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Testing Lepton Nonuniversality In Tau Neutrino Scattering

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TESTING LEPTON NONUNIVERSALITY IN TAU NEUTRINO SCATTERING

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ABSTRACT

Recently hints of lepton flavor non-universality emerged in the BaBar and LHCb experiments. In this paper we propose tests of lepton universality in ν_τ scattering. To parametrize the new physics we adopt an effective Lagrangian approach and consider the neutrino deep inelastic scattering processes $\nu_\tau + N \to \tau + X$ and $\nu_\mu + N \to \mu + X$ where we assume the largest new physics effects are in the τ sector. We also consider an explicit leptoquark model in our calculations. In order to make comparison with the standard model and also in order to cancel out the uncertainty of the parton distribution functions, we consider the ratio of total and differential cross sections of tau-neutrino to muon-neutrino scattering. We find new physics effects that can be possibly be observed at the proposed Search for Hidden Particles (SHiP) experiment at CERN.

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CHAPTER 1

INTRODUCTION

The flavor sector of standard model (SM) has many puzzles. A key property of the SM gauge interactions is that they are lepton flavor universal. Evidence for violation of this property would be a clear sign of new physics (NP) beyond the SM. In the search for NP, the second and third generation quarks and leptons could be special because they are comparatively heavier and are expected to be relatively more sensitive to NP. As an example, in certain versions of the two Higgs doublet models (2HDM) the couplings of the new Higgs bosons are proportional to the masses and so NP effects are more pronounced for the heavier generations. Moreover, the constraints on new physics, especially involving the third generation leptons and quarks, are somewhat weaker allowing for larger new physics effects.

Interestingly, there have been some reports of nonuniversality in the lepton sector from experiments. Recently, the BaBar Collaboration with their full data sample has reported the following measurements [1, 2]:

$$\begin{aligned} R(D) &\equiv \frac{\mathcal{B}(\bar{B} \to D^+ \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D^+ \ell^- \bar{\nu}_\ell)} = 0.440 \pm 0.058 \pm 0.042 \ , \\ R(D^*) &\equiv \frac{\mathcal{B}(\bar{B} \to D^{*+} \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D^{*+} \ell^- \bar{\nu}_\ell)} = 0.332 \pm 0.024 \pm 0.018 \ , \end{aligned} \tag{1.1}$$

where $\ell = e, \mu$. The SM predictions are $R(D) = 0.297 \pm 0.017$ and $R(D^*) = 0.252 \pm 0.003$ [1, 3], which deviate from the BaBar measurements by 2σ and 2.7σ, respectively. (The BaBar Collaboration itself reported a 3.4σ deviation from SM when the two measurements of Eq. (1.1) are taken together.) This measurement of lepton flavor nonuniversality, referred to as the $R(D^{(*)})$ puzzles, may be providing a hint of the new physics (NP) believed to exist beyond the SM. There have been numerous analyses examining NP explanations of the $R(D^{(*)})$ measurements [4, 5].

In another measurement the LHCb Collaboration recently measured the ratio of decay rates for $B^+ \to K^+ \ell^+ \ell^-$ ($\ell = e, \mu$) in the dilepton invariant mass-squared range 1 GeV^2 $\le q^2 \le 6$ GeV^2 [6]. They found

$$R_K \equiv \frac{\mathcal{B}(B^+ \to K^+ \mu^+ \mu^-)}{\mathcal{B}(B^+ \to K^+ e^+ e^-)} = 0.745^{+0.090}_{-0.074} \ (\text{stat}) \pm 0.036 \ (\text{syst}) \ , \tag{1.2}$$

is a 2.6σ difference from the SM prediction of $R_K = 1 \pm O(10^{-4})$ [7]. In addition, we note that the three-body decay $B^0 \to K^* \mu^+ \mu^-$ by itself offers a large number of observables in the kinematic and angular distributions of the final-state particles, and it has been argued that some of these distributions are less affected by hadronic uncertainties [8]. Interestingly, the measurement of one of these observables shows a deviation from the SM prediction [9]. However, the situation is not clear whether this anomaly is truly a first sign of new physics [10].

The tau neutrino, ν_τ, was discovered by the DONuT experiment [11] which measured the charged-current (CC) interaction cross section of the tau neutrino. The DONuT central-value results for a ν_τ scattering cross section show deviation from the standard model predictions by about 40% but with large experimental errors; thus, the measurements are consistent with the standard model. The third generation lepton has been explored relatively less than the other two generations and in particular there has not been much investigation of ν_τ properties. One of the predictions of the Standard Model (SM) is that gauge bosons couple to the three generations of leptons universally. A careful test of this prediction is very important and observation of nonuniversality in the interactions of the lepton families would be an important discovery.

In previous publications there was new physics in ν_τ scattering for quasi-exclusive, resonant and DIS scattering [12]. Those papers were more focused on the error in the extraction of neutrino mixing angles in presence of new physics. In this work we study just ν_τ interaction with focus on observables that may be measured at a ν_τ scattering experiment. There is a proposed Search for Hidden Particles (SHiP) experiment at CERN [13] which is expected to have a large sample of ν_τ which could be used to probe new physics in ν_τ scattering. In previous work new physics tensor interactions were not considered. In this work we will be interested at neutrino energies where the DIS component of the scattering process is dominant.

The paper is organized in the following manner.

- Chapter 2 briefly presents an overview of Standard Model.
- In Chapter 3 a description of gauge theory is given.
- Chapter 4 gives a brief description about physics beyond the standard model and introduce an effective Hamiltonian with new physics operators.
- Chapter 5 discusses the DIS process with new physics effects.
- Chapter 6 fixes the constraints of non-standard interaction coupling constants from τ decays. We will consider the decays $\tau \to \pi\nu_\tau$ and $\tau \to \pi\pi\nu_\tau$ which are well measured..
- Chapter 7 present a final results and analyses.
- We present our conclusion in Chapter 8.

CHAPTER 2

THE STANDARD MODEL OVERVIEW

2.1 The particles

The current standard model(SM) of particle is a partially unified quantum gauge field theory for the electromagnetic and weak interactions, which exhibits a broken SU(2)×U(1) symmetry, together with the SU(3) symmetric quantum chromodynamics for the strong interaction. The Stadard Model of particle physics comprises 12 vector boson fields, 45 Weyl fermion fields, and a complex doublet scalar Higgs fields. Our universe is made of those fundamental spin-1/2 fermions, the leptons and quarks, which are listed below.

Generation	First	Second	Third
Leptons	e^-,ν_e	μ^-,ν_μ	τ^-,ν_τ
Quarks	d,u	s,c	b,t

Those fermions interact with each other through the exchange of the spin-1 verctor bosons and through gravitation as described by general relativity, and also through the exchange of some spin-0 scalar Higgs, which play a crucial role in generating mass. The 12 vector bosons $A^a_\mu(x)$ are gauge fields of the local symmetry

$$G = SU(3)_C \times SU(2)_L \times U(1)_Y, \tag{2.1}$$

which are listed below.

mediator	Force	Mass
γ	Eletromagnetic Interaction	0
$W^\pm$,Z^0	Weak Interaction	80.385± 0.015GeV,91.187± 0.0021GeV
g	Strong Interaction	0

The leptons carry "lepton number" $L = +1$, but zero "baryon number", while the quarks have "baryon number" $B = +1/3$, but zero "lepton number". Each particle has an associated antiparticle with the same mass but opposite quantum numbers. The net lepton number and net baryon number appear to be conserved in all the interactions. What's more, in Standard Model, "lepton flavors" is conserved. Therefore, we have three kinds of "lepton number", "electron number" L_e, "muon number" L_μ, and "tau number" L_τ, which are conserved individually.

The weak interaction can mix the quarks in all possible ways consistent with charge conservation. There is thus no analog of the conservation of L_e or L_μ for quarks. But the thing special for quarks is that each of the quarks can exist in three forms distinguished by the so-called "color" quantum number associated with the strong interaction which can take the values "red", "blue", or "green". Mesons which are made of quark and anti-quark and baryons which are made of three quarks have to be color singlet.
There are no known interactions in Standard Model that mix quarks with leptons and hence the total quark number is conserved. This is referred to as baryon number conserved since $B = 3[N(q) - N(\bar{q})]$. This rule is crucial to the stability of protons and hence of matter itself, but it has no deep theoretical basis. Indeed, in new physics, we shall find the interaction violates this rule, thereby giving the proton a finite lifetime.

2.2 The forces

In the standard model these fundamental particles undergo four known types of gauge interaction–gravitation, electromagnetism, and the weak and strong forces.

According to Newton's nonrelativistic theory the gravitational potential energy between particle of mass m_1 and m_2 separated by a distance r is

$$V(r) = -\frac{G_N m_1 m_2}{r}. \tag{2.2}$$

A dimensionless measure of the strength of the gravitational coupling is give by $\frac{G_N m^2}{\hbar c}$, and if we insert a typical mass, such as that of the proton, we find that $\frac{G_N m_p^2}{\hbar c} \approx 10^{-40}$ is so extraordinary small that gravity can safely be neglected in most practical aspects of particle physics. The gravitational force is carried by graviton. Since the gravitational force is of infinite range, these gravitons must be massless, and because in general relativity the quantum field represents fluctuations of the rank-2 metric of space time $g_{\mu\nu}$, it must have spin 2.
According to Coulomb's law, the interaction potential between particles with charges Q_1 and Q_2 respectively, separated by a distance r is

$$V(r) = \frac{Q_1 Q_2}{4\pi\epsilon_0 r}. \tag{2.3}$$

A convenient dimensionless measure of the electromagnetic coupling is $\alpha \equiv \frac{e^2}{4\pi\epsilon_0 \hbar c} \approx \frac{1}{137}$. The quantum of the electromagnetic field is of course the photon γ. Since this force is also of infinite range, the photon must be massless, which is consistent with the Standard Model. As we should see later, photon represents the U(1) gauge-invariant rank-1 electromagnetic potential A_μ, it must have spin 1.

We don't have a classical equation to describe weak interaction and strong interaction. The weak interaction is of very short range. The strength is given by the Fermi constant G_F. It is a universal interaction in the sense that all quarks and leptons have the same overall weak coupling strength. The dimensionless coupling for a particle of typical hadronic mass m_p is $\frac{G_F m_p^2 c^4}{(\hbar c)^3} \approx 10^{-5}$. In fact, the weak interaction coupling g is comparable to e(see eq.(3.47)). But according to the Feynman rule $G_F \sim \frac{g^2}{M_W^2} \sim \frac{e^2}{M_W^2}$, it is obvious that the reason why weak interaction is so much weaker than electromagnetic interaction is that the mediator is very massive. Parity is not conserved in weak interaction because these W bosons couple only to the left-handed chiral projections of the quark and lepton fields.

Finally, we come to the strong force that bind quarks together to form hadrons, and hadrons into complex neclei. This force is associated with the color coupling. As we will mention later, the nature of the strong force can be deduced simply by demanding that it should obey an exact local SU(3) color gauge symmetry, analogous to the U(1) invariance of electromagnetism. The massless quanta of this quantum chromodynamics(QCD) color field are called "gluons". Since the gluons carry color they can interact with each other. The strength of the coupling $g_s(M_Z) = 1.214$ gives the probability of a quark or gluon emitting a gluon, and it is convenient to introduce $\alpha_s \equiv \frac{g_s^2}{4\pi}$, indicating the strength of strong interaction. The value of coupling constants of three fundamental force are given in the Table 2.1.

In summary, in order to account for all the known types of interactions, we need the spin-2 graviton and the 12 spin-1 gauge bosons. The standard model of particles is thus gravitation, together with the above $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariant strong and electroweak interactions. After the spontaneous symmetry breaking as a result of the Higgs coupling, we are left with $SU(3)_C \times U(1)_{EM}$ as exact gauge symmetries, and the gluons and the photon as massless particles. In the next chapter, we will present more details about gauge theory.

Electric interaction	Weak interaction	Strong interaction
$g_e = e = \sqrt{4\pi\alpha} = 0.302822$	$g_W = \frac{g_e}{sin\theta_W}$=0.6295, $g_Z = \frac{g_W}{cos\theta_W} = 0.7180$	$g_s = 1.214$

Table 2.1: The coupling constant of three fundamental force at M_Z scale

CHAPTER 3

GAUGE THEORY

All of the quantum field theories that have been successful in describing the fundamental interactions of nature are "gauge theories," that is to say that they are invariant under gauge transformations of the field potentials. This property has long been recognized in classical electromagnetism. It also turned out to be the key to the development of QCD for the strong color interaction, and to the formulation of the unified electroweak theory, too. This chapter gives a brief introduction to such theories. And we will see how to obtain interaction Lagrangian from the requirement of local gauge invariance.

3.1 QED

Quantum electrodynamics(QED) is the quantum theory of the interactions of charged particles. Its free-field Lagrangian density

$$\mathscr{L}_0 = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x) \tag{3.1}$$

is invariant under global phase transformation

$$\begin{aligned} \psi(x) \to \psi'(x) &= \psi(x)e^{-i\alpha}, \\ \bar{\psi}(x) \to \bar{\psi}'(x) &= \bar{\psi}(x)e^{i\alpha}, \end{aligned} \tag{3.2}$$

where α is a real number. According to Noether's theorem, this invariance ensures current conservation. The current

$$\begin{aligned} s^\mu(x) &= \frac{\partial \mathscr{L}}{\partial \psi_{r,\mu}} \delta\psi_r \\ &= q\bar{\psi}(x)\gamma^\mu \psi(x) \end{aligned} \tag{3.3}$$

satisfies $\partial_\mu s^\mu(x) = 0$, so that the charge

$$Q = \int d^3x s^0 x = q \int d^3x \psi^\dagger(x)\psi(x) \tag{3.4}$$

is conserved.
Now, let us extent to a set of local phase transformations

$$\begin{aligned} \psi(x) \to \psi'(x) &= \psi(x)e^{-iqf(x)}, \\ \bar{\psi}(x) \to \bar{\psi}'(x) &= \bar{\psi}(x)e^{iqf(x)}. \end{aligned} \tag{3.5}$$
$$\tag{3.6}$$

To restore invariance, we need to introduce the covariant derivative D_μ and a gauge boson $A_\mu(x)$

$$D_\mu\psi(x) = [\partial_\mu + iqA_\mu(x)]\psi(x), \tag{3.7}$$

where the gauge boson transforms under a gauge transformation in the usual way:

$$A_\mu(x) \to A'_\mu(x) + \partial_\mu f(x). \tag{3.8}$$

Then, we obtain the familiar $U(1)$ local gauge invariant Lagrangian of QED:

$$\begin{aligned} \mathscr{L} &= \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x), \\ &= \mathscr{L}_0 - \mathscr{L}_I, \end{aligned} \tag{3.9}$$

where

$$\mathscr{L}_I = q\bar{\psi}\gamma^\mu\psi(x)A_\mu(x) \tag{3.10}$$

which is the interaction of the charged particle and gauge boson, photon. For completeness, we have to include the Lagrangian in the absence of charge

$$\mathscr{L}_F = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x), \tag{3.11}$$

where

$$F^{\mu\nu}(x) \equiv \partial^\nu A^\mu(x) - \partial^\mu A^\nu(x). \tag{3.12}$$

Therefore, the total Lagrangian is

$$\mathscr{L} = \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{3.13}$$

3.2 QCD

Quantum chromodynamics(QCD) is a quantum field theory for describing strong interaction. We can formulate QCD as a gauge theory with symmetry group $SU(3)$ in analogy to QED. But instead, we have an SU(3) color charge(red, green and blue) conservation rather than electric charge. Just as the spin wavefunctions are acted on by spin operators which can be represented by two-dimensional Hermitian matrices, the color wavefunctions are acted on by 'colour operators' which can be represented by three-dimensional Hermitian matrices. Apart from the unit matrix, there are eight linearly independent Hermitian matrices in three dimensions. These are conveniently chosen to be

$$\hat{F}_i = \frac{1}{2}\lambda_i \;\; (i = 1, 2, ..., 8), \tag{3.14}$$

where λ_i $(i = 1, 2, ..., 8)$ are eight linearly independent Hermitian matrices in three dimension, given as follow

$$\begin{aligned} \lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \\ \lambda_5 &= \begin{pmatrix} 0 & 0 & -i \\ 1 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \end{aligned} \tag{3.15}$$

The operators $\hat{F}_i (i = 1, 2, ..., 8)$ are called colour operators and satisfy commutation relations of the form

$$[\hat{F}_i, \hat{F}_j] = i f_{ijk} \hat{F}_k. \tag{3.16}$$

The structure constant f_{ijk} are numbers which are totally antisymmetric.

