crucial for teachers to support all students’ needs for autonomy, relatedness, and competence in the classroom (Deci & Ryan, 2000; Jones, Uribe-Florez, & Wilkins, 2011; Reeve & Jang, 2006; Ross & Bergin, 2011; Ryan & Deci, 2000a). Meeting reluctant problem solvers’ needs could potentially allow them to feel more self-determined (Ross & Bergin, 2011) and less reluctant to engage in problem solving. Adopting pedagogical strategies and implementing them in the classroom can meet the needs for autonomy and relatedness. Meeting the need for competence, however, is more complex. Experiencing success in problem solving allows students to develop high self-efficacy (Bandura & Schunk, 1981) and meets their need for competence (Ross & Bergin, 2011).

This chapter begins with a discussion of the case study method utilized in the study, followed by the research design, including a restatement of the purpose of the study and the research question. The research design is followed by the research context. Next, sections describing the instruments and data sources, procedures, data analysis, and limitations are provided. Finally, a summary concludes the chapter.

Case Study Method

This study used a qualitative approach to examine the perceptions of reluctant problem solvers related to mathematical tasks without words and word problems. The sections that follow describe the type and design of the study. The first section defines the study as an exploratory case study, and the second section discusses the multiple holistic case design of the study.
Exploratory Case Study

An exploratory case study approach was utilized to explore, in depth, the experiences of participants related to mathematical tasks without words and word problems. An exploratory case study approach is used when the intervention utilized is not known to generate specific outcomes (Yin, 2009). In describing case studies, Patton (2002) expressed the following:

Case data consist of all the information one has about each case: interview data, observations, the documentary data (e.g., program records or files, newspaper clippings), impressions and statements of others about the case, and contextual information – in effect, all the information one has accumulated about each particular case goes into that case study. (p. 449)

I chose to use a case study approach for three reasons. First, the study was exploratory in nature and focused on a situation in which the outcomes were unknown (Baxter & Jack, 2008). Second, the study examined contemporary events and did not require control of behavioral events (Yin, 2009). Third, I chose a case study approach because I wanted to examine, in depth, each participant’s experiences in sessions with mathematical tasks without words and/or word problems throughout the study. The three characteristics described were best suited for a case study research method.

Yin (2009) stated that two data sources are utilized especially in a case study method: “direct observation of the events being studied and interviews of the persons involved in the events.” Further, Yin (2009) stated, “The case study’s unique strength is its ability to deal with a full variety of evidence – documents, artifacts, interviews, and observations.” (p.11). Toward collecting a full variety of evidence, each session in the current study was composed of four parts: choosing which task to work, working it, providing a written reflection, and responding to
questions about the task. Examining each part in every session provided me with insight into the participant’s perceptions that I might not have gained by simply examining one isolated part of a session or even one isolated session. I focused on a limited number of participants, which allowed me to examine their experiences comprehensively, rather than using a large number of participants and examining only one session or part of a session. According to Patton (2002), “Qualitative methods typically produce a wealth of detailed data about a much smaller number of people and cases” (p. 227).

**Multiple Holistic Case Design**

The study is an analysis of two individuals serving as the multiple cases in the study. I chose to examine multiple individuals who were classified as reluctant problem solvers, focusing on their perceptions of mathematical tasks without words and word problems. “The evidence from multiple cases is often considered more compelling, and the overall study is therefore regarded as being more robust (Herriott & Firestone, 1983)” (Yin, 2009, p. 53). This study examined the perceptions of reluctant problem solvers from a global approach to allow all perceptions to be exposed. As a result, the study used holistic cases (Yin, 2009).

**Research Design**

This study utilized an exploratory multiple-case study approach, as described in the previous section. The following paragraphs describe the components of the research design including the research question, propositions, and units of analysis. It is important to note that the research design allowed the needs for autonomy and relatedness to be met. The study explored whether or not mathematical tasks without words held the potential for meeting the need for competence for reluctant problem solvers.
Research Question

The purpose of this study was to examine reluctant problem solvers and their perceptions of mathematical tasks without words and/or word problems. This study aimed to answer one central research question: How do reluctant problem solvers perceive mathematical tasks without words and word problems?

Propositions

Due to the exploratory nature of the study, there were no propositions identified. Yin (2009) stated that it is not a requirement to declare propositions in a case study; further, exploratory case studies do not necessitate propositions.

At the same time, some studies may have a legitimate reason for not having any propositions. This is the condition – which exists in experiments, surveys, and the other research methods alike – in which a topic is the subject of ‘exploration.’ Every exploration, however, should still have some purpose. Instead of propositions, the design for an exploratory study should state this purpose, as well as the criteria by which an exploration will be judged successful. (Yin, 2009, p. 28)

The purpose of this study was to explore any and all perceptions of reluctant problem solvers related to mathematical tasks without words and word problems. I did not want to direct attention to any particular aspect to be examined by naming a proposition, but rather allow all perceptions to be exposed and examined. The criteria for deeming this study successful was finding answers to the research question.

Units of Analysis

As previously discussed, this study used a multiple holistic case design. This study was an analysis of individuals, namely reluctant problem solvers, focused on their perceptions of
mathematical tasks without words and word problems. Therefore, the unit of analysis in this study was the individual. Since there were two individuals or participants in the study, there were two units of analysis or cases. Yin (2009) discussed studies in which individuals served as the case:

In each situation, an individual person is the case being studied, and the individual is the primary unit of analysis. Information about the relevant individual would be collected, and several such individuals or ‘cases’ might be included in a multiple-case study. (p. 29)

My goal was to compile and analyze all of the data collected concerning the individuals to determine the perceptions of the two reluctant problem solvers related to mathematical tasks without words and word problems.

**Research Context**

I chose a school that was located in north Mississippi in a college town with outlying rural areas. The school educated approximately six hundred students per school year in grades four and five. There were 14 fourth grade teachers and 14 fifth grade teachers at the school, as well as teachers of special education, gifted, music, art, physical education, etc. The school was approximately 50% Caucasian American, 40% African American, and 10% were classified as Other Ethnicity. Approximately 60% of students qualified for free or reduced lunches. I selected the school for convenience of location.

I elected to utilize one specific fifth grade mathematics class. The participants for the study were selected from that one classroom taught by a single teacher, whom I will refer to as the host teacher. The host teacher participated in an externally funded project that aimed to help teachers strengthen their content knowledge and learn the best practices for teaching mathematics. The host teacher had been observed by project staff and found to implement
standards-based instruction as described by the National Council of Teachers of Mathematics (NCTM, 2000). I selected this particular teacher because of her involvement in the grant project and the observations previously completed in her classroom. The host teacher’s involvement in the grant project enhanced her understanding and implementation of standards-based instruction in her classroom. I selected her classroom because she utilized standards-based instruction as opposed to traditional instruction, as defined in Chapter I. Given the nature of each type of instruction, students in a classroom utilizing standards-based instruction are generally exposed to more hands-on activities than students in a classroom utilizing traditional instruction. I reasoned that it would be easier to identify students displaying behaviors of reluctant problem solvers in a standards-based classroom where they had opportunities to work in groups and engage in tasks and activities as opposed to students in a traditional classroom where the opportunities for working in groups and engaging in activities would likely be limited. For this reason, I saw the host teacher’s use of standards-based instruction in her classroom as advantageous for the purpose of this study.

**Instruments and Data Sources**

I utilized three instruments in this study to examine the perceptions of reluctant problem solvers related to mathematical tasks without words. These instruments included: participant reflections, interview protocol, and the researcher. Additionally, the transcriptions from sessions with the participants served as data sources. Each instrument will be described in the following sections.

**Participant Reflection Prompts**

Participants were asked to write a reflection describing their ideas and thoughts related to the featured task at the conclusion of each session. I provided a generic prompt to which the
participants responded (see Appendix A). The purpose of these written reflections was to obtain information from participants before they responded to interview questions. Further, the written reflections were completed individually before the interview was conducted to ensure that participants’ perceptions were recorded before their partners had an opportunity to influence their responses during the interview. Additionally, by writing their thoughts, participants could take time to think, edit, and record their perceptions in a response without the anxiety of the interview process.

**Interview Protocol**

The interview protocol consisted of eight questions that I asked of the participants upon completion of the task (see Appendix B). The questions pertained to the task, feelings about the task, and participants’ experiences in mathematics. The semi-structured interview allowed me to probe participants if responses to interview questions were brief.

**The Researcher**

Due to my role as researcher, I served as an instrument in the study (Creswell, 2009; Gall et al., 2007). My background is given to establish credibility. I am a Caucasian female pursuing a Doctorate of Education with an emphasis in elementary education. I currently hold a Master of Education in Curriculum and Instruction with emphasis in elementary education. I have three years of teaching experience in public schools including two years of teaching sixth grade mathematics and one year of teaching fourth grade mathematics and science. In addition to elementary teaching experience, I have taught elementary mathematics methods courses at my university of study. I currently work on an externally funded grant housed at the university. My work includes professional development with in-service middle grades teachers focusing on their development of mathematical content knowledge as well as their understanding of standards-
based instruction. I have studied qualitative research in a class at the university and served as a researcher on studies utilizing qualitative research methods. These professional experiences have prepared me and qualify me for serving as the researcher and instrument in the current study.

Procedures

This section explains the procedures utilized during the study. First, I describe the selection of the students to participate in the study. Next, I explain the details of the problem-solving sessions, followed by the procedures used in the problem-solving sessions. Last, a summary for the section is provided.

Selection of Participants

Upon approval of the Institutional Review Board (IRB), I utilized purposeful sampling to select students for the study. Specifically, I chose to use criterion sampling to select cases that met the criteria (Patton, 2002) of being reluctant problem solvers. Fifth grade students at the chosen school in the host teacher’s mathematics classroom served as the selection pool. The students were selected from the classroom given that they displayed behaviors of reluctant problem solvers and were therefore classified as such.

Students were selected based on two tools: the Observation Guide and the Behavior Frequency Chart. The role of each tool is described in the following paragraphs, followed by the selection process, including consideration of the markings on the Behavior Frequency Chart and the field notes on the Observation Guide. Finally, the last section details the study as it was narrowed to two participants or cases: Jackson and Destiny.

Observation Guide. Observations from a pilot study, discussions with the host teacher for the pilot study, personal classroom teaching experience, and the literature all informed the
development of the Observation Guide found in Appendix C. To aid the reader, the behaviors used to identify students in the current study are shown in the Observation Guide.

I observed in the mathematics classroom for three non-consecutive days during one week using the Observation Guide (Appendix C), which focused on observable behaviors of reluctant problem solvers, to identify students to participate in the study. I identified behaviors of reluctant problem solvers displayed by students in the classroom and made field notes on the Observation Guide.

**Behavior Frequency Chart.** During the time that I observed in the classroom, I asked the host teacher to complete the Behavior Frequency Chart (See Appendix D) for the students in her classroom stating how often students displayed given behaviors. I developed the Behavior Frequency Chart after creating the previously described Observation Guide using the same behaviors listed in the guide. The host teacher evaluated each student in the classroom by marking behaviors exhibited by students. Before the teacher returned the Behavior Frequency Chart to me, I observed in the classroom. I did not examine the teacher’s Behavior Frequency Chart before observing in the classroom so that the results would not influence my observations.

**Selection Process.** In my observations, nine students displayed behaviors of reluctant problem solvers. The host teacher identified four students with behaviors of reluctant problem solvers on the Behavior Frequency Chart. Names of students identified by the Observation Guide and the Behavior Frequency Chart were recorded in two different lists in descending order of frequency of occurring behaviors. All four of the students on the teacher’s list appeared on my list as well and were identified as potential reluctant problem solvers. Additionally, the top two students on my list, not on the teacher’s list, were also identified as potential reluctant problem solvers. In total, six students were identified as potential reluctant problem solvers and were
asked to participate in problem-solving sessions. Once the students were selected, I read the Oral Assent Form (See Appendix E) to them and they agreed to participate in the next part of the study.

**Two Cases.** Initially, six students were identified as potential reluctant problem solvers and were selected to participate in the problem-solving sessions. Once they were identified, their names were listed in alphabetical order, and then paired with a letter A, B, C, D, E, or F in alphabetical order as well. Midway through the sessions, Student D withdrew from the study, leaving five students, A, B, C, E, and F.

During the study, I realized that two of the students, Students C and F, displayed the highest frequency of behaviors of reluctant problem solvers as listed on the Observation Guide and Behavior Frequency Chart. Based on statements made and behaviors displayed during the sessions, the other three students, Students A, B, and E, were not as reluctant as Students C and F were to engage in the mathematics. For example, on multiple occasions, Student A and Student E made comments similar to the following: “I like to challenge myself.” For the reasons listed, I focused my attention on two students, Students C and F, who became the two cases or units of analysis in the study. In the following sections, I provide the backgrounds of Participant C, Jackson, and Participant F, Destiny, as well as the rationale for their selection as participants in the study.

**Jackson (Participant C).** I identified Jackson, a male, African American fifth grade student, as a potentially reluctant problem solver based on my observations in his classroom and his teacher’s assessment of his behavior in the classroom. Additionally, his behaviors in the problem-solving sessions confirmed that he was a reluctant problem solver. Jackson’s behaviors in the classroom are discussed here. Since my observations occurred before I read his teacher’s
notes, they are discussed first. Many of Jackson’s behaviors reflected the behaviors listed on the Observation Guide.

Jackson displayed three behaviors classified as “Delays engaging in the math task,” two behaviors classified as “Relies on group members to do the work for them or tell them what to do,” and four behaviors classified as “Off task during math class” on the Observation Guide. As an example, on one occasion the teacher gave the class a task to work in groups. The teacher distributed papers to Jackson’s group, and each of his group members picked up a paper and began to read the problem. Jackson looked at another paper under his desk. One member got up and walked across the room to retrieve a ruler for each group member. The other three group members picked up a ruler and placed it on the paper. Jackson picked up a ruler in one hand and a pencil in the other hand. Then he stopped. As his group members began working the problem, Jackson sat quietly. A few minutes later, he looked at his classmate’s paper. Finally, Jackson wrote on his own paper. This instance exemplified the behaviors “Delays engaging in the math task” and “Off task during math class.”

In addition, three of Jackson’s behaviors fit into the “Distracts themselves and their classmates” section and two of his behaviors represented the “Quiet when the rest of the group is working” section on the Observation Guide. Jackson often distracted himself and his classmates. A few times during my observations, Jackson’s teacher asked him to get on task. In one instance, he was searching through his backpack while his group members were working on a problem. On that particular day, the teacher was examining student work when they completed the problem. Jackson finished the problem after most of his classmates had. When he took the problem to his teacher to examine, she said that he did not follow directions and that was because he had been off task. This incidence exemplified the behavior “Distracts themselves and their
classmates.” In another instance, the class had completed a task and was transitioning to a class discussion. Jackson had not contributed to his group while they were working the task. The minute that Jackson’s teacher initiated a class discussion, he got up to sharpen his pencil. When he returned to his desk, he started writing on a piece of paper. He stopped for a minute and then erased something that he had written. Next, he wrote on his paper again. As he was writing and erasing, his classmates were discussing the problem. He did not look up at the teacher or his classmates for several minutes. This behavior exemplified the “Quiet when the rest of the group is working” section because Jackson was quiet most of the time that his group was working the problem and the entire time that his classmates were discussing strategies and solutions to the problem that they worked in class. He did not participate in the discussion or acknowledge his classmates as they contributed to the discussion.

For the most part, Jackson did not engage in the discussion during group work. Occasionally he contributed to the discussion, but not often. During whole group discussion, he spoke only when asked a direct question and did not speak otherwise. Jackson’s lack of contribution during discussions indicated that he was “quiet when the rest of the group is working” as labeled on the Observation Guide. After I observed his behaviors in the classroom, I examined his teacher’s Behavior Frequency Chart. His teacher indicated that Jackson “delays engaging in the math task.” Jackson was classified as a potentially reluctant problem solver based on his behaviors during my observations and his teacher’s marks on the Behavior Frequency Chart.

I asked Jackson if he was interested in participating in the problem-solving sessions, and he said that he was interested. Before each of his sessions, I read him the Oral Assent Form and
he signed and dated the form. He participated in five sessions throughout the study. Jackson’s behaviors in the problem-solving sessions are discussed in Chapter IV.

**Destiny (Participant F).** I identified Destiny, a female, African American fifth grader, as a potentially reluctant problem solver based on my observations and her teacher’s notes. Later, her behaviors in the problem-solving sessions confirmed that she was a reluctant problem solver. Destiny’s behaviors in the classroom are discussed here. I observed before I read her teacher’s notes, so my observations are discussed first.

I classified five of Destiny’s behaviors in the “Delays engaging in the math task” section on the Observation Guide. In one instance, Destiny drew in her journal, coloring letters and shapes while her group members began working on a task assigned to them by the teacher. She spent several minutes drawing before a group member asked her a question about the problem and she looked at her paper. Destiny sat quietly for a minute and then shrugged her shoulders. Her behavior in this incident exemplified the behaviors “Delays engaging in the math task” and “off task.”

Four of her behaviors exemplified the section “Quiet when the rest of the group is working” on the Observation Guide. Frequently, I noted that Destiny did not contribute to discussions within her group or whole class discussions. She typically watched the other members of her group as they worked. Occasionally, she commented on the group’s work, however, her comments were infrequent. Additionally, in another instance, the whole class was discussing a problem that they had worked in class. Destiny’s gaze was focused on the window. When the teacher called her name to answer a question, Destiny did not give a response. The teacher told Destiny to pay attention to the discussion and said that she would come back to Destiny for a response later in the discussion.
My observations allowed me to consider classifying Destiny as a potential reluctant problem solver, but I needed to examine the Behavior Frequency Chart completed by her teacher. Upon inspection, the chart affirmed my classification. Destiny’s teacher indicated that she “delays engaging in the math task.” In addition, she is “quiet when the rest of the group is working.” Based on the teacher’s marks and my observations, Destiny was identified as a potential reluctant problem solver.

I asked Destiny if she was interested in participating in the study, and she agreed to participate. Before each session, I read the Oral Assent Form to her, and she signed and dated it. She participated in four problem-solving sessions. Destiny’s behaviors in the problem-solving sessions are discussed in Chapter IV.

**Problem-Solving Session Details**

The problem-solving sessions took place in a classroom that was not occupied by a class during the current school year. It was located on the fourth grade hall of the school. There was little traffic near the classroom, which made it a quiet place to work. Figure 1 provides a visual layout of the classroom.
Students met with me in pairs during each session. Having the students work in pairs allowed the need for relatedness to be met. Additionally, in a pilot study, I found that having students work in pairs was the best method for finding the information required to inform the research questions. In the current study, students did not work with the same partner for every task, so pairs varied. Originally, students were paired so that they did not work with any other student more than once. The pairings were random and established before students were identified to participate in the sessions to remove any bias that might occur from pairing students after initial classroom observations. The random pairings in the study allowed the reluctant problem solvers to work with less-reluctant problem solvers at times as well as with other reluctant problem solvers at times.
Since Student D withdrew from the study after his first two sessions, his partners for the last two sessions, Student B and Destiny (Student F) were left without a partner according to the original plan. Student D’s last two sessions, Session 2B and Session 4C, were edited to provide his partners with a new partner. First, for Session 4C, I replaced Student D with Jackson (Student C). I chose to allow Jackson (Student C) and Destiny (Student F), the two most reluctant problem solvers, to work together in the last session of the study after working together in a previous session at the beginning of the study. For Session 2B, I replaced Student D with Student E because she also displayed some behaviors of a reluctant problem solver in previous sessions, though she was not as reluctant as Jackson (Student C) or Destiny (Student F).

Table 1 provides the details for each session including: dates the sessions occurred, session names, pairing of students, and the mathematical tasks without words and word problems used. The two reluctant problem solvers, Jackson (Student C) and Destiny (Student F), are shown in the table in bold print.
<table>
<thead>
<tr>
<th>Date</th>
<th>Session</th>
<th>Pairs</th>
<th>Mathematical Task without Words</th>
<th>Word Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>December 6</td>
<td>1A</td>
<td>A and B</td>
<td>Bead Problem</td>
<td>Sports Problem</td>
</tr>
<tr>
<td>December 7</td>
<td>1B</td>
<td>C and F</td>
<td>Pattern Block Quilts Task</td>
<td>Tom’s Meals Problem</td>
</tr>
<tr>
<td>December 7</td>
<td>1C</td>
<td>D and E</td>
<td></td>
<td>72 words</td>
</tr>
<tr>
<td>December 11</td>
<td>2A</td>
<td>A and C</td>
<td>Four 4’s Problem</td>
<td>Birthday Party Tables</td>
</tr>
<tr>
<td>edited</td>
<td>2B</td>
<td>B and  D</td>
<td></td>
<td>Problem</td>
</tr>
<tr>
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<td>E and F</td>
<td></td>
<td>58 words</td>
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<tr>
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<td>A and D</td>
<td></td>
<td></td>
</tr>
<tr>
<td>December 12</td>
<td>3B</td>
<td>B and  F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>December 12</td>
<td>3C</td>
<td>C and E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>December 13</td>
<td>4A</td>
<td>B and C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>edited</td>
<td>4B</td>
<td>A and  E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>December 13</td>
<td>4C</td>
<td>F and  D</td>
<td></td>
<td></td>
</tr>
<tr>
<td>December 14</td>
<td>4A</td>
<td>B and C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>December 13</td>
<td>4B</td>
<td>A and E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>December 14</td>
<td>4C</td>
<td>F and  C</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Edited Sessions:**

<table>
<thead>
<tr>
<th>Date</th>
<th>Session</th>
<th>Pairs</th>
<th>Mathematical Task without Words</th>
<th>Word Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>December 13</td>
<td>2B</td>
<td>B and E</td>
<td>What’s the Area?</td>
<td>Lemonade Problem</td>
</tr>
<tr>
<td>December 14</td>
<td>4C</td>
<td>F and C</td>
<td></td>
<td>62 words</td>
</tr>
</tbody>
</table>

*Note.* The participants identified as reluctant problem solvers, Participants C and F, are shown in bold print. Student D withdrew after his first two sessions in the study. His final two sessions indicate with “D” that he withdrew from the study and the sessions are labeled as “edited” to indicate that the pairs for the sessions changed before the sessions were conducted.
For the purpose of this research, word problems were defined as problems with 50 or more words in the problem. The number of words in each word problem used in the study is provided in the table. I asked the host teacher to review the problems to ensure that students had not previously worked them and that they were appropriate for the grade level.

