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Constant Velocity Combustion Scramjet Analysis

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University of Mississippi

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CONSTANT VELOCITY COMBUSTION SCRAMJET ANALYSIS

A Thesis
presented in partial fulfillment of requirements
for the degree of Master of Science
in the Department of Mechanical Engineering
The University of Mississippi

by
LAKSHMI SRIKANTH TIRUVEEDULA
May 2016
ABSTRACT

The supersonic combustion ramjet or scramjet, is the most suitable engine cycle for sustained hypersonic flight in the atmosphere. The present work deals with the performance of a scramjet engine by parametrically analyzing the performance of the ideal scramjet using the engine parameters: specific thrust, fuel-to-air ratio, thrust specific fuel consumption, thermal efficiency, propulsive efficiency, overall efficiency and thrust flux. The objective of the work is to determine the desirable performance terms of the ideal scramjet, by varying three different candidate fuels and three different candidate materials for the combustion chamber. The engine parameters are related by the lower heating value ($h_{PR}$) of the fuel and the maximum service temperature ($T_{max}$) of the material. This convenient mathematical equations are development for ideal scramjet performance. The knowledge offered on this work has not been achieved by using others within the scientific literature.
NOMENCLATURE

$a_o$ = freestream speed of sound, m/s

$A_2$ = diffuser (engine inlet) exit area, cm$^2$; combustor entrance area

$A_4$ = combustor exit area, cm$^2$

$A^*/A$ = area ratio

$c_p$ = specific heat at constant pressure, kJ/(kg K)

$F$ = thrust, N

$F/\dot{m}_o$ = specific thrust, N/(kg/s)

$f$ = fuel-to-air ratio

$g_c$ = Newton’s constant, (kg m)/(N s$^2$)

$h_{PR}$ = fuel lower heating value, kJ/kg
\( m_o \) = mass flow rate of air, kg/s

\( M_2 \) = combustor entrance Mach number

\( M_4 \) = combustor exit Mach number

\( M_o \) = Mach number at freestream flight conditions

\( M_9 \) = Mach number at engine nozzle exit

\( P \) = pressure, Pa

\( P_o \) = free-stream static pressure, Pa

\( R \) = gas constant for air, kJ/(kg K)

\( S \) = thrust-specific fuel consumption, mg/(N s)

\( s \) = entropy, kJ/(kg K)

\( T \) = temperature, K

\( T_{\text{max}} \) = material temperature limit, K
\( T_0 \) = freestream ambient temperature, K

\( T_{to} \) = freestream total temperature, K

\( T_{t2} \) = combustor entrance total temperature, K

\( T_{t4} \) = combustor exit total temperature, K

\( T_9 \) = temperature at engine nozzle exit, K

\( V_0 \) = velocity at freestream conditions, m/s

\( V_2 \) = velocity at combustor entrance, m/s

\( V_4 \) = velocity at combustor exit, m/s

\( V_9 \) = engine nozzle exit velocity, m/s

\( \gamma \) = ratio of specific heats

\( \rho \) = density, kg/m\(^3\)

\( \tau \) = \( T_{t4} / T_{t2} \)
\( \tau_r = \) freestream total temperature to static temperature ratio

\( \tau_\lambda = \frac{T_{\text{max}}}{T_0} \)

\( \eta_T = \) thermal efficiency

\( \eta_P = \) propulsive efficiency

\( \eta_o = \) overall efficiency

Subscripts

0 = freestream conditions

2 = combustor entrance conditions

4 = combustor exit conditions

9 = nozzle exit conditions

t = stagnation (total) conditions
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTERS</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>vii</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>viii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xviii</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Air-Breathing Engines</td>
<td>2</td>
</tr>
<tr>
<td>1.2.1 Brayton Cycle</td>
<td>3</td>
</tr>
<tr>
<td>1.2.2 Ramjet</td>
<td>4</td>
</tr>
<tr>
<td>1.2.3 Scramjet</td>
<td>5</td>
</tr>
</tbody>
</table>
1.3 Historical Development ................................................................. 9