ijk	123	147	156	246	257	345	367	458	678
f_{ijk}	1	1/2	-1/2	1/2	1/2	1/2	-1/2	$\sqrt{3}/2$	$\sqrt{3}/2$

Table 3.1: Some of the coefficients in the commutation relations (3.16). All the non-zero coefficients can be deduced from these by using the fact that f_{ijk} is totally antisymmetry

Because the matrices λ_j are not all diagonal, and thus they are not all commute with each other, this interaction can annihilate quarks of one colour r,g,b and create quarks of a differnet colour. Hence by colour conservation, the quanta of the gauge fields $A_j^\mu(x)$-called gluons - must also have non-zero colour charges. This is contrast to QED, where the interactions are transmitted by photons which have zero eletric charge. So, the gluons can interact with themselves. Those kinds of groups are called non-abelian group.
We next find a set of symmetry transformations which lead to the conservation of the colour charges, rather than electric charge. In order to do this, we start from the free quark Lagrangian density

$$\begin{aligned} \mathscr{L}_0 &= [\bar{\psi}_r^f(x)(i\partial\!\!\!/ - m_f)\psi_r^f(x) + \bar{\psi}_g^f(x)(i\partial\!\!\!/ - m_f)\psi_g^f(x) + \bar{\psi}_b^f(x)(i\partial\!\!\!/ - m_f)\psi_b^f(x)], \\ &= \bar{\Psi}^f(x)(i\partial\!\!\!/ - m_f)\Psi^f(x), \end{aligned} \tag{3.17}$$

where$\Psi^f(x) = \begin{pmatrix} \psi_r^f(x) \\ \psi_g^f(x) \\ \psi_b^f(x) \end{pmatrix}$. And the Lagrangian density is invariant under the global transformation

$$\begin{aligned} \Psi^f(x) \to \Psi^{f\prime}(x) &= U(\alpha)\Psi^f(x) \equiv e^{i\alpha_i\lambda_i/2}\Psi^f(x), \\ \bar{\Psi}^f(x) \to \bar{\Psi}^{f\prime}(x) &= \bar{\Psi}^f(x)U^\dagger(\alpha) \equiv \bar{\Psi}^f(x)e^{-i\alpha_i\lambda_i/2}, \end{aligned} \tag{3.18}$$

where λ_i are the matrices (3.15) and the α_i are eight arbitrary real numbers. Now using the Eq.(3.3),we can get eight conserved currents

$$S_i^\mu(x) = \frac{1}{2}\bar{\Psi}^f(x)\gamma^\mu\lambda_i\Psi^f(x) \ \ (i = 1, 2, ..., 8). \tag{3.19}$$

And eight corresponding conserved charges

$$\hat{F}_i \equiv \int d^3\mathbf{x}S_i^0(x) = \frac{1}{2}\int d^3\mathbf{x}\Psi^{f\dagger}(x)\lambda_i\Psi^f(x) \ \ (1 = 1, 2..., 8), \tag{3.20}$$

which are called colour charges also satisfying the commutation relation (3.16). We next generalize the SU(3) transformations (3.18) from global to local phase transformations. In order to retain invariance under local phase transformations, we shall have to introduce gauge fields $A_j^\mu(x)$, ang this will automatically generate the interactions. The local transformations are

$$\begin{aligned} \Psi^f(x) \to \Psi^{f\prime}(x) &= exp[ig_s\lambda_j\omega_j(x)/2]\Psi^f(x), \\ \bar{\Psi}^f(x) \to \bar{\Psi}^{f\prime}(x) &= \bar{\Psi}^f(x)exp[-ig_s\lambda_j\omega_j(x)/2]. \end{aligned} \tag{3.21}$$

Here $\omega_j(x)$ are arbitary real differentiable functions and g_s is a real coupling constant given in the Table 2.1. Then the lagrangian denstity goes to

$$\mathscr{L}_q(x) = \bar{\Psi}^f(x)(i\not{D} - m_f)\Psi^f(x) = \mathscr{L}_0 + \mathscr{L}_I \tag{3.22}$$

where

$$\begin{aligned} D_\mu\Psi^f(x) &= [\partial^\mu + ig_s\lambda_j A_j^\mu(x)/2]\Psi^f(x), \\ \mathscr{L}_I &= -\frac{1}{2}g_s\bar{\Psi}^f(x)\gamma_\mu\lambda_j\Psi^f(x)A_j^\mu(x). \end{aligned} \tag{3.23}$$

And

$$A_j^\mu(x) \to A_j^{\mu\prime}(x) \equiv A_j^\mu(x) - \partial^\mu\omega_i(x) - g_s f_{ijk}\omega_j(x)A_k^\mu(x), \tag{3.24}$$

where f_{ijk} are the structure constants of SU(3). Their values are in the Table 3.1.
Now let us write another part of QCD Lagrangian density containing terms which describe the gluons when no quarks are present.

$$\mathscr{L}_G = -\frac{1}{4}G_{i\mu\nu}(x)G_i^{\mu\nu}(x), \tag{3.25}$$

where

$$G_i^{\mu\nu}(x) \equiv F_i^{\mu\nu}(x) + g_s f_{ijk} A_j^\mu(x) A_k^\nu(x),$$
$$F_i^{\mu\nu}(x) \equiv \partial^\nu A_i^\mu(x) - \partial^\mu A_i^\nu(x) \tag{3.26}$$

We can see that comparing to QED, we have an additional term contains the structure constant f_{ijk} and strong interaction coupling constant g_s. Then quantum chromodynamics is the theory defined by the Lagrangian density

$$\mathscr{L} = \bar{\Psi}^f(x)(i\not{D} - m_f)\Psi^f(x) - \frac{1}{4}G_{i\mu\nu}(x)G_i^{\mu\nu}(x). \tag{3.27}$$

Now, let us summarize some important physical properties of QCD which can be inferred directly from the Lagrangian density (3.27). We start by considering the gluons. On substituting Eq.(3.26) in Eq.(3.25) one obtains

$$\mathscr{L}_G = -\frac{1}{4}F_{i\mu\nu}(x)F_i^{\mu\nu}(x) + g_s f_{ijk} A_{i\mu}(x) A_{j\nu}(x) \partial^\mu A_k^\nu(x) \tag{3.28}$$
$$-\frac{1}{4}g_s^2 f_{ijk} f_{ilm} A_j^\mu(x) A_k^\nu(x) A_{l\mu}(x) A_{m\nu}(x) \tag{3.29}$$

for the purely gluonic contribution to the Lagrangian density. The first term in Eq.(3.29) is the Lagrangian density for eight non-interacting massless spin 1 gluon fields. As already noted, these particles have non-zero colour charges and hence cannot be observed as isolated free particles because of colour confinement. In contrast, the second and third terms represent interactions of gluon field amongst themselves. In perturbation theory, they generate three- and four-line vertices. These gluon self-interactions are the triking features of the theory. They arise because the gluons, which transmit the interaction between colour charges, themselves have non-zero colour charges.

3.3 Weak Interaction

We shall now apply the approach outlined in the last section to formulate the theory of weak interaction as a gauge theory. To begin with, we shall assume that all leptons are massless. Then the free-lepton Lagrangian density is

$$\begin{aligned}\mathscr{L}_0 &= i[\bar{\psi}_l(x)\not{\partial}\psi_l(x) + \bar{\psi}_{\nu l}(x)\not{\partial}\psi_{\nu l}(x)],\\ &= i[\bar{\Psi}_l^L(x)\not{\partial}\Psi_l^L(x) + \bar{\psi}_l^R(x)\not{\partial}\psi_l^R(x) + \bar{\psi}_{\nu l}^R(x)\not{\partial}\psi_{\nu l}^R(x)],\end{aligned} \tag{3.30}$$

where $\Psi_l^L(x) = \begin{pmatrix} \psi_{\nu l}^L(x) \\ \psi_l^L(x) \end{pmatrix}$. For the two-component left-handed fields which we call weak isospinor, the global SU(2) transformations are

$$\begin{aligned} \Psi_l^L(x) \to \Psi_l^{L\prime}(x) &= U(\alpha)\Psi_l^L(x) \equiv e^{i\alpha_j\tau_j/2}\Psi_l^L(x), \\ \bar{\Psi}_l^L(x) \to \bar{\Psi}_l^{L\prime}(x) &= \bar{\Psi}_l^L(x)U^\dagger(\alpha) \equiv \bar{\Psi}_l^L(x)e^{-i\alpha_j\tau_j/2}, \end{aligned} \tag{3.31}$$

where τ_j are Pauli spin matrices

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{3.32}$$

with the commutation relations $[\tau_i, \tau_j] = 2i\epsilon_{ijk}\tau_k$ and $\alpha \equiv (\alpha_i, \alpha_j, \alpha_k)$ are three real numbers. ϵ_{ijk} is the usual completely antisymmetric tensor.

The one-component right-handed fields which we called weak isoscalar are singlet under any SU(2) transformation. Now let us consider the global U(1) transformations for weak isospinor and weak isoscalar. The free-lepton Lagrangian density is invariant under the transformations

$$\begin{aligned} \Psi_l^L(x) \to \Psi_l^{L\prime}(x) &= e^{-i\beta/2}\Psi_l^L(x), \\ \psi_l^R(x) \to \psi_l^{R\prime}(x) &= e^{-i\beta}\psi_l^R(x), \\ \psi_{\nu l}^R(x) \to \psi_{\nu l}^{R\prime}(x) &= \psi_{\nu l}^R(x), \\ \bar{\Psi}_l^L(x) \to \bar{\Psi}_l^{L\prime}(x) &= \bar{\Psi}_l^L(x)e^{i\beta/2}, \\ \bar{\psi}_l^R(x) \to \bar{\psi}_l^{R\prime}(x) &= \bar{\psi}_l^R(x)e^{i\beta}, \\ \bar{\psi}_{\nu l}^R(x) \to \bar{\psi}_{\nu l}^{R\prime}(x) &= \bar{\psi}_{\nu l}^R(x). \end{aligned} \tag{3.33}$$

Eq.(3.31) leads to the three conserved currents

$$J_i^\alpha(x) = \frac{1}{2}\bar{\Psi}_l^L(x)\gamma^\alpha\tau_i\Psi_l^L(x), \;\; i = 1, 2, 3, \tag{3.34}$$

which are called weak isospin currents. And the three corrsponding conservd charges

$$I_i^W = \int d^3\mathbf{x}J_i^0(x) = \frac{1}{2}\int d^3\mathbf{x}\Psi_l^{L\dagger}(x)\tau_i\Psi_l^L(x), \;\; i = 1, 2, 3, \tag{3.35}$$

which are called weak isospin charges. Eq.(3.33) can give us one conserved current, which is weak hypercharge curren as follow

$$\begin{aligned} J_Y^\alpha(x) &= -\frac{1}{2}\bar{\Psi}_l^L(x)\gamma^\alpha\Psi_l^L(x) - \bar{\psi}_l^R(x)\gamma^\alpha\psi_l^R(x), \\ &= s^\alpha(x)/e - J_3^\alpha(x), \end{aligned} \tag{3.36}$$

where $s^\alpha(x) = -e\bar{\psi}_l(x)\gamma^\alpha\psi_l(x)$ is the electromagnetic current. And the corresponding charge

$$Y = \int d^3\mathbf{x} J_Y^0(x), \tag{3.37}$$

is called the weak hypercharge. We see from Eq.(3.36) that Y is related to the electric charge Q and the weak isocharge I_3^W by $Y = Q/e - I_3^W$.

We next generalize the above SU(2) and U(1) transformations from global to local phase transformations. Let us start with the SU(2) transformations and replace the global transformations (3.31) by the local phase transformations

$$\begin{aligned}\Psi_l^L(x) \to \Psi_l^{L\prime}(x) &= e^{ig\tau_j\omega_j(x)/2}\Psi_l^L(x),\\ \bar{\Psi}_l^L(x) \to \bar{\Psi}_l^{L\prime}(x) &= \bar{\Psi}_l^L(x)e^{-ig\tau_j\omega_j(x)/2},\end{aligned} \tag{3.38}$$

where $\omega_j(x),\ \ j = 1, 2, 3$, are three arbitrary real differential functions of x, and g is a coupling constant. And the corresponding local transformations for U(1) is

$$\begin{aligned}\psi(x) \to \psi'(x) &= exp[ig'Yf(x)]\psi(x),\\ \bar{\psi}(x) \to \bar{\psi}'(x) &= \bar{\psi}(x)exp[-ig'Yf(x)],\end{aligned} \tag{3.39}$$

where g' is a coupling constant, $f(x)$ is an arbitrary real differentiable function, and $Y = -\frac{1}{2}, -1, 0$ is the weak hypercharge associated with the fields $\Psi_l^L(x), \psi_l^R(x)$ and $\psi_{\nu l}^R(x)$ respectively. To restore the invariance of Lagrangian density under local SU(2) × U(1) gauge transformations, the leptonic Lagrangian density goes to

$$\mathscr{L}^L = i[\bar{\Psi}_l^L(x)\not{D}\Psi_l^L(x) + \bar{\psi}_l^R(x)\not{D}\psi_l^R(x) + \bar{\psi}_{\nu l}^R(x)\not{D}\psi_{\nu l}^R(x)],$$

where we replace the ordinary derivatives by covariant derivatives

$$\begin{aligned}D^\mu\Psi_l^L(x) &= [\partial^\mu + ig\tau_i W_i^\mu(x)/2 - ig'B^\mu(x)/2]\Psi_l^L(x),\\ D^\mu\psi_l^R(x) &= [\partial^\mu - ig'B^\mu(x)]\psi_l^R(x),\\ D^\mu\psi_{\nu l}^R(x) &= \partial^\mu\psi_{\nu l}^R(x),\end{aligned} \tag{3.40}$$

with the transformations of the gauge fields,

$$\begin{aligned}W_i^\mu(x) \to W_i^{\mu\prime}(x) &= W_i^\mu(x) - \partial^\mu\omega_i(x) - g\epsilon_{ijk}\omega_j(x)W_k^\mu(x),\\ B^\mu(x) \to B^{\mu\prime}(x) &= B^\mu(x) - \partial^\mu f(x),\end{aligned} \tag{3.41}$$

where the gauge fields $W_i^\mu(x)$ are invariant under U(1) gauge transformations and $B^\mu(x)$ is is invariant under SU(2) gauge transformations.