**Problem-Solving Session Procedures**

At the beginning of each session, I turned on a video recorder to document the work of the students and record their thought processes that they discussed as they worked. Each pair was presented with two tasks, a word problem and a mathematical task without words. Both tasks were non-routine for the students and required students to engage in problem solving to find a solution. Students were asked to choose one task to complete. Additionally, manipulatives were provided for the pair if they chose to use them. For each task, students could choose any of the following manipulatives to support their work: two-color counters, base-ten blocks, colored cubes, and pattern blocks. Additionally, highlighters were supplied for students who chose to work the Triangles Problem. Allowing students to choose which task to work and which manipulatives to use allowed their need for autonomy to be met. The students were given as much time as they needed to complete the task, and the average time was twenty minutes. While students were working, I prompted the pairs to work together and discuss their strategies aloud. Additionally, I redirected questions from students to their partners instead of to me, and I asked probing questions of students pertaining to the task.

Upon completion of the task, the students were immediately asked to respond individually in writing to the Participant Reflection Prompt (see Appendix A).
Responding individually in writing allowed students to record their initial thoughts without being influenced by their partner’s thoughts. Students required as much as five minutes to complete this written reflection. When they completed the written reflections, I interviewed them in pairs using the interview protocol (see Appendix B).

The pilot study influenced my decision regarding interviewing the students in pairs. In the pilot study, the participants who were interviewed individually generally did not provide as much information as participants who were interviewed in pairs or in threes. When participants were interviewed with a partner, however, they were able to answer a question and listen to their partner answer the same question. Listening to their partners appeared to trigger participants to answer the same question again, including more information in their responses. For this reason, I chose to interview students in pairs in the current study.

Student responses were captured via video. Additionally, an audio recorder was turned on as a means of providing a back-up recording of the interview. Interviews lasted approximately fifteen minutes. Depending on student responses during the interviews, I asked additional questions when I felt that they were needed to probe students for more detailed responses or to explain their responses. At the conclusion of the interview, I turned off the video recorder and audio recorder. The interview concluded the session and students returned to their classroom.

Summary

Originally, I planned to meet with each of the six students four times. Student D withdrew from the study, after completing two sessions, and the numbers were altered. Students A, B, and F completed four sessions and Students C and E completed five
sessions. I met with each student one to four times a week for a period of two weeks. Since students met with me multiple times, the example of a word problem on the interview protocol was omitted after each student had been interviewed once. Students C (Jackson) and F (Destiny) were identified as the students who displayed the highest frequency of behaviors of reluctant problem solvers and were chosen as the two cases to be analyzed for the study. The other three students were identified as less-reluctant problem solvers. I use the phrase “less-reluctant problem solvers” to indicate that the other three students are reluctant problem solvers, but not as reluctant as the two participants, Jackson and Destiny, in the study.

**Data Analysis**

Upon completion of the problem-solving sessions, I transcribed the audio recordings. Transcribing the recordings of sessions allowed me to re-live participants’ experiences of working the tasks. I hoped to understand the participants’ perceptions and experiences by listening to the recordings many times and typing the transcriptions.

As previously described, I decided to focus only on the sessions in which one or both of the reluctant problem solvers, Jackson (Participants C) and Destiny (Participant F), participated. Table 2 identifies the sessions with one or both participants, and their letters (C and F) are shown in bold print. The student who initiated the choosing of the task is shown with an asterisk (*) before their letter in the *Pairs* column. In the following pages, I use the phrase “initiated the choosing of the task.” This does not mean that one student chose the task and the pair worked it. Instead, “initiated the choosing of the task” indicates that one student initially selected the task, and then the pair discussed the task and agreed to work it instead of the other task presented during the session. The task that
the pair chose to work is shown in bold print. The students in Sessions 2A and 2C chose
to work the same task, the Pattern Block Quilts Task. Those two sessions are grouped
together in the table and indicated with two asterisks (**).
Table 2

Session Details for Participants C and F

<table>
<thead>
<tr>
<th>Date</th>
<th>Session</th>
<th>Pairs</th>
<th>Mathematical Task without Words</th>
<th>Word Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>December 7</td>
<td>1B</td>
<td>*C and F</td>
<td>Bead Problem (Krutik &amp; Rudnick 1995)</td>
<td>Sports Problem (Dougherty &amp; Slovin, 2006)</td>
</tr>
<tr>
<td>**December 11</td>
<td>2A</td>
<td>*A and C</td>
<td>Pattern Block Quilts Task (Hartweg, 2003)</td>
<td>Tom’s Meals Problem (Billstein, Libeskind, &amp; Lott, 2010)</td>
</tr>
<tr>
<td>**December 11</td>
<td>2C</td>
<td>E and *F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>December 12</td>
<td>3B</td>
<td>B and *F</td>
<td>Four 4’s Problem (Carver, 1994)</td>
<td>Birthday Party Tables Problem (Dougherty &amp; Slovin, 2006)</td>
</tr>
<tr>
<td>December 12</td>
<td>3C</td>
<td>C and *E</td>
<td>Four 4’s Problem</td>
<td>Birthday Party Tables Problem</td>
</tr>
<tr>
<td>December 14</td>
<td>4A</td>
<td>B and *C</td>
<td>Triangles Problem (Rachlin et al., 2001)</td>
<td>Egg Problem (Dougherty &amp; Slovin, 2006)</td>
</tr>
<tr>
<td>December 14</td>
<td>4C</td>
<td>F and *C</td>
<td>What’s the Area? (Dougherty &amp; Slovin, 2006)</td>
<td>Lemonade Problem (Dougherty &amp; Slovin, 2006)</td>
</tr>
</tbody>
</table>

Note. The reluctant problem solvers, Jackson (Participant C) and Destiny (Participant F) are shown in bold print. The student who initiated the choosing of the task or problem for each session is indicated with an asterisk (*). The task or problem chosen is shown in bold print. In Sessions 2A and 2C, the participants chose the same task, the Pattern Block Quilts Task. Those two sessions are indicated with two asterisks (**).
I sought to maintain objectivity and sensitivity by discarding any biases or expectations regarding the data. Before the data could be analyzed, it needed to be organized so that all of the information from one session was compiled into the same document. According to Patton (2002),

Once the raw case data have been accumulated, the researcher may write a case record. The case record pulls together and organizes the voluminous case data into a comprehensive, primary resource package. The case record includes all the major information that will be used in doing the final case analysis and writing the case study. (p. 449)

I completed many steps to create two case records for the two reluctant problem solvers in the study, Jackson (Participant C) and Destiny (Participant F). First, I focused on Jackson. I identified all of the sessions in which he participated: Sessions 1B, 2A, 3C, 4A, and 4C. I watched the video from each of his sessions and wrote a summary of the problem-solving session as well as a summary of the interview. I created a document or session description for each session containing the progression of the session. For example, the session description for Session 1B was composed of a summary of the problem-solving session, Jackson’s written reflection prompt, a summary of the interview and the interview transcription. All of the session descriptions concerning Jackson, five session descriptions, were then compiled into a larger document, a case record. “The case record is used to construct a case study appropriate for sharing with an intended audience, for example, scholars, policymakers, program decision makers, or practitioners” (Patton, 2002, p. 450). I completed the same procedure to create a case record for Destiny (Participant F). I created session descriptions for each of her four sessions including
Sessions 1B, 2C, 3B, and 4C. Finally, I compiled all of the session descriptions for Destiny into a larger document, a case record.

I examined both case records to study each participant’s perceptions and experiences during the study and to write two case study narratives capturing the outcomes of the participants’ experiences.

The case study is a readable, descriptive picture of or story about a person, program, organization, and so forth, making accessible to the reader all the information necessary to understand the case in all its uniqueness. The case story can be told chronologically or presented thematically (sometimes both). The case study offers a holistic portrayal, presented with any context necessary for understanding the case. (Patton, 2002, p. 450)

After writing the case studies, I used content analysis to analyze the data again. I sought to determine if any patterns and themes existed within each case and between the two cases. Patton (2002) states,

More generally, however, content analysis is used to refer to any qualitative data reduction and sense-making effort that takes a volume of qualitative material and attempts to identify core consistencies and meanings. Case studies, for example, can be content analyzed. The core meanings found through content analysis are often called patterns or themes. Alternatively, the process of searching for patterns or themes may be distinguished, respectively, as pattern analysis or theme analysis. (p. 453)
I performed the cross-case analysis to make connections between the two cases, and further, to discuss transferability and implications of the patterns and themes discovered. The findings are discussed in Chapter IV.

**Limitations**

As with any study, there are limitations that should be mentioned. Three primary limitations are listed. First, there is no previously set framework for determining whether or not a student might be considered to be a reluctant problem solver, though the completion of a pilot study provided characteristics for the current study utilized in selecting participants. Second, due to the qualitative nature of the study, the results are not generalizable. Thick and robust descriptions of participants will be provided, however, so that the findings will be transferable. In qualitative research, it is common to substitute “transferability” for generalization when discussing findings (Patton, 2002). The findings will lead to suggestions for future research. Third, the timing of the study served as a limitation. The sessions occurred during December, the last month of school in the fall semester. At the initiation of the study, students had been in the host teacher’s classroom for four months. The class had established classroom norms and expectations. Additionally, it is possible that the standards-based methods used in the classroom had already begun to alter the participant’s perceptions of word problems or mathematical tasks without words.

**Summary**

In the Standards and Accountability era (Lambdin & Walcott, 2007), it is increasingly necessary for all students to meet expectations as defined by national standards (e.g., CCSSI, 2010, NCTM, 2000). Teachers must find methods to engage all
students, even the students who are reluctant. With the goal of engaging reluctant
students, it is necessary to determine methods that have proven successful in capturing
the attention of those students. This study sought to examine the perceptions of reluctant
problem solvers related to mathematical tasks without words and word problems. The
participants included fifth grade students who were classified as reluctant in their
mathematics classroom at a public elementary school. Data collection consisted of
participants completing mathematical tasks without words, writing reflections, and
answering questions in an interview. Upon completion of data collection, I analyzed the
data, resulting in the creation of two case studies and in a cross-case analysis. The
findings are reported in Chapter IV.
CHAPTER IV: FINDINGS

Introduction

In the age of standards, accountability, and expectations, it is essential for all students to learn mathematics in a way that encourages them to persist in solving problems, even when the problems are difficult (CCSSM, 2010; NCTM, 2000). Students who are reluctant to engage in the mathematics, however, have a slight disadvantage when compared to their less-reluctant or non-reluctant peers. Reluctant problem solvers need motivation to engage in problem solving. Research must be conducted, however, to determine what type of problems and tasks might be used to motivate reluctant problem solvers. To this end, researchers must examine reluctant problem solvers’ perceptions of such problems and tasks.

This chapter presents two cases or reluctant problem solvers: Jackson (Participant C) and Destiny (Participant F). Jackson and Destiny’s experiences were captured through data collection, and then the data were analyzed. The findings are reported in this chapter. The first two sections in the chapter present each case including descriptions of sessions in which the participant was partnered with less-reluctant problem solvers. The next section presents the sessions in which the two participants or cases were partnered together. The participants’ experiences are documented through summaries and dialogue from each of the sessions. Individual sessions are divided into two sections: a problem-solving session, including written reflections, and an interview. Once the experiences of each case from all sessions have been captured, within-case analyses for each case are
reported, followed by a cross-case analysis. Finally, a section responding to the research question and a summary conclude the chapter. Throughout the chapter, mathematical tasks without words are given the abbreviation “MTW” and word problems are given the abbreviation “WP.”

Jackson

Jackson completed five sessions throughout the study. He was paired with less-reluctant problem solvers, Students A, B and E, during three of the sessions. In two sessions, he was paired with Destiny, the other reluctant problem solver. The following sections describe his experiences in sessions with less-reluctant problem solvers. Jackson’s own words from the sessions are used throughout the text.

Sessions with Less-Reluctant Problem Solvers

Jackson worked with three of his classmates who were less-reluctant problem solvers during Session 2A, Session 3C, and Session 4A. Session details were shown in Chapter III in Table 2. In the paragraphs that follow, I describe each session, including task selection, a discussion of the problem-solving session, and a discussion of the interview.

Session 2A. Jackson worked with a male, less-reluctant problem solver during this session. I gave the pair a choice between the Meals Problem (WP) and the Pattern Block Quilts Task (MTW). Jackson’s partner said that he wanted to work the Pattern Block Quilts Task (MTW), but he did not know how to find the area. He took the pattern blocks and began placing them on the quilts on the paper. I reminded the boys that they had to choose the same problem to work together. Jackson’s partner said that he liked the
Jackson agreed to work the Pattern Block Quilts Task (MTW).

**Problem-solving session.** Jackson and his partner began by placing pattern blocks on the quilts on the paper, and I reminded them to work together. Jackson’s partner asked if they were supposed to count how many blocks were inside the quilt when they were finished. I asked Jackson what he thought about his partner’s question, and Jackson nodded his head, “Yes.”

The pair began by placing different pattern blocks on the quilts. I asked them how they would count the blocks when they were finished, and Jackson’s partner demonstrated by counting all of the blocks on his quilt. I questioned the pair about the area of each pattern block, including the hexagon, trapezoid, rhombus, and triangle. Jackson’s partner answered the first question, so I directed the next question to Jackson.

**Interviewer:** I asked you if the red trapezoid would take up as much room as the green triangle. What do you think? Do they take up the same amount of room?

**Jackson:** No.

**Partner:** No.

**Interviewer:** Ok.

**Jackson:** The trapezoid is bigger.

**Interviewer:** The trapezoid is bigger? So can you count them both as one?

**Jackson:** I’ve got like a plan. If you like, these two can be like that, or this one can go right there.

**Interviewer:** So, can you count them both as one when you’re counting?
Partner: I think you can. I’m not sure.

Through further questioning, Jackson’s partner realized that he could not count each pattern block as one unit. Instead, the pair had to consider the area of each block when counting. Jackson’s partner asked many questions concerning the area of the trapezoid and the area of the triangle while Jackson continued to place pattern blocks on the quilts. Jackson’s partner suggested removing all of the blocks from the paper and using only the trapezoids to fill the quilts. He added that he would need triangles to fill the small spaces. I asked Jackson what he was thinking, and he said that he was trying to determine where to place the blocks. Jackson’s partner began placing trapezoids on one of the quilts, and Jackson continued to use all of the blocks on his quilt. Jackson’s partner explained what he was doing, and Jackson replied, “This is like a tessellation.”

After using trapezoids to fill part of the quilt, Jackson’s partner decided to use triangles instead of trapezoids. Jackson and his partner completely filled one of the quilts with triangles, and Jackson said, “It maybe a, look like a star.” Later, Jackson picked up the white rhombus and stated, “If it’s like a line you could put this one.” He thought that if the quilt was a narrow shape, more like a line, the dimensions of the white rhombus might be better to fill the quilt.

I asked the pair what the problem had asked them to do, and Jackson’s partner responded by counting the triangles on the first quilt. They found an area of thirty-six triangles for the first quilt and recorded their answer on the paper. For the second quilt, Jackson suggested, “Maybe you can use triangles for this one.” His partner said, “Probably can, let’s see.” The pair made a hypothesis for the area of the second quilt in relation to the area they had found for the first quilt.
Partner: Quilt B’s gonna have the greatest area.

Interviewer: Oh, you think so?

Partner: Mmhmm.

Interviewer: What makes you think?

Partner: Because it, cause it look like it’s gonna take up a lot of triangles.

Jackson: And it’s bigger.

The pair continued to place triangles on the second quilt until it was completely filled, and then they counted the triangles. As they were almost finished counting, Jackson said, “It’s equal.” They found that the second quilt also took thirty-six triangles to fill it, so it was equal. I asked if it looked like they were going to be equal, and Jackson said, “No. They may be the same height.” The pair wrote down the area for the second quilt and then placed the pattern blocks back in the bag. I handed them the Participant Reflection Prompt and asked them to record their responses to the prompt. Jackson’s response is shown in Figure 2.
**Participant Reflection Prompt**

Today I chose the **greatest area** problem to work on because ... it looks fun to do putting the shapes onto it.

Please include in your response:
- What you liked about the problem
- What you didn’t like about the problem
- If you could choose again, would you choose the same problem? Why or why not?

- I liked putting shapes on it to find the area.
- I didn’t hate anything about it.
- I wouldn’t choose it again because he will get the answer again.

*Figure 2. Jackson’s Participant Reflection Prompt for Session 2A.*
**Interview.** Jackson expressed a desire to work the Pattern Blocks Quilt Task (MTW) because “You build shapes. (pause) And tessellate ‘em.” Further, he stated, “I wanted to work it because you like get to build shapes onto them and find the area.”

When asked to describe a word problem, Jackson expressed his opinion of what a word problem is.

Um, it don’t have no pictures. It don’t have no pictures. It just you (pause) um (pause) it just not give you like well it give you words cause it’s a word problem but it’s like it don’t give you no like I said last time, it don’t give you much.

I asked Jackson how the task that he just worked, the Pattern Block Quilts Task (MTW), was like or not like a word problem. He explained in the following dialogue.

Jackson: It’s like it say which shape covers the greatest, which has the greatest area so all you gotta do is build shapes on ‘em it don’t give you like what to do like like a word problem give you information what to do and this one just got this one um longer and this one short.

Interviewer: What’s long and what’s short?

Jackson: The word problem is longer than the picture problem.

Jackson said that if he had the opportunity to choose again, he would pick the Pattern Blocks Quilt Task (MTW) again.

Jackson: Yeah, I pick [the Pattern Block Quilts Task (MTW)] because it look fun cause you can build shapes and everything so I’d pick this one.
Interviewer: After solving the problem, did you wish you had chosen the other one?

Partner: No.

Jackson: No.

Interviewer: Why not?

Partner: Because

Jackson: Because

Partner: it looked funner.

Jackson: [The Pattern Block Quilts Task (MTW)] more easy than [the Meals Problem (WP)] gonna have like like you gonna have to add, multiply, divide.

Partner: Uh

Jackson: And that one

Partner: Why would you go for that one when you could go for the easy one?

Jackson: And you can do [the Meals Problem (WP)] next time.

Later, Jackson repeated the reason why he enjoyed the Pattern Block Quilts Task (MTW). He stated, “It’s easy and you just build shapes to get the area and make sure your um blocks are in the right place.”

Session 3C. Jackson and his partner, a female, chose between the Tables Problem (WP) and the Four 4s Problem (MTW) to work during this session. They each read the problem in front of them and then traded and read the other problem. They did not understand what the Four 4s Problem (MTW) was asking them to do, so I talked them
through the example problem. During the discussion, I asked the pair what would be equal to four fourths. Jackson said that it would equal one. His partner did not say anything, and then Jackson added that it would be either one or zero. After we talked through the example problem, I asked them to look at both of the problems again and decide together which one they wanted to work. Jackson’s partner pointed to the Four 4s Problem (MTW) and said that she wanted to work that one. Jackson read through the Tables Problem (WP) and said that the answer was thirty. I asked him if he was sure, and he said he was. I reminded the pair to choose a problem to work together and then they could discuss it. Together, Jackson and his partner chose to work the Four 4s Problem (MTW).

**Problem-solving session.** Jackson asked if he could use the solution for 17 that was in the example. I told him that a solution for 17 was already given to them, so 17 was not one of the numbers they were asked to find. I reminded him that the problem asked him to find solutions for the numbers from one to 10. I gave both of the participants a piece of paper so that they could each write on separate papers, but I reminded them to work together and use only four 4s in their problem. Jackson asked if he could multiply 1 times 10.

Jackson: Can you do 1 times 10?

Interviewer: What numbers can you use?

Jackson: 1 times 10.

Partner: 1 through 10.

Interviewer: Just four 4s. You can only use four 4s, and you want to use four 4s to equal one, four 4s to equal two.
I reminded them that they could use any operation but had to use four 4s to equal every number from one to 10. They each returned to their own papers to work. Jackson stopped writing and looked at his partner’s paper. His partner said that she had found a solution and explained it to him.

Partner: Four minus 4 equals 0 (pause)

Jackson: That’s one.

Partner: Four plus 4 plus, 8. Eight divided by 4 equals 2. No. Eight divided by, yeah.

Jackson: Eight divided by what?

Partner: Four equals 2.

Interviewer: Does that work?

Jackson: Yes.

I suggested that they try to find a solution for another number. The dialogue below shows Jackson’s partner discussing a process aloud as Jackson worked.

Partner: You can’t do that. Oh, I did mess it up. No I didn’t. I wonder if that’s one way I have to do.

Jackson: Mmhmm.

Partner: Four plus 4 is 8. Eight minus 4. No, it has to go 4 minus 4 is 0 and 4 plus 4 equals 8. Oh, I messed it up.