1.4 Materials and Fuels................................................................. 10

   1.4.1 Materials................................................................. 12

   1.4.2 Fuels................................................................. 14

1.5 Parametric Thermodynamic Analysis.......................................... 14

1.6 This Work................................................................. 15

2. STATEMENT OF PROBLEM............................................................ 17

   2.1 Definition of the Problem.................................................... 17

   2.2 T-s Diagram................................................................. 18

      2.2.1 Situations for \( V_2 \leq V_o(M_2 \leq M_o) \) ......................... 20

   2.3 Materials and Fuel selection.............................................. 21

3. ANALYSIS....................................................................................... 24

   3.1 Assumptions of Cycle Analysis............................................. 24

   3.2 Parametric Analysis for Ideal Mass Flow Rate Scramjet.............. 25

      3. 2. A Engine Performance Expressions.................................. 29
3.3 Parametric Analysis for Non-Ideal Mass Flow Rate Scramjet.............. 34

3. 3. A. Engine Performance Expressions.............................................. 39

4. RESULTS.......................................................................................... 45

4.1 Performance parameters for constant velocity versus constant
Mach number................................................................. 46

4.2 Performance parameters for ideal and non-ideal mass flow rate........ 60

4.3 Performance parameters for different scramjet fuels....................... 76

4.3.1 Fuel-to-air ratio................................................................. 78

4.3.2 Specific thrust................................................................. 82

4.3.3 Thrust-specific fuel consumption......................................... 86

4.3.4 Propulsive and Overall efficiencies...................................... 90

4.3.5 Thrust flux................................................................. 97

4.3.6 Overall desirable fuel selection analysis.............................. 101

4.4 Performance parameters for different scramjet materials.................. 103

4.4.1 Fuel-to-air ratio................................................................. 105
4.4.2 Specific thrust................................................................. 109

4.4.3 Thrust-specific fuel consumption................................. 113

4.4.4 Thermal efficiency....................................................... 117

4.4.5 Propulsive and Overall efficiencies.............................. 121

4.4.6 Thrust flux................................................................. 128

4.4.7 Overall desirable material selection analysis.................132

5. CONCLUSIONS........................................................................134

REFERENCES..............................................................................137

VITA.....................................................................................141
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>Important Scramjet Technology programs worldwide</td>
</tr>
<tr>
<td>2-1</td>
<td>Selected materials and corresponding maximum service temperatures ($T_{\text{max}}$).</td>
</tr>
<tr>
<td>2-2</td>
<td>Selected fuels and corresponding lower-heating values</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>3</td>
</tr>
<tr>
<td>1-2</td>
<td>5</td>
</tr>
<tr>
<td>1-3</td>
<td>6</td>
</tr>
<tr>
<td>1-4</td>
<td>6</td>
</tr>
<tr>
<td>1-5</td>
<td>8</td>
</tr>
<tr>
<td>1-6</td>
<td>8</td>
</tr>
<tr>
<td>1-7</td>
<td>14</td>
</tr>
<tr>
<td>2-1</td>
<td>19</td>
</tr>
<tr>
<td>3-1</td>
<td>25</td>
</tr>
<tr>
<td>3-2</td>
<td>35</td>
</tr>
<tr>
<td>4-1</td>
<td>48</td>
</tr>
<tr>
<td>4-2</td>
<td>49</td>
</tr>
</tbody>
</table>

1-1 The ideal Brayton cycle

1-2 Schematic of a ramjet engine

1-3 Specific impulse and operating Mach numbers for different flight systems

1-4 Flight Mach number for various propulsion systems

1-5 Boeing X-51A Attached to the Rocket Engine and Boeing B52

1-6 Representative scramjet engine

1-7 Maximum service temperature of various materials

2-1 T-s diagram comparison for ramjet (0-t2-4-9-0) and scramjet (0-2-4-9-0)

3-1 Heat in and heat out of ideal scramjet combustion chamber

3-2 Heat in and heat out of non-ideal scramjet combustion chamber

4-1 Specific Thrust for constant Mach number versus constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$)