We can write the Lagrangian density (3.40) in the form

$$\mathscr{L}^L = \mathscr{L}_0 + \mathscr{L}_I, \tag{3.42}$$

where $\mathscr{L}_0$ is the free-lepton Lagrangian density and

$$\mathscr{L}_I = -gJ_i^\mu(x)W_{i\mu}(x) - g'J_Y^\mu(x)B_\mu(x) \tag{3.43}$$

represents the interaction of the weak currents and the weak hypercharge current with the gauge fields $W_{i\mu}(x)$ and $B_\mu(x)$. In order to make the interaction to be in accord with IVB theory, we shall write the weak isospin currents $J_1^\mu(x)$ and $J_2^\mu(x)$ in terms of the charged leptonic currents

$$\begin{aligned} J^\mu(x) &= 2[J_1^\mu(x) - iJ_2^\mu(x)] = \bar{\psi}_l(x)\gamma^\mu(1-\gamma^5)\psi_{\nu l}(x), \\ J^{\mu\dagger}(x) &= 2[J_1^\mu(x) + iJ_2^\mu(x)] = \bar{\psi}_{\nu l}(x)\gamma^\mu(1-\gamma^5)\psi_l(x). \end{aligned} \tag{3.44}$$

And introduce the non-Hermitian gauge field

$$W_\mu(x) = \frac{1}{\sqrt{2}}[W_{1\mu}(x) - iW_{2\mu}(x)] \tag{3.45}$$

and its adjoint $W_\mu^\dagger(x)$ in place of $W_{1\mu}(x)$ and $W_{2\mu}(x)$. In the remaining two terms of $\mathscr{L}_I$, we write $W_{3\mu}(x)$ and $B_\mu(x)$ as linear combination of two different Hermitian fields $A_\mu(x)$ and $Z_\mu(x)$, defined by

$$\begin{aligned} W_{3\mu}(x) &= cos\theta_W Z_\mu(x) + sin\theta_W A_\mu(x), \\ B_\mu(x) &= -sin\theta_W Z_\mu(x) + cos\theta_W A_\mu(x), \end{aligned} \tag{3.46}$$

where the angle θ_W is the weak angle and in order to include electromagnetic interaction in this Lagrangian, we required

$$gsin\theta_W = g'cos\theta_W = e. \tag{3.47}$$

Then we obtain the final expression for the interaction Lagrangian density

$$\mathscr{L}_I = -s^\mu(x)A_\mu(x) - \frac{g}{2\sqrt{2}}[J^{\mu\dagger}(x)W_\mu(x) + J^\mu(x)W_\mu^\dagger(x)] - \frac{g}{cos\theta_W}[J_3^\mu(x) - sin^2\theta_W s^\mu(x)/e]Z_\mu(x). \tag{3.48}$$

The first term in Eq.(3.48) is the familiar interaction of QED. The second term is just the IVB interaction Lagramgian density. The quanta of the gauge field $W(x)$ are just the $W^\pm$ vector bosons. The third term represents a neutral current

$$\begin{aligned} J_3^\mu(x) &- sin^2\theta_W s^\mu(x)/e \\ &= \frac{1}{4}\bar{\psi}_{\nu l}(x)\gamma^\mu(1-\gamma^5)\psi_{\nu l}(x) - \frac{1}{4}\bar{\psi}_l(x)\gamma^\mu[(1-4sin^2\theta_W) - \gamma^5]\psi_l(x) \end{aligned} \tag{3.49}$$

coupled to a real vector field $Z_\mu(x)$. And good agreement between theory and experiment is obtained by taking

$$sin^2\theta_W = 0.23122 \pm 0.00015. \tag{3.50}$$

The Lagrangian density (3.42), which we have considered, describes the free leptons and their interactions with the gauge fields. The complete Lagrangian density must also have terms which describe these gauge bosons when no leptons are present. These terms must also be SU(2)×SU(1) gauge-invariant. We shall for the moment assume the gauge bosons to have zero masses. In analogy to the electromagnetic case, a U(1) gauge-invariant Lagrangian density for the $B^\mu(x)$ field is given by

$$\mathscr{L}_B = -\frac{1}{4}B_{\mu\nu}(x)B^{\mu\nu}(x), \tag{3.51}$$

where

$$B^{\mu\nu}(x) \equiv \partial^\nu B^\mu(x) - \partial^\mu B^\nu(x). \tag{3.52}$$

And in analogy to the SU(3) case in Eq.(3.26), the Lagrangian density for the $W_i^\mu(x)$ fields is

$$\mathscr{L}_G = -\frac{1}{4}G_{i\mu\nu}(x)G_i^{\mu\nu}(x), \tag{3.53}$$

where

$$B_i^{\mu\nu}(x) \equiv F_i^{\mu\nu}(x) + g\epsilon_{ijk}W_j^\mu(x)W_k^\nu(x), F_i^{\mu\nu}(x) \equiv \partial^\nu W_i^\mu(x) - \partial^\mu W_i^\nu(x). \tag{3.54}$$

Combining expressions(3.51)and(3.53), we obtain the complete SU(2)×U(1) gauge-invariant Lagrangian density for the gauge bosons

$$\begin{aligned}
\mathscr{L}^B &= -\frac{1}{4}B_{\mu\nu}(x)B^{\mu\nu}(x) - \frac{1}{4}G_{i\mu\nu}(x)G_i^{\mu\nu}(x), \\
&= -\frac{1}{4}B_{\mu\nu}(x)B^{\mu\nu}(x) - \frac{1}{4}F_{i\mu\nu}(x)F_i^{\mu\nu}(x) \\
&+ g\epsilon_{ijk}W_{i\mu}(x)W_{j\nu}(x)\partial^\mu W_{k\nu}(x) \\
&- \frac{1}{4}g^2\epsilon_{ijk}\epsilon_{ilm}W_j^\mu(x)W_k^\nu(x)W_{l\mu}(x)W_{m\nu}(x), \\
&= \mathscr{L}_0^B + \mathscr{L}_I^B,
\end{aligned} \tag{3.55}$$

where $\mathscr{L}_0^B$ represents the free gauge fields, and $\mathscr{L}_I^B$ represents interactions of gauge bosons. In terms of the fields $A^\mu(x)$,$Z^\mu(x)$ and $W^\mu(x)$, $\mathscr{L}_0^B$ becomes

$$\mathscr{L}_0^B = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{1}{2}F^\dagger_{W\mu\nu}(x)F_W^{\mu\nu}(x) - \frac{1}{4}Z_{\mu\nu}(x)Z^{\mu\nu}(x), \tag{3.56}$$

where $F^{\mu\nu}(x)$ is the electromagnetic field tensor (3.12), $F_W^{\mu\nu}(x)$ is the corresponding tensor for the $W^\mu(x)$ field

$$F_W^{\mu\nu}(x) \equiv \partial^\nu W^\mu(x) - \partial^\mu W^\nu(x), \tag{3.57}$$

and

$$Z^{\mu\nu}(x) \equiv \partial^\nu Z^\mu(x) - \partial^\mu Z^\nu(x), \tag{3.58}$$

is similar associated with the Z^μ field. Eq.(3.56) is thus the free-field Lagrangian density for mass zero, spin 1 γ, $W^\pm$ and Z^0 bosons. In contrast the third and fourth terms in Eq.(3.55) represent interactions of the gauge bosons amongst themselves. In perturbation theory, these terms generate three- or four-line line vertices. These boson self-interactions are one of the most striking features of the theory. They arise because the W_i^μ fields, which transmit the interactions between the weak isospin currents, themselves are weak isospin vectors carrying weak isospin charge.

CHAPTER 4

PHYSICS BEYOND THE STANDARD MODEL

In the chapter 2, a brief introduction to the Standard Model was given. In this chapter, let us talk a little bit about the physics beyond the Standard Model and give the effective Hamiltonian for ν_τ nucleon deep inelastic scattering in the presence of NP. At the beginning, we should know why we wish to go beyond it.

There are lots of reasons for us to believe that the standard model is not the whole story. Undoubtedly, the most compelling reason for our dissatisfaction is that the standard model does not include gravity. Except that, the standard model itself, although experimentally well confirmed, has several unsatisfactory, or at least, unnatural features. The Higgs spontaneous symmetry breakdown mechanism, which is crucial to the success of the standard model, requires an inelegant and arbitrary addition to the Lagrangian. And before spontaneous symmetry breakdown, we cannot distinguish each leptons and quarks with the same electric charge. Since their only difference is their mass. Therefore, even worse is the fact that the theory offers no explanation for family replication. Related to this is our complete ignorance as to the origin of the parameters in the mass matrix, all of which seem quite arbitrary. The standard model is based on the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge

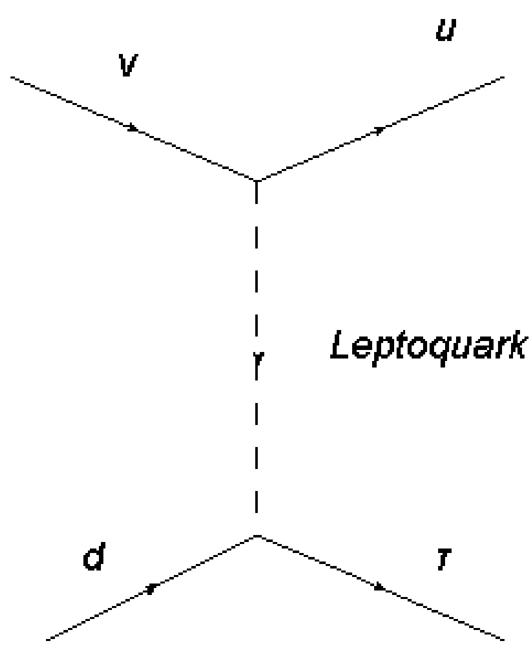

Figure 4.1: The Feynman diagram for the process $\nu_\tau + d \to \tau + u$ mediated by leptoquark.

group. This gauge group has three factors, and hence has three independent coupling constants. It offers a glimpse of unification in the breakdown of $SU(2)_L \times U(1)_Y \to U(1)_Q$, but this does not take us very far. Also, it is strange that one of these factors(SU(2)) is only related to the left handed states. Thus, it is natural for us to extend the gauge group to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_Y$, which means in addition to the 12 known gauge bosons of the standard model(the eight gluons, $W^{\pm}$,Z^0 and γ), we have three more gauge bosons similar to $W^{\pm}$ and Z^0. Let us call it $W'^{\pm}$ and Z'^0. Those kinds of new gauge bosons can mediated non-standard interaction like $V + A$ interaction.

What's more, in the new physics there might be gauge bosons, called leptoquarks, which link the quarks and leptons. These leptoquarks will mediate new interactions that violate baryon number (B) and lepton number (L) conservation. The Feynman diagram for the process $\nu_\tau + d \to \tau + u$ mediated by leptoquarks is depicted as Fig.(4.1).

Evidently, these new interaction must be sufficiently weak to have eluded detection so far, which means that $W'^{\pm}$, Z'^0, leptoquarks and some other new gauge bosons must be very massive. In the chapter 5, we will investigate effects of model independent NP operators on the deep inelastic neutrino nucleon scattering through charged current. Their interaction coupling constants will be discussed in Chapter 6. Next, we will introduce the effective Hamiltonian with model independent NP operators.

In the presence of NP, the effective Hamiltonian for the scattering process $\nu_\tau + N \to \tau + X$ can be written in the form [14],

$$\begin{aligned} \mathcal{H}_{eff} &= \frac{4G_F V_{qq'}}{\sqrt{2}} \Big[(1+V_L)\,[\bar{q}'\gamma_\mu P_L q]\;[\bar{l}\gamma^\mu P_L \nu_l] + V_R\,[\bar{q}'\gamma^\mu P_R q]\;[\bar{l}\gamma_\mu P_L \nu_l] \\ &\quad + S_L\,[\bar{q}' P_L q]\,[\bar{l} P_L \nu_l] + S_R\;[\bar{q}' P_R q]\;[\bar{l} P_L \nu_l] + T_L\,[\bar{q}'\sigma^{\mu\nu} P_L q]\;[\bar{l}\sigma_{\mu\nu} P_L \nu_l] \Big], \end{aligned} \tag{4.1}$$

where $G_F = 1.1663787(6) \times 10^{-5} GeV^{-2}$ is the Fermi coupling constant, $V_{qq'}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, $P_{L,R} = (1 \mp \gamma_5)/2$ are the projectors of negative/positive chiralities. We use $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$ and assume the neutrino to be always left chiral.To introduce non-universality the NP couplings are in general different for different lepton flavors. We assume the NP effect is mainly through the τ lepton. The SM effective Hamiltonian corresponds to $S_L = S_R = V_L = V_R = T_L = 0$.

The Hamiltonian in the presence of only scalar and tensor operators can be written as,

$$\mathcal{H}_{\rm eff} = \frac{G_F V_{qq'}}{\sqrt{2}} \left[\bar{l}(1-\gamma_5)\nu_l \; \bar{q}'(A + B\gamma_5) q + T_L \; \bar{l}\sigma_{\mu\nu}(1-\gamma_5)\nu_l \; \bar{q}'\sigma^{\mu\nu}(1-\gamma_5) q \right], \tag{4.2}$$

where $A = S_R + S_L$ and $B = S_R - S_L$ with S_L and S_R are the left and right handed scalar couplings and T_L is the tensor coupling.

We will first employ a model independent approach and treat the scalar and tensors coupling one at a time. Since, in many realistic models both the scalar and tensor couplings may be present, we will consider an explicit leptoquark model where both the scalar and tensor couplings are present.

The Hamiltonian in the presence of only $V \pm A$ operators was considered in our previous work [12]. There the effective Hamiltonian was written in terms of a W' model, which could arise in extensions of the SM [15], as

$$\mathcal{L} = \frac{g}{\sqrt{2}} V_{f'f} \bar{f}' \gamma^\mu (g_L^{f'f} P_L + g_R^{f'f} P_R) f W'_\mu + \ h.c.. \tag{4.3}$$

Integrating out the W' leads to

$$\begin{aligned}
\mathcal{L} &= \frac{g^2}{2M_W^2} V_{f'f} \left[\bar{f}' \gamma^\mu (\frac{M_W^2}{M_{W'}^2} g_L^{f'f} P_L + \frac{M_W^2}{M_{W'}^2} g_R^{f'f} P_R) f \right] \left[g^{l,\nu_l} \bar{l} \gamma_\mu P_L \nu_l \right] + \ h.c., \\
\mathcal{L} &= \frac{4G_F V_{f'f}}{\sqrt{2}} \left[\bar{f}' \gamma^\mu (\frac{M_W^2}{M_{W'}^2} g_L^{f'f} P_L + \frac{M_W^2}{M_{W'}^2} g_R^{f'f} P_R) f \right] \left[g_{\nu_l,l} \bar{l} \gamma_\mu P_L \nu_l \right] + \ h.c..
\end{aligned} \tag{4.4}$$

Comparing Eq. 4.4 with Eq. 4.1 we have the following relations

$$\begin{aligned}
V_L &= \frac{M_W^2}{M_{W'}^2} g_L^{f'f} g^{l,\nu_l}, \\
V_R &= \frac{M_W^2}{M_{W'}^2} g_R^{f'f} g^{l,\nu_l}.
\end{aligned} \tag{4.5}$$

In the following, I will use model independent approach rather than W' model. We just call $V_{L,R}$ the couplings of $(V \pm A)$ NP operators.

CHAPTER 5

DEEP INELASTIC NEUTRINO NUCLEON SCATTERING

There are three main reasons why we look for new physics effects on deep inelastic τ neutrino nuclear scattering. The first one is that in deep inelastic scattering, the incident particle has high energy. According to quantum theory a high-energy beam has a short associated wavelength $\lambda \approx \hbar c/E$. Hence the incident particle can interact with quarks inside the nucleon directly and some nuclear effects can be ignored. The second reason is due to the small cross section in neutrino scattering, where the effects of new physics involved can be easily observed. The last one has to do with the mass of τ neutrino. Since τ neutrino is much heavier that the other two kinds of neutrino, it may have a relatively large coupling constant with new gauge bosons. In the following, more details about theoretical calculations of the cross section in the DIS process (5.1) will be presented.

$$\nu_\tau(k_1) + N(p_1) \rightarrow \tau^-(k_2) + X(p_n). \tag{5.1}$$

In the parton model, the inclusive scattering is viewed as due to incoherent elastic scattering from point-like constituents of the nucleon, the quarks, depicted in Fig.(5.1). The interaction process becomes

$$\nu_\tau(k_1) + q(\xi p_1) \rightarrow \tau^-(k_2) + q'(p_2). \tag{5.2}$$

The final-state partons then recombine somehow into hadronic states. Each of the spin-1/2 quarks is hypothesized to carry a fraction of the original nucleon momentum ξp with $0 \leq \xi \leq 1$. And the probability density that the quark q has a momentum constituting an ξth part of the nucleon momentum is $f(\xi)$, which is called parton distribution function.