Jackson: I’m gonna go to the other numbers. Umm, four. I think I got it.

Partner: Can you use the same symbols twice?

Interviewer: Sure.

Jackson: I got it. For number two. Four.
Partner: Oh man.

Jackson: I got it.

Partner: For what?

Jackson: Three. Use a minus sign.

Partner: (looks at Jackson’s paper) You have to use four 4s. Not three 4s.

Jackson’s partner found another solution and explained it to him. Jackson continued to work and glanced at his partner’s paper every few minutes. I asked him how he was doing.

Interviewer: How’s it going, Jackson?

Jackson: I’m looking. I’m seeing if number three is going to be division or addition.

Interviewer: Ok. So what do you think you can do?

Jackson: Four times 4 is 16.

Partner: Doesn’t it have to be like 4 plus 4 equals 8. Eight divided by 4 equals 2.

Interviewer: Do what now?

Partner: Like this, 4 plus 4 divided 4 plus 4.

Jackson: You gotta use nothing but fours?

Interviewer: Nothing but fours.

Partner: And then 4 plus 4 is 8. Eight divided 4. Or 4 divided 4?

Interviewer: It’s, well using the order of operations, what do you think would come first?

Partner: Oh! Ok.
Jackson: Can you put um
Partner: Parentheses
Jackson: Parentheses
Interviewer: Mmhmm.
Partner: Now I get it.
Interviewer: Does that make it better?
Jackson: Yes.
Partner: Yes.
Jackson: Four plus 4 equals 8.
Partner: Four divided by 4 equals 1. One
Jackson: Divided by
Partner: One plus 4. Five. One and 5 minus 4 equals 1. Yes, that’s right.
And four. Four divided by 4 equals 1. One plus 4 equals 5. Five
plus 4, huh? Five plus 4 equals 9. Oh. So this goes over here.
Jackson: I’m fixing to go with the other ones.
Interviewer: Just pick, or, you can just start doing something with the fours,
Jackson, and then see what it would equal.

Jackson and his partner checked their previous solutions using the order of
operations and were able to use it when trying to find new solutions to the problem. For
the sake of time, I had to stop the pair and ask them to respond to the Participant
Reflection Prompt. Jackson’s work on the Four 4s Problem is shown in Figure 3.
Jackson’s response to the Participant Reflection Prompt is shown in Figure 4.
Figure 3. Jackson’s work on the Four 4s Problem during Session 3C.
**Figure 4.** Jackson’s Participant Reflection Prompt for Session 3C.

Participant Reflection Prompt

Today I chose the problem problem to work because...

Please include in your response:
- What you liked about the problem
- What you didn’t like about the problem
- If you could choose again, would you choose the same problem? Why or why not?

Doing math makes

Couldn’t figure out the problems

I would choose another problem because it was hard
**Interview.** Jackson expressed that he had selected the Four 4s Problem (MTW) instead of the other problem because he thought that it would be easier to work. His explanation is provided.

Jackson: Uh (pause) it maybe easier and you might can like if you mess up right there and it was three you could put it on the problem right there like that.

Interviewer: Ok. What made it easier?

Jackson: Um it made it easier because um we can do parentheses.

Jackson claimed that he liked it because he could do mathematics in the problem and that included using parentheses, exponents, multiplication, and division. However, he did not like that he could not find all of the answers.

During the interview, Jackson continued to work on the problem and exclaimed that he had a solution. The dialogue below articulates the instance.

Interviewer: What made you want to work this task or not want to work it?

Jackson: And I just got my ans- answer for number three.

Interviewer: Did you? You can write it down.

Partner: Wha- what was the question?

Interviewer: What made you want to work this task or not want to work it?

Partner: I wanted to work it because I thought re- both of them, that this will be more challenging a little bit more and that it's difficult.

Interviewer: Ok um

Jackson: I think I did it wrong. Four minus 4 plus 4 equals 4, and minus 4 and it’s like 4 fraction 4.
Interviewer: Ok?

Jackson: And it was three but I need five.

Interviewer: It equals five? Five 4s. Ok. Well maybe if the challenge was five 4s instead of four 4s. Um, Jackson, what made you want to work this task or not want to work it?

Jackson: I wanted to work it because like it got a lot of math and it will be um, um it will be easier for me to learn new stuff about this.

After discussing the Four 4s Problem (MTW), I questioned the pair regarding word problems.

Interviewer: Ok. So how would you describe a word problem?

Jackson: A word problem is, it don’t have no pictures, it just give you directions what to do.

I asked Jackson if the Four 4s Problem (MTW) was a word problem, and he said that it was not. He explained, “This is not cause it got like pictures, and this is not like a word problem because it got like these and it don’t give you like a word lots of words like the word problem.” He indicated that he would like to work another problem like the Four 4s problem because “it’s like kind of (pause) challenging and stuff. And, it’s hard to find the answers.” When asked about the Tables Problem (WP), Jackson said that he did not wish that he had chosen to work it instead of the Four 4s Problem (MTW). However, he did read the Tables Problem (WP) again and provide his thoughts about the problem as recorded.

Jackson: This [Tables Problem (WP)] you (pause) got a lot of

Partner: A lot of words.
Interviewer: A lot of words?
Partner: Yes. I don’t like one with a lot of words.
Jackson: This one is 30 though.
Interviewer: It is what?
Jackson: Thirty.
Interviewer: Thirty? You think so?
Jackson: See, it say what
Partner: Cameron said
Jackson: The least
Partner: last night I finish making
Jackson: The least what
Partner: This
Jackson: What the least number of card tables they need then they, they
needed 30.
Interviewer: I don’t know. I think that if we could spend some time on that
problem
Jackson: Fifteen?
Interviewer: It might. That was a quick answer. I think if we could spend some
time on that problem we could maybe see what the answer was.

Jackson read the word problem and then immediately arrived at a solution without
actually working the problem or providing a justification for his solution. When
questioned, he suggested another number as the solution. However, he had not spent time
working the problem, and he did not provide an explanation to accompany his solution.
Jackson described what he would tell his classmates about his experience working the Four 4s Problem (MTW).

Um this is a hard problem and you can only use fours and be sure to um if, if it’s not, if it’s on number six, if you on number six, and you couldn’t get number five and if the um it equaled five, put it right there

Further, he said that his classmates might like to work the problem because it would be challenging, and if they like math, they might like to work it.

Session 4A. I gave Jackson and his partner, a male, a choice of the Egg Problem (WP) or the Triangles Problem (MTW). They looked at both of the problems, and then Jackson said that he had worked the Triangles Problem (MTW) in class. I replied that they might have worked a similar one, but they had not worked the Triangles Problem (MTW). Jackson began working the Triangles Problem (MTW) while his partner read the Egg Problem (WP). I reminded them to choose a problem together before beginning to work. Jackson read the other problem, and his partner asked Jackson which one he wanted to work. Jackson chose the Triangles Problem (MTW) and his partner agreed to work it.

Problem-solving session. I gave Jackson and his partner another piece of paper so that they could both write on their own paper. I also gave them several highlighters of different colors to use if they felt the colors would aid them in keeping track of the triangles they counted. Before writing on his paper, Jackson’s partner asked what “equilateral” meant, and I questioned the boys to help them figure it out.

Partner: What’s this? Equal?

Interviewer: Equilateral. What does that mean?
Partner: What do equilateral mean again? What that word mean?

Jackson: Equilateral?

Partner: Yes.

Jackson: Just um, counting the squares. I meant counting the uh triangles. But like [teacher] said, do like these.

Interviewer: What’s an equilateral triangle?

Jackson: Triangles. Like

Interviewer: What’s an equilateral triangle? All of the what are equal?

Jackson: Sides!

Interviewer: Did you hear it, [partner]?

Partner: Yeah.

I reminded the boys to work together and share their strategies for solving the problem. Jackson’s partner said that there were 25 triangles in all. Jackson continued counting, and his partner asked if they could work on the same paper. Jackson did not answer the question, instead he said that he had two answers: 15 and 25. Jackson’s partner asked if I wanted them to work together on Jackson’s paper. I replied that I wanted them to work together.

Jackson placed his head on the table, and his partner moved his chair closer to Jackson to work together. Jackson continued to count and then exclaimed that he had three answers. He and his partner discussed the problem:

Jackson: Seventy-five. I got three answers.

Partner: Dude, I know what you do.

Jackson: What?
Partner: I think you do five times five.
Jackson: Cause like this five, ten
Partner: Shouldn’t you do five times five?
Jackson: Look, five, ten, 15. And 25 is inside. Threes, count by threes is 75.
Partner: Shouldn’t you do like five times five?
Interviewer: Should you?
Partner: Do we, we have to multiply or add?
Interviewer: I don’t know.
Jackson: I think it’s either 15, 25, or 75.
Interviewer: How many triangles have you found so far?
Partner: It’s 25 triangles in all.
Interviewer: Ok. And
Jackson: I got 15 because five
Partner: Dude, I’m talking ‘bout
Jackson: Five, 10, 15 or 20
Partner: I know, I did it like this
Jackson: Is 25 inside
Partner: No, I’m doing it like this.
Jackson: or three, six, nine
Partner: No like this I’m counting, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 25
Jackson: But if it was like three, so 3, 6, 9, 12, 15, 18, 21, 14, 27, 30.
Partner: I don’t get this. I wish we did the [Egg Problem (WP)].
Interviewer: Why?
Partner: This one kinda frustrating.
Interviewer: Let me ask you this, how many small triangles are there? 25 triangles, you said?
Partner: No it’s 25 triangles in all.
Interviewer: Can you make, so that’s of the little triangles, just one. Can you make bigger triangles with
Jackson: Oh!
Interviewer: Like can you make a triangle out of two small triangles? Or three small triangles? Or four small triangles? You can use the highlighter if that would help.
Jackson: Oh! (picked up a highlighter)
Partner: Man, I wish we could’ve did this Egg Problem (WP).
Jackson: Uh, this how you do
Partner: Let me see,
Jackson: Three sides
Partner: Let me see, let me see. Dude, let me see. Oh, I see what you did.

Jackson used the highlighter while drawing on his paper, and his partner watched.

Then, his partner picked up a highlighter to use on his own paper. His partner held up his paper and asked me if the shape he had highlighted was a triangle. I asked him if it could be a triangle, and he said, “Yes.” At that moment, Jackson stopped counting and said, “You still get 25 no matter how you do it.” His partner then asked if they could cross colors with the highlighter, and I said, “Yes.” Jackson stopped writing and watched his
partner work. Then he picked up a highlighter and returned his attention to his own paper. Again, he said that he thought the answer was 25. I replied that he had 25 small triangles and asked if he had counted the larger triangles. Before Jackson could respond, his partner said that he had made a mistake. Jackson pointed to his partner’s paper and told him to fix his line. His partner asked where he needed to draw his line, and Jackson said that he had it right. Jackson’s partner had found a larger triangle made up of four small triangles.

I asked Jackson’s partner if he could find any more triangles that would be the same size as the larger triangle that he had found. His partner asked for a new paper, so I gave both boys a new piece of paper with the problem on it. The boys worked on the new sheet of paper, referring occasionally to their work on their first paper. Jackson found four triangles and then counted to 25.

Jackson: Three (pause). That’s 4, 5, 6, 7, 8, 9, 10, 11, 12, 13. It’s 25.

Interviewer: What about the bigger triangles, Jackson?

Jackson: I don’t see no bigger triangles.

Interviewer: What about the ones that [partner] highlighted?

Jackson: It’s five, which he highlighted.

As his partner continued to work, Jackson sat back in his chair and watched. Finally, he placed his head down on the table. I told the pair that there were more than 25 triangles, because there were some larger triangles like the ones that Jackson’s partner had found. However, we did not have time for the pair to continue working. At that time, I distributed the Participant Reflection Prompt and asked them to record their responses.
Jackson’s work on the Triangles Problem (MTW) is shown in Figure 5. Jackson’s response is shown in Figure 6.
Figure 5. Jackson’s work on the Triangles Problem (MTW) during Session 4A.
Participant Reflection Prompt

Today I chose the ______ triangles ______ problem to work because ... it looked challenging and easy but it's not.

Please include in your response:
• What you liked about the problem
• What you didn’t like about the problem
• If you could choose again, would you choose the same problem? Why or why not?

I like the problem because it was challenging and hard to find the answer and count triangles.

It was hard, difficult to find the answer.

I would choose another problem because this one is really difficult to find.

Figure 6. Jackson’s Participant Reflection Prompt for Session 4A.
Jackson stated that he chose the Triangle Problem (MTW) because it looked easy. He said, “It looked easy and you get to count squares. Equilateral squares. I meant triangles.” I questioned him further by asking what made it look easy. Additionally, I asked what he liked and/or did not like about the problem.

Interviewer: Ok. What made it look easy, Jackson?

Jackson: Because cause this uh triangles it just like counting them but it was hard, difficult.

Interviewer: Why was it hard?

Jackson: Difficult because you got you had to count the big squares, I meant big triangles.

Interviewer: What, if anything, did you like about the task you just completed?

Partner: Uh that I like it a little because

Jackson: Cause you get to count.

Partner: Yeah, uh huh. And we already went over it in class.

Interviewer: What, if anything, did you not like about the task?

Jackson: Finding the answer.

Partner: It was

Jackson: Finding the answer.

Partner: Yeah. Finding the answer.

Interviewer: Why?

Jackson: Because it was like difficult because I had a lot of answers.

Partner: And

Jackson: To see which one it is
Partner: Uh, it wasn’t, it was kind of easy and hard

Jackson: I think

Partner: When I

Jackson: It’s 45

Partner: Yes.

Interviewer: You think it was what?


When I asked what made Jackson want to work the task, he said that they had done something similar in class, and he added, “We wanted to work it because we thought it was like easy from like doing instead of that word problem so we just chose that so it can be more challenging to us.” After he mentioned word problems, I asked him to describe a word problem. He said, “It’s lots of words details, words, details, information and it has numbers, information about what to do and so you gotta do a lot of mathematics.” Later, he stated, “. . . a word problem gotta have like, a word problem is more than twenty uh words but uh uh not a word, picture though is like not more than twenty words.” This was the first time that Jackson had included a specific requirement for the number of words that a word problem or mathematical task without words must contain.

Returning to the Triangles Problem (MTW), I asked Jackson if it was a math problem. I also asked which problem he would have chosen if he could choose again.

Interviewer: Ok. Is [the Triangles Problem (MTW)] a math problem?

Partner: Yes.

Jackson: It’s math because you gotta count or add or subtract or divide.
Interviewer: After solving the problem, did you wish you had chosen the [Egg Problem (WP)]?

Jackson: Yes.

Interviewer: After working it?

Jackson: Yes.

Partner: I would’ve had chosen

Jackson: Cause [the Triangles Problem (MTW)] was like difficult.

Further, Jackson explained his perception of the Triangles Problem (MTW) before he started working it and then after he worked it.

Jackson: It looked easy, but it’s kind of

Partner: It’s difficult.

Jackson: It’s really hard.

Partner: Mmhmm.

I asked Jackson what he would tell his classmates about the Triangles Problem (MTW) and he responded, “This is difficult, hard, challenging, so um study it, and don’t don’t just like count the little the um little squares, count the big squares too. I mean triangle!” Jackson’s partner agreed that their classmates would think that the Triangles Problem was difficult to work.

**Destiny**

Destiny completed four sessions throughout the study. In two sessions, she was paired with less-reluctant problem solvers. In her other two sessions, she was paired with Jackson, the other reluctant problem solver. The following sections describe her
experiences in sessions with less-reluctant problem solvers. Destiny’s own words from the sessions appear in the text.

**Sessions with Less-Reluctant Problem Solvers**

Destiny worked with two of her classmates who were less-reluctant problem solvers during Session 2C and Session 3B. Session details were shown in Chapter III in Table 2. In the paragraphs that follow, I describe each session, including task selection, a discussion of the problem-solving session, and a discussion of the interview.

**Session 2C.** Destiny and her partner, a female, were given the choice between the Meals Problem (WP) and the Pattern Block Quilts Task (MTW). Destiny was quick to initiate the choosing of the task. She chose the Pattern-Block Quilts Task (MTW), and her partner agreed to work it.

**Problem-solving session.** Destiny and her partner began by placing the pattern blocks on the two quilts on the paper. As they placed shapes, I asked how they would find the area once they had covered the quilts with blocks. Destiny said they would count all of the shapes, but her partner shook her head as if to say that they would not. Destiny nodded her head up and down and said that they needed to count all of the shapes. Then, she questioned herself and said that was wrong because it was perimeter. Again, she changed her mind and returned to her original response of counting the shapes. She said, “I say count the shapes, cause that’s what you do when you, um you do it in class like count the squares.” I asked her if in class when she counted the squares on the inside of a shape if the squares were all the same shape. She did not understand what I meant, so I asked her instead if the green triangle took up the same amount of area as the red trapezoid, as demonstrated in the following exchange.
Interviewer: Let me ask you this, would a triangle take up the same amount of area as a trapezoid?

Destiny: No.

Partner: Hmm?

Interviewer: Okay, the green triangle, would that take up the same amount of area as the red trapezoid?

Destiny: (Shook her head “no”)

Partner: Yes.

Interviewer: Does it?

Partner: No.

Interviewer: Put it, put the triangle on top of the trapezoid. Does it take up the same amount of space as that trapezoid?

Destiny: (Shook her head “no”)

Partner: No. Three.

Interviewer: Three of them?

Destiny: (Nodded her head “yes”)

Partner: Yes. I think we have to choose one shape and do only those.

Destiny: Ok.

Destiny’s partner placed three triangles on top of the trapezoid, and they said that three triangles would be equal to one trapezoid. The pair continued to fill both quilts on the paper with all shapes of pattern blocks. Destiny’s partner stopped her and told her to use only the triangles to fill the shapes. They each worked on separate quilts and filled them with triangles. Suddenly, Destiny’s partner stopped using triangles and started
filling her shape completely with trapezoids. Though her shape was filled with trapezoids, she said that the trapezoids did not work. Destiny continued to fill her shape with triangles until it was completely filled.

Once both shapes were filled I asked the pair how they could compare the area for the two quilts, since one was filled with trapezoids and the other was filled with triangles. Destiny’s partner said they would do so by counting the blocks. Destiny replied with another suggestion, and the pair discussed how they might compare the shapes.

Destiny: So, so we just, I count these, she count those and we just multiply together to get the area?

Interviewer: What do you think?

Partner: Don’t they both have to be the same shapes?

Interviewer: What do you think?

Partner: I guess so.

Interviewer: Wait. Is there a way to compare with what you have?

Partner: Yes.

Interviewer: How?

Partner: With triangles.

Interviewer: What do you mean?

Partner: Like, three triangles equals one trapezoid.

Interviewer: So can you count, can you count the trapezoids that you have, and then decide how many triangles that would equal?

Partner: Thirty-six, I guess.

Destiny: Mmhmm.
Interviewer: How’d you get that?

Partner: Twelve times 3.

Interviewer: Why’d you do 12 times 3?

Partner: Cause there are 12 trapezoids and three triangles equals one trapezoid.

I asked the girls about the area of the other quilt. They both appeared to stare at the blocks as if to count, and I suggested that they move each block as they counted to keep track of them. Destiny’s partner counted the triangles aloud as Destiny counted quietly. Her partner counted by threes as if she was counting trapezoids, and she counted 36 triangles. Destiny counted 36 triangles also. They agreed that the area for both quilts was equal to 36 triangles. I asked what the original question for the task was, and they said it asked which shape had the greatest area. Destiny’s partner said both quilts, and it was because they both had 36 triangles. Immediately after solving the problem, I asked Destiny and her partner to complete the Participant Reflection Prompt. Destiny’s response is shown in Figure 7.
Figure 7. Destiny’s Participant Reflection Prompt for Session 2C.
**Interview.** Destiny initiated the choosing of the task in the current session. She explained her choice in the following excerpt.

**Interviewer:** What made you select this task instead of the other one?

**Destiny:** The other problem seemed difficult to me.

**Interviewer:** The other one seemed difficult? What made it seem difficult?

**Destiny:** It’s um, it’s like it had a whole bunch of numbers that you had to do and then sometimes I get confused when we do like half cups and all that.

Destiny chose the Pattern Block Quilts Task (MTW) because the Meals Problem (WP) appeared to be more challenging to her, as indicated in the dialogue. She later described her favorite part about working the Pattern Block Quilts Task (MTW). “That, um, all you had to do was just use the shape to figure out the area so it wasn’t hard at all.” Additionally, she expressed, “I wanted to work it because it seemed easy and not hard or frustrating.” When asked what made the task easy, she indicated:

Because all you have to do is just put the shapes, well you had to pick a simple shape to make it work, and in [the Meals Problem (WP)] you had to do a lot of stuff to get the answer.

Destiny said that the Meals Problem (WP) looked difficult to her, however, when asked about word problems in general, she said that they give her clues or hints to help her solve. Further, they contain more words than mathematical tasks without words which do not have as many words or sentences. She described the Pattern Block Quilt Task (MTW) by saying that “. . . it didn’t have any numbers. It just had one sentence instead of a whole bunch of sentences like telling you what to do and all that.”
After working the task and discussing it during the interview, Destiny expressed conflicting thoughts about the problems. The following dialogue demonstrates Destiny’s conflicting statements during the interview.

Interviewer: If you were given another opportunity, would you like to work another task like this one? Why or why not?

Partner: I say yes and no, because yes because it’s not that difficult but it has shapes and then I would not want to work this problem again because like I would like just to work [the Meals Problem (WP)] that has a lot of numbers to figure it out.