4-2 Fuel-to-air ratio for constant Mach number versus constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$)
<table>
<thead>
<tr>
<th>Page</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-3</td>
<td>Thrust-specific fuel consumption for constant Mach number versus constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ((M_2 = 0, 1, 2, 3))</td>
</tr>
<tr>
<td>4-4</td>
<td>Thermal efficiency for constant Mach number versus constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ((M_2 = 0, 1, 2, 3))</td>
</tr>
<tr>
<td>4-5</td>
<td>Propulsive efficiency for constant Mach number versus constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ((M_2 = 0, 1, 2, 3))</td>
</tr>
<tr>
<td>4-6</td>
<td>Overall efficiency for constant Mach number versus constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ((M_2 = 0, 1, 2, 3))</td>
</tr>
<tr>
<td>4-7</td>
<td>Thrust flux for constant Mach number versus constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ((M_2 = 0, 1, 2, 3))</td>
</tr>
<tr>
<td>4-8</td>
<td>Combustor area ratio for constant Mach number versus constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ((M_2 = 0, 1, 2, 3))</td>
</tr>
<tr>
<td>4-9</td>
<td>Combustor exit Mach number for constant velocity combustion scramjet versus freestream Mach numbers for various combustion Mach numbers ((M_2 = 0, 1, 2, 3))</td>
</tr>
<tr>
<td>4-10</td>
<td>Combustor total pressure ratio for constant Mach number versus constant velocity combustion scramjet versus freestream Mach numbers for various combustion Mach numbers ((M_2 = 0, 1, 2, 3))</td>
</tr>
<tr>
<td>4-11</td>
<td>Specific thrust for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at free stream Mach numbers for various combustion Mach numbers ((M_2 = 0, 1, 2, 3))</td>
</tr>
<tr>
<td>4-12</td>
<td>Fuel-to-air ratio for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ((M_2 = 0, 1, 2, 3))</td>
</tr>
<tr>
<td>Section</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>4-13</td>
<td>Thrust-specific fuel consumption for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$)</td>
</tr>
<tr>
<td>4-14</td>
<td>Thermal efficiency for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$)</td>
</tr>
<tr>
<td>4-15</td>
<td>Propulsive efficiency for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$)</td>
</tr>
<tr>
<td>4-16</td>
<td>Overall efficiency for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$)</td>
</tr>
<tr>
<td>4-17</td>
<td>Thrust flux for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$)</td>
</tr>
<tr>
<td>4-18</td>
<td>Area ratio across the combustor for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$)</td>
</tr>
<tr>
<td>4-19</td>
<td>Combustor exit Mach number for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$)</td>
</tr>
<tr>
<td>4-20</td>
<td>Total temperature ratio for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$)</td>
</tr>
<tr>
<td>4-21</td>
<td>Entropy change for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$)</td>
</tr>
<tr>
<td>4-22</td>
<td>Combustor velocity ratio for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$)</td>
</tr>
<tr>
<td>4-23</td>
<td>Fuel-to-air ratio for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.</td>
</tr>
</tbody>
</table>
4-24 Fuel-to-air ratio for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels

4-25 Fuel-to-air ratio for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

4-26 Specific thrust for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

4-27 Specific thrust for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

4-28 Specific thrust for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

4-29 Thrust-specific fuel consumption for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels

4-30 Thrust-specific fuel consumption for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels

4-31 Thrust-specific fuel consumption for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels

4-32 Propulsive efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

4-33 Propulsive efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

4-34 Propulsive efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels

4-35 Overall efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels

4-36 Overall efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels
4-37 Overall efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels

4-38 Thrust flux for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels

4-39 Thrust flux for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels

4-40 Thrust flux for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels

4-41 Fuel-to-air ratio for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

4-42 Fuel-to-air ratio for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

4-43 Fuel-to-air ratio for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

4-44 Specific thrust for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials

4-45 Specific thrust for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials

4-46 Specific thrust for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials

4-47 Thrust-specific fuel consumption for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

4-48 Thrust-specific fuel consumption for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

4-49 Thrust-specific fuel consumption for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

4-50 Thermal efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials
| 4-51 | Thermal efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials | 119 |
| 4-52 | Thermal efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials | 120 |
| 4-53 | Propulsive efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials | 122 |
| 4-54 | Propulsive efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials | 123 |
| 4-55 | Propulsive efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials | 124 |
| 4-56 | Overall efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials | 125 |
| 4-57 | Overall efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials | 126 |
| 4-58 | Overall efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials | 127 |
| 4-59 | Thrust flux for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials | 129 |
| 4-60 | Thrust flux for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials | 130 |
| 4-61 | Thrust flux for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials | 131 |
CHAPTER 1
INTRODUCTION

1.1 Introduction

The supersonic combustion ramjet or scramjet is the most suitable engine for sustained hypersonic flight in atmosphere. For years, the hydrocarbon fueled conventional ramjet engine was under development for high speed missile applications. The performance of a conventional ramjet is limited to as high as Mach 5 [1]. Only scramjet propulsion engines have the potential of achieving a maximum Mach number of 15 theoretically; hence it is the most likely engine for hypersonic air-breathing flight [2]. The recently tested X-51A, which was built on the experience of previous hypersonic flight tests, NASA-Air force NAVY X-15 and NASA’s X-43A which reached Mach 10 under scramjet power in 2004, proved that the scramjet engine can achieve a sustained hypersonic flight by flying at hypersonic speed for about 200 seconds [3].