In order to calculate the cross section of the process (5.2), we must find the matrix element $S_{fi} =< f|S|i >$ at the second order, where the operator S is given by the following general expression

$$S = T(e^{-i\int \mathcal{H}_I(x)dx}). \tag{5.3}$$

The expression for the S-matrix element in the momentum space is given as

$$S_{fi} = \delta_{fi} + (2\pi)^4\delta^4(\Sigma p'_f - \Sigma p_i)\prod_i(\frac{1}{2VE_i})^{1/2}\prod_f(\frac{1}{2VE'_f})^{1/2}\prod_l(2m_l)^{1/2}\mathcal{M}, \tag{5.4}$$

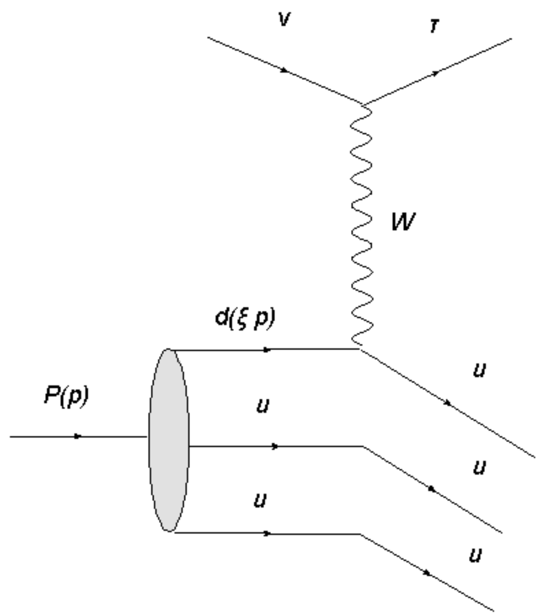

Figure 5.1: Inelastic neutrino nucleon scattering in parton model.

where $\mathcal{M}$ is the Feynman amplitude. The differential cross-section is the transition rate into this group of final states for one scattering center and unit incident flux, which is given as

$$d\sigma = (2\pi)^4\delta^4(\Sigma p'_f - \Sigma p_i)\frac{1}{4E_1E_2v_{rel}}(\prod_l(2m_l))(\prod_f\frac{d^3\mathbf{p}'_f}{(2\pi)^3 2E'_f})|\mathcal{M}|^2. \tag{5.5}$$

In the deep inelastic scattering we calculate the differential cross section in the rest frame of nucleon, where the four momentums are defined as follow $k_1^\mu = \big(E_\nu,\ \ 0,\ \ 0,\ \ E_\nu\big)$, $k_2^\mu = \big(E_\tau,\ \ p_\tau sin\theta,\ \ 0,\ \ p_\tau cos\theta\big)$, $p_1^\mu = \big(M,\ \ 0,\ \ 0,\ \ 0\big)$. And as indicated in Eq.(5.12), we integrate cross-section with respect to the scaling variables which are defined as follows

$$\begin{aligned} x &= -\frac{q^2}{2M\nu}, \\ y &= \frac{\nu}{E_\nu}, \\ \nu &= p_1\cdot q/M = E_\nu - E_\tau, \end{aligned} \tag{5.6}$$

where x is the Bjorken variable and y is the inelasticity with q is the four-momentum transfer. Through a long calculation, then Eq.(5.5) goes to

$$\frac{d\sigma}{dxdy} = \frac{1}{32\pi ME_\nu}\int\frac{d\xi}{\xi}f(\xi)|\bar{\mathcal{M}}(\xi)|^2\delta(\xi - x). \tag{5.7}$$

The derivation is included in Appendix (A). The physical regions for x and y are obtained by Albright and Jarlskog [16, 17]

$$\frac{m_\tau^2}{2M(E_\nu - m_\tau)} \leq x \leq 1, \tag{5.8}$$

and

$$A - B \leq y \leq A + B, \tag{5.9}$$

where

$$A = \frac{1}{2}\left(1 - \frac{m_\tau^2}{2ME_\nu x} - \frac{m_\tau^2}{2E_\nu^2}\right) \Big/ \left(1 + \frac{xM}{2E_\nu}\right), \tag{5.10}$$

$$B = \frac{1}{2}\left[\left(1 - \frac{m_\tau^2}{2ME_\nu x}\right)^2 - \frac{m_\tau^2}{E_\nu^2}\right]^{\frac{1}{2}} \Big/ \left(1 + \frac{xM}{2E_\nu}\right). \tag{5.11}$$

5.1 Scalar and Tensor Interactions

In this section we discuss deep inelastic neutrino nucleon Scattering with the various types of interactions. We first present the total and differential cross sections for the deep inelastic scattering (DIS) process (5.2) with scalar and tensor interactions. The total differential cross section thus can be written as follows

$$\frac{d\sigma_{\rm tot}}{dxdy} = \frac{d\sigma_{\rm SM}}{dxdy} + \frac{d\sigma_{\rm S}}{dxdy} + \frac{d\sigma_{\rm T}}{dxdy} + \frac{d\sigma_{\rm SM,ST}}{dxdy} + \frac{d\sigma_{\rm S,T}}{dxdy}. \tag{5.12}$$

According to the effective Hamiltonian in Eq.(4.2), we can obtain the Feynman amplitudes as follow

$$\begin{aligned}
M_{SM} &= (\frac{-iG_F V_{qq'}}{\sqrt{2}})[\bar{u}_\tau(k_2)\gamma^\mu(1-\gamma^5)u_{\nu_\tau}(k_1)][\bar{u}_{q'}(p_2)\gamma_\mu(1-\gamma^5)u_q(\xi p_1)], \\
M_{ST} &= (\frac{G_F V_{qq'}}{\sqrt{2}})[[\bar{u}_\tau(k_2)(1-\gamma^5)u_{\nu_\tau}(k_1)][\bar{u}_{q'}(p_2)(A+B\gamma^5)u_q(\xi p_1)] \\
&\quad + T_L[\bar{u}_\tau(k_2)\sigma^{\mu\nu}(1-\gamma^5)u_{\nu_\tau}(k_1)][\bar{u}_{q'}(p_2)\sigma_{\mu\nu}(1-\gamma^5)u_q(\xi p_1)],
\end{aligned} \tag{5.13}$$

where $A = S_R + S_L$,$B = S_R - S_L$. S_L and S_R are the coupling constants for left-handed and right-handed NP operators. T_L are the coupling constant for tensor NP operators. Plugging those Feynman amplitude into Eq.(5.7), we then obtain

$$
\begin{aligned}
\frac{d\sigma_{\rm SM}}{dxdy} &= \frac{G_F^2 M E_\nu}{\pi}\left(y(xy+\frac{m_\ell^2}{2ME_\nu})F_1+(1-y-\frac{Mxy}{2E_\nu}-\frac{m_\ell^2}{4E_\nu^2})F_2\right.\\
&\left.+(xy(1-\frac{y}{2})-y\frac{m_\ell^2}{4ME_\nu})F_3-\frac{m_\ell^2}{2ME_\nu}F_5\right),\\
\frac{d\sigma_{\rm S}}{dxdy} &= \frac{G_F^2 M E_\nu}{4\pi}(A^2+B^2)y(xy+\frac{m_\ell^2}{2ME_\nu})F_1,\\
\frac{d\sigma_{\rm T}}{dxdy} &= \frac{8G_F^2 M E_\nu}{\pi}T_L^2\left(y(xy+\frac{m_\ell^2}{2ME_\nu})F_1+2(1-y-\frac{Mxy}{4E_\nu}-\frac{m_\ell^2}{8E_\nu^2})F_2-\frac{m_\ell^2}{ME_\nu}F_5\right),\\
\frac{d\sigma_{\rm SM,ST}}{dxdy} &= 0,\\
\frac{d\sigma_{\rm S,T}}{dxdy} &= \frac{2G_F^2 M E_\nu}{\pi}T_L(B-A)\left(xy(1-\frac{y}{2})-y\frac{m_\ell^2}{4ME_\nu}\right)F_3.
\end{aligned}
\tag{5.14}
$$

In Appendix (B), we present the details of the derivation of $d\sigma_{SM}/dxdy$ as an example. The form factors F_i are given as

$$
\begin{aligned}
F_1 &= \sum_{q,\bar{q}} f_{q,\bar{q}}(\xi,Q^2)V_{q,q'}^2,\\
F_2 &= 2\sum_{q,\bar{q}} \xi f_{q,\bar{q}}(\xi,Q^2)V_{q,q'}^2,\\
F_3 &= 2\sum_{q} f_{q}(\xi,Q^2)V_{q,q'}^2-2\sum_{\bar{q}} f_{\bar{q}}(\xi,Q^2)V_{\bar{q},\bar{q}'}^2,\\
F_5 &= 2\sum_{q,\bar{q}} f_{q,\bar{q}}(\xi,Q^2)V_{q,q'}^2,
\end{aligned}
\tag{5.15}
$$

where $f_{q,\bar{q}}(\xi,Q^2)$ are the parton distribution functions. Their explicit forms in this case are presented in Appendix (C).

We also can write the differential cross sections (5.14) in terms of different variables(t,ν), where $t=q^2$. Using Eq.(5.6), the transformation has the form

$$
\frac{d\sigma}{dxdy} = 2ME_\nu \nu \frac{d\sigma}{dtd\nu}. \tag{5.16}
$$

Then the differential cross section can be written as

$$
\begin{aligned}
\frac{d\sigma_{\rm SM}}{dtd\nu} &= \frac{G_F^2}{8\pi E_\nu^2}(2(m_\ell^2-t)W_1+(4E_\nu(E_\nu-\nu)-(m_\ell^2-t))W_2 \\
&\quad -\frac{1}{M}(2E_\nu t+\nu(m_\ell^2-t))W_3-\frac{2m_\ell^2E_\nu}{M}W_5), \\
\frac{d\sigma_{\rm S}}{dtd\nu} &= \frac{G_F^2}{16\pi E_\nu^2}(A^2+B^2)(m_\ell^2-t)W_1, \\
\frac{d\sigma_{\rm T}}{dtd\nu} &= \frac{G_F^2}{\pi E_\nu^2}T_L^2(2(m_\ell^2-t)W_1+(8E_\nu(E_\nu-\nu)-(m_\ell^2-t))W_2-\frac{4m_\ell^2E_\nu}{M}W_5), \\
\frac{d\sigma_{\rm SM,ST}}{dtd\nu} &= 0, \\
\frac{d\sigma_{\rm S,T}}{dtd\nu} &= \frac{G_F^2}{4\pi ME_\nu^2}T_L(B-A)(-2E_\nu t-(m_\ell^2-t)\nu)W_3,
\end{aligned}
\tag{5.17}
$$

where the (time-reverse invariant) structure functions are [17]

$$
\begin{aligned}
W_1(q^2,\nu) &= \frac{F_1(x)}{M}, \\
W_2(q^2,\nu) &= \frac{F_2(x)}{\nu}, \\
W_3(q^2,\nu) &= \frac{F_3(x)}{\nu}, \\
W_5(q^2,\nu) &= \frac{F_5(x)}{\nu}.
\end{aligned}
\tag{5.18}
$$

We also define some Lorentz invariant variables in terms of the four-momenta of the incoming neutrino (k_1), target nucleon (p_1) and produced charged lepton (k_2) in the laboratory frame

$$Q^2 = -q^2 = -t, \tag{5.19}$$

$$W^2 = (p_1+q)^2. \tag{5.20}$$

Q^2 is the magnitude of the momentum transfer and W is the hadronic invariant mass. The physical regions of these variables are given by [18]

$$W_{\rm cut} \leq W \leq \sqrt{s}-m_\ell, \tag{5.21}$$

in the DIS region with $W_{\rm cut} = 1.4GeV \sim 1.6GeV$, and

$$Q_-^2(W) \leq Q^2 \leq Q_+^2(W), \tag{5.22}$$

where $s = (k_1 + p_1)^2$ and

$$Q^2_{\pm}(W) = \frac{s - M^2}{2}(1 \pm \bar{\beta}) - \frac{1}{2}\left[W^2 + m_\ell^2 - \frac{M^2}{s}\left(W^2 - m_\ell^2\right)\right], \tag{5.23}$$

with $\bar{\beta} = \lambda^{\frac{1}{2}}(1, m_\ell^2/s, W^2/s)$ and $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$. In the lab frame, $s = M^2 + 2ME_\nu$.

5.2 Explicit Leptoquark Model

Here we will discuss an explicit case for the leptoquark model. Many extensions of the SM, motivated by a unified description of quarks and leptons, predict the existence of new scalar and vector bosons, called leptoquarks, which decay into a quark and a lepton. These particles carry nonzero baryon and lepton numbers, color and fractional electric charges. The most general dimension four $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant Lagrangian of leptoquarks satisfying baryon and lepton number conservation was considered in Ref [19]. As the tensor operators in the effective Lagrangian get contributions only from scalar leptoquarks, we will focus only on scalar leptoquarks and consider the case where the leptoquark is a weak doublet or a weak singlet. The weak doublet leptoquark, R_2 has the quantum numbers $(3, 2, 7/6)$ under $SU(3)_c \times SU(2)_L \times U(1)_Y$ while the singlet leptoquark S_1 has the quantum numbers $(\bar{3}, 1, 1/3)$.

The interaction Lagrangian that induces contributions to $\nu_\tau + N \to \tau + X$ process is [20]

$$\begin{aligned} \mathcal{L}_2^{\rm LQ} &= \left(g_{2L}^{ij}\, \overline{u}_{iR} R_2^T L_{jL} + g_{2R}^{ij}\, \overline{Q}_{iL} i\sigma_2 \ell_{jR} R_2\right), \\ \mathcal{L}_0^{\rm LQ} &= \left(g_{1L}^{ij}, \overline{Q}_{iL}^c i\sigma_2 L_{jL} + g_{1R}^{ij}, \overline{u}_{iR}^c \ell_{jR}\right) S_1, \end{aligned} \tag{5.24}$$

where Q_i and L_j are the left-handed quark and lepton $SU(2)_L$ doublets respectively, while u_{iR}, d_{iR} and ℓ_{jR} are the right-handed up, down quark and charged lepton $SU(2)_L$ singlets. Indices i and j denote the generations of quarks and leptons and $\psi^c = C\overline{\psi}^T = C\gamma^0\psi^*$ is a charge-conjugated fermion field. The fermion fields are given in the gauge eigenstate basis and one should make the transformation to the mass basis. Assuming the quark mixing matrices to be hierarchical, and considering only the leading contribution we can ignore the effect of mixing. After performing the Fierz transformations, one finds the general Wilson coefficients at the leptoquark mass scale contributing to the $\nu_\tau + N \to \tau + X$ process:

$$\begin{aligned} S_L &= \frac{1}{2\sqrt{2} G_F V_{ud}}\left[-\frac{g_{1L}^{13} g_{1R}^{13*}}{2M_{S_1}^2} - \frac{g_{2L}^{13} g_{2R}^{13*}}{2M_{R_2}^2}\right], \\ T_L &= \frac{1}{2\sqrt{2} G_F V_{ud}}\left[\frac{g_{1L}^{13} g_{1R}^{13*}}{8M_{S_1}^2} - \frac{g_{2L}^{13} g_{2R}^{13*}}{8M_{R_2}^2}\right]. \end{aligned} \tag{5.25}$$

It is clear from Eq. (5.25) that the weak singlet leptoquark and the weak doublet doublet can add constructively or destructively to the Wilson's coefficients of the scalar and tensor operators in the effective Hamiltonian. In this section we will also consider the possibilities where both the scalar and the tensor operators are present and are of similar sizes. In the most general case both the singlet and doublet leptoquarks are present and so both the scalar and tensor operators appear in the effective Hamiltonian. As there is limited experimental information, including both the singlet and the doublet leptoquarks will allow us more flexibility in fitting for the Wilson's coefficients, but this will come with the price of less precise predictions for the various observables. We can, therefore, consider the simpler cases when only a singlet or a doublet leptoquark are present. In these cases, from Eq. 5.25 the coefficients of scalar operators and the tensor operators have the same magnitudes. One can now consider two further cases:

Case. (a): In this case only the weak doublet scalar leptoquark R_2 is present. In this case the Wilson's coefficients are

$$\begin{aligned} S_L &= \frac{1}{2\sqrt{2}G_F V_{cb}}\left[-\frac{g_{2L}^{13}g_{2R}^{13*}}{2M_{R_2}^2}\right], \\ T_L &= \frac{1}{2\sqrt{2}G_F V_{cb}}\left[-\frac{g_{2L}^{13}g_{2R}^{13*}}{8M_{R_2}^2}\right], \end{aligned} \tag{5.26}$$

Case. (b): In this case only the singlet leptoquark is present and the relevant Wilson's coefficients are

$$\begin{aligned} S_L &= \frac{1}{2\sqrt{2}G_F V_{cb}}\left[-\frac{g_{1L}^{33}g_{1R}^{23*}}{2M_{S_1}^2}\right], \\ T_L &= \frac{1}{2\sqrt{2}G_F V_{cb}}\left[\frac{g_{1L}^{33}g_{1R}^{23*}}{8M_{S_1}^2}\right]. \end{aligned} \tag{5.27}$$

The relations above are valid at the leptoquark mass scale. We have to run them down to the τ mass scale using the scale dependence of the scalar and tensor currents at the leading logarithm approximation

$$\begin{aligned} S_L(m_\tau) &= \left[\frac{\alpha_s(m_t)}{\alpha_s(m_\tau)}\right]^{\frac{\gamma_S}{2\beta_0^{(5)}}}\left[\frac{\alpha_s(m_{\rm LQ})}{\alpha_s(m_t)}\right]^{\frac{\gamma_S}{2\beta_0^{(6)}}} S_L(m_{\rm LQ}), \\ T_L(m_\tau) &= \left[\frac{\alpha_s(m_t)}{\alpha_s(m_\tau)}\right]^{\frac{\gamma_T}{2\beta_0^{(5)}}}\left[\frac{\alpha_s(m_{\rm LQ})}{\alpha_s(m_t)}\right]^{\frac{\gamma_T}{2\beta_0^{(6)}}} T_L(m_{\rm LQ})\,, \end{aligned} \tag{5.28}$$

where the anomalous dimensions of the scalar and tensor operators are $\gamma_S = -6C_F = -8$, $\gamma_T = 2C_F = 8/3$ respectively and $\beta_0^{(f)} = 11 - 2n_f/3$ [21] and n_f is the number of active quark flavours [22, 23]. One can use the equations above to run down couplings from a chosen value of $m_{\rm LQ}$ to the tau mass, m_τ.