Interviewer: Ok, the word problem where it gives you the numbers?

Partner: Yes.

Interviewer: Ok.

Destiny: Um, I would, I wouldn’t work [the Pattern Block Quilts Task (MTW)] again because I probably would know the answer again. I would work [the Meals Problem (WP)] because it gives clues and it’s not, it’s not really difficult, but it’s ok but um. I would work [the Meals Problem (WP)] because it would be kind of easy or hard.

Interviewer: So you said you wouldn’t work that one again because you already know the answer, but what about something similar to it?

Destiny: Mmm yeah.

In particular, Destiny’s conflicting thoughts were demonstrated when she spoke about the Meals Problem (WP). At the beginning of the interview, she declared that she
did not want to work it because it was too difficult. However, by the end of the interview, she said, “it would be kind of easy or hard.” In response to the Pattern Block Quilts Task (MTW), she indicated that she would encourage her classmates to work it because it was easy; they would just need to find the area of both shapes and compare them.

**Session 3B.** Destiny and her partner, a male, were asked to choose to work the Tables Problem (WP) or the Four 4s Problem (MTW). Similar to Session 2C, Destiny initiated the choosing of the task during this session. After they each examined both problems, Destiny pointed to the Tables Problem (WP) and her partner agreed. Her partner indicated that he knew she was going to choose the Tables Problem (WP), though he did not reveal why he thought that.

**Problem-solving session.** Destiny re-read the problem, and her partner simply suggested that they multiply 5 times 6. They both said that it would equal 30. Knowing that their solution was incorrect, I asked the pair to draw a picture of the problem to help them solve. Destiny drew a picture of tables and people as her partner watched. She stopped after every table to count how many people could be seated around the tables.

I questioned the pair concerning placement of the tables to prompt their thinking. Destiny’s partner took the paper from her, wrote on it and then said “Bam!” Interested, I asked how the tables would be placed in the room. Destiny looked at the problem again and then mouthed some thoughts quietly under her breath. Her partner wrote on the paper again, and Destiny reminded him that the tables had to seat one person on each side.

Destiny: It said to seat one person on each side.

Interviewer: One person on each side? And they need to be arranged how?

Destiny: In a long row.
Interviewer: In a long row? So what does that mean if they’re arranged in a long row?

Destiny: Like, like, a long, like end to end.

Interviewer: Mmhmm. So why don’t you draw a picture of maybe two of the tables. What would they look like if they were arranged end to end?

Destiny watched as her partner drew a picture of two tables. Neither participant said anything. Again, I recommended that they determine how many people could sit around those two tables. I provided Destiny with a separate piece of paper so that she could draw a picture also. The pair established that six people could sit at two tables, and I asked them how many tables they would need for 30 people. Destiny attempted to talk through the problem without drawing a picture. She stated that the tables were touching, counted people, but then lost track and began drawing a picture. As she was drawing, her tables were not touching, so I reminded her that she said earlier that they had to touch. She stopped, crossed out her drawing and then started drawing again.

To prompt discussion again, I asked the pair how many people could sit on each side of the table.

Interviewer: How many people can sit on each side of the table?

Destiny: Three.

Partner: Four.

Destiny: Three. Cause three on this side, three on that side.

Partner: No. How bout somebody right here? Yeah three, three can.

Destiny: Cause three
Partner: One table gonna have three, two tables gonna have three and one of ‘em gonna have two.

Destiny: This table gonna have six cause three on this side three on this side, then this table gonna have six cause three on this side, three on this side. And then when you put ‘em all together it equals 30.

Interviewer: How many tables, you said that table has six people. How many small tables is that made of? Is that just one table?

Destiny: That’s mainly one table, for all of them to sit.

Interviewer: Ok. Look at the problem. How many does it say can sit on each side of the table?

Destiny: So I’m supposed to make it like six little blocks to make six tables?

Partner: Yes.

Interviewer: What do you think?

Destiny: I’m just gonna make some.

Destiny began drawing another picture including six small tables. Relating this problem to his real world, Destiny’s partner asked what happened if this restaurant had booth tables, and further, what happened if this restaurant did not allow 30 people at a table. I told him that it was allowed at this restaurant. I directed their thinking back to the problem by asking how many people they needed to sit total and how many they had seated already. Destiny’s partner stated that they needed to seat 30 people.

I invited Destiny to explain her drawing and she added by sixes. When asked about the ends of the table, she said that it was too late. She had already seated everyone. Her partner suggested that they take one table off, and Destiny found that she could count
by threes instead of sixes when using smaller tables. Her partner said that there would be six tables and five chairs at each. Next, Destiny added by fives and stopped when she got to 30. She started counting again, adding two, two and one, and then adding two and three. Finally, she stopped and exclaimed, “This is so difficult!”

Destiny turned her paper to the back and began counting again. She wrote on her paper as she counted. Her partner worked on his own paper while she worked. He asked her how many people could be at a table. Before she could answer, he said that he was going to draw stick people and told her to put five people at a table and use six tables. She said, “Agreed,” and returned to her drawing. Then she declared that she had a solution. She counted one person on the first end then four on the side, then four more on each side, until she got to the other end of the row of tables. Then she added one more person on that end.

Destiny smiled when she finished and said, “Ooooh, that was hard!” Then she showed her partner and he said, “That’s what I tried to tell you.” She said that he had not, and that he had actually shown her something else. She said that her solution was seven tables, an incorrect solution. Once the discussion concluded, I gave them the Participant Reflection Prompts and asked them to write their responses to the prompts. Page one of Destiny’s work on the Tables Problem (WP) is shown in Figure 8. Page two of her work is shown in Figure 9. Destiny’s response is shown in Figure 10.
Figure 8. Page 1 of Destiny’s work on the Tables Problem (WP) for Session 3B.
Figure 9. Page 2 of Destiny’s work on the Tables Problem (WP) for Session 3B.
Participant Reflection Prompt

Today I chose the ___ problem to work because...

Please include in your response:
- What you liked about the problem
- What you didn’t like about the problem
- If you could choose again, would you choose the same problem? Why or why not?

The thing I liked about this problem was that it was challenging almost easy. The thing I didn’t like was that nothing because it was okay.

If I could choose again I would probably pick the same or maybe the other one.

Figure 10. Destiny’s Participant Reflection Prompt for Session 3B.
**Interview.** Destiny explained her reasoning for selecting the Tables Problem (WP) in a statement.

“Um, I wanted this task because it was kinda hard but it was kinda easy at the same time, cause I did have the answer at first but then um it wasn’t right cause you had to fix it so.”

Similar to her responses at the end of the interview in Session 2C, Destiny began the interview in Session 3B by responding that she liked the problem because it was hard and easy. When asked why it was easy, she stated, “. . . cause you could probably already know the answer cause it was 5 times 6, but then you had to change it up because you had to have some on the end for everybody to sit.” In addition, she expressed, “Um (pause) the thing I liked about that it was challenging and it was kinda hard and easy it was easy to do.” Later, Destiny said, “I wanted to work the [Tables Problem (WP)] because [the Four 4s Problem (MTW)] seemed really, kinda hard but I don’t, I don’t know but it probably was. But [the Tables Problem (WP)] was challenging and so.” Destiny speaks about problems being “hard” or “easy” without providing information to support her reasoning.

After Destiny mentioned the Four 4s Problem (MTW), her partner said that it did not make sense. Destiny added that it did not give enough information, and she wanted to work the Tables Problem (WP) because it had more information and details. Destiny provided a description of a word problem:

It has information that tells you what you need to do and how like [the Tables Problem (WP)] said what’s the least number of card tables that you need so you had to figure that out instead of doing the other stuff.
Destiny specified that a word problem provides a question that indicates what you need to do to solve. When asked if she wanted to work another word problem like the Tables Problem (WP), Destiny said “Probably.” However, she said she would not want to work the Four 4s Problem (MTW) because it did not make sense to her. Realizing that Destiny and her partner did not understand what the Four 4s Problem (MTW) was asking them to do, I decided to discuss it with them. I read the example and asked questions of the pair to facilitate their thinking about the problem. The dialogue below occurred just after the pair began to understand the example problem in the Four 4s Problem (MTW).

Partner: Seve-, Oh! That’s what you mean?
Destiny: Oh! Well then I would pick this problem.
Interviewer: You would pick it, why?
Destiny: Because [the Four 4s Problem (MTW)] so easy, but I just didn’t understand it for a second.
Interviewer: Ok, so maybe you just didn’t understand the directions?
Destiny: Mmhmm.
Interviewer: But now that you understand, what do you think about the problem?
Destiny: It’s um easy.
Partner: It say, it say, it say to use fours to get all numbers for one through 10.
Interviewer: So it seems? How does it seem now that you understand the problem, [partner]?
Partner: Yes ma’am.
Interviewer: No, how does it seem now that you understand the problem?

Destiny: Easy.

Partner: Um.

Interviewer: Does it seem like something you would want to work or not want to work?

Destiny: I would wanna work, I would have worked it.

Interviewer: Why?

Partner: Because I I didn’t know what they was talking about four got to equal 17 but now since you tell us uh so um what you have to do that’s why.

Destiny: Ok, I understand it. It’s like 4 times 4 equals 16 plus one whole equals 17.

Interviewer: And so you would just use that to find the other numbers, like use four 4s to find all the other numbers. Ok, so now if you were given another opportunity, would you like to work a task like that one?

Destiny: Yes.

Partner: Mmhmm.

Interviewer: Why?

Destiny: Because that one seems easy, but I just didn’t understand it cause it didn’t seem right.

Interviewer: So after you solved the word problem, do you wish you had chosen the other problem?

Partner: Mmhmm.
Destiny: Yes.

Interviewer: Why?

Partner: Because since you had told us what to do, I thought uh we had to do something that gave equals to 17. Now I said I don’t nothing in a four equals to 17.

Destiny: But you had to make it equal 17, but you just it was kinda difficult how it was set up.

Once the pair understood what the Four 4s Problem (MTW) was asking them to do, they saw it in a new light. Their newly altered, positive perception of the problem allowed them to say that they would encourage their classmates to work the problems.

**Jackson and Destiny’s Sessions Together**

Jackson and Destiny worked together in two sessions. One of their sessions, Session 1B, occurred at the beginning of the study and was the first session for each of them. Jackson and Destiny also worked together in Session 4C at the end of the study. Session 4C was Destiny’s last session, however, Jackson participated in one session after Session 4C with less-reluctant problem solver. This section discusses both sessions in which Jackson and Destiny work together, Session 1B and Session 4C. In the paragraphs that follow, I describe each session, including task selection, a discussion of the problem-solving session, and a discussion of the interview.

**Session 1B.** Destiny and Jackson were given the choice of the Sports Problem (WP) or the Bead Problem (MTW). Destiny read the Sports Problem (WP) first. Next, she looked at the Bead Problem (MTW) and pushed it away, but then she pulled it back towards her to give it a second look. Jackson looked at both problems as well. When he
looked at the Bead Problem (MTW), he asked if he needed to count how many were inside the box. I said that he had to find the number of beads on the chain, including the beads inside the box and the beads outside the box. Destiny sat quietly while he asked questions and then while he read the Sports Problem (WP). Jackson pointed to the Sports Problem (WP) and said, “This one.” Destiny immediately agreed and then stated of the Bead Problem (MTW), “This one seem difficult.”

**Problem-solving session.** Destiny and Jackson read the Sports Problem (WP) again and started to work it on separate pieces of paper. I told the pair to make sure to work together. Instead, Jackson wrote on his paper, stopped to read the problem, and then wrote on his paper again. Destiny also wrote on her paper. After I encouraged the pair to work together again, they started to do so by sharing strategies for solving. Destiny listened while Jackson shared his strategy for solving, and then she looked back at her own work and restated the information in the problem. Their discussion is provided.

Jackson: I’m thinking of doing like group is 30, 19 is football, soccer is 17, and 10 on both teams equals 10. And it says 30, 30 it’s 30 boys (pause) in a group.

Destiny: Ok. This is the whole group, the whole. Thirty is the whole group, and there are 19 on the football team, 17 on the soccer team and 10 on both teams. Ok. Five boys play basketball and nine play football and (pause). Ten.

Interviewer: Are you working together?

Destiny: Ok.
After being encouraged to work together, Destiny asked Jackson what he found for the answer and how he found it. He said that he had gotten an answer of 17, an incorrect answer. Destiny listened as he told her his process.

Destiny: How many did you get?
Jackson: Seventeen.
Destiny: What’d you do?
Jackson: Three boys play basketball, five boys play soccer and basketball, nine boys play basketball. All the boys are on at least one team, how many boys play basketball only?
Destiny: So you added?
Jackson: Three plus 5 plus 9. Three of the boys play basketball, five of the boys play basketball and nine play basketball.
Destiny: Ok. Ok.
Jackson: I’m done.

After explaining, he placed his head down on the table. Destiny then returned her attention to her own paper, wrote on it and arrived at same solution that Jackson had reached. As Destiny was working, Jackson started explaining his work and then Destiny added her solution.

Jackson: I did like. It said like three of the boys play basketball, football and soccer, so I put three under basketball. Five boys play soccer and basketball so I put five right there under basketball. And nine play football and basketball, then I put a nine right there. All the boys
are on at least one team. How many boys play basketball only?

Then I added them up and got 17.

Interviewer: Are those the boys that play basketball only?

Jackson: Mmhmm (yes).

Interviewer: Or do they play something else with basketball?

Jackson: No, it’s all it says.

Destiny: Um, 10 on both teams. Like it had 30 in a group, 19 football, 17 soccer, and 10 on both teams, then five play basketball and soccer.

And all the boys are on one team. It will either (pause). Seventeen on the soccer team. That’s what it add up to: 3 plus 5 is 8 plus 9 is 17.

I asked Destiny where she had found the numbers that she used to solve. She explained how she found the numbers in the problem and further, why she used them. I asked Destiny what the numbers represented, and she stated what each number represented.

Interviewer: So those numbers that you wrote down, what do they represent?

The numbers that you added, what is the first number?

Destiny: Three

Interviewer: And what does it represent?

Destiny: Three of the boys play basketball, football and soccer.

Interviewer: So what does the three represent? Basketball, football, and soccer?

Destiny: Mmhmm.
I followed that by asking what the question in the problem asked, attempting to draw attention to an error in the pair’s strategy for solving. Destiny responded to my questions.

Interviewer: And what is the question asking?
Destiny: Umm, how many boys play, wait (pause) In a group of 30 boys, 19 are on the football team, 17 are on the soccer team, and ten are on both teams. Three of the boys play basketball, football, and soccer. Five boys play soccer and basketball, and nine play football and basketball. All the boys are at least on one team. How many boys play basketball only?

Interviewer: So how many boys play basketball only? So can they play another sport?
Destiny: Mmhmm.

Interviewer: If they play basketball ONLY, can they play another sport?
Destiny: Well, no. Like, just basketball.

Neither Destiny or Jackson realized that they had added the boys who played sports other than basketball as well the boys who played basketball only. Jackson said that he found a different solution, and Destiny asked him how he had solved the problem.

He placed his head on the table and discussed the problem.

Jackson: If it was wrong, it would be 63. It says one it’s on one team. On at least one team.

Destiny: What? What’d you do? Sixty-three on one team?

Jackson: If it was wrong, I added 19, 17, ten, three, five, nine.
Destiny: And you got 63?

Jackson: Nine plus 7. Nine plus 9 is 18 plus 7

(pause)

Destiny: So you added all these numbers up together?

Jackson: Fifteen.

Destiny: Um, if it was

Jackson: It would be 15. Nineteen on the football team. 17, 17 on the soccer team and ten were (pause). Ok, add 20.

Both participants looked down at their papers and continued to work. After a long pause of silence, Destiny proclaimed that she had an answer.


Jackson: That’s what I got.

Interview: Y’all both got 17? Ok, alright, and how’d you get 17?

Jackson: We added three, five, nine.

Destiny: That’s what I did. Cause I went back and checked and it said um, three of the boys play basketball, then five boys play basketball and then nine plays basketball.

They worked the problem using the same method that they had previously used and arrived at the same solution, 17. Every solution that the pair discussed, including their final solution, was incorrect. They were both asked to complete the Participant Reflection Prompt. Jackson’s work for the Sports Problem (WP) is shown in Figure 11. Destiny’s work for the Sports Problem (WP) is shown in Figure 12. Jackson’s response to
the Participant Reflection Prompt is shown in Figure 13. Destiny’s response is to the prompt is shown in Figure 14.
Figure 11. Jackson’s work for the Sports Problem (WP) during Session 1B.
Figure 12. Destiny’s work for the Sports Problem (WP) during Session 1B.
Figure 13. Jackson’s Participant Reflection Prompt for Session 1B.
Figure 14. Destiny’s Participant Reflection Prompt for Session 1B.

Participant Reflection Prompt

Today I chose the ___________ problem to work because...

Please include in your response:
- What you liked about the problem
- What you didn’t like about the problem
- If you could choose again, would you choose the same problem? Why or why not?

because it seems like an easy problem to work on. I think this problem was well organized for me to answer correctly. The thing I didn’t like about the problem is that at first you only needed to answer, basketball. If I could choose a different problem, it will be the same because its easy to work and write out.
Interview. Destiny and Jackson were interviewed immediately following completion of their written reflections. When asked why he wanted to work the Sports Problem (WP) instead of the Bead Problem (MTW), Jackson said, “Because I ha- I love sports.” I asked, “You love sports?” He replied, “Sports problems.” Destiny described her reasoning for choosing the Sports Problem (WP) as well.

Destiny: Because I chose this problem was that it seemed like a well-organized um problem to be worked easily and um the other problem it would be kinda hard to figure out how many was in the box.

Interviewer: You think it might be kinda hard?

Destiny: Mmhmm.

Earlier, Jackson expressed that he had chosen the problem because he loved sports. I asked a question concerning the problem context.

Interviewer: Ok. Um alright let me ask you this: do you think and just, I don’t know, what if I had given you a word problem about sports but then say that other problem had had a picture and was about sports? What do you think might have happened?

Jackson: Umm, I would think that happened is that, like when you, I would more likely pick the picture one.

Interviewer: Why do you think?

Destiny: Yeah, I think the same what he said. It would be more details about it so it could be not hard to explain just a little or probably.
Interviewer: Ok so if I had given you a word problem and a problem with pictures, both about sports, you both think that you would pick which one?

Destiny: Picture.

Returning to the Sports Problem (WP), I asked what they liked about the problem. They provided the statements below to answer my question.

Jackson: Umm, figuring out the problem.

Interviewer: What do you mean?

Jackson: Like, if you like, you can read the paragraph and see um the question it gives you so to you can like put ‘em in a group like this and, and if you if it say how many boys play basketball only you can just see how many basketball was in it and how many numbers was um on the basketball like if it’s three you put three, put a five put a five, and put a nine put a nine.

Destiny: Ok the thing I liked about this problem was that it gave you a lot of information like what how many boys was on the football team, soccer team, basketball, and other sports.

Jackson described his procedure for solving the problem instead of stating what he liked about the problem. Later, he said that there was not anything that he disliked about the problem. I asked him if he would have chosen the word problem if it had been about something that seemed girly, like lipstick. He replied that he would “still work it.” Further, I asked him to describe word problems in general. He said, “It’s like, it don’t
have no pictures or nothing but it shows like this [Sports Problem (WP)] and it give you numbers and stuff to see um to like demonstrate it.”

Though Destiny previously stated that a picture problem might provide more details, she later said that she liked the Sports Problem (WP) because it gave her a lot of information and it was not frustrating to her. Destiny said that both types of problems give information.

Umm, a word problem it gives you information but if you look at a picture, it gives you more details and information about the picture so you can work it right, but maybe like a word problem would give you more information.

I asked the pair how the Bead Problem (MTW) was like or not like a word problem, and Destiny and Jackson both agreed that it was not a word problem.

Interviewer: Ok. What about the Bead Problem (MTW)? How is it like or not like a word problem?

Jackson: It’s not because it got a picture in it um, it shows details within it but it don’t have like many words as this one. Cause this only says how many beads are on the chain shown? So you gone have to figure out what’s in the box and what’s in the outside.

Interviewer: (To Destiny) What do you think?

Destiny: Umm

Interviewer: (To Jackson) You want to hand it so she can see it closer?

Destiny: Yeah, like he said it don’t have as many words it’s just telling you how many beads are on the chain shown and it would be hard to
Both participants agreed that the Bead Problem (MTW) did not have as many words. However, they did not indicate how many words a word problem should have. Destiny said that if she were given the choice between the two problems again, she would choose to work the word problem.

I think that I would work this problem again because it gives you a lot of information that you need to know, and then when it has all the numbers and you get confused, all you have to read is the question they’re asking you mainly like how many boys play basketball only.

Further, Destiny said that she did not wish to have worked the Bead Problem (MTW) because it did not provide enough information. A later response from Destiny indicated that the Sports Problem (WP) was the better choice because it gave her the needed information and a clear question.

Similar to Destiny’s response, Jackson expressed interest in working another problem comparable to the Sports Problem (WP). He also stated that he would also work another problem similar to the Bead Problem (MTW). He reasoned, “We can like work it so we can know like we can know so we can figure it out.” However, he said that he did not wish he had chosen to work the Bead Problem (MTW) instead of the Sports Problem (WP).

Finally, I asked the pair what they would tell their classmates about the Sports Problem (WP).
Interviewer: Alright, what would you tell other students about your experiences in working this task?

Destiny: Um I would tell them that it’s a really nice problem that gives you all the information you need, has a lot of details, and um, it describes like how many boys the play uh whatever, and then how many then the question would ask you how many boys play basketball only so you will know that you’re only looking for basketball.