Over the years, significant technical challenges have been overcome in establishing the scramjet into reality. The problem faced by the scramjet is that it cannot operate at subsonic speeds and must be accelerated up to supersonic speeds by another vehicle such as a ramjet or a turbojet in order for the air to be compressed in the inlet and ignited in the combustor [4]. Another major challenge being faced is to find the materials that can withstand the high operating temperatures, which require particular attention to overcome the overall thermal management of the engine/air
frame [5]. The fuel cooling the engine is a viable way of maintaining thermal balance over a range of flight; this fuel should have a high lower-heating value (h_{PR}) to obtain the maximum thrust during the combustion of fuel.

The purpose of the present work is to give a better understating of the performance of the ideal scramjet with respect to operating temperature (T_{max}) of the material and the lower-heating value (h_{PR}) of the fuel by integrating them into the seven performance parameters of the ideal scramjet engine as a function of free stream Mach number (M_o) with combustion Mach number (M_2) as a parameter.

1.2 Air-Breathing Engines

Propulsion is defined as “the act of propelling, the state of being propelled, a propelling force or impulse” [4]. The study of propulsion is concerned with vehicles such as automobiles, trains, ships, aircraft and spacecraft. Methods formulated to produce a thrust force for the propulsion of a vehicle in flight are based on the principles of jet propulsion, which are divided into two broad categories: air-breathing and non-air-breathing [4]; which are further classified as ramjet, scramjet, turbojet, and rocket propulsion. Our main view is based on the air-breathing engines. The commonly used type of an air-breathing propulsion is ramjet propulsion as at a flight velocity of approximately Mach 3, where compressors are no longer needed to compress the incoming air to increase the pressure [6]. The ramjet engine becomes inefficient to accomplish hypersonic air-breathing vehicle above Mach 5. This spawned the idea of supersonic combustion ramjet which is called as a scramjet. The fundamental difference between the ramjet and scramjet is that the combustion in ramjet is at subsonic speeds, whereas the combustion in scramjet is at supersonic speeds [7]. Like the ramjet, the ideal scramjet is also based on the Brayton cycle [4].
1.2.1 Brayton Cycle

The ideal Brayton cycle is a model used in thermodynamics for an ideal gas turbine power cycle. It is composed of the following four processes [4] :

1. Isentropic compression
2. Constant-pressure heat addition
3. Isentropic expansion
4. Constant-pressure heat rejection

![Figure 1-1. The ideal Brayton cycle.](image)

The highest temperature in the cycle occurs at the end of the combustion process, and it is limited by the maximum operating temperature which the materials can withstand. This also limits the pressure ratios that can be used in the cycle.
1.2.2 Ramjet

The schematic diagram of a scramjet is as shown in the Figure 1-2, consists of an inlet, a combustion chamber and a nozzle. It does not have a compressor or a turbine as the combustion occurs at subsonic velocities. The operation of the ramjet depends upon the inlet, as it has to decelerate the incoming air to raise the pressure difference in the combustor [4]. The higher the velocity of incoming air, the higher the pressure rise; thus the ramjet operates best at high supersonic velocities. The ramjet has the virtue of maximum simplicity with no need for turbo machinery and maximum tolerance to high-temperature operation and minimum mass per unit thrust at suitable high Mach numbers [9].

The actual supersonic vehicle, which operates with a ramjet engine, has reached a maximum speed of Mach 3 [10]. At high supersonic velocities, a large pressure rise is developed that is more than sufficient to operate the ramjet. The inlet has to decelerate the supersonic air stream to low subsonic speeds, this produces a high temperature rise. At some flight speed, the temperature will approach the maximum limit for the wall materials and the cooling methods; thus making it difficult to burn the fuel in the airstream [4]. This makes the ramjet fallible to operate at high Mach numbers beyond about Mach 5.
1.2.3 **Scramjet**

A scramjet propulsion system is a hypersonic air-breathing engine in which the flow of air stream and combustion occurs at supersonic velocities within the engine [11]. Scramjet engines operate on the same principle as the ramjets, but do not decelerate the flow to low subsonic velocities. The inlet decelerates the flow to a lower Mach number for combustion, after which the flow is accelerated to a very high Mach number through the exit nozzle. Supersonic combustion denotes better performance as the flight Mach number can be greater that Mach 5 [12]. The relationship of engine specific impulse and Mach number for different air-breathing engines is
Figure 1-3. Specific impulse and flight Mach numbers for different flight systems [13].