In the simplified scenario with the presence of only one type of leptoquark, namely R_2 or S_1, the scalar S_L and tensor T_L Wilson coefficients are no longer independent: one finds that at the scale of leptoquark mass $S_L(m_{\rm LQ}) = 4 \pm T_L(m_{\rm LQ})$. Then, using Eq. (5.28), one obtains the relation at the τ mass scale,

$$S_L(m_\tau) \simeq \pm 8.93\, T_L(m_\tau)\,. \tag{5.29}$$

for a leptoquark mass of 1 TeV [20]

5.3 $V \pm A$ interactions

There are some models that can lead to $V \pm A$ NP operators, like W' model in Eq.(4.3). In this work, we just use the model independent approach. Therefore we just care about V_L and V_R only. Then, generally the effective Hamiltonian can be written as

$$\mathcal{H}_{eff} = \frac{4G_F V_{qq'}}{\sqrt{2}}[(1+V_L)[\bar{\tau}\gamma^\mu P_L \nu_\tau][\bar{q'}\gamma_\mu P_L q] + V_R[\bar{\tau}\gamma^\mu P_L \nu_\tau][\bar{q'}\gamma_\mu P_R q]]. \tag{5.30}$$

The differential cross section with respect to the variables (x, y) is given in [24]. One can give it in terms of the momentum transfer using Eq.(5.16) as follows

$$\begin{aligned} \frac{d\sigma_{\rm SM+V}}{dq^2 d\nu} &= \frac{G_F^2}{8\pi E_\nu^2}((|a'|^2+|b'|^2)(m_\ell^2-q^2)W_1 \\ &+\frac{1}{2}(|a'|^2+|b'|^2)(4E_\nu^2-4E_\nu\nu-(m_\ell^2-q^2))W_2 \\ &-\frac{1}{M}Re[a'b'^*](2E_\nu q^2+\nu(m_\ell^2-q^2))W_3 - \frac{1}{M}(|a'|^2+|b'|^2)m_\ell^2 E_\nu W_5), \\ \frac{d\sigma_{\rm SM+V}}{dxdy} &= \frac{G_F^2 M E_\nu}{\pi}\Bigg(y(xy+\frac{m_\ell^2}{2ME_\nu})\frac{1}{2}(|a'|^2+|b'|^2)F_1 + (1-y-\frac{Mxy}{2E_\nu}-\frac{m_\ell^2}{4E_\nu^2})\frac{1}{2}(|a'|^2+|b'|^2)F_2 \\ &+(xy(1-\frac{y}{2})-y\frac{m_\ell^2}{4ME_\nu})Re[a'b'^*]F_3 - \frac{m_\ell^2}{2ME_\nu}\frac{1}{2}(|a'|^2+|b'|^2)F_5\Bigg), \end{aligned} \tag{5.31}$$

where the definitions are

$$\begin{aligned} a' &= 1+V_L+V_R, \\ b' &= 1+V_L-V_R. \end{aligned} \tag{5.32}$$

CHAPTER 6

NEW PHYSICS CONSTRAINTS

In order to investigate the new physics effects on τ neutrino nucleon deep inelastic scattering, we have to know the possible value of the new physics coupling constants S_L, S_R and T_L. This chapter will discuss this issue.

Because pion is a pseudoscalar, the scalar couplings S_L and S_R will be constrained by τ one-pion decay. While the tensor coupling T_L can be constrained by the two-pion decay channel of τ, $\tau(p) \to \pi^-(p_1) + \pi^0(p_2) + \nu_\tau(p_3)$.

6.1 $\tau^-(k_1) \to \nu_\tau(k_2) + \pi^-(q)$

The hadronic current of the bound state π can be parametrized as

$$\langle 0|\bar{d}\gamma^\mu(1-\gamma^5)u|\pi(q)\rangle = -i\sqrt{2}f_\pi q^\mu, \tag{6.1}$$

$$T_L\langle 0|\bar{d}\sigma^{\mu\nu}(1-\gamma^5)u|\pi(q)\rangle = 0, \tag{6.2}$$

where f_π is the decay constant. The SM decay rate is

$$\Gamma^\pi_{\rm SM} = \frac{1}{8\pi}G_F^2|V_{ud}|^2 f_\pi^2 m_\ell^3\left(1-\frac{m_\pi^2}{m_\ell^2}\right)^2\delta_{\tau/\pi}\,. \tag{6.3}$$

Here $\delta_{\tau/\pi} = 1.0016 \pm 0.0014$ [25] is the radiative correction. Further, the SM branching ratio can also be expressed as [26]

$$Br^{SM}_{\tau^-\to\pi^-\nu_\tau} \;=\; 0.607 Br(\tau^- \to \nu_\tau e^-\bar{\nu}_e) = 10.82 \pm 0.02\%\,, \tag{6.4}$$

while the measured $Br(\tau^- \to \pi^-\nu_\tau)_{exp} = (10.91 \pm 0.14)\%$ at 2 σ. [27]. In the presence of the scalar NP operators, the decay rate is

$$\Gamma^\pi_{\rm LQS} = \frac{1}{8\pi(m_u+m_d)^2}G_F^2|V_{ud}|^2 B^2 f_\pi^2 m_\pi^4 m_\ell\left(1-\frac{m_\pi^2}{m_\ell^2}\right)^2, \tag{6.5}$$

where

$$\langle 0|\bar{d}(A-B\gamma^5)u|\pi(q)\rangle = \frac{i\sqrt{2}f_\pi m_\pi^2}{m_u+m_d}B. \tag{6.6}$$

Since we cannot make a antisymmetric tensor with only q^μ, tensor NP operators cannot make contribution to this decay channel. Thus, the total branching ratio can be written as follows

$$BR^{\pi}_{\rm tot} = BR^{\pi}_{\rm SM}\left(1 + (r^{\pi}_{\rm S})^2\right), \tag{6.7}$$

where

$$(r^{\pi}_{\rm S})^2 \quad = \quad \frac{BR^{\pi}_{\rm LQS}}{BR^{\pi}_{\rm SM}}, \tag{6.8}$$

with

$$r^{\pi}_{\rm S} = \frac{Bm^2_{\pi}}{m_\ell(m_u + m_d)}. \tag{6.9}$$

Note, the interference term of the SM and scalar terms vanishes.

If the right handed coupling is fixed to be zero $S_R = 0$, the allowed values of the left handed coupling will be $0 < |S_L| < 9.25$. On the other hand, when $S_L = 0$, the experimental constraints will give $0 < |S_R| < 9.25$. The allowed region of the couplings are given Fig. 6.1 for the measured $\tau^- \to \pi^- \nu_\tau$ within the 2σ level. Once we take the couplings to be real values, and in another case we assume the right-handed component vanishes and take the left-handed coupling complex. The SM expectation for the branching ratio is allowed within the experimental range. Here we use the constituent up and down quark mass $m_u = 336$ MeV and $m_d = 340$ MeV [28].

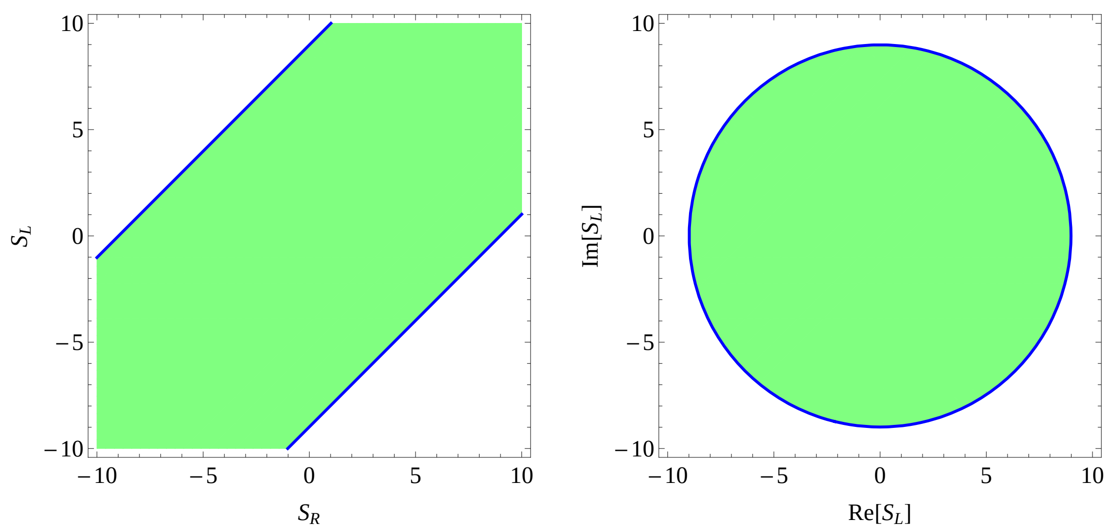

Figure 6.1: The constraints on the scalar NP operator couplings. The colored region is allowed. The constraint is from $\tau^- \to \pi^- \nu_\tau$. Left panel: we treat S_L and S_R as real couplings. Right panel: we take $S_R = 0$ and treat S_L as a complex coupling. We will get the same figure when we do $S_L = 0$ and S_R is a complex coupling.

6.2 $\tau(p) \to \pi^-(p_1) + \pi^0(p_2) + \nu_\tau(p_3)$

Here we consider three-body decays of τ. The process is

$$\tau(p) \to \nu_\tau(p_3) + \pi^-(p_1) + \pi^0(p_2). \tag{6.10}$$

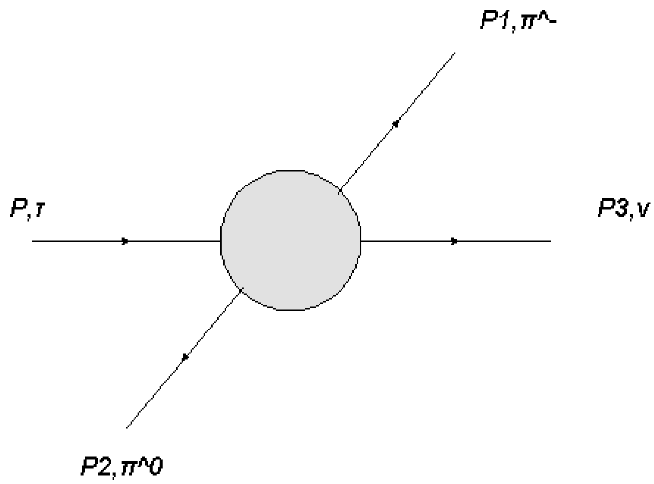

Figure 6.2: τ Three-body decay.

In the rest frame of π^- and π^0,

$$p = \left(E, \quad \vec{P}\right), p_1 = \left(E_1, \quad \vec{P_1}\right), p_2 = \left(E_2, \quad -\vec{P_1}\right), p_3 = \left(E_3, \quad \vec{P}\right),$$
$$k = p_1 - p_2 = \left(E_1 - E_2, \quad 2\vec{P_1}\right), q = p_1 + p_2 = \left(E_1 + E_2, \quad 0\right),$$

and we define two Lorentz invariant variables,

$$\begin{aligned} m_{12}^2 &= (p_1 + p_2)^2 = q^2, \\ m_{23}^2 &= (p_2 + p_3)^2. \end{aligned} \tag{6.11}$$

Then, the decay rate can be expressed as (derivation details in Appendix (D))

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32 m_\tau^3} X dm_{12}^2 dm_{23}^2, \tag{6.12}$$

with

$$X = \frac{1}{2} \sum_{spin} |M_{SM} + M_{LQT}|^2 = \frac{1}{2} \sum_{spin} |M_{SM}|^2 + \frac{1}{2} \sum_{spin} |M_{LQT}|^2, \tag{6.13}$$

where the cross terms are zero, and

$$M_{SM} = \frac{-iG_F V_{ud}}{\sqrt{2}} \langle \pi^- \pi^0 | \bar{d} \gamma^\mu (1 - \gamma^5) u | 0 \rangle \bar{u}_{\nu_\tau} \gamma_\mu (1 - \gamma^5) u_\tau, \tag{6.14}$$

$$M_{LQT} = \frac{G_F V_{ud}}{\sqrt{2}} T_L \langle \pi^- \pi^0 | \bar{d} \sigma^{\mu\nu}(1-\gamma^5) u | 0 \rangle \bar{u}_{\nu_\tau} \sigma_{\mu\nu}(1-\gamma^5) u_\tau. \tag{6.15}$$

It is not hard to show that,

$$\langle \pi^- \pi^0 | \bar{d} \gamma^\mu (1-\gamma^5) u | 0 \rangle = \sqrt{2} F(q^2) k^\mu, \tag{6.16}$$

$$\langle \pi^- \pi^0 | \bar{d} \sigma^{\mu\nu} (1-\gamma^5) u | 0 \rangle = \sqrt{2} F_T(q^2)(k^\mu q^\nu - q^\mu k^\nu). \tag{6.17}$$

$$\langle \pi^- \pi^0 | \bar{d} (1-\gamma^5) u | 0 \rangle = 0, \tag{6.18}$$

The form factor $F(q^2)$ is given in [29]. The origin of $\sqrt{2}$ comes from the wavefunction of $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$. Considering the isospin symmetry, $u\bar{u} = d\bar{d} = \phi$, so $\pi^0 = \sqrt{2}\phi$. Using the equations of motion, we have

$$F_T = -i \frac{(m_u + m_d)}{q^2} F. \tag{6.19}$$

When we contract q^μ on the each side of Eq.(6.16), we can see that the scalar NP operators cannot contribute since $k \cdot q = 0$. By averaging the spin, we can get

$$X_{SM} = 4 G_F^2 V_{ud}^2 F^2(q^2)[2(k \cdot p)(k \cdot p_3) - k^2 (p \cdot p_3)], \tag{6.20}$$

and,

$$\begin{aligned} X_{LQT} &= 16 G_F^2 V_{ud}^2 T_L^2 (m_u + m_d)^2 \frac{F^2(q^2)}{q^4} [(k \cdot q)^2 (-p \cdot p_3) + 2(k \cdot q)((k \cdot p_3)(p \cdot q) \\ &\quad + (k \cdot p)(p_3 \cdot q)) - 2q^2 (k \cdot p)(k \cdot p_3) + k^2 (q^2 (p \cdot p_3) - 2(p \cdot q)(p_3 \cdot q))]. \end{aligned} \tag{6.21}$$

All these $X's$ can be expressed in terms of m_{12}^2 and m_{23}^2 because

$$E_1 = \frac{m_{12}^2 - m_2^2 + m_1^2}{2m_{12}}, \tag{6.22}$$

$$E_2 = \frac{m_{12}^2 - m_1^2 + m_2^2}{2m_{12}}, \tag{6.23}$$

$$E_3 = \frac{M^2 - m_{12}^2 - m_3^2}{2m_{12}}, \tag{6.24}$$

$$E = \frac{M^2 + m_{12}^2 - m_3^2}{2m_{12}}, \tag{6.25}$$

$$2\vec{p_1} \cdot \vec{p} = m_{23}^2 - m_2^2 - m_3^2 - 2E_2 E_3, \tag{6.26}$$

$$|\vec{p}| = \sqrt{E^2 - M^2}, \tag{6.27}$$

$$|\vec{p_1}| = \sqrt{E_1^2 - m_1^2}, \tag{6.28}$$

with $M = m_\tau = 1.77$ GeV, $m_1 = m_{\pi^-} = 0.140$ GeV, $m_2 = m_{\pi^0} = 0.135$ GeV, $m_3 = m_{\nu_\tau} = 0$.