Interviewer: (To Jackson) What would you tell other students?

Jackson: That it’s a easy problem so

Interviewer: Why is it easy?

Jackson: Because you only look for basketball.

Interviewer: Ok.

Jackson: And you only look for basketball so I’ll tell ‘em it’s three on basketball, so it’s basketball three, five play soccer, and basketball that’s basketball and nine play football and basketball that’s basketball so nine, five, three.

Destiny and Jackson both thought the Sports Problem (WP) was easy to work, however, their solution was incorrect.

Session 4C. Destiny and Jackson chose between the Lemonade Problem (WP) and the What’s the Area Problem (MTW) to work during this session. Both participants read one problem, switched papers, and read the second problem. Jackson pointed to the Lemonade Problem (WP) and said, “This one.” Destiny told him to “Hold up,” while she
continued to examine the problems. I reminded them that they must select only one problem to work together, and Jackson said that he thought the Lemonade Problem (WP) would be easier because it asked which container of lemonade would be the better buy. After Jackson commented, Destiny pointed to the Lemonade Problem (WP) and said, “This one.” Jackson, again, said that it looked easy.

**Problem-solving session.** With a problem selected, the pair read the Lemonade Problem (WP) again. Jackson asked if it said that the ounces were not for sale. I told him that the 32-ounce container sells for $4.42, and if you buy one, you get one for free. The other container is 84 ounces and is not on sale. I encouraged him to work with Destiny. Jackson quickly said that he had an answer for the problem, and they discussed it.

Jackson: I got the answer though.

Interviewer: You got the what?

Jackson: Answer.

Destiny: You could buy it, one the 32 ounces because you get another one for free. And this one

Jackson: Which um, four ounces, see this six so it’s not for sale, it’s gonna be more than $6.49. But it’s buy one get one free so $4.42 then if it’s um, it’s two of ‘em so $4.42 plus $4.42 equals 8 (pause)

Interviewer: Jackson, you can write it down, you can write on that sheet.

Jackson: Like this. $6.49 is not for sale. So that means, so if this one like, if this one like $4.42 and it’s get one free sale, so if you were to add that up but it do not count, if this one don’t count. I’m just adding that up so (pause) it gonna be this $8.84 so that’s my answer.
Destiny: Oh, if you add, if this one wasn’t on buy one get one free, she just got two, it would equal up to that. I see what you’re saying.

Jackson: And this one, this don’t cause it says it’s not on sale. I’m done.

Jackson discussed the container that was on sale, but he added the price twice instead of representing one free container. He then rationalized to Destiny by saying that they would not get anything for free when they bought the 84-ounce container.

When Jackson proclaimed that he was finished with the problem, I asked him what he did to solve the problem.

Interviewer: So what’s, what did you do?

Jackson: The 32. See, like this. It’s like, the $6.49 is not for sale so the 84 ounces doesn’t say like get something for free it’s just like $6.49. But the other one, the 32 ounces is just $4.42, but it say get one for free so that’s how, this, like that four, you add that up, you get eight, eight, four. Eight hundred. $8.84. That’s how you, how you gonna get. Since um that’s it, so the 32 ounces gonna be the better buy.

Interviewer: Okay, so how many ounces are you going to get for the $8.84?

Jackson: How many ounces?

Destiny: 84 ounces.

Jackson: 84 ounces.

(pause)

Jackson: See it would be the ounces 84, but it’s not on sale. But
Interviewer: You were saying $8.84, and my question is, how many ounces of lemonade do you get for $8.84?

Destiny: You can, he said like you add $4.42 lasts like together again. Then you get $8.84. Then, but it’s a buy one get one free so he bought one for $4.42 he get the other one for free.

Jackson: And so if you would’ve if that would’ve cost money, it would’ve been more and plus that’s how much money it is

Destiny: because it says buy one get one free, you had to add both of those up, it would just be $8.84.

Jackson: It gone be twice the size of the $6.49.

Interviewer: What size is it though?

Jackson: Thirty-two.

Destiny: Thirty-two.

Interviewer: If you buy one, get one free. How much have you gotten?

Jackson: Oh, that’s the wrong answer.

Interviewer: Is it?

Jackson: Mmhmm.

Jackson said the 32-ounce container was the better buy. I attempted to ask questions that would prompt the pair to look at the unit rate of the lemonade. Still, they focused on the numbers 84 and 32, the number of ounces in the two containers, instead of trying to find the unit rate.

Jackson focused on the fact that the larger container was on sale. I asked the pair how many ounces they got in each container, and Jackson said that it would be the 84-
ounce container, but it was not on sale. I asked him how many ounces he would get for $8.84. Destiny explained to him that he added the price of that container twice, and he did not have to since they would get the second for free after they bought the first one.

Destiny and Jackson took turns explaining their thoughts about the problem. I asked them how many ounces they would have if they bought one and got one for free. Jackson determined that his answer was incorrect. The pair continued to discuss the prices and found that they would have 64 ounces of lemonade for $4.42, and the other container provided 84 ounces of lemonade. I asked how they could compare the containers if they were not the same size. Jackson continued to refer to the container that was on sale as the better buy. With further prompting, he said that three containers containing 32 ounces each would equal 96 ounces.

Destiny and Jackson both sat back in their chairs and looked at the problem again. Jackson said that the better buy could not be the container with fewer ounces. He said that the buyer would get one free, which would make 64 ounces total. He said that the other container was 84 ounces in just one container. Destiny sat quietly and watched while Jackson worked the problem with his head resting on the table.

I asked the pair another question to prompt their thinking. The discussion led to Destiny determining the correct answer.

Interviewer: If you wanted to buy 84 ounces, which would be cheaper? To buy the one that comes in 84 ounces? Or to buy several of the ones that are 32 ounces?

Jackson: $6.49.

Interviewer: You think so?
Jackson: You can get several of these?

Interviewer: That’s what I’m saying. If you want to buy 84 ounces, would it be cheaper to get the one that comes in 84 ounces for $6.49, or to get several of the ones that

Jackson: Several

Interviewer: are 32 ounces and buy one get one

Jackson: Several because

Interviewer: Are you guessing? Or are you sure.

Destiny: No. No because

Jackson: Yes, cause you get one for free

Destiny: But you can get the 84 ounces, cause when she say several she mean like

Interviewer: Like more than one. Like here’s one jug of 32 ounces. Here’s another jug of 32 so that’s 64. Here’s another jug so that’s three which you said would be

Destiny: Ninety-six.

Interviewer: Ninety-six. So my question is would it be cheaper to buy more than one of those jugs, or just the one for 84.

Destiny: Just the one for 84.

Interviewer: Are you sure? Or are you guessing:

Destiny: I’m not guessing.

Destiny found the correct answer to be that the 84-ounce container is the better buy. Jackson continued to talk through the problem.
Jackson: I thought it would be like that one because you get like if you buy two of ‘em, you get two for free.

Interviewer: Ok. And how much is that? How much have you spent?

Jackson: Um

Interviewer: If you buy two and get two for free, how much did you spend?

Jackson: So you get $8.84. So you get two more for free. So two, two for free. See if you buy one container you get one for free. Buy two you get two for free. Buy three, you get three for free. Cause you bought one and you got another one and it says one for free, so you get two. Four, eight, 12, 16. That’s 16. (pause) Twenty. So I think that’s the answer. I think 84 ounces.

Jackson continued to discuss the container that was on sale. He counted some numbers, and then said that the larger container, not on sale, was the better buy. I gave Jackson and Destiny the Participant Reflection Prompt, and they wrote their responses to the prompts. The pair worked the problem on the same piece of paper, and that paper is shown in Figure 15. Jackson’s response to the Participant Reflection Prompt is shown in Figure 16. Destiny’s response to the prompt is shown in Figure 17.
Figure 15. Jackson and Destiny’s work on the Lemonade Problem (WP) during Session 4C.
Figure 16. Jackson’s response to the Participant Reflection Prompt for Session 4C.
Today I chose the **lemonade** problem to work because...

Please include in your response:
- What you liked about the problem
- What you didn’t like about the problem
- If you could choose again, would you choose the same problem? Why or why not?

I didn’t like this problem. The thing I liked about this problem was “Nothing” because it was hard and difficult. The thing I didn’t like about this problem was it was boring and hard. If I could choose, I would pick the other one because this one was challenging.

*Figure 17. Destiny’s response to the Participant Reflection Prompt for Session 4C.*
Before interviewing the pair, I asked them to look at the other problem, the What’s the Area Problem, (MTW). Jackson read the problem aloud as Destiny gathered pattern blocks. Destiny said, “You can use like different kinds of shapes to count.” Both participants began placing blocks on top of the shapes on the paper. They discussed different blocks that could be used to make larger shapes or blocks.

Jackson: A rhombus and a triangle and a trapezoid will make a hexagon.

Destiny: All of these rhombuses and two of these can make a hexagon.

Interviewer: Two of what can make a hexagon?

Destiny: Trapezoids.

Interviewer: So if the trapezoid is nine, what would be the area of a hexagon?

Destiny: To use both of these?

Interviewer: Do both of those make a hexagon?

Destiny: Yes.

Interviewer: So what would be the area of the hexagon?

Destiny: Eighteen. Cause the trapezoid is nine. So one of these is nine and the other one is nine. And then you just add them both up.

Though Destiny arrived at a correct answer of 18 for the hexagon, Jackson counted and arrived at 16 for the hexagon. I asked the pair to write down their thoughts and then compare their solutions. Jackson asked if he could trace the shape and I said that he could. I asked Jackson to look at Destiny’s work and explain how she found her solution. This time, he counted the sides and said that the solution was 13. I asked Jackson how Destiny had arrived at her solution. Destiny explained her reasoning while Jackson listened, and then he understood how she got her answer.
Interviewer: Did she add the sides, or Destiny, what did you add?

Destiny: These. You said that one was nine and the other was nine too, and when you put them on the hexagon, 9 plus 9 equals 18.

Interviewer: What do you think about that? (pause) Let me ask you this

Jackson: Oh!

Interviewer: What are you saying “Oh” about?

Jackson: That she meant the trapezoid is nine and this, these two equals nine, so this one’s 18. Eighteen.

Destiny and Jackson determined the area for the hexagon, and they already knew the area for the trapezoid. I asked the pair if they could figure out the area of the rhombus, and Destiny reasoned, “But you gotta figure out what the triangle is because the triangle can make a rhombus with just like two.” She found that three triangles could make a trapezoid, and the trapezoid had an area of nine. “Cause three triangles, I was gonna say three triangles equals one trapezoid.” She was able to state that she would divide the trapezoid’s area of 9 by 3 to see that each triangle had an area of 3.

However, as she was working, Jackson declared that he knew what was happening. He explained that each triangle had an area of six. Destiny was working and did not hear her partner say that the triangle had an area of six. I asked her to look at his solution. She suddenly questioned him about his solution, and the pair discussed their two different solutions.

Jackson: See like the triangle, I think it do like this, the triangle is 6 plus 3 equals 9 plus 4 equals 13 plus 5 equals 18.

Interviewer: Ok. So you think the triangle is six?
Jackson: Mmhmm

Interviewer: How many triangles does it take to equal the trapezoid?

Jackson: Three.

Interviewer: So what would that be?

Jackson: Um, see like this, 6 plus 3 equals 9.

Interviewer: Look, look at what she had. Can you count the triangles if they’re six? Can you count them?

Jackson: One, two, three, four, five, six

Interviewer: No, just the ones that she has on top of the trapezoid.

Jackson: Three

Interviewer: So if they were six, what would the trapezoid equal?

Jackson: Nine.

Interviewer: Destiny, are you listening to what he’s saying?

Destiny: (nods “yes”) 

Interviewer: If the triangles were six, what would the trapezoid equal?

Jackson: Twelve.

Destiny: Hold up, you said the triangle was six

Interviewer: Jackson said the triangle was six. So I’m looking at your trapezoid with three triangles on top of it. If the triangles are six, what would the trapezoid equal?

Jackson: Six.

Destiny: Nine, right?

Jackson: It would equal six.
Interviewer: Count the triangles.

Jackson: Three

Destiny: One, two, three

Interviewer: So if one is six, then the second one would be what?

Destiny: Six. Cause this is six, this one, well all three of them are six.

Interviewer: So what would that equal if you have three of them and they all three equal six, what would the trapezoid equal?

Destiny: Eighteen.

Interviewer: Does it say the trapezoid equals 18?

Destiny: No. I’m confused.

Jackson: Like this, this can be six.

Destiny had the correct answer when she said that the triangle was equal to three.

After the discussion with Jackson, though, she said that she was confused. Destiny had placed three triangles on top of a trapezoid, so I asked Jackson to look at her blocks.

Interviewer: Jackson, look at what Destiny has built. If the trapezoid is nine, what do the triangles equal?

Jackson: Three

Interviewer: How do you know?

Destiny: Three triangles.

Interviewer: Count, so count. If you have

Destiny: One, two, three

Interviewer: So what’s the area of each one?

Destiny: Three.
Interviewer: And then what would be the area of the trapezoid?
Destiny: Nine? I mean like
Jackson: Do 3 plus 3. Three, if you put 3 right there plus 6 is 9.
Interviewer: Ok. So you have three. If one triangle is three, the second triangle is three, that gives you what?
Destiny: Six.
Interviewer: And the third triangle is three, that gives you what?
Destiny: Um, nine.
Interviewer: And is that what it says the trapezoid equals?
Destiny: Yes.
Interviewer: So what does the triangle equal?
Destiny: Six. Right?
Interviewer: What did you just add up?
Destiny: Oh, three! Oh, I got confused!

After determining that the triangle was equal to three, Destiny said that to find the rhombus, they would just need to put two triangles on top of the rhombus. I asked what the area for the rhombus was and Jackson said that it was 13. Destiny placed two triangles on top of the rhombus and said that the area was two. I asked her if each triangle equaled one. She replied that the triangle was three, so the area of the rhombus would be six. Jackson agreed that it would equal six. Upon completion of the What’s the Area Problem (MTW), I interviewed Destiny and her partner. Jackson and Destiny’s work is shown in Figure 18.
Figure 18. Jackson and Destiny’s work on the What’s the Area Problem (MTW) during Session 4C.
Interview. Jackson chose to work the Lemonade Problem (WP) because he thought that it might be easier to work than the What’s the Area Problem (MTW). Jackson said, “Because this one looks like more easier and I chose it because it’s like the better better ounce like let’s, we gonna see the um which the answer and it looked easier.” Later, he added, “It looked easier because it said the which, what size is the better ounce, better buy.” Destiny chose to work the Lemonade Problem (WP) for the same reason. She said that she chose to work it because “… it seemed kind of easy cause it was just telling you all the information, and like it gave you a question that you need to know which one is the better buy. But then it started getting difficult so.” She described the problem as easy at first and then difficult. Her responses in the dialogue provided below explain in more detail how she felt about the problem.

Interviewer: What made it seem easier at first?
Partner: Because
Destiny: Um
Partner: It looked easier because it said the which what size is the better ounce? Better buy? So that’s like
Destiny: It seemed easy because it was just like because it was saying that 32 ounces is $4.42 and then you get another one free. But then, the other one’s 84 or 82 ounces and you get it for $6.49. But then we, I thought it could be like the um 32 ounces.
Interviewer: So what made it then seem difficult? You said it got difficult. What made it get difficult?
Destiny: Because one of ‘em you can just buy one for $4.42 and get another one free, and then other one was a higher cost of the other one.

Jackson agreed that the problem was challenging, “. . . and it was challenging. Like sort of because it’s like 40 it’s like get one get one free but I realized that 84 ounce that is not on sale and it’s the like the bigger buy.” He added that he liked the problem because it was challenging. Immediately after her partner’s comment, Destiny made a similar statement saying, “Umm (pause) the thing I liked about this problem was it felt easy to me but it was challenging so.” She said that there was not anything she disliked about the problem. Jackson said that he disliked it because he could not find the answer, which led Destiny to say that she did not like “how difficult it was.”

I asked the pair why they wanted to work the Lemonade Problem (WP).

Responses included:

Jackson: I wanted to work it because it was easy and it was fun to work cause it’s like it’s challenging to you and you can like uh challenging like hard, difficult so it can be like in the future you might have one of these questions and and you might know it now.

Destiny: Um, really uh, what was the question um

Interviewer: What made you want to

Destiny: Oh

Interviewer: Work it or not want to work it?

Destiny: I wanted to work it because it was looking kind of easy but that wa- [What’s the Area Problem (MTW)] looked easy, but it seemed
hard. But [the Lemonade Problem (WP)] um, it was just saying like giving you the information that you needed.

Interviewer: Ok, the area problem, what did you say it looked easy but it seemed hard? And then the lemonade problem what?

Destiny: It um

Partner: [The What’s the Area Problem (MTW)] was kind of easy.

Destiny: [The Lemonade Problem (WP)] was hard, well not hard but like

Partner: Difficult

Destiny: Difficult was kind-, had a lot of stuff to it.

Both participants described the Lemonade Problem (WP) as being difficult, with very little explanation about why it was difficult. I asked them to describe word problems in general, and Jackson stated:

Um, lots of words, details, what to do, um, gives you information, and it has numbers, and a lot (pause) it’s like [the Lemonade Problem (WP)] but [the What’s the Area problem (MTW)] not. It could be cause it don’t have no pictures, but I think it’s that one.

Destiny responded, “Um, it got numbers, a lot of information, details, um (pause) words.” When asked if the Lemonade Problem (WP) was a word problem, Jackson said, “I think it is because it got lots of words and what to do and it’s like what size is the best buy so you gotta figure out with mathematics and it got number, words, details, and information.” Destiny agreed, “I think it is a word problem because it have words. It give you numbers like and then it give you the question that you needed to answer.”
Alternatively, Destiny described the What’s the Area Problem (MTW) as a “picture problem” and later she added, “A few words but it’s a picture.” She said that the word problem had more information. Jackson replied, “It just tells you what to do like if the area of the trapezoid is nine, what is the other of uh what I meant what’s the area of each other shape other shapes?”

I asked the pair if they would choose to work another problem like the Lemonade Problem (WP) and if they would choose to work another problem like the What’s the Area Problem? (MTW) They explained their choices.

**Interviewer:** If you were given another opportunity, would you like to work another task like the Lemonade Problem (WP)? Why or why not?

**Jackson:** Yes. It got like it’s like in the future you can like work it and it by it may be like another one of these and you can just easily work it out now since you know it.

**Destiny:** Um, I wouldn’t wanna work this problem no more, but I will pick the other problem because um it will seem it looks hard, but then it seemed to get easier cause you was just using triangles to figure out the area. So then you have to figure out the um number like the area for the triangle, the trapezoid, the rhombus, and the hexagon.

**Interviewer:** So it looked hard but

**Destiny:** It seemed easy.

**Interviewer:** So you would not choose to work another one like the lemonade problem?

**Destiny:** (shook her head “no”)
Interviewer: Jackson, you said what?

Jackson: I said I would choose this one again cause in the future um you may know it now so you may get a easier answer to do it.

Interviewer: After you solved the Lemonade Problem (WP), did you wish you had chosen the other one?

Destiny: Yes.

Interviewer: Why?

Destiny: Because, that, that problem it wasn’t hard. It was kinda but it wasn’t hard all the way cause all you have to do is just put the triangle and figure out what the what’s the area of the triangle. And then when you use it, it was just like three so then you just count when you put it on the trapezoid and then you just count it up and then you’ll get like trapezoid goes three triangle so three plus three plus three.

Jackson: I would choose this problem again cause it may be difficult, but you still can get the answer whether it’s hard or not so you can just imagine how it looks like and other stuff add decimal, money, and get the answer what’s the better ounce.

Interviewer: So you wouldn’t choose the uh, would you, would you wanna work one another one like the area problem or no?

Jackson: Mmhmm. Um, this one?

Interviewer: The [What’s the Area Problem (MTW)].

Jackson: I can.
Jackson: Yes, I would choose that one because it looks more easier to do cause 3 plus 6 minus 3, um (pause) and plus 12. Three plus 6 equals 9. Nine minus 3 equals 6. And 6 plus 12 equals 18.

After explaining which problem they would choose to work again, I asked Jackson and Destiny to describe what they would tell their classmates about working both problems. Their thoughts concerning the Lemonade Problem (WP) are recorded below.

Jackson: It’s like easy I meant not like kind of easy but remember to do um addition in imagine in your head um what it looked like and um don’t just pick the the first one like the 32-ounce because it might not be um it may not be the answer, because it just say buy one get one free. But the 82, 84-ounce, it may see the 84-ounce is not on sale and it’s $6.49 so um remember to do that.

Destiny: Um, I would say for this problem that it’s kind of difficult but it can um it probably not be difficult for them cause it’s um kind of easy just think in your head not, not like just pick a answer that’s like the 32 ounces, just you buy one for a lower price and then get another one free.

Next, they shared their thoughts regarding the What’s the Area Problem (MTW). They both included a description of how they solved the problem as well.

Jackson: It’s like area, area, it’s just you gotta find out what’s um outside and it’s like how many like how many squares can go in there. If this equals three so 3 plus, 3 plus 3 equals um 6 so 3 plus 6 plus 9 I meant 3 plus 6 equals 9. Nine plus 3 plus 6 equals 18.
Interviewer: But if you weren’t telling them how to work it, if you weren’t
telling all those numbers and how to work it, what would you tell
them just about when you worked it?

Jackson: Like, it was, it was easy to do because all you doing is finding the
area and adding ‘em up to see what’s the answer so that’s why.