Let's work on the SM case first and set

$$\begin{aligned} A_1 &= k \cdot p = E(E_1 - E_2) - 2\vec{p_1} \cdot \vec{p} \\ &= E(E_1 - E_2) + 2E_2E_3 + m_2^2 + m_3^2 - m_{23}^2, \end{aligned} \tag{6.29}$$

$$\begin{aligned} A_2 &= k \cdot p_3 = E_3(E_1 - E_2) - 2\vec{p_1} \cdot \vec{p} \\ &= E_3(E_1 - E_2) + 2E_2E_3 + m_2^2 + m_3^2 - m_{23}^2, \end{aligned} \tag{6.30}$$

$$\begin{aligned} A_3 &= k^2 = (E_1 - E_2)^2 - 4\vec{p_1}^{\,2} \\ &= (E_1 - E_2)^2 - 4(E_1^2 - m_1^2), \end{aligned} \tag{6.31}$$

$$\begin{aligned} A_4 &= p \cdot p_3 = E_1E_3 - \vec{p}^{\,2} \\ &= E_1E_3 - E^2 + M^2. \end{aligned} \tag{6.32}$$

Then,

$$X_{SM} = 4G_F^2 V_{ud}^2 F^2(q^2)[2A_1A_2 - A_3A_4], \tag{6.33}$$

Now, let us integrate X_{SM} by m_{23}^2 within the limits

$$(m_{23}^2)_{max} = (E_2 + E_3)^2 - \left(\sqrt{E_2^2 - m_2^2} - \sqrt{E_3^2 - m_3^2}\right)^2, \tag{6.34}$$

$$(m_{23}^2)_{min} = (E_2 + E_3)^2 - \left(\sqrt{E_2^2 - m_2^2} + \sqrt{E_3^2 - m_3^2}\right)^2, \tag{6.35}$$

where $m_1 = m_2 = m_\pi$, $m_{12}^2 = q^2$ and $M = m_\tau$. One can get

$$\Gamma_{SM} = \frac{4G_F^2 m_\tau^5}{96(2\pi)^3} \frac{cos\theta_c}{2} \int_{q_{min}^2}^{q_{max}^2} \frac{dq^2}{m_\tau^2} F^2(q^2) \left(1 - \frac{q^2}{m_\tau^2}\right)^2 \left(1 + 2\frac{q^2}{m_\tau^2}\right) \left(1 - \frac{4m_\pi^2}{q^2}\right)^{3/2}. \tag{6.36}$$

Now, we can integrate over m_{12}^2 within the limits

$$\begin{aligned} q_{max}^2 &= (m_{12}^2)_{max} = (M - m_1)^2, \\ q_{min}^2 &= (m_{12}^2)_{min} = (m_1 + m_2)^2. \end{aligned} \tag{6.37}$$

We find that $\Gamma_{SM} = 5.53 \times 10^{-13}$ GeV. The total decay rate of τ is $\Gamma_{tot} = 2.27 \times 10^{-12}$ GeV, so that $BR(\tau^- \to \nu_\tau + \pi^- + \pi^0)$ is 24.3% in our calculations which is close to the experimental result $(25.52 \pm 0.09)\%$ [30]. Using the CVC hypothesis, it is predicted that $B(\tau^- \to \pi^-\pi^0\nu_\tau) = (24.75 \pm 0.38)\%$ [31].

Now let's work on the tensor NP operator case, we set

$$B_1 = k \cdot q = E_1^2 - E_2^2, \tag{6.38}$$
$$B_2 = p \cdot p_3 = EE_3 - \vec{p}^2, \tag{6.39}$$
$$B_3 = k \cdot p_3 = E_3(E_1 - E_2) - 2\vec{p} \cdot \vec{p_1}, \tag{6.40}$$
$$B_4 = p \cdot q = E(E_1 + E^2), \tag{6.41}$$
$$B_5 = k \cdot p_3 = E(E_1 - E_2) - 2\vec{p} \cdot \vec{p_1}, \tag{6.42}$$
$$B_6 = P_3 \cdot q = E_3(E_1 + E_2), \tag{6.43}$$
$$B_7 = q^2 = (E_1 + E_2)^2, \tag{6.44}$$
$$B_8 = k^2 = (E_1 - E_2)^2 - 4\vec{p_1}^2. \tag{6.45}$$

Then,

$$X_{LQT} = 8G_F^2 V_{ud}^2 T_L^2 F_T^2(q^2)[-B_1^2 B_3 + 2B_1(B_3 B_4 + B_1 B_6) - 2B_7 B_1 B_5 + B_8(B_7 B_2 - 2B_4 B_6)], \tag{6.46}$$

Similarly, we can get $\Gamma_{LQT} = 2.82 \times 10^{-12} T_L^2$ GeV.

From the constraint $\tau^- \to \pi^- \pi^0 \nu_\tau$, we find that $0.04 < |T_L| < 0.105$ within the 2σ level. If we take the tensor coupling to be complex, the contour plot in Fig. 6.3 shows the allowed region of the real and imaginary components of the coupling for the measured $\tau \to \pi^- \pi^0 \nu_\tau$ within the 2σ level. The SM expectation for the branching ratio is not allowed within the experimental range at the 2σ level but it is allowed at higher standard deviation level.

In the explicit model, $S_L(m_{\rm LQ}) = \pm 4T_L(m_{\rm LQ})$, one can obtain the constraint on S_L and T_L from $\tau^- \to \nu_\tau \pi^-$ and $\tau^- \to \pi^- \pi^0 \nu_\tau$ at the same time. The allowed regions of the real and imaginary components are shown in the contour plot in Fig. 6.4.

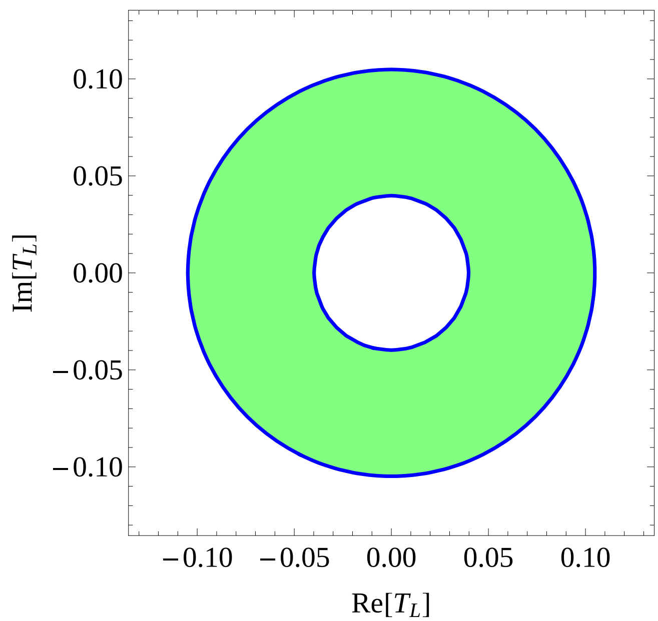

Figure 6.3: The allowed region for the real and imaginary components of the complex leptoquark coupling T_L. The constraint on T_L is from $\tau^- \to \pi^-\pi^0\nu_\tau$.

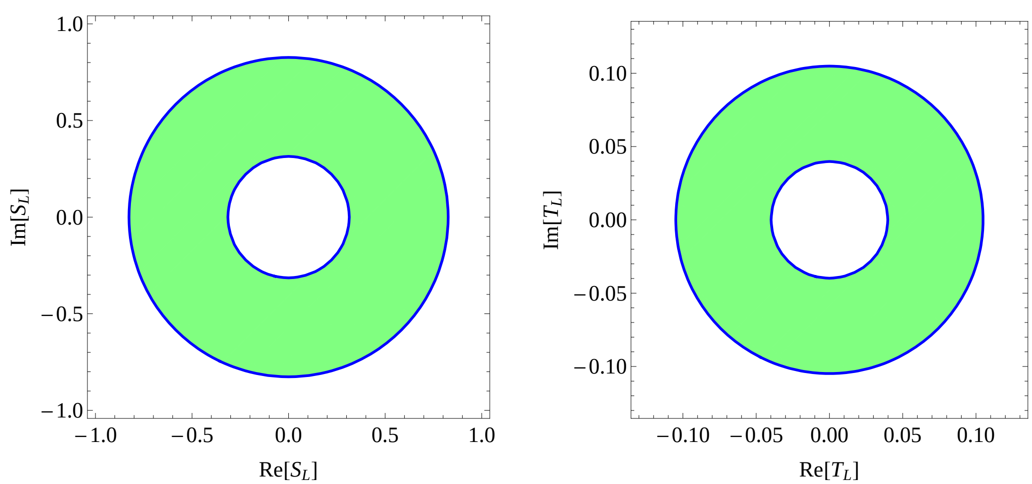

Figure 6.4: The allowed regions for the real and imaginary components of the leptoquark running couplings $S_L(m_\tau)$ and $T_L(m_\tau)$ for $S_L(m_{LQ}) = \pm 4\ T_L(m_{LQ})$. The constraint on $S_L(m_\tau)$ is from $\tau^- \to \pi^-\nu_\tau$ and $T_L(m_\tau)$ is from $\tau^- \to \pi^-\pi^0\nu_\tau$.

CHAPTER 7

NUMERICAL ANALYSIS

In this section the sensitivity of the neutrino cross-section experiments to the scalar, tensor and $V \pm A$ NP operators will be discussed. We will study the ratio of the total cross section, $d\sigma/dxdy$, and $d\sigma/dt$ for the tau-neutrino to the muon-neutrino scattering. Also, we will show the results of the total cross section, $d\sigma/dxdy$, and $d\sigma/dt$ for the process $\nu_\tau + N \to \tau + X$.

7.1 Scalar Tensor And Leptoquark Results

The ratio of the total cross section is shown in Fig. 7.1 while the ratio of the differential cross sections are given in Figs. (7.2, 7.3). One can see that it is not difficult to distinguish between the new physics and the SM total cross section results. And, the impact of the new physics is clearly detectable in the ratio of the differential cross sections. In Fig. 7.4, we show the results in the light of the particular model $S_L(m_{\rm LQ}) = \pm 4\ T_L(m_{\rm LQ})$. It is again easy to observe the effect of the new physics. The new physics effect is the same in the total cross section, $d\sigma/dxdy$, and $d\sigma/dt$ for the process $\nu_\tau + N \to \tau + X$, as shown in Figs. (7.5, 7.6, 7.7, 7.8).

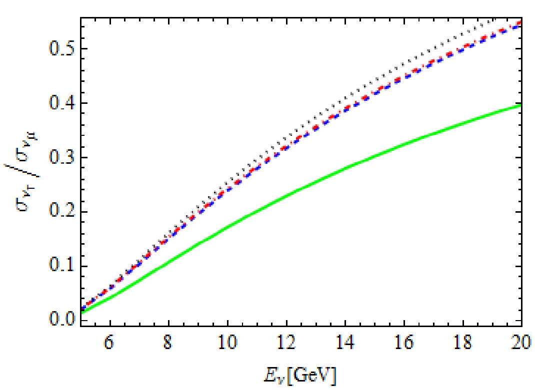

Figure 7.1: Scalar and tensor model: The ratio between the total cross section of $\nu_\tau + N \to \tau + X$ to $\nu_\mu + N \to \mu + X$ in the scalar and tensor model. The green solid line corresponds to the standard model prediction $S_R = S_L = T_L = 0$. The blue dashed, black dotted and red dotdashed lines correspond to $(S_R, S_L, T_L) = (1.49, -0.73, 0.067)$, $(1.98, 0.04, -0.079)$, $(-1.87, 0.32, 0.077)$.

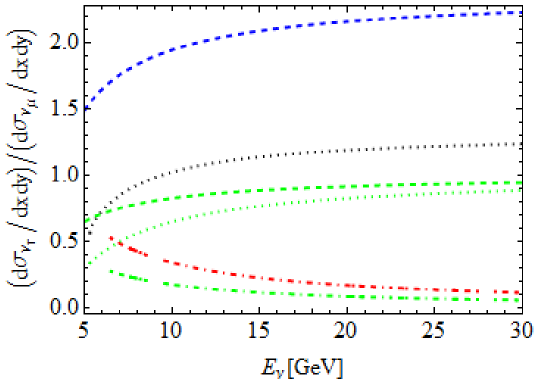

Figure 7.2: Scalar and tensor model: The ratio between the differential cross section $(d\sigma/dxdy)$ of $\nu_\tau + N \rightarrow \tau + X$ to $\nu_\mu + N \rightarrow \mu + X$ in the scalar and tensor model. The green lines correspond to the standard model predictions with $S_R = S_L = T_L = 0$. The blue, black, and red lines correspond to $(S_R, S_L, T_L) = (1.64, -1.95, -0.07), (2, -2, -0.09), (2, -1.9, -0.078)$. The blue and green dashed lines correspond to the maximum values $(x, y) = (0.95, A+B)$. The black and green dotdashed lines correspond to the halfway values $(x, y) = (0.475, (A+B)/2)$. The red and green dotted lines correspond to the minimum values $(x, y) = (\frac{m_\ell^2}{2M(E_\nu - m_\ell)}, A - B)$.

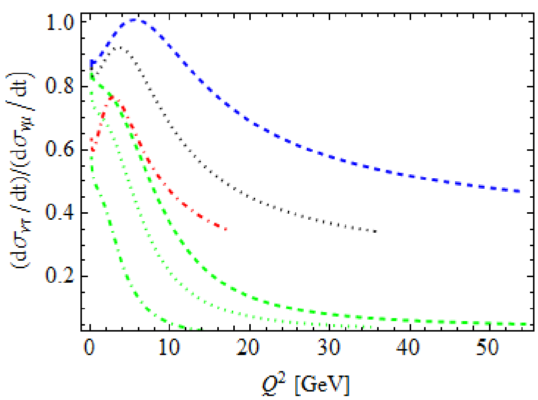

Figure 7.3: Scalar and tensor model: The ratio between the differential cross section $(d\sigma/dt)$ of $\nu_\tau + N \rightarrow \tau + X$ to $\nu_\mu + N \rightarrow \mu + X$ in the scalar and tensor model. The green dashed, dotted and dotdashed lines correspond to the standard model predictions with $S_R = S_L = T_L = 0$ at $E_\nu = 30, 20, 10$, respectively. The blue dashed, black dotted, and red dotdashed lines correspond to $(S_R, S_L, T_L) = (-1.45, 1.89, -0.056), (1.1, -1.8, 0.078), (-1.89, 1.67, -0.085)$ at $E_\nu = 30, 20, 10$, respectively. The physical regions of the momentum transfer taken to be $Q^2_-(W_{cut}) \leq Q^2 \leq Q^2_+(W_{cut})$.

7.2 $V \pm A$ model results

The sensitivity of the experiments to the ratio of the total cross section, $d\sigma/dxdy$, $d\sigma/dt$ are shown in Figs. (7.9, 7.10, 7.11), respectively. The figures show that it is difficult to distinguish the presence of the $V \pm A$ NP operators in the neutrino cross section measurements. The

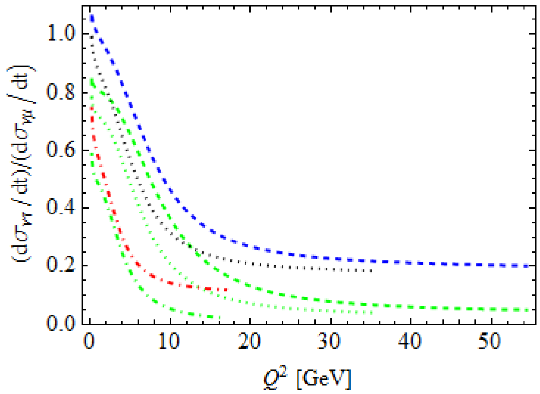

Figure 7.4: Explicit scalar and tensor model: The ratio between the differential cross section $(d\sigma/dt)$ of $\nu_\tau + N \to \tau + X$ to $\nu_\mu + N \to \mu + X$ in the explicit scalar and tensor model with $S_L(m_{\rm LQ}) = \pm 4\ T_L(m_{\rm LQ})$ at $m_{\rm LQ} = 250$ GeV. The green dashed, dotted and dotdashed lines correspond to the standard model predictions with $S_R = S_L = T_L = 0$ at $E_\nu = 30, 20, 10$, respectively. The blue, black, and red lines correspond to $(Re[S_L(m_{LQ})], Im[S_L(m_{LQ})], Re[T_L(m_{LQ})], Im[T_L(m_{LQ})]) = (0.47, 0.42, 0.12, 0.11)$ at $E_\nu = 30, 20, 10$, respectively. The physical regions of the momentum transfer taken to be $Q^2_-(W_{cut}) \leq Q^2 \leq Q^2_+(W_{cut})$.

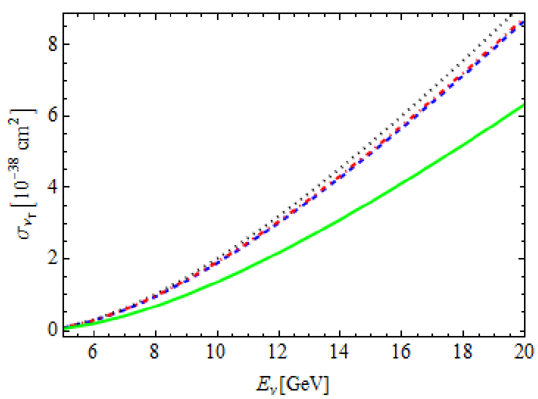

Figure 7.5: Scalar and tensor model: The total cross section of $\nu_\tau + N \to \tau + X$ in the scalar and tensor model. The green solid line corresponds to the standard model prediction $S_R = S_L = T_L = 0$. The blue dashed, black dotted and red dotdashed lines correspond to $(S_R, S_L, T_L) =$ (1.49, -0.73, 0.067), (1.98, 0.04, -0.079), (-1.87, 0.32,0.077).

new physics effect is the same in the total cross section, $d\sigma/dxdy$, and $d\sigma/dt$ for the process $\nu_\tau + N \to \tau + X$, as shown in Figs. (7.12, 7.13, 7.14).