Destiny: Okay, um, I would tell them that that problem is it look hard but
it’s easy but um all you have to do is just figure out the area of
each um each shape but you already know it’s the um trapezoid is
nine so if you put if you use the triangle and on each one you could
probably figure out what’s the um area.

Interviewer: Ok so if you weren’t talking about the numbers, what would you
tell them about when you worked it?

Destiny: Umm

Jackson: I would say that it does like deal with area and perimeter but you
just you um finding the area, and it may be hard, it may be hard to
you if you’re a fourth grader it may be not because if you’re
learning that so it’s not.

Destiny: Umm, I would tell them that you’re just trying a number like trying
a shape to figure out what the area um instead of like using
numbers you can just put one shape on to make it um equal.

Last, I asked Destiny and Jackson if their classmates would enjoy working the
What’s the Area Problem (MTW). Destiny said that she thought they would enjoy
working it. Jackson agreed that they might enjoy it because he thought that it might be
easy for them to work. Finally, the interview was complete. This session completed
Jackson and Destiny’s sessions together.

**Within-Case Analyses**

Each case was analyzed separately to determine patterns, if any, existing within
the case, across sessions, including problem-solving sessions, written reflections, and
interviews. The paragraphs that follow present the within-case analysis for Jackson and
the within-case analysis for Destiny. General patterns within each case are described,
however, Jackson and Destiny’s perceptions of mathematical tasks without words and
word problems are reserved for a later section in this chapter, *Responses to the Research
Question.* Upon completion of the within-case analyses, a summary concludes the
section.

**Jackson**

Jackson participated in five problem-solving sessions throughout the study:
Sessions 1B, 2A, 3C, 4C, and 4A. In two of the sessions, he was paired with Destiny, the
other reluctant problem solver. In the other three sessions, he was paired with Students A,
B, and E, all less-reluctant problem solvers. During the study, three main patterns
emerged within his case and are described in the following paragraphs.

**Initial Selection of the Tasks.** Jackson initiated the choosing of the task in three
of his five problem-solving sessions. He initiated the choosing of the task in both of his
sessions with Destiny, the other reluctant problem solver. He also initiated the choosing
of the task in a session with one of the less-reluctant problem solvers, Student B. In the
three instances that he initiated the choosing of the task, he chose one mathematical task
without words, the Triangles Problem (MTW), and two word problems, the Sports
Problem (WP) and the Lemonade Problem (WP). In the other two instances, Jackson’s less-reluctant problem solver partners, Student A and Student E chose the tasks.

**Motive for Choosing the Tasks.** Jackson demonstrated three main reasons for choosing a particular task. He chose tasks because he thought they were easy, fun, or the context appealed to him. The three reasons are discussed in the following paragraphs.

**Easy.** In four of his five problem-solving sessions, Jackson stated that he chose to work the task because it looked easy. In his Participant Reflection Prompts and his responses to the interview, Jackson stated that he chose to work the Sports Problem (WP), Pattern Block Quilts Task (MTW), Lemonade Problem (WP), and Triangles Problem (MTW) because they looked easy. For example, during Session 4A, Jackson described the Triangles Problem (MTW) during the interview, “It looked easy and you get to count squares. Equilateral squares. I meant triangles.” In the remaining session, he described the problem as easy after working it but not before working it.

**Fun.** Jackson chose to work the Pattern Block Quilts Task (MTW) and Sports Problem (WP) because he said that they looked fun. On the Participant Reflection Prompt during Session 2A, Jackson said of the Pattern Block Quilts Task (MTW), “Today I chose the greatest area problem to work because it looks fun to do putting the shapes onto it. I liked putting shapes on it to find the area.” He added during the interview that, “It looked, it, it was fun. Building shapes on them.”

**Problem Context.** Throughout the study, Jackson chose to work two word problems: the Sports Problem (WP) and the Lemonade Problem (WP). In both instances, Jackson indicated that he chose to work the problems because he liked the topic of the problem. He explained in his Participant Reflection Prompt from Session 1B that his love
of sports prompted him to work the Sports Problem (WP). Jackson stated, “Today I chose
the sports problem to work because I love sports and it was kinda easy and it was fun. I
like the problem because I loved sports.”

Jackson’s responses to questions in the interview after completing the Participant
Reflection Prompt reaffirmed his motive for selecting the Sports Problem (WP).

Interviewer: Alright, what made you select this task, the Sports Problem (WP),
instead of the other one, the Bead Problem (MTW)?

Jackson: Because I – I love sports.

Later in the interview he added, “Well I said that I had loved sports (pause) so (pause) I
just like sports.”

In Session 4C, Jackson reaffirmed his desire to work a problem in relation to a
topic he enjoyed. He indicated on the Participant Reflection Prompt, “Today I chose the
lemonade problem to work because it was about lemonade . . .” He added in the
interview, “Because this one looks like more easier and I chose it because it’s like the
better ounce, like, let’s we gonna see the um which the answer it looked easier.”

Table 3 provides an overview of Jackson’s problem-solving sessions throughout
the study including his initial perceptions and reasons for choosing particular tasks. Also,
the table shows his perceptions after working the tasks.
Table 3

Overview of Jackson’s Perceptions of the Tasks

<table>
<thead>
<tr>
<th>Session</th>
<th>Partner</th>
<th>Participant that Initiated Choosing</th>
<th>Task Chosen</th>
<th>Jackson’s Initial Perception of the Task</th>
<th>Jackson’s Perception after Working the Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>1B</td>
<td>F</td>
<td>Jackson</td>
<td>Sports Problem (WP)</td>
<td>“I love sports” “kinda easy” “fun”</td>
<td>“easy problem” “I loved sports” “didn’t hate nothing about it”</td>
</tr>
<tr>
<td>2A</td>
<td>A</td>
<td>A</td>
<td>Pattern Block Quilts Task (MTW)</td>
<td>“looks fun to do putting the shapes onto it” “get to build shapes”</td>
<td>“I liked putting shapes on it to find the area” “easy”</td>
</tr>
<tr>
<td>3C</td>
<td>E</td>
<td>E</td>
<td>Four 4s Problem (MTW)</td>
<td>“you can do mathematics” “maybe easier” “would be easier for me to learn new stuff about this”</td>
<td>“couldn’t figure out the problem” “it was hard” “couldn’t find the answers” “more challenging”</td>
</tr>
<tr>
<td>4A</td>
<td>B</td>
<td>Jackson</td>
<td>Triangles Problem (MTW)</td>
<td>“it looked easy” “cause you got to count” “it was like easy from like doing instead of that word problem”</td>
<td>“challenging and hard to find the answer and count triangles” “it was like difficult because I had a lot of answers”</td>
</tr>
<tr>
<td>4C</td>
<td>F</td>
<td>Jackson</td>
<td>Lemonade Problem (WP)</td>
<td>“it was about lemonade” “easy to work” “challenging” “it looked easier to find the answer”</td>
<td>“it was mathematics and easy to find answer and we got to see when one is the best buy”</td>
</tr>
</tbody>
</table>
Personality opposite of his partner’s personality. During the sessions, Jackson’s personality shifted at times depending upon the personality of his partner. When paired with Destiny, the other reluctant problem solver, in Sessions 1B and 4C Jackson acted as the leader of the two. For instance, Jackson initiated the choosing of the task during both sessions in which he was paired with Destiny. However, when paired with less-reluctant problem solvers, he allowed them to choose the task in two out of three instances, Sessions 2A and 3C.

Jackson suggested strategies for solving the problems when paired with Destiny, as described earlier in this chapter. When Jackson’s partner was a less-reluctant problem solver, however, he tended to wait for his partner to develop a strategy to solve the problem. When paired with Student A during Session 2A, Jackson waited for his partner to suggest a strategy for solving the Pattern Block Quilts Task (MTW). During session 3C with Student E, another less-reluctant problem solver, Jackson sat quietly while his partner persisted in working the problem. Jackson attempted to solve the problem, however, he stopped and said that the problem was difficult.

Destiny

Destiny participated in four problem-solving sessions during the study: Sessions 1B, 2C, 3B, and 4C. Destiny was paired with Jackson, the other reluctant problem solver in two of her sessions. In the other two sessions, she was paired with Students B and E, less-reluctant problem solvers. Throughout the study, three main patterns emerged within Destiny’s case and are discussed in the following paragraphs.

Initial selection of the tasks. In half of her problem-solving sessions, Destiny initiated the choosing of the task or problem to complete. In both instances that she
initiated the choosing of the task, she was paired with less-reluctant problem solvers, Students B and E. During those two sessions, she chose one mathematical task without words and one word problem. In the other two instances she was paired with Jackson, the other reluctant problem solver partner, and he initiated the choosing of the task.

**Motive for choosing the tasks.** Destiny demonstrated two main reasons for choosing a certain task. The first reason was that the tasks were either easy or the other task offered in the session seemed difficult. The second reason she chose particular tasks was because she thought that they were organized and gave her the information that she needed. The two reasons are discussed in the following paragraphs.

**Easy or the other problem seemed difficult.** In all four of her sessions, Destiny expressed a desire to work the chosen tasks, the Sports Problem (WP), the Pattern Block Quilts Task (MTW), the Tables Problem (WP), and the Lemonade Problem (WP), because they seemed easy to work or because she thought that the other task seemed difficult to work. Destiny chose to work the Tables Problem (WP) and the Lemonade Problem (WP) because she thought that they looked easy. Destiny said of the Tables Problem (WP) during Session 3B, “I wanted this task because it was kinda hard, but it was kinda easy at the same time. Cause I did have the answer at first but then um it wasn’t right cause you had to fix it.” Destiny described her reason for choosing the Lemonade Problem (WP) during Session 4C.

I chose that one cause it seemed kind of easy cause it was just telling you all the information, and like it gave you a question that you need to know which one is the better buy. But then it started getting difficult.
Destiny chose the Pattern Block Quilts Task (MTW) and the Sports Problem (WP) because she thought that the other tasks offered during those sessions looked difficult. For example, when asked why she selected the Sports Problem (WP), Destiny said:

Because I chose this problem was that it seemed like a well organized um problem to be worked easily, and um the [Bead] problem, it would be kinda hard to figure out how many was in the box.

In a later session, Destiny described her reason for choosing the Pattern Blocks Quilt Task (MTW) instead of the Meals Problem (WP). Destiny said the following:

The [Meals Problem (WP)] seemed difficult to me. (Pause) It’s um, it’s like it had a whole bunch of numbers that you had to do and then sometimes I get confused when we do like half cups and all that.

At the conclusion of the sessions, Destiny described three of the four problems as being “easy” for her to work. The only problem that Destiny described as being “difficult,” “hard,” and “boring” after working it was the Lemonade Problem (WP).

**Well-organized and provided ample information.** Destiny primarily chose problems that she felt gave her information and told her what she needed to do to solve. She described the Sports Problem (WP) as a “well-organized” problem. Also, she stated that the Sports Problem (WP), the Tables Problem (WP), and the Lemonade Problem (WP) gave her the information that she needed to solve the problem. For example, Destiny said of the Sports Problem (WP), “Ok the thing I liked about this problem was that it gave you a lot of information, like what, how many boys was on the football team, soccer team, basketball and other sports.” In a later session, Destiny said of the Four 4s
Problem, “It didn’t give enough information for me.” Further, she described her reasoning for choosing the Tables Problem (WP) instead of the Four 4s Problem (MTW), “I wanted to work this [Tables Problem (WP)] because it had more information than this [Four 4s Problem (MTW)] and it had a lot of details and stuff that you needed to know.”

Table 4 provides an overview of Destiny’s problem-solving sessions throughout the study including her initial perceptions and reasons for choosing particular tasks. Also, the table shows her perceptions after working the tasks.
Table 4

*Overview of Destiny’s Perceptions of the Tasks*

<table>
<thead>
<tr>
<th>Session</th>
<th>Partner</th>
<th>Participant that Initiated Choosing</th>
<th>Task Chosen</th>
<th>Destiny’s Initial Perception of the Task</th>
<th>Destiny’s Perception after Working the Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C</td>
<td>C</td>
<td>Sports Problem (WP)</td>
<td>“well organized problem”</td>
<td>“easy to work”</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“easy problem”</td>
<td>“gave you a lot of information”</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“gives you all the information you need”</td>
<td>“really nice problem”</td>
</tr>
<tr>
<td>2</td>
<td>E</td>
<td>Destiny</td>
<td>Pattern Block Quilts Task (MTW)</td>
<td>“fun”</td>
<td>“fun”</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“easy”</td>
<td>“use the shape to figure out the area so it wasn’t hard at all”</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>“not frustrating”</td>
<td>“doesn’t give clues”</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>“the other problem seemed difficult”</td>
<td>“easy”</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>Destiny</td>
<td>Tables Problem (WP)</td>
<td>“kinda hard, but it was kinda easy at the same time”</td>
<td>“challenging”</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“already know the answer”</td>
<td>“easy to do”</td>
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<td></td>
<td>“[the other problem] didn’t give enough information”</td>
<td>“you could probably already know the answer”</td>
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<td></td>
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<td></td>
<td>cause it was five times six but then you had to change it up</td>
</tr>
<tr>
<td>4</td>
<td>C</td>
<td>C</td>
<td>Lemonade Problem (WP)</td>
<td>“seemed kind of easy cause it was just telling you all the information”</td>
<td>“difficult”</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“gave you a question that you need to know”</td>
<td>“boring and hard”</td>
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<td></td>
<td></td>
<td></td>
<td>“challenging”</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“seemed kind of easy . . . but then it started getting difficult”</td>
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</tbody>
</table>
Personality influenced by her partner’s personality. Destiny’s personality seemed to change depending upon her partner’s personality. In essence, Destiny took on the same the personality as partner. For example, if Destiny was paired with a reluctant problem solver, she displayed more behaviors of a reluctant problem solver. She allowed her partner to choose the task and begin finding a strategy for solving the problem. Also, she did not challenge her partner when he presented a solution to her. Alternatively, if she was paired with a less-reluctant problem solver, she was less reluctant when working the problem. Further, when paired with a soft-spoken partner, Destiny was more soft-spoken. However, when paired with a somewhat loud partner, Destiny spoke in a louder voice.

Summary

This section discussed the within-case analysis for Jackson and the within-case analysis for Destiny. Both Jackson and Destiny exhibited consistent behaviors across their sessions throughout the study. Each case demonstrated patterns in selecting a task as well as patterns in actions displayed during the sessions.

Cross-Case Analysis: Jackson and Destiny

The patterns within each case were examined and compared across the two cases. Patterns emerged across the cases and are discussed in this section. Patterns existed within the selection of the tasks including the motives for selecting certain tasks. Patterns also existed concerning the personalities of the reluctant problem solvers during the sessions. Additional patterns existed across the cases related to the research question. Those patterns, however, will be discussed in the next section, Responses to the Research Question.
**Initial Selection of the Tasks**

Both participants initiated the choosing of the task in at least half of their sessions as previously discussed. Jackson initiated the choosing of the task in three of his five sessions, while Destiny initiated the choosing of the task in two of her four sessions. In both of Jackson and Destiny’s sessions together, Jackson initiated the selection of the task and Destiny agreed to work the task that he selected. Both participants initiated the selection of the task when paired with one particular less-reluctant problem solver, Student B. Even when the partner initiated the selection of the task, however, the pair then worked collaboratively to discuss which task to select and then to complete it.

**Chose the Easier Task**

In the majority of the sessions, both participants stated that they chose a particular task because it looked easier than the other task offered in the same session. Jackson chose four of the five tasks that he worked because to him, they seemed easy. Destiny chose two of the four tasks that she worked because she said that they seemed easy. In her other two sessions, she said that she chose the tasks that she worked because the other tasks offered during those sessions seemed difficult. In essence, Destiny chose all four tasks that she worked because she felt that they were the easier tasks of the two offered in each session.

In nine of their ten combined sessions, Jackson and Destiny chose the task that they thought would be easier to work. Therefore, it was important for Jackson and Destiny to know that they could complete the task and have their need for competence met (Deci & Ryan, 2000).
**Personality Shifts**

As previously discussed in the within-case analyses, both Jackson and Destiny’s personalities during the problem-solving sessions changed depending upon the personality of their partner. Their personalities did not change, however, in the same manner. Jackson took on the opposite personality of his partner. If his partner was a reluctant problem solver, Jackson was less reluctant and took on more of a leadership role. If Jackson’s partner was less-reluctant, Jackson displayed more behaviors of a reluctant problem solver.

Alternatively, Destiny took on the same personality as her partner’s personality. For example, if Destiny’s partner was a reluctant problem solver, Destiny displayed more behaviors of a reluctant problem solver. When Destiny’s partner was a less-reluctant problem solver, Destiny was less reluctant to engage in problem solving.

**Summary**

This section briefly discussed patterns that existed between the cases that were not necessarily related to the research question. Patterns existing between the cases included initiating the selection of the task in certain sessions, selecting the majority of the tasks because they appeared to be easier than the other task in the same session, and shifting personalities depending upon partners. This section did not include a discussion of patterns existing between cases related to the research question. Those patterns are discussed in the next section, *Responses to Research Question*.

**Response to the Research Question**

This study sought to answer one central research question: How do reluctant problem solvers perceive mathematical tasks without words and word problems? Patterns
existed within each case and between cases that answer the research question. However, to avoid duplication, they were not presented prior to this section. In this section, responses to the research question including patterns within cases and between the two cases are presented.

The following paragraphs discuss the reluctant problem solvers’ perceptions of mathematical tasks without words and word problems. Each section is broken into two parts: perceptions before working the task and perceptions after working the task. Generally, participants viewed the tasks in one way before working the tasks, but their perceptions were altered after working the tasks. For this reason, I separated the participants’ views into those two parts. Following the presentation of participants’ perceptions a summary concludes the section.

Perceptions of Mathematical Tasks without Words

Jackson chose to work three mathematical tasks without words during the study. In addition, he examined a fourth mathematical task without words after working a word problem in one of his sessions. The four mathematical tasks without words that Jackson worked were the Pattern Block Quilts Task (MTW), the Four 4s Problem (MTW), the Triangles Problem (MTW), and the What’s the Area Problem (MTW).

Destiny chose to work one mathematical task without words during the study. In addition, she examined two mathematical tasks without words after working word problems in two of her sessions. The three mathematical tasks without words that she worked were the Pattern Block Quilts Task (MTW), the What’s the Area Problem (MTW), and the Four 4s Problem (MTW). Both reluctant problem solvers’ perceptions are presented.
Before working the mathematical tasks without words. Before working the tasks, Jackson and Destiny both described the mathematical tasks without words as looking hard to work, having few words, and containing pictures in the tasks. Their thoughts are provided in this section.

Seemed difficult to work. Before working the tasks, Destiny described mathematical tasks without words as appearing to be difficult to work. She said, “The Bead Problem (MTW), it would be kinda hard to figure out how many was in the box.” She later added, “Because the Bead Problem (MTW), it don’t show enough details or information to it just, it just says ‘How many beads are on the chain shown?’ ”

Destiny also described the Four 4s Problem (MTW) as appearing difficult to work. “I wanted to work the [Tables Problem (WP)] because this [Four 4s Problem (MTW)] seemed re- kinda hard, but I don’t, I don’t know but it probably was. But this one, it was challenging.” Later she added, “Um, [the Four 4s Problem (MTW)] didn’t give enough information for me.” Finally, she said, “Cause that [Four 4s Problem (MTW)], it just don’t make sense to me.” The mathematical tasks without words appeared more challenging to the participants because they felt that the tasks did not provide enough information.

Few words. Jackson and Destiny provided many statements claiming that the mathematical tasks without words had fewer words than the word problems. Jackson described the Bead Problem (MTW) as having few words in his statement below.

It got a picture in it, um, it shows details within it, but it don’t have like many words as [the Sports Problem (WP)]. Cause this only says how many beads are on
the chain shown? So you gonna have to figure out what’s in the box and what’s in the outside.

Destiny also felt that the Bead Problem (MTW) had few words. After Jackson’s statement, she added,

Yeah, like he said it don’t have as many words. It’s just telling you how many beads are on the chain shown and it would be hard to figure out what’s in the box, but you already know what’s outside the box.

In Session 2A, Jackson said, “The word problem is longer than the picture problem.” In another session, I asked Jackson how the Four 4s Problem (MTW) was like or not like a word problem. Jackson responded, “[The Four 4s Problem (MTW)] is not cause it got like pictures, and [it] is not like a word problem because it got like these and it don’t give you like a word, lots of words like the word problem.”

Destiny described the Pattern Block Quilts Task (MTW) as having few words, “It didn’t have any numbers. It just had one sentence instead of a whole bunch of sentences like telling you what to do and all that.” Later, Jackson claimed that the Pattern Block Quilts Task (MTW) had fewer words than the Meals Problem (WP).

Jackson: It’s like, it say, “Which shape covers the greatest, which has the greatest area?” So all you gotta do is build shapes on ‘em. It don’t give you like what to do like, like a word problem give you information, what to do and this one just got this one um longer and this one short.

Interviewer: What’s long and what’s short?

Jackson: The word problem is longer than the picture problem.
As stated, both Jackson and Destiny affirmed in multiple statements made throughout the study that mathematical tasks without words have fewer words than word problems.

**Contain pictures.** Both participants indicated that the mathematical tasks without words contained pictures. Jackson said of the Four 4s Problem (MTW), “it got like pictures.” When asked if the Lemonade Problem (WP) was a word problem, he said, “It could be cause it don’t have no pictures.” His statement indicated that word problems do not contain pictures.

Jackson and Destiny also described the What’s the Area Problem (MTW) problem as a picture problem and not a word problem.