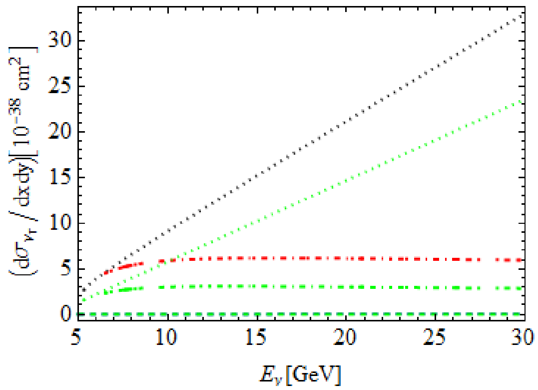

Figure 7.6: Scalar and tensor model: The differential cross section $(d\sigma/dxdy)$ of $\nu_\tau + N \to \tau + X$ in the scalar and tensor model. The green lines correspond to the standard model predictions with $S_R = S_L = T_L = 0$. The blue, black, and red lines correspond to (S_R, S_L, T_L)=(1.64,-1.95,-0.07),(2,-2,-0.09),(2,-1.9,-0.078). The blue and green dashed lines correspond to the maximum values $(x, y) = (0.95, A + B)$. The black and green dotdashed lines correspond to the halfway values $(x, y) = (0.475, (A+B)/2)$. The red and green dotted lines correspond to the minimum values $(x, y) = (\frac{m_\ell^2}{2M(E_\nu - m_\ell)}, A - B)$.

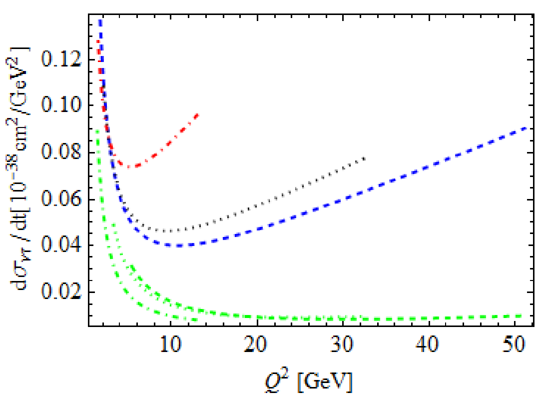

Figure 7.7: Scalar and tensor model: The differential cross section $(d\sigma/dt)$ of $\nu_\tau + N \to \tau + X$ in the scalar and tensor model. The green dashed, dotted and dotdashed lines correspond to the standard model predictions with $S_R = S_L = T_L = 0$ at $E_\nu = 30, 20, 10$, respectively. The blue dashed, black dotted, and red dotdashed lines correspond to (S_R, S_L, T_L)=(-1.45, 1.89 , -0.056),(1.1 , -1.8 , 0.078),(-1.89 , 1.67, -0.085) at $E_\nu = 30, 20, 10$, respectively. The physical regions of the momentum transfer taken to be $Q_-^2(W_{cut}) \leq Q^2 \leq Q_+^2(W_{cut})$.

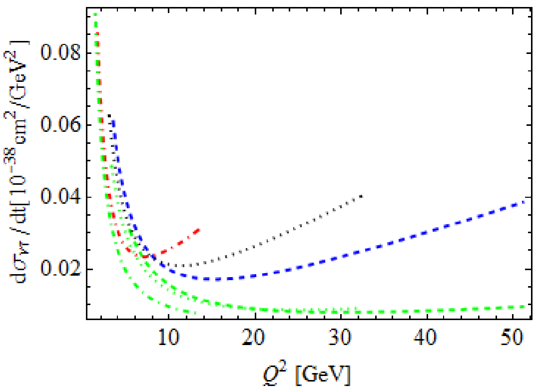

Figure 7.8: Explicit scalar and tensor model: The differential cross section $(d\sigma/dt)$ of $\nu_\tau + N \to \tau + X$ in the explicit scalar and tensor model with $S_L(m_{\rm LQ}) = \pm 4\, T_L(m_{\rm LQ})$ at $m_{\rm LQ} =$ 250 GeV. The green dashed, dotted and dotdashed lines correspond to the standard model predictions with $S_R = S_L = T_L = 0$ at $E_\nu = 30, 20, 10$, respectively. The blue, black, and red lines correspond to ($Re[S_L(m_{LQ})]$,$Im[S_L(m_{LQ})]$, $Re[T_L(m_{LQ})]$, $Im[T_L(m_{LQ})]$)=(0.47, 0.42, 0.12, 0.11) at $E_\nu = 30, 20, 10$, respectively. The physical regions of the momentum transfer taken to be $Q^2_-(W_{cut}) \leq Q^2 \leq Q^2_+(W_{cut})$.

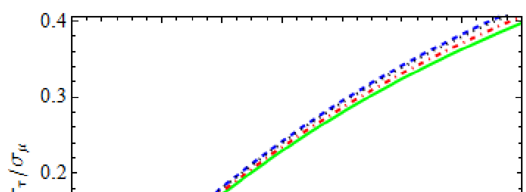

Figure 7.9: $V \pm A$ model: The ratio between the total cross section of $\nu_\tau + N \to \tau + X$ to $\nu_\mu + N \to \mu + X$ in the $V \pm A$ model. The green lines correspond to the standard model predictions $V_L = V_R$=0. The blue, black, and red lines correspond to (V_L, V_R)=(0.027,0.020),(0.022,0.021),(0.011,0.006).

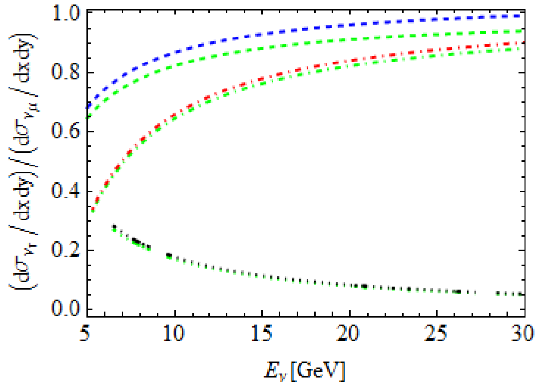

Figure 7.10: $V \pm A$ model: The ratio between the differential cross section $(d\sigma/dxdy)$ of $\nu_\tau + N \to \tau + X$ to $\nu_\mu + N \to \mu + X$ in the $V \pm A$ model. The green lines correspond to the standard model predictions $V_L = V_R$=0. The blue, black, and red lines correspond to (V_L, V_R)=(0.027,0.020),(0.022,0.021),(0.011,0.006). The blue and green dashed lines correspond to the maximum values $(x, y) = (0.95, A + B)$. The red and green dotdashed lines correspond to the halfway values $(x, y) = (0.475, (A + B)/2)$. The black and green dotted lines correspond to the minimum values $(x, y) = (\frac{m_\ell^2}{2M(E_\nu - m_\ell)}, A - B)$.

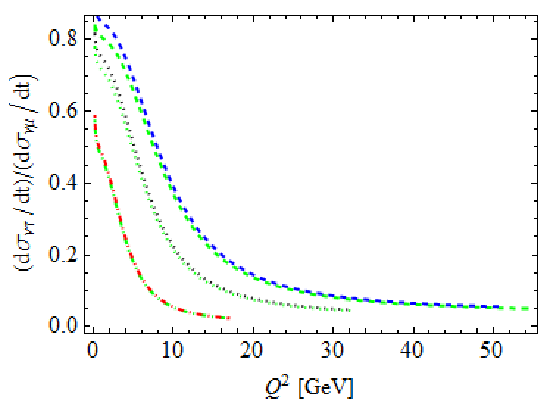

Figure 7.11: $V \pm A$ model: The ratio between the differential cross section $(d\sigma/dt)$ of $\nu_\tau + N \to \tau + X$ to $\nu_\mu + N \to \mu + X$ in the $V \pm A$ model. The green lines correspond to the standard model predictions $V_L = V_R$=0. The blue, black, and red lines correspond to (V_L, V_R)=(0.027,0.020),(0.022,0.021),(0.011,0.006) at $E_\nu = 30, 20, 10$, respectively. The physical regions of the momentum transfer taken to be $Q_-^2(W_{cut}) \le Q^2 \le Q_+^2(W_{cut})$

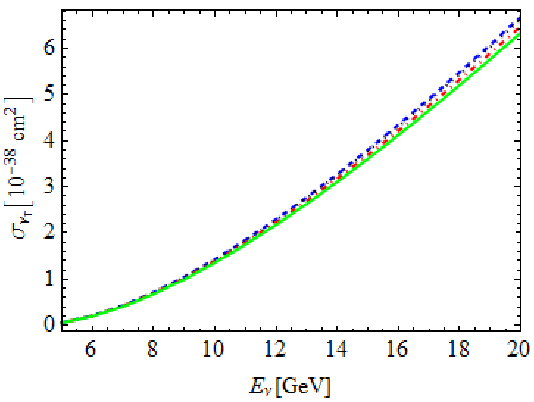

Figure 7.12: $V \pm A$ model: The total cross section of $\nu_\tau + N \to \tau + X$ in the $V \pm A$ model. The green lines correspond to the standard model predictions $V_L = V_R$=0. The blue, black, and red lines correspond to (V_L, V_R)=(0.027,0.020), (0.022,0.021), (0.011,0.006).

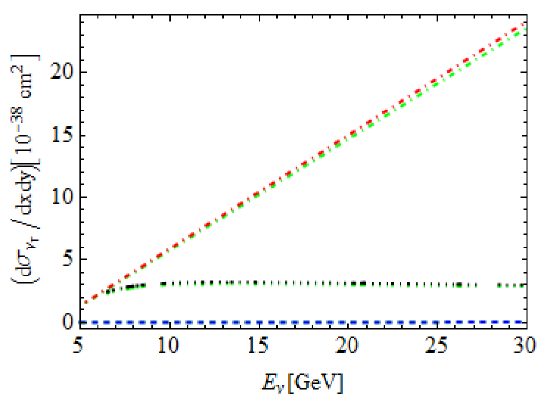

Figure 7.13: $V \pm A$ model: The differential cross section $(d\sigma/dxdy)$ of $\nu_\tau + N \to \tau + X$ in the $V \pm A$ model. The green lines correspond to the standard model predictions $V_L = V_R$=0. The blue, black, and red lines correspond to (V_L, V_R)=(0.027,0.020), (0.022,0.021), (0.011,0.006). The blue and green dashed lines correspond to the maximum values $(x, y) = (0.95, A + B)$. The red and green dotdashed lines correspond to the halfway values $(x, y) = (0.475, (A + B)/2)$. The black and green dotted lines correspond to the minimum values $(x, y) = (\frac{m_\ell^2}{2M(E_\nu - m_\ell)}, A - B)$.

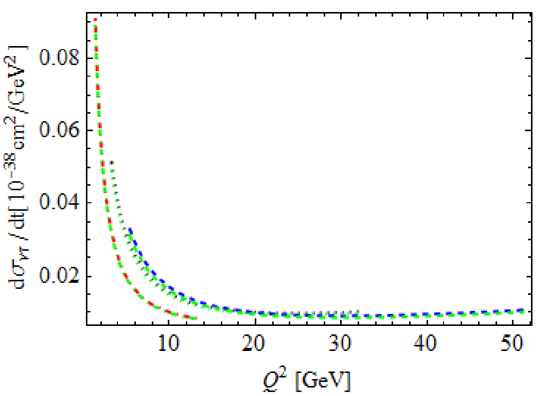

Figure 7.14: $V \pm A$ model: The differential cross section $(d\sigma/dt)$ of $\nu_\tau + N \to \tau + X$ in the $V \pm A$ model. The green lines correspond to the standard model predictions $V_L = V_R$=0. The blue, black, and red lines correspond to (V_L, V_R)=(0.027,0.020), (0.022,0.021), (0.011,0.006) at $E_\nu = 30, 20, 10$, respectively. The physical regions of the momentum transfer taken to be $Q^2_-(W_{cut}) \leq Q^2 \leq Q^2_+(W_{cut})$

CHAPTER 8

CONCLUSION

We discussed the ratio of the total and differential cross sections of the two deep inelastic scattering processes $\nu_\tau + N \to \tau + X$ and $\nu_\mu + N \to \mu + X$ as a test of lepton nonuniversality in the neutrino cross-section experiments. In the ratio of cross sections, the uncertainty of the parton distribution functions cancelled out leading to precise results. We introduced two new physics models, scalar, tensor model and $V \pm A$ model. In the scalar and tensor model, scalar and tensor states were included with real and complex couplings. An explicit case was discussed in the leptoquark model where the couplings run down from the leptoquark mass scale to the tau mass scale with the left-handed scalar and tensor couplings (S_L, T_L) are no longer independent, $S_L(m_{\rm LQ}) = \pm 4T_L(m_{\rm LQ})$ at $m_{\rm LQ} = 1$ TeV.

The constraint on the free parameters of the scalar leptoquark state (S_L, S_R) was taken from the tau decay $\tau^-(k_1) \to \nu_\tau(k_2) + \pi^-(q)$. While the constraint on the tensor coupling was considered from the tau three body decay mode $\tau(p) \to \pi^-(p_1) + \pi^0(p_2) + \nu_\tau(p_3)$. Here we considered the couplings are real and complex values.

The results showed that it is difficult to observe the new physics effect in the $V \pm A$ model in the ratio of the total and differential cross sections. The new physics impact is clearly detectable in the ratio of the differential cross sections. And it is not difficult to distinguish between the new physics and the SM results in the ratio of total cross section. The same observation is found in the total and differential cross sections of the process $\nu_\tau + N \to \tau + X$ in the scalar, tensor and $V \pm A$ models.

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LIST OF APPENDICES

APPENDIX A: DIFFERENTIAL CROSS SECTION

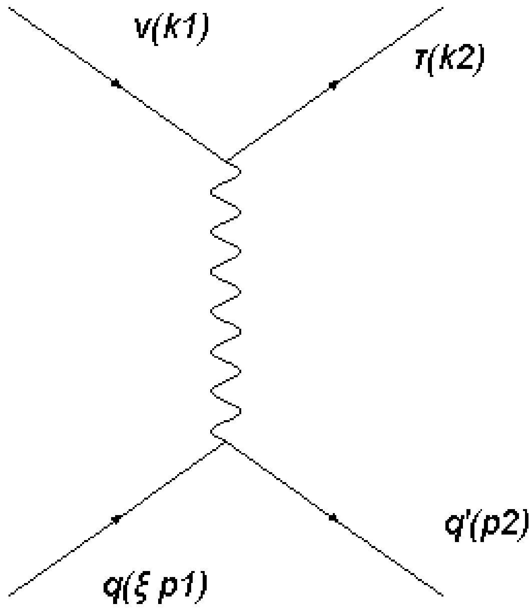

Figure A.1: Feynman diagram of DIS in the parton approximation.