Interviewer: What about the What’s the Area Problem [(MTW)]?
Jackson: It’s not a word problem.
Destiny: It’s
Jackson: It’s just a picture problem.
Destiny: Yeah, it’s kinda like a picture problem.
Jackson: Because
Destiny: It got
Jackson: It got pictures.
Destiny: A few words but it’s a picture.

As discussed, the participants indicated that mathematical tasks without words generally contain pictures.

**After working the mathematical tasks without words.** After working the mathematical tasks without words, Jackson and Destiny described them as fun. If they
found solutions to the tasks, they thought the tasks were easy. If they did not find solutions to the task, they thought the tasks were hard. Their perceptions after working the mathematical tasks without words as described below.

**Fun.** Jackson and Destiny both said that the Pattern Block Quilts Task (MTW) was fun. Jackson worked it in Session 2A with Student A and then described it in his written reflection: “Today I chose the [Pattern Block Quilts Task (MTW)] to work because it looks fun to do putting the shapes onto it. I liked putting shapes on it to find the area.” He later said in the interview, “It looked it, it was fun. Building shapes on ‘em.” I asked Jackson if he would choose the same task to work if he was given another opportunity. The discussion is shown below.

Jackson: Yeah, I pick this one because it look fun cause you can build shapes and everything so I’d pick this one.

Interviewer: After solving the problem, did you wish you had chosen the other one?

Partner: No.

Jackson: No.

Interviewer: Why not?

Partner: Because

Jackson: Because

Partner: It looked funner.

Destiny worked the same problem in Session 2C with Student E and said in her written reflection, “The [thing] I liked about the problem was that it was fun.” Both participants described the Pattern Block Quilts Task (MTW) as being fun.
If no solution was found, it was hard. Jackson described mathematical tasks without words as being difficult if he did not find a solution. When asked about the Four 4s Problem (MTW), Jackson said in his written reflection that he did not like it because he “Couldn’t figure out the problems.” He also wrote, “I would choose another problem because it was hard.” Later in the interview he said that he “Couldn’t find all the answers” and “it was like difficult.”

Jackson also described the Triangles Problem (MTW) as difficult after working it. In his written reflection, he stated, “Today I chose the triangles problem to work because it look challenging and easy, but it’s not.” During the interview, he said that initially, it looked easy. I asked him what made it look easy and he said, “Because cause this uh triangles it just like counting them, but it was hard, difficult.” I asked him why it was hard and he responded, “Difficult because you got, you had to count the big squares. I meant big triangles.” Later he said, “It looked easy, but it’s kind of (pause) it’s really hard.”

After working both problems, the Four 4s Problem (MTW) and the Triangles Problem (MTW), Jackson said that they were difficult. In both instances, Jackson and his partner were not able to correctly complete the tasks.

If a solution was found, it was easy. Jackson and Destiny both described the Pattern Block Quilts Task (MTW) as being easy to work. In Session 2A Jackson and his partner, Student A, found a solution to the Pattern Block Quilts Task (MTW). In the interview, Jackson said, “This [Pattern Block Quilts Task (MTW) more easy than this [Meals Problem (WP)] gonna have like, like you gonna have to add, multiply, divide.”
Later he added, “It’s easy and you just build shapes to get the area and make sure your um blocks are in the right place.”

Destiny worked the Pattern Block Quilts Task (MTW) in Session 2C with Student E. I asked her if there was anything that she liked about the problem, and she responded, “That um all you had to do was just use the shape to figure out the area so it wasn’t hard at all.” Destiny elaborated on her response when asked to do so.

Interviewer: What made you want to work this task or not want to work it?

Destiny: I wanted to work it because it seemed easy and not hard or frustrating.

Interviewer: What made it seem easy?

Destiny: Because all you have to do is just put the shapes, well you had to pick a simple shape to make it work, and in this [Meals Problem (WP)] you had to do a lot of stuff to get the answer.

When asked if she would choose to work the Pattern Block Quilts Problem again if given another chance, Destiny said, “I choose this problem cause it seemed easy to do, easy to work, and it doesn’t give clues, but all you have to do is use the shape to find out the area.”

Destiny and Jackson worked the Lemonade Problem (WP) during Session 4C. When they completed the problem, I asked them to work the What’s the Area (MTW) problem together. They were able to find a correct solution, and later described the problem during the interview.

Interviewer: What made you want to work this [What’s the Area Problem (MTW)] or not want to work it?
Jackson: I wanted to work it because it was easy and it was fun to work, cause it’s like, it’s challenging to you and you can like uh challenging like hard, difficult so it can be like in the future you might have one of these questions and and you might know it now.

Destiny: Um, really uh, what was the question um (pause)

Interviewer: What made you want to

Destiny: Oh

Interviewer: Work it or not want to work it?

Destiny: I wanted to work [What’s the Area Problem (MTW)] because it was looking kind of easy but that wa- that [Lemonade Problem (WP)] looked easy but it seemed hard but this [What’s the Area (MTW)] um, it was just saying like giving you the information that you needed.

Interviewer: Ok, the What’s the Area Problem (MTW), what did you say it looked easy but it seemed hard? And then the Lemonade Problem (WP) what?

Destiny: It um

Jackson: This [What’s the Area Problem (MTW)] was kind of easy.

Destiny: [The Lemonade Problem (WP)] was hard, well not hard but like

Jackson: Difficult

Destiny: Difficult was kind, had a lot of stuff to it.

Interviewer: [Jackson], you said the What’s the Area Problem (MTW) was kind of easy?
Jackson: This one. (Pointed to the What’s the Area Problem.)

At the end of the interview, I asked Jackson and Destiny if they thought that their classmates would like to work the What’s the Area Problem (MTW). The pair answered the question with the following responses.

Jackson: Just working it. Cause it may be easy to them.
Destiny: Working it probably.

Interviewer: Would they, do you think they would think it was hard? Would they think it was easy?

Jackson: This one?

Interviewer: The What’s the Area Problem (MTW)?
Destiny: Probably.

Jackson: That it would be easy because I think they’re learning it.
Destiny: Yeah.

Destiny also thought that the Four 4s Problem was easy (MTW) once she understood what the problem was asking her to do. As described in an earlier section, Destiny and her partner did not originally understand the Four 4s Problem (MTW) and Destiny said, “Cause that one, it just don’t make sense to me.” Once I talked Destiny and her partner through the example problem, however, she thought that it was easy.

Destiny: Oh! Well then I would pick this [Four 4s Problem (MTW)].

Interviewer: You would pick it, why?

Destiny: Because this [Four 4s Problem (MTW)] so easy, but I just didn’t understand it for a second.

Interviewer: Ok, so maybe you just didn’t understand the directions?
Destiny:  Mmhmm.

Interviewer:  But now that you understand, what do you think about the [Four 4s Problem (MTW)]?

Destiny:  It’s um easy.

Later in the interview, Destiny said, again, “Because [the Four 4s Problem (MTW)] seems easy, but I just didn’t understand it cause it didn’t seem right.” I asked her what she would tell her classmates about the Four 4s Problem (MTW), and Destiny replied, “I would tell them do this because it seems very easy.”

**Perceptions of Word Problems**

Jackson worked two word problems during his sessions in the study. In both sessions, he was paired with Destiny. They chose to work the Sports Problem (WP) and the Lemonade Problem (WP). In addition to the two word problems already listed, Destiny worked a third word problem when paired with a different partner. The third word problem that she worked was the Tables Problem (WP). Their perceptions are presented in the following paragraphs.

*Before working the word problem.* Before working the word problems, Jackson and Destiny described the word problems as appearing easy to work, giving the information needed to solve and telling you what to do to solve. Also, the participants said that word problems did not contain pictures. Their statements are provided in the paragraphs that follow.

*Easy.* Both Jackson and Destiny described the Sports Problem (WP) as appearing easy to work. Destiny wrote in her written reflection, “Today I chose the Sports Problem (WP) to work because it seem like a easy problem to work. I think this problem was well
organized for me to answer correctly.” Later, in the interview she stated, “Because I chose this problem was that it seemed like a well organized um problem to be worked easily. . .” Destiny also described the Tables Problem (WP) as appearing easy to work. She said, “I wanted this task because it was kinda hard but it was kinda easy at the same time. . .”

Jackson said that the Lemonade Problem (WP) appeared easy to work. In his written reflection, he wrote, “Today I chose the lemonade problem to work because it was about lemonade and it looks easy to work and challenging.” In the interview, he added that he chose the problem, “Because this one looks like more easier, and I chose it because it’s like the better, better ounce like let’s, we gonna see the um which the answer and it looked easier.” Destiny agreed that the Lemonade Problem (WP) appeared easy. She stated, “The thing I liked about this problem was it felt easy to me but it was challenging.”

*Provides ample information.* Destiny described what she liked about the Sports Problem (WP): “Ok, the thing I liked about this problem was that it gave you a lot of information like what how many boys was on the football team, soccer team, basketball and other sports.” When asked if she would choose to work the same problem again or a different problem, Destiny said, “I think that I would work [the Sports Problem (WP)] again because it gives you a lot of information that you need to know and then when it has all the numbers and you get confused, all you have to read is the question they’re asking you mainly like how many boys play basketball only.” When asked what she would tell her classmates about the Sports Problem (WP), Destiny said, “I would tell them that it’s a really nice problem that gives you all the information you need, has a lot
of details, and um, it describes like how many boys the play uh whatever, and then how many then the question would ask you how many boys play basketball only so you will know that you’re only looking for basketball.”

Destiny also stated, “A word problem it gives you information, but if you look at a picture, it gives you more details and information about the picture so you can work it right, but maybe like a word problem would give you more information.” Her reasoning for selecting the Tables Problem (WP) over the Four 4s Problem (MTW) was as follows: “I wanted to work [the Tables Problem (WP)] because it had more information than [the Four 4s Problem (MTW)] and it had a lot of details and stuff that you needed to know.”

Jackson described the Lemonade Problem (WP) during Session 4C: “[The Lemonade Problem (WP) is] like what size is the best buy, so you gotta figure out with mathematics and it got number, words, details, and information. Destiny said of the same problem, “I chose that [Lemonade Problem (WP)] cause it seemed kind of easy cause it was just telling you all the information and like it gave you a question that you need to know which one is the better buy.” Destiny later added that it “has lots of information.”

*Tells you what to do.* Destiny stated that the Tables Problem (WP) told her what she needed to do to solve it. “It has information that tells you what you need to do and how like [Tables Problem (WP)] said what’s the least number of card tables that you need, so you had to figure that out instead of doing the other stuff.” Jackson discussed the Pattern Block Quilts Task (MTW) and then proceeded into a discussion of word problems in Session 2A: “It’s like it say ‘Which shape covers the greatest which has the greatest area’ so all you gotta do is build shapes on ‘em. It don’t give you like what to do
like, like a word problem give you information what to do and this one just got this
[Meals Problem (WP)] um longer and this [Pattern Block Quilts Task (MTW)] short.”

During Session 3C, Jackson said of word problems in general, “[they] just give
you directions what to do.” Jackson later said the Lemonade Problem (WP) is a word
problem and tells you how to solve it. He stated, “I think [the Lemonade Problem (WP)]
is because it got lots of words and whats to do. Destiny later said, “I think [the Lemonade
Problem (WP)] is a word problem because it has words. It give you numbers, like and
then it give you the question that you needed to answer.”

No pictures. In Session 1B, Jackson described a word problem “It’s like, it don’t
have no pictures or nothing, but it shows like this [Sports Problem (WP)] and it give you
numbers and stuff to see um to like demonstrate it.” He later said that the Bead Problem
(MTW) could not be a word problem: “It’s not because it got a picture in it. . .”

Jackson described a word problem again in Session 2A: “It don’t have no
pictures. It don’t have no pictures. It just you (pause) um (pause) it just not give you like,
well it give you words cause it’s a word problem . . .” In Session 3C, Jackson said, “A
word problem is, it don’t have no pictures.” Later, I asked him if the Tables Problem
(WP) was a word problem and he said, “[The Four 4s Problem (MTW)] is not cause it got
like pictures.”

After working the word problem. After working the word problems, Jackson and
Destiny described them in two ways. If they were not able to find a solution to the
problem, they described the word problems as being difficult to work. If they were able
to find a solution to the word problem, even an incorrect solution, they described the
word problems as being easy to work. Their perceptions are provided below using their own words.

*If no solution was found, it was hard.* Jackson and Destiny worked together to solve the Lemonade Problem (WP). Destiny’s perception of the problem after working it was not pleasant. In her written response, Destiny said that she did not like anything about the problem. She said that she did not like the problem because “it was boring and hard.” I asked Jackson if there was anything that he did not like about the task, and he said, “I would say nothing, or couldn’t find the answer.” When asked if she would chose to work the same problem again, Destiny said, “I wouldn’t wanna work this [Lemonade Problem (WP)] no more, but I will pick the other [What’s the Area Problem (MTW)] because um it will seem it looks hard, but then it seemed to get easier cause you was just using triangles to figure out the area so then you have to figure out the um number like the area for the triangle, the trapezoid, the rhombus, and the hexagon.”

*If a solution was found, it was easy.* Jackson and Destiny found a solution to the Sports Problem (WP), though their solution was incorrect. In her written reflection, Destiny said of the problem, “it’s easy to work and write out.” When asked what Jackson would tell his classmates about the Sports Problem (WP), he said, “That like, it’s a easy problem.”

After working the Tables Problem (WP), Destiny said in her written reflection that she liked the problem because “it was challenging, almost easy.” During the interview, she added, “I wanted this [Tables Problem (WP)] because it was kinda hard but it was kinda easy at the same time, cause I did have the answer at first but then um it wasn’t right cause you had to fix it so.” She also said, “The thing I liked about [the
Tables Problem (WP)] it was challenging and it was kinda hard and easy it was easy to do.” When asked what she would tell her classmates about the problem, she said, “I would tell them do this because it seems very easy.”

Jackson discussed the Lemonade Problem (WP) in his written reflection: “It was mathematics, and easy to find answer and we got to see when one is the best buy.” He added, “I wanted to work it because it was easy and it was fun to work, cause it’s like it’s challenging to you and you can like uh challenging like hard, difficult so it can be like in the future you might have one of these questions and and you might know it now.” In his responses, Jackson described the Lemonade Problem (WP) as easy and as difficult.

Summary

In this section, I presented the findings from the study in response to the research question. I described Jackson and Destiny’s perceptions of both mathematical tasks without words and word problems, including their perceptions before and after working the tasks. Before working the mathematical tasks without words, the participants described them as having few words, seeming difficult to work, and containing pictures. After working the mathematical tasks without words, the participants described them as fun. Additionally, if they did not find a solution, they described the tasks as hard; if they did find a solution, they described the tasks as easy. Before working word problems, the participants said that they were easy, provided all of the information necessary to solve, provide directions for solving, and did not contain pictures. After working the word problems, the participants said that they were hard to work if no solution was found, but they were easy to work if a solution was found.
Summary

The two cases described in this chapter were Destiny, a female participant, and Jackson, a male participant. Both participants were identified as reluctant problem solvers based on my observations of their behaviors recorded on the Observation Guide as well as their teacher’s classification of their behaviors on the Behavior Frequency Chart.

Destiny participated in four sessions during the study and Jackson participated in five sessions. Destiny and Jackson completed two of their sessions together. Each session required the pair involved to choose a mathematical task without words or a word problem to complete together, write a reflection about the problem, and respond to interview questions. This chapter provided summaries of the problem-solving sessions, including written reflections and interviews. Additionally, this chapter provided within-case analyses for each case and a cross-case analysis identifying patterns existing across both cases. Finally, this chapter provided a section responding to the research question: How do reluctant problem solvers perceive mathematical tasks without words and word problems? Discussions and implications based on the findings in this chapter are described in Chapter V.
CHAPTER V: DISCUSSION AND IMPLICATIONS

Introduction

In the age of standards and accountability (CCSSI, 2010; Lambdin & Walcott, 2007), it is crucial for students to engage in practices and develop skills as described in the Standards for Mathematical Practice when learning mathematics (CCSSI, 2010). Students are expected to persevere even when mathematical tasks are challenging (CCSSI, 2010). It is more common for students to attempt new mathematical tasks and persevere to solve them when they have already experienced success in mathematics (Akin & Kurbanoglu, 2011; Hoffman, 2010; Lee, 2009). Some students, however, are reluctant to engage in mathematical tasks. Reluctant problem solvers are less inclined to attempt new tasks and even take measures to avoid engaging in the tasks (Holbert & Barlow, 2012/2013), providing them with fewer opportunities to feel successful in mathematics. In the current age of standards and accountability, it is undesirable for students to be reluctant toward mathematics because they are expected to meet high standards (CCSSI, 2010). Consequently, teachers must find ways to encourage reluctant problem solvers to engage in mathematics problem solving. Therefore, it is necessary to identify which types of mathematical tasks are appealing to reluctant problem solvers.

The purpose of this qualitative research study was to examine reluctant problem solvers’ perceptions of mathematical tasks without words and word problems. I utilized an exploratory case study approach (Yin, 2009). Specifically, I used a multiple, holistic case study approach and captured participants’ experiences and perceptions using multiple instruments, including written
reflections and interviews (Yin, 2009). This study sought to answer one research question: How do reluctant problem solvers perceive mathematical tasks without words and word problems?

This chapter begins with a brief discussion of the self-determination theory, including the need for competence related to the study, followed by a section on reluctant problem solvers. Finally, a discussion of implications and recommendations is provided. A summary concludes the chapter.

**Self-Determination Theory**

Meeting the three needs of students according to the self-determination theory, autonomy, relatedness, and competence (Deci & Ryan, 2000; Jones et al., 2011; Ross & Bergin, 2011; Ryan & Deci, 2000a, 2000b), holds the potential for helping reluctant problem solvers to become less reluctant to engage in mathematics. In the current study, the participants’ needs for autonomy and relatedness were met by design. The exploratory nature of the study allowed for the meeting of the need for competence to be examined in reluctant problem solvers.

**Competence**

As described in Chapter IV, the reluctant problem solvers’ perceptions of the tasks were different before and after working the tasks. Their perceptions were likely based on their feelings of self-efficacy toward the tasks and competence from past experiences. The following sections discuss the reluctant problem solvers’ levels of self-efficacy related to the tasks they worked in the study, including high self-efficacy, false self-efficacy, and low self-efficacy.

*High self-efficacy.* In most instances, the reluctant problem solvers chose the tasks that they believed to be easier to work. Their desire to choose the easier tasks indicated two things: their need to have high self-efficacy toward tasks to attempt to work them and their avoidance of challenging tasks. By avoiding challenging tasks, however, reluctant problem solvers do not give
themselves the opportunities to embrace the struggle that occurs when problem solving, therefore they do not necessarily learn to persevere through the struggle when problem solving to reach a solution. This is troublesome because mathematics students are expected to persevere when problem solving (CCSSI, 2010). The following paragraphs discuss the aspects of the mathematical tasks without words and word problems that the reluctant problem solvers chose to work, signifying that they had the self-efficacy needed to attempt to work the problem. Also, discussed are the effects of the use of probing questions during the problem-solving sessions.

**Mathematical tasks without words.** In some sessions, the reluctant problem solvers chose to work the mathematical tasks without words simply because they appeared easier to work than the word problem offered in the session. This occurred when the word problem contained fractions and measurements, such as the Meals Problem (WP) and the reluctant problem solvers did not have the self-efficacy to attempt to solve.

In other sessions, however, the reluctant problem solvers appeared to have the self-efficacy needed to complete mathematical tasks without words when the tasks appeared to be fun to work. In general, participants indicated that tasks were fun when they seemed to be puzzle-like in nature and did not appear to be mathematics problems. In many instances, when asked if the mathematical tasks without words tasks were mathematics problems, the participants said that they did not look like mathematics problems. In separate sessions, both Jackson and Destiny chose to work the Pattern Block Quilts Task (MTW) because it appeared fun to work, because they could use pattern blocks when working it. The findings suggest that mathematical tasks without words that appear fun allow reluctant problem solvers to have higher self-efficacy toward the tasks because of their puzzle-like nature. The participants’ feeling of success increased their self-efficacy (Hoffman, 2010) and finally, met their need for competence (Ross &
Bergin, 2011). The findings support the work of Bandura and Schunk (1981) who stated that individuals who are exposed to tasks and experience success working those tasks have a higher self-efficacy. When the reluctant problem solvers worked mathematical tasks without words and successfully found solutions, they described them as easy and fun to work.

Additionally, Jackson demonstrated a high self-efficacy toward the Triangles Problem (MTW) because he thought that it looked similar to one that he had previously worked in his mathematics class. Though he had not worked the exact same problem, his connection to the previously worked, similar problem allowed him to have high self-efficacy toward the Triangles Problem (MTW), thus driving him to choose to work it. Unlike word problems that the participants worked often in their mathematics class, the mathematical tasks without words might have appeared more difficult to solve if the participants had not worked similar tasks and were not able to recall strategies that might be useful in solving the tasks. Jackson was not able to reach one correct solution while working the Triangles Problem (MTW), and stated after working it that it was difficult. Jackson’s struggle through the problem showed that having high self-efficacy toward a problem does not necessarily lead to success in solving a problem, contrary to the findings of Hoffman (2010). Because Jackson did not experience success when working the Triangles Problem (MTW), his need for competence was not met, though he had had a high level self-efficacy toward the task at the beginning of the session.

Word problems. In many problem-solving sessions, the reluctant problem solvers chose the word problems instead of the mathematical tasks without words. Participants stated that the word problems, in general, appeared easy. This perhaps indicated that they had prior exposure to working and solving word problems and that they had high levels of self-efficacy toward word problems (Bandura & Schrunk, 1981). The reading comprehension aspect of word problems,
however, posed an obstacle at times. Assuming that some reluctant problem solvers avoid engaging in mathematics tasks because they struggle with reading or have low efficacy toward word problems, mathematical tasks without words could provide them with an opportunity to engage in mathematics without the need for reading (Holbert & Barlow, 2012/2013). The removal of the reading aspect could potentially allow them to feel successful, raising their self-efficacy and meeting their need for competence. (Deci & Ryan, 2000; Ross & Bergin, 2011). Based on the actions of reluctant problem solvers in the current study, however, reading did not pose a problem, because they had adopted other strategies to help them solve word problems that did not require the use of their reading comprehension skills. Therefore, the results do not support the hypothesis of Holbert and Barlow (2012/2013). Additional work is needed to explore the topic. The following sections discuss the reluctant problem solvers’ perceptions of word problems that led them to have high self-efficacy toward the problems.

Provided information. Participants said they chose to work word problems because they provided the details, information, numbers, hints, and clues that they needed to solve the problem. Also, they said that the word problems provided them with a question to answer. In some responses, participants said that the word problems told them how to solve the problem. Instead of viewing word problems as challenging because of the requirement to read and comprehend, the reluctant problem solvers found word problems to be more appealing because they seemed to provide more information than the mathematical tasks without words. Thus, the aspect of reading and comprehension was not an issue for the reluctant problem solvers.

Quick operation. When solving the word problems, in some cases, the reluctant problem solvers initially identified the numbers in the problem, chose an operation to perform, and found a solution, usually incorrect, until they were asked probing questions to prompt their thinking
about the problem again. Similar to the findings of Xin et al. (2008), this occurred when Jackson and Destiny worked the Sports Problem (WP). Participants in Xin et al.’s study and the current study, at times, did not problem solve or consider the reasonableness of their solutions. Instead, Jackson used his personal background knowledge, guessed which operation to use, and found an incorrect solution for the Sports Problem (WP). Jackson’s use of his background knowledge was similar to the work of participants in Pape’s (2004) study.

**Effect of probing questions.** The reluctant problem solvers demonstrated more persistence when solving problems (CCSSI, 2010) and higher self-efficacy when asked probing questions while working (Alter et al., 2011). For example, Destiny and her partner were able to solve the Tables Problem (WP) when asked probing questions and encouraged to create a drawing to represent the tables in the problem (Abdullah et al., 2012). Her work during the session indicated that the use of probing questions to prompt her thinking combined with her use of drawings to solve the problem increased her self-efficacy. Additionally, the use of probing questions while Jackson and Destiny worked the Pattern Block Quilts Task (MTW), during separate sessions, facilitated their thinking and helped them to reach a correct solution, raising their self-efficacy toward the task. Finally, during Jackson and Destiny’s work on the Lemonade Problem (WP), the use of probing questions, again, helped them to successfully solve the problem, meeting their need for competence (Ross & Bergin, 2011).

Jackson and Destiny worked the Pattern Block Quilts Task with different partners, but in both sessions, the use of probing questions facilitated their thinking and helped them to reach the correct solution to the task. In a later session, Jackson and Destiny worked the Lemonade Problem (WP) together and the use of probing questions, again, helped them to successfully
solve the problem (Alter et al., 2011), increasing their self-efficacy and meeting their need for competence (Ross & Bergin, 2011).

**False sense of self-efficacy.** Upon completion of the problem-solving sessions, I did not tell the reluctant problem solvers if their solutions to the tasks were correct or incorrect. After some sessions, participants were certain that their answer was correct, though it was incorrect. The strategy of listing numbers from the problems and performing an operation with them, as previously described, led to incorrect solutions, but the participants did not know that their solutions were incorrect. Instead, they had a sense of success because they found a solution to the problems. Their sense of success led to an increase in self-efficacy and met their need for competence. However, theirs was a false sense of success and a false increase in self-efficacy, because the feelings were based on incorrect solutions.

**Low self-efficacy.** In one session, Destiny displayed low self-efficacy when faced with the Meals Problem (WP). The inclusion of fractions and measurement in the problem led her to believe that it would be too challenging for her to complete, thus indicating that she had low self-efficacy toward the problem, and it would not meet her need for competence. Witt (2012) stated that even the presence of digits could trigger mathematics anxiety in students. The mere existence of fractions and measurement in the problem appeared to trigger mathematics anxiety (Akin & Kurbanoglu, 2011; Chinn, 2009; Waddlington & Waddlinton, 2008) in Destiny, leading her to choose to work the mathematical task without words instead. Her self-efficacy toward the Meals Problem (WP) was not enough to motivate her to attempt the problem with measurements and/or fractions. Potentially experiencing success in working problems about measurements and fractions could build her self-efficacy and meet her need for competence, though her reaction
after working the Lemonade Problem (WP) demonstrated that success might not always lead to a feeling of competence.

At times, participants appeared to have low self-efficacy (Hoffman, 2010) and high mathematics anxiety (Hoffman, 2010; Lee, 2009) toward mathematical tasks without words, including Jackson’s experience working on the Triangles Problem (MTW) and the Four 4s Problem, (MTW). Unlike their descriptions of word problems, participants did not say that the mathematical tasks without words provided clues or told them how to work the tasks, indicating low levels of self-efficacy if clues were not apparent. If participants had not been exposed to mathematical tasks without words similar to those utilized during the study, or had not had experiences of success in working similar tasks, they would not have the self-efficacy required to participate in the problem-solving process to solve the tasks (Bandura & Schunk, 1981). Without enough self-efficacy needed to engage in problem solving for specific tasks, the participants avoided engaging in the tasks or spent less time persisting in finding a solution when they did engage in problem solving for the tasks (Akin & Kurbanoglu, 2011; Bandura & Schunk, 1981; Hoffman, 2010; Pajares & Miller, 1994). Self-efficacy and mathematics anxiety are related (Ashcraft & Moore, 2009; Hoffman, 2010) and might have worked together against the reluctant problem solvers with low levels of self-efficacy and high levels of mathematics anxiety when faced with the challenge of the mathematical tasks without words.

If after working the tasks participants were not able to find a solution, were not certain that their solution was correct, or spent much time struggling to find a solution, they found the task to be difficult and did not appear to have a feeling of success after completing it. For example, though Jackson and Destiny reached the correct solution to the Lemonade Problem (WP), Destiny still described the problem as “difficult.” Further, though Jackson reached several
solutions to the Triangles Problem (MTW), he described it as challenging also. This struggle likely decreased the participants’ self-efficacy toward the problem and hindered the meeting of their need for competence.

**Expectation of Struggle when Problem Solving**

Regardless of the type of task worked, reluctant problem solvers indicated that the tasks were easy if they found a solution and difficult if they did not find a solution or struggled to find a solution. In general, these findings support the work of Bandura and Schunk (1981) who stated that individuals who are exposed to tasks and, in particular, experience success in working tasks, develop higher levels of self-efficacy related to those tasks. Additionally, experiencing success when working tasks meets the need for competence (Deci & Ryan, 2000; Ross & Bergin, 2011). Alternatively, when students experience failure, their levels of self-efficacy decrease and their mathematics anxiety increases (Ashcraft & Moore, 2009; Hoffman, 2010) and they develop negative feelings toward mathematics (Akin & Kurbanoglu, 2011) and even avoid mathematics (Akin & Kurbanoglu, 2011; Hoffman, 2010) or do not persevere when attempting to problem solve.

If the reluctant problem solvers struggled to find a solution, they were not as persistent in working the tasks and said that they were difficult to work. The findings supported the work of Bandura and Schunk (1981) and Akin and Kurbanoglu (2011) who found that self-efficacy affects persistence level when working mathematics tasks. The lack of persistence and frustration on the mathematical tasks without words deemed difficult by the reluctant problem solvers could indicate low levels of self-efficacy toward those particular tasks. The findings suggest that the reluctant problem solvers do not expect to struggle when problem solving, making them sometimes unwilling to persevere when tasks are challenging. This is not desirable, because
students are expected to “make sense of problems and persevere when solving them” (CCSSI, 2010, p. 6). Persevering involves struggling.

**Effects of Partner’s Self-Efficacy**

The reluctant problem solvers displayed different levels of self-efficacy toward the tasks when paired with different partners. Jackson appeared to take on the opposite role of his partner. For example, if his partner was less-reluctant and appeared to have high self-efficacy, Jackson displayed more characteristics of a reluctant problem solver and appeared to have low self-efficacy. Destiny, however, took on the same role as her partner. If her partner was a less-reluctant problem solver, with high self-efficacy, she displayed fewer characteristics of a reluctant problem solver, and appeared to have high self-efficacy as well. If her partner was another reluctant problem solver, she displayed more behaviors of a reluctant problem solver and lower self-efficacy. The findings indicated that the reluctant problem solvers’ levels of self-efficacy changed depending upon the self-efficacy of their partner, however, not in the same manner. For this reason, more research should be conducted to determine which characteristics of partners best motivate reluctant problem solvers to become less reluctant and engage in the mathematics.

**Implications and Recommendations**

The current study explored two reluctant problem solvers’ perceptions of mathematical tasks without words and word problems. The following sections discuss the implications derived from the findings and suggest recommendations for future research.

**Struggle**

It is important to note that the reluctant problem solvers did not expect to have to struggle when problem solving for word problems or mathematical tasks without words. “Students need
to know that a challenging problem will take some time and that perseverance is an important aspect of the problem-solving process and of doing mathematics” (NCTM, 2000, p. 186). Holbert and Barlow (2012/2013) hypothesized that the reluctant problem solvers would be attracted to mathematical tasks without words because they eliminated the reading requirement present in word problems. The participants in the current study, however, had adopted strategies that they used to avoid struggling with word problems, as previously described. The participants’ strategies in solving word problems helped them to reach a solution and feel successful (though sometimes a false success), without having to rely on reading and comprehension skills. These findings suggest two implications. First, because students have adopted strategies for avoiding the struggle of problem solving in word problems, it is essential to expose reluctant problem solvers to mathematical tasks without words, despite their initial perceptions of the tasks, because they did not display strategies for avoiding the struggle of problem solving when working mathematical tasks without words. Second, reluctant problems solvers need to be made aware of the flaws behind their strategies for solving word problems. They need to learn new strategies for solving problems that can be implemented when they face challenging problems and will lead them to correct solutions, meeting their need for competence.

**Initial Perceptions**

The findings from this study imply that when given a choice, reluctant problem solvers will choose the perceived easiest task or assignment to complete. In the current study, however, reluctant problem solvers perceptions’ of the tasks that they worked changed once they began working. The reluctant problem solvers found that they could not judge a task without actually beginning to work it because appearances might be deceiving. Given these findings, reluctant problem solvers should begin by attempting to understand the problem, the first step in the
problem-solving process (Polya, 1957). Then, the reluctant problem solvers could find that by understanding the problem, they are better prepared to devise and carry out a plan for solving the problem (Polya, 1957). This process could allow reluctant problem solvers to meet the expectation that they “make sense of problems and persevere in solving them” (CCSSI, 2010, p.6).

**Distinct Name for Tasks**

During the interview, I asked participants the following question: “If given another opportunity, would you like to work another task like this one? Why or why not?” The intent of the question was to determine if participants enjoyed working the type of task they had chosen enough to work another task of the same type, or if they would switch to the other type. For example, if participants had chosen to work a mathematical task without words during one session, the question sought to determine if they would want to work another mathematical task without words during another session. The participants, however, did not seem to interpret the question as asked. In many instances, the participants answered the question saying that they would not want to work the task again because they had just worked it. They interpreted the question to be asking if they wanted to work the same task again, instead of asking if they wanted to work the same type of task again. When conducting future interviews with participants, it would be helpful to provide students with names for each type of task. For example, it would be helpful to explicitly tell participants that one task is a word problem and the other is a mathematical task without words. During the interview, asking participants if they would rather work a word problem or a mathematical task without words might elicit the type of answers that were intended in the question previously mentioned, but not received.
**Problem Context**

In two instances, Jackson chose a particular task because the problem context was relevant and/or important to him. For example, Jackson chose to work the Sports Problem (WP) because he liked sports. Additionally, Jackson chose to work the Lemonade Problem (WP) because he liked lemonade and wanted to see which container of lemonade in the problem was the better buy. This finding indicates that it is necessary to consider problem context when working with reluctant problem solvers. For future research, both the mathematical task without words and the word problem offered during one session should have the same problem context (for example, both tasks might pertain to sports) removing that as a variable.

**Pairings**

Findings from the study imply that pairing reluctant problem solvers with different partners could help motivate them to display fewer characteristics of reluctant problem solvers. In the current study, findings varied between the two cases, so more research should be conducted to determine what characteristics of partners support reluctant problem solvers’ engagement in mathematics. Including more participants in the study could potentially provide more insight into the question.

**Research Timing**

Research should be conducted as close to the beginning of the school year as possible before classroom norms are established. Once students have been in school for a few months, the teacher has had time to share his or her expectations of students in the classroom, including expectations for participating and engaging in the mathematics. Students who are reluctant to engage in problem solving, potentially, would display more characteristics of reluctant problem solvers at the beginning of the school year before norms are established, therefore making it
easier to identify them. Later in the school year, reluctant problem solvers might have become less reluctant depending upon their experiences in the mathematics classroom. Consequently, research on reluctant problem solvers should be conducted near the beginning of the school year. Additionally, observations in the classroom should be conducted before and after conducting problem-solving sessions to examine the affects of the problem-solving sessions in relation to student engagement in the classroom.

**Summary**

Potentially, meeting students’ needs for autonomy, relatedness, and competence (Deci & Ryan, 2000; Ryan & Deci, 2000b) could help reluctant problem solvers (Holbert & Barlow, 2012/2013) become less reluctant to engage in mathematics. This study met participants’ needs of autonomy and relatedness as previously described, and explored the potential of mathematical tasks without words and word problems for meeting the need of competence. To determine how to meet reluctant problem solvers’ need for competence, this qualitative study explored their perceptions of certain mathematical tasks without words and word problems. The findings indicated that reluctant problem solvers are drawn to mathematical tasks without words or word problems when they have the self-efficacy needed to work them. Additionally, the reluctant problem solvers’ perceptions of the tasks being easy or difficult based on their struggle when solving indicated that they do not desire to be challenged, a natural occurrence when problem solving (NCTM, 2000). Instead, the reluctant problem solvers had adopted strategies for circumventing problem solving when faced with word problems, making the word problems more appealing since participants had not adopted similar strategies to avoid problem solving when faced with mathematical tasks without words. Therefore, mathematical tasks without
words should be useful in supporting students’ engagement in problem solving and, potentially, persistence in problem solving since the participants thought that the tasks were fun.

The literature lacked research on mathematical tasks without words, specifically on reluctant problem solvers’ perceptions of mathematical tasks without words. The literature also lacked research on reluctant problem solvers’ perceptions of word problems. Therefore, the current study adds to the literature by providing the perceptions of reluctant problem solvers pertaining to mathematical tasks without words and word problems. Additionally, the study provides a link between reluctant problem solvers, the self-determination theory, and self-efficacy, which does not currently exist in the literature.

The goal of this study was to answer the research question: How do reluctant problem solvers perceive mathematical tasks without words and word problems? Answers to this question provided insight into the perceptions of reluctant problem solvers, which can then be used to conduct more research to determine how to help these students become less reluctant in the mathematics classroom and engage in problem solving. “By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom” (NCTM, 2000, p. 52). Essentially, the motive for helping students become less reluctant in the mathematics classroom is to help them become problem solvers, to achieve success in mathematics in this era of standards and accountability, and to achieve success in life.
LIST OF REFERENCES
REFERENCES


LIST OF APPENDICES
APPENDIX A: PARTICIPANT REFLECTION PROMPT
Participant Reflection Prompt

Today I chose the ____________________________ problem to work because . . .

Please include in your response:
- What you liked about the problem
- What you didn’t like about the problem
- If you could choose again, would you choose the same problem? Why or why not?
APPENDIX B: INTERVIEW PROTOCOL
Interview Protocol

Date: 

Time: 

Place: 

Interviewer: Sydney Margaret Holbert

Interviewee:

Interview Questions:

1. What made you select this task instead of the other one?

2. What, if anything, did you like about the task that you just completed?

3. What, if anything, did you not like about the task that you just completed? Was anything frustrating?

4. What made you want to work this task or not want to work it?

5. How would you describe a word problem?

6. Here is an example of a word problem: “Ms. Strong wants to make peach tarts for her friends who are coming for dinner. She needs two-thirds of a peach to make each tart. She bought a basket of peaches containing 10 peaches. What is the greatest number of tarts that she can make with the 10 peaches that she bought?”

   How was this task like or not like a word problem?

7. If given another opportunity, would you like to work another task like this one? Why or why not?

8. After solving the problem, did you wish you had chosen the other one?

9. Did you feel like this task was any different from the tasks that you normally solve in math class? If yes, how is it different?
10. What would you tell other students about your experiences in working this task?
APPENDIX C: OBSERVATION GUIDE
Observation Guide  
**Behaviors of Reluctant Problem Solvers**

List observed behaviors in the box next to each behavior in the appropriate column.

<table>
<thead>
<tr>
<th>Date</th>
<th>Student</th>
<th>Student</th>
<th>Student</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Delays engaging in the math task</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Relies on group members to do the work for them or tell them what to do</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Asks the teacher to tell them how to begin working</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Distracts themselves and their classmates</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Complains that math is hard or they cannot do the math</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Off task during math class</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Quiet when the rest of the group is working</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX D: BEHAVIOR FREQUENCY CHART
## Behavior Frequency Chart

For each student, please label each behavior with one of the following numbers to identify the frequency of the behavior:

<table>
<thead>
<tr>
<th>Student</th>
<th>Behavior</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Delays engaging in the math task</td>
<td>0 never</td>
</tr>
<tr>
<td></td>
<td>Relies on others to do the work for them/tell them what to do</td>
<td>1 sometimes</td>
</tr>
<tr>
<td></td>
<td>Asks the teacher to tell them how to begin working</td>
<td>2 often</td>
</tr>
<tr>
<td></td>
<td>Distracts themselves and their classmates</td>
<td>3 usually</td>
</tr>
<tr>
<td></td>
<td>Complains that math is hard or they cannot do the math</td>
<td>4 always</td>
</tr>
<tr>
<td></td>
<td>Off task during math class</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Quiet when the rest of the group is working</td>
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**Total**

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<tr>
<td></td>
<td>Off task during math class</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Quiet when the rest of the group is working</td>
<td></td>
</tr>
</tbody>
</table>

**Total**

212
APPENDIX E: ORAL ASSENT FORM
APPENDIX E

CHILD ASSENT FORM

Oral Assent Script with Record of Child’s (Aged 9-12) Response

I would like to ask you to help me with a research project that I am doing through The University of Mississippi. If you agree, you would work some mathematics tasks while being videotaped, and then you would answer some questions that I will ask you about the tasks in an interview. You will be video and audio recorded during the interview. It will take about 30 minutes.

What questions do you have about what you will do?

Will you do this?

Name: ______________________________________________  Date: __________________

Response: ☐ YES  ☐ NO
VITA

Sydney Margaret Holbert was born to John and Dianne Holbert of Jackson, Mississippi on September 22, 1984. She has one older sister, Joanna, and one younger sister, Elizabeth. Sydney Margaret grew up on the Mississippi Gulf Coast and attended schools in Long Beach, Mississippi, where she graduated from Long Beach High School in 2002. She attended Mississippi College in Clinton, Mississippi, where she earned a Bachelor of Science degree in Elementary Education.

Sydney Margaret taught sixth grade mathematics for two years at Lovett Elementary School in Clinton, Mississippi, before moving to Oxford, Mississippi, where she taught fourth grade mathematics and science for one year at Della Davidson Elementary School. During her year teaching in Oxford, she attended graduate school at The University of Mississippi. In 2010, Sydney Margaret earned a Master of Education in Curriculum and Instruction degree from The University of Mississippi.

Upon completion of her master’s degree, Sydney Margaret entered the doctoral program at The University of Mississippi. While taking classes, she tutored student athletes at The University of Mississippi and spent three years serving as a graduate research assistant on the externally funded grant, Project PrIME, housed at the university. Through Project PrIME, she had the opportunity to work with over one hundred teachers in North Mississippi, teaching them during summer institutes and teaching their students in demonstration lessons during the academic school year. She also taught the mathematics methods course to pre-service teachers at
The University of Mississippi. All of her work centered around improving the quality of mathematics instruction in Mississippi.

Sydney Margaret was inducted into Phi Kappa Phi Honor Society in 2010. She received the Lamar Memorial Scholarship in 2011 and again in 2012. She received the Graduate Achievement Award in Curriculum and Instruction and the Outstanding Doctoral Student in Elementary Education Award in 2012. In 2013, she received the Graduate Achievement Award in Teacher Education. Sydney Margaret served as a senator on the Graduate Student Council, member of the Social and Philanthropic Committee on the Graduate Student Council, and mentor for the Graduate Women’s Group Mentorship Program. She also served as a manuscript referee for *Teaching Children Mathematics* published by the National Council of Teachers of Mathematics.

Sydney Margaret’s research interests included exploring the benefits of demonstration lessons with in-service teachers in a school setting and in a summer institute setting. Additionally, her research interests involved working with reluctant problem solvers and exploring their perceptions and experiences in mathematics. She co-authored three manuscripts, published during her doctoral career, and presented at numerous state and national conferences. Sydney Margaret is a member of many professional organizations related to mathematics education.