In the following, we will derive the Eq.(5.7) by using Eq.(5.5), which is given as follow

$$d\sigma = (2\pi)^4\delta^4(\Sigma p'_f - \Sigma p_i)\frac{1}{4E_1E_2v_{rel}}(\prod_l(2m_l))(\prod_f\frac{d^3\mathbf{p}'_f}{(2\pi)^3 2E'_f})|M|^2.$$

In the parton approximation and ignoring all the m_l since it will be cancelled, we can obtain

$$d\sigma = \int f(\xi)d\xi(2\pi)^4\delta^4(\Sigma p'_f - \Sigma p_i)\frac{1}{4E_1E_2v_{rel}}(\prod_f\frac{d^3\mathbf{p}'_f}{(2\pi)^3 2E'_f})|M(\xi)|^2, \tag{A.1}$$

where

$$E_1E_2v_{rel} = [(\xi p_1 k_1)^2 - m_1^2 m_2^2]^{1/2} = \xi p_1 k_1. \tag{A.2}$$

In the rest frame of nucleon, we have $k_1^\mu = \big(E_\nu,\ 0,\ 0,\ E_\nu\big)$, $k_2^\mu = \big(E_\tau,\ p_\tau sin\theta,\ 0,\ p_\tau cos\theta\big)$, $p_1^\mu = \big(M,\ 0,\ 0,\ 0\big)$, $q^\mu = k_1^\mu - k_2^\mu = \big(E_\nu - E_\tau,\ -p_\tau sin\theta,\ 0,\ E_\nu - p_\tau cos\theta\big)$. Plugging Eq.(A.2) into Eq.(A.1), we can obtain

$$\begin{aligned} d\sigma &= \int f(\xi)d\xi(2\pi)^4\delta^4(k_2 + p_2 - K_1 - \xi p_1)\frac{|M(\xi)|^2}{4\xi p_1 k_1}\frac{d^3\vec{k_2}}{(2\pi)^3 2E_\tau}\frac{d^3\vec{p_2}}{(2\pi)^3 2E'_2} \\ &= \frac{1}{64\pi^2 p_1 k_1 E_\tau E'_2}\int\frac{f(\xi)}{\xi}d\xi|M(\xi)|^2\delta^4(k_2 + p_2 - K_1 - \xi p_1)d^3\vec{k_2}d^3\vec{p_2} \\ &= \frac{1}{32\pi^2 p_1 k_1}\int\frac{f(\xi)}{\xi}d\xi|M(\xi)|^2\frac{\delta(p_2^0 - q^0 - \xi p_1^0)}{2E'_2}\frac{d^3\vec{k_2}d^3}{E_\tau};(\vec{p_2} = \xi\vec{p_1} + \vec{q}). \end{aligned} \tag{A.3}$$

By using the property of δ function, we know

$$\begin{aligned}
\frac{\delta(p_2^0 - q^0 - \xi p_1^0)}{2E_2'} &= \frac{\delta(p_2^0 - q^0 - \xi p_1^0)}{2p_2^0} \\
&= \theta(p_2^0)\delta[(p_2^0)^2 - (\xi p_1^0 + q^0)^2] \\
&= \theta(\xi p_1^0 + q^0)\delta[(p_2)^2 - (\xi p_1 + q)^2] \\
&= \theta(\xi p_1^0 + q^0)\delta(2M\nu\xi + q^2) \\
&= \theta(\xi p_1^0 + q^0)\frac{\delta(\xi - x)}{2M\nu}, \qquad \text{(A.4)}
\end{aligned}$$

where the definitions of x,y and ν are in the Eq.(5.6). Then, plugging Eq.(A.4) into Eq.(A.3), we obtain

$$d\sigma = \frac{1}{32\pi^2 p_1 k_1}\int \frac{f(\xi)}{\xi} d\xi |M(\xi)|^2 \frac{\delta(\xi - x)}{2M\nu}\frac{d^3\vec{k_2}}{E_\tau}. \qquad \text{(A.5)}$$

And we know that

$$\int \frac{d^3\vec{k_2}}{E_\tau} = \int \frac{p_\tau^2 d\Omega dp_\tau}{E_\tau} = 2\pi \int p_\tau dcos\theta dE_\tau. \qquad \text{(A.6)}$$

According to Eq.(5.6),we know

$$\begin{aligned}
x &= \frac{2E_\nu E_\tau - m_\tau^2 - 2p_\tau E_\nu cos\theta}{2M\nu}, \\
y &= 1 - \frac{E_\tau}{E_\nu}. \qquad \text{(A.7)}
\end{aligned}$$

Therefore, we can obtain the relation

$$\begin{aligned}
dxdy &= \begin{bmatrix} \frac{\partial x}{\partial cos\theta} & \frac{\partial x}{\partial E_\tau} \\ \frac{\partial y}{\partial cos\theta} & \frac{\partial y}{\partial E_\tau} \end{bmatrix} dcos\theta dE_\tau, \\
&= \frac{p_\tau}{M\nu} dcos\theta dE_\tau. \qquad \text{(A.8)}
\end{aligned}$$

Thus we have,

$$\int \frac{d^3\vec{k_2}}{E_\tau} = 2\pi M\nu \int dxdy. \qquad \text{(A.9)}$$

Finally, we can obtain Eq.(5.7) by plugging Eq.(A.9) into Eq.(A.5).

$$\frac{d\sigma}{dxdy} = \frac{1}{32\pi M E_\nu}\int \frac{d\xi}{\xi} f(\xi)|\bar{\mathcal{M}}(\xi)|^2 \delta(\xi - x).$$

APPENDIX B: DERIVATION OF $d\sigma_{SM}/dxdy$

Let us start with Eq.(5.7)

$$\frac{d\sigma_{SM}}{dxdy} = \frac{1}{32\pi M E_\nu} \int \frac{d\xi}{\xi} f(\xi) |\bar{\mathcal{M}}_{SM}(\xi)|^2 \delta(\xi - x),$$

where,

$$\begin{aligned} \bar{\mathcal{M}}_{SM}(\xi)|^2 &= \frac{1}{2}\Sigma_{spin}|M_{SM}(\xi)|^2, \\ M_{SM}(\xi) &= (\frac{-iG_F V_{qq'}}{\sqrt{2}})[\bar{u}_\tau(k_2)\gamma^\mu(1-\gamma^5)u_{\nu_\tau}(k_1)][\bar{u}_{q'}(p_2)\gamma_\mu(1-\gamma^5)u_q(\xi p_1)]. \end{aligned} \tag{B.1}$$

Now assume

$$\begin{aligned} L^\mu &= \bar{u}_{\tau\alpha}(k_2)(\gamma^\mu(1-\gamma^5))_{\alpha\beta}u_{\nu_\tau\beta}(k_1), \\ W(\xi)_\mu &= \bar{u}_{q'\alpha}(p_2)(\gamma_\mu(1-\gamma^5))_{\alpha\beta}u_{q\beta}(\xi p_1) \end{aligned} \tag{B.2}$$

By using the equtions

$$\begin{aligned} \Sigma_{spin}u_\beta(p)\bar{u}_\alpha(p) &= (\not{p}+m)_{\beta\alpha}, \\ \Sigma_{spin}\nu_\beta(p)\bar{\nu}_\alpha(p) &= (\not{p}-m)_{\beta\alpha}, \end{aligned} \tag{B.3}$$

we obtain

$$\begin{aligned} L^{\mu\nu} = \Sigma_{spin}L^\mu L^{*\nu} &= (\not{k}_2+m_\tau)_{\sigma\alpha}(\gamma^\mu(1-\gamma^5))_{\alpha\beta}(\not{k}_1)_{\beta\rho}(\gamma^\mu(1-\gamma^5))_{\rho\sigma}, \\ &= Tr[(\not{k}_2+m_\tau)\gamma^\mu(1-\gamma^5)(\not{k}_1)\gamma^\mu(1-\gamma^5)], \\ &= 4[-2g^{\mu\nu}k_1\cdot k_2 + 2i\epsilon^{\mu\nu\rho\sigma}k_{1\rho}k_{2\sigma} + 2(k_1^\mu k_2^\nu + k_2^\mu k_1^\nu)], \\ W(\xi)_{\mu\nu} = \Sigma_{spin}W_\mu W_\nu^* &= (\not{p}_2)_{\sigma\alpha}(\gamma_\mu(1-\gamma^5))_{\alpha\beta}(\xi\not{p}_1)_{\beta\rho}(\gamma_\mu(1-\gamma^5))_{\rho\sigma}, \\ &= Tr[\not{p}_2\gamma_\mu(1-\gamma^5)\xi\not{p}_1\gamma_\mu(1-\gamma^5)], \\ &= 4\xi[-2g_{\mu\nu}p_1\cdot q + 2i\epsilon_{\mu\nu\rho\sigma}p^{1\rho}q^\sigma + 2(p_{1\mu}q_\nu + q_\mu p_{1\nu}) + 4\xi p_{1\mu}p_{1\nu}]. \end{aligned} \tag{B.4}$$

Plugging Eq.(B.4) into Eq.(B.1), we can obtain

$$\begin{aligned} \bar{\mathcal{M}}_{SM}(\xi)|^2 &= \frac{G_F^2 V_{qq'}^2}{4} L^{\mu\nu}W_{\mu\nu}, \\ \frac{d\sigma_{SM}}{dxdy} &= \frac{G_F^2 V_{qq'}^2}{128\pi M E_\nu} \int \frac{d\xi}{\xi} f(\xi) L^{\mu\nu}W(\xi)_{\mu\nu}\delta(\xi - x), \\ &= \frac{G_F^2 V_{qq'}^2 y}{16\pi}[A + B - C + E]f(x), \end{aligned} \tag{B.5}$$

where

$$
\begin{aligned}
A &= 16 k_1 \cdot k_2 = 16 M E_\nu (xy + \frac{m_\tau^2}{2ME_\nu}),\\
B &= 8\frac{2(k_1\cdot p_1)(k_2\cdot p_1) - p_1^2(k_1\cdot k_2)}{p_1\cdot q} = 16(ME_\nu)^2(1-y-Mxy/(2E_\nu)-m_\tau^2/(4E_\nu^2))/(p_1\cdot q),\\
C &= 16\frac{(k_1\cdot q)(k_2\cdot p_1)-(k_1\cdot p_1)(k_2\cdot q)}{p_1\cdot q} = -32(ME_\nu)^2[xy(1-y/2)-ym_\tau^2/(4ME_\nu)]/(p_1\cdot q),\\
E &= 16\frac{(k_1\cdot q)(k_2\cdot p_1)+(k_1\cdot p_1)(k_2\cdot q)-(k_1\cdot k_2)(p_1\cdot q)}{p_1\cdot q} = 16(ME_\nu)\frac{m_\tau^2}{p_1\cdot q}. \qquad \text{(B.6)}
\end{aligned}
$$

Then, we can easily get Eq.(5.7)

$$
\frac{d\sigma_{\rm SM}}{dxdy} = \frac{G_F^2 ME_\nu}{\pi}\left(y(xy+\frac{m_\ell^2}{2ME_\nu})F_1 + (1-y-\frac{Mxy}{2E_\nu}-\frac{m_\ell^2}{4E_\nu^2})F_2 \;+(xy(1-\frac{y}{2})-y\frac{m_\ell^2}{4ME_\nu})F_3 - \frac{m_\ell^2}{2ME_\nu}F_5\right)
$$

where the from factors F_i, $(i=1,2,3,5)$ are given in Eq.(5.15).

APPENDIX C: PARTON DISTRIBUTION FUNCTION

The quark distribution function can be usefully decomposed into valence quarks and sea quarks. The presence of the valence quarks is already indicated by the original quark model. Like protons and neutrons have valence quarks of (uud) and (udd), respectively. The sea quarks correspond to those quarks pairs produced by the gluons. Therefore τ neutrino cannot only interact with d quark in nucleon, but also can interact with s,$\bar{u}$, and $\bar{c}$ quarks in the following process

$$\nu_\tau + d \to \tau^- + u(c), \nu_\tau + s \to \tau^- + c(u), \nu_\tau + \bar{u} \to \tau^- + \bar{d}(\bar{s}), \nu_\tau + \bar{c} \to \tau^- + \bar{s}(\bar{d}).$$

Using the isospin symmetry, we have

$$\begin{aligned} u_p(x) &= d_n(x) \equiv u(x), \\ u_n(x) &= d_p(x) \equiv d(x), \\ s_p(x) &= s_n(x) \equiv s(x) \\ . \end{aligned} \tag{C.1}$$

And The hadronic weak current has the form

$$J_h^\mu = \bar{u}\gamma^\mu(1-\gamma^5)(d\ \ cos\theta_C + s\ \ sin\theta_C) + \bar{c}\gamma^\mu(1-\gamma^5)(-d\ \ sin\theta_C + s\ \ cos\theta_C), \tag{C.2}$$

where θ_C is Cabibbo angle. Thus, if we only consider about light quarks(u,d,s) and the nucleon has equal number of protons and neutrons, we can write function $f_{q,\bar{q}}(\xi, Q^2)$ in Eq.(5.15) as follow

$$\begin{aligned} f_q(\xi, Q^2) &= \frac{u+d}{2} cos^2\theta_C + s\ \ sin^2\theta_C, \\ f_{\bar{q}}(\xi, Q^2) &= \frac{\bar{u}+\bar{d}}{2}. \end{aligned} \tag{C.3}$$

APPENDIX D: τ THREE-BODY DECAY

The decay rate Γ can be expressed as

$$\Gamma = \frac{1}{2E_i}\int\prod_f \frac{d^3p_f}{(2\pi)^3 2E_f}(2\pi)^4\delta^4(p_i - \Sigma_f p_f)|M_{fi}|^2. \tag{D.1}$$

In the process of τ three-body decay

$$\tau(p) \to \nu_\tau(p_3) + \pi^-(p_1) + \pi^0(p_2),$$

there are three particles in the final state. For the convenience, we use the rest frame of τ. Then, the four-momentum has the forms as $p^\mu = \big(m_\tau,\ \ \vec{0}\big)$, $p_1^\mu = \big(E_1,\ \ \vec{p_1}\big)$, $p_2^\mu = \big(E_2,\ \ \vec{p_2}\big)$, $p_3^\mu = \big(E_3,\ \ \vec{p_3}\big)$. The decay rate then can be written as

$$\begin{aligned}\Gamma &= \frac{1}{2m_\tau}\int \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}(2\pi)^4\delta^4(p - p_1 - p_2 - p_3)|M_{fi}|^2, \\ &= \frac{1}{2m_\tau(2\pi)^5}\int \frac{d^3p_1 d^3p_2 d^3p_3}{8E_1E_2E_3}\delta^4(p - p_1 - p_2 - p_3)|M_{fi}|^2. \end{aligned} \tag{D.2}$$

The phase integral is

$$R_3 = \int \frac{d^3p_1 d^3p_2 d^3p_3}{8E_1E_2E_3}\delta^4(p - p_1 - p_2 - p_3). \tag{D.3}$$

We first integrate Eq.(D.3) over p_3 with the result

$$\begin{aligned} R_3 &= \int \frac{d^3p_1 d^3p_2}{8E_1E_2E_3}\delta(m_\tau - E_1 - E_2 - E_3), \\ \vec{p} &= \vec{p_1} + \vec{p_2} + \vec{p_3}. \end{aligned} \tag{D.4}$$

The next step is to integrate over the angles. First integrate over the directions $\vec{p_2}$ relative to a fixed direction of $\vec{p_1}$, then over the directions of $\vec{p_1}$ itself. Assume θ is the angle between $\vec{p_1}$ and $\vec{p_2}$ and set $cos\theta = \xi$. Then the first angular integration gives $2\pi p_2^2 dp_2 d\xi$. After first integration, no direction is distinguished any more. Therefore the second angular integration gives $4\pi p_1^2 dp_1$. Hence we obtain

$$R_3 = \pi^2\int \frac{p_1^2 p_2^2}{E_1E_2E_3}dp_1 dp_2 d\xi\,\delta(m_\tau - E_1 - E_2 - E_3). \tag{D.5}$$

Next, we need to change variables from $p_1, p_2.\xi$ to E_1, E_2, E_3. According to Einstein's mass-energy relation and conservation of momentum, we have the formula for each energy as follow

$$\begin{aligned} E_1 &= \sqrt{p_1^2 + m_1^2}, \\ E_2 &= \sqrt{p_2^2 + m_2^2}, \\ E_3 &= \sqrt{p_3^2 + m_3^2} = \sqrt{(p_1 + p_2)^2 + m_3^2} = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\xi + m_3^2}. \end{aligned} \tag{D.6}$$

We now transform the variables by means of

$$dp_1dp_2d\xi = dE_1dE_2dE_3|\frac{\partial(E1E2E3)}{\partial(p_1p_2\xi)}|^{-1}, \tag{D.7}$$

where the Jacobian is

$$\frac{\partial(E1E2E3)}{\partial(p_1p_2\xi)}|^{-1} = \frac{E_1E_2E_3}{p_1^2p_2^2}. \tag{D.8}$$

Inserting this into the integral (D.5) yields

$$\begin{aligned} R_3 &= \pi^2 \int dE_1dE_2dE_3\delta(m_\tau - E_1 - E_2 - E_3), \\ &= \pi^2 \int dE_1dE_3; \;\; m_\tau = E_1 + E_2 + E_3. \end{aligned} \tag{D.9}$$

As in the rest frame of τ we have

$$\begin{aligned} m_{12}^2 &= (p_1+p_2)^2 = (p-p_3)^2 = m_\tau^2 + m_3^3 - 2m_\tau E_3, \\ m_{23}^2 &= (p_2+p_3)^2 = (p-p_1)^2 = m_\tau^2 + m_1^3 - 2m_\tau E_1, \end{aligned} \tag{D.10}$$

the phase integral can be written as

$$R_3 = \frac{\pi^2}{4m_\tau^2}\int dm_{12}^2dm_{23}^2. \tag{D.11}$$

Inserting this into Eq.(D.2), we can obtain Eq.(6.12)

$$d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32m_\tau^3}Xdm_{12}^2dm_{23}^2,$$

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