Figure 1-4. Flight Mach number for various propulsion systems [13].
depicted in Figure 1-3 for two different combustor fuels [13]. It can be seen that, for Mach numbers greater that 6 only scramjet and rocket propulsion systems are applicable. The advantage of scramjet over rockets is that scramjets have higher specific impulse levels [6]. Also ramjets have greater efficiency, but cannot operate at higher free stream Mach numbers like scramjets.

The theoretically possible flight Mach number of various propulsion systems are as shown in Figure 1-4. The curve gives the approximate operating altitudes at a given flight Mach number. The chart is shown for two primary fuel options: hydrogen and hydrocarbons. Theoretically, hydrogen can yield much higher Mach number than hydrocarbons.

As the scramjet operates at supersonic flow conditions, it requires additional support to go to supersonic flight conditions, thus an air-breathing vehicle must accelerate the scramjet to its initial operational flight conditions to start the combustion process [14]. While testing the Boeing X-51A, it was lifted up to an altitude of 50K ft by a Boeing B-52 as shown in Figure 1-5 and then it is accelerated up to a Mach 5 by using a rocket booster to provide enough speed for the scramjet combustion process to start.

The scramjet is composed of three basic components: a converging inlet, where incoming air is compressed; a combustor, where gaseous fuel is burned with atmospheric oxygen to produce heat; and a diverging nozzle, where the heated gas is accelerated to produce thrust. Unlike a typical jet engine, such as a turbojet or turbofan, a scramjet does not use rotating fan-like components to compress the air; rather the speed of air craft causes it to compress air in the inlet. Due to the nature of this design, scramjet operation is limited to hypersonic velocities. To accomplish the
Figure 1-5. Boeing X-51A Attached to the Rocket Engine and Boeing B52 [3].

Figure 1-6. Representative scramjet engine [15].
function of a compressor, scramjet engines rely heavily on the shape of aircraft. A concept known as “air frame –integrated scramjet” became the standard for most designs [15]. The front section is designed so that the shockwaves produced are displaced into the engine.

1.3 Historical Development

The first design of an operational ramjet engine equipped aircraft was Rene – Leduc’s demonstrator. Conceptually the design began in 1920’s, was patented in 1934 [6]. The first supersonic flight was displayed in WWII in 1946. After WWII, the U.S. and U.K. took on several military technologies through operation “paperclip” to put more importance on their own weapons development, including jet engines.

America’s rocket-propelled aircraft program, which began in 1940’s, produced the first aircraft to break the ‘sound barrier’- the supersonic XS-1 in 1947. NASA’s most famous hypersonic demonstrator remains the X-15, which was designed to reach speeds up to Mach 6 has set a record of Mach 6.7 in October 1967 [11]. The SR-71 Blackbird first flew under J58 engine power in December, 1964. These flights were designed for sustained operations at speeds of Mach 3. Records show that 32 SR-71’s were built until they concluded their operations in 2001 [10]. NASA first flew its X-43A scramjet successfully on March 27, 2004; after it is separated from its mother craft and booster, it achieved a speed equivalent to Mach 7, breaking the previous speed record for level flight of an air-breathing aircraft. The third X-43A flight set a new speed record of nearly Mach 10. The Boeing X-51A wave rider flew successfully for approximately 200 seconds at about Mach 5, setting a new world record for duration at hypersonic flight [12].
However, a second flight test failed after it failed its transition to its primary fuel JP-7, thus failing to reach full power [3].

The HySHOT team from the University of Queensland conducted leading-edge experiments of scramjet technology and successfully tested its first scramjet engine in 2001 [16]. Four practical scramjet engine tests were conducted in the HySHOT program and supersonic combustion was achieved in HySHOT-II and III flights [16]. Not only the U.S. and Australia, but also Brazil, England, France and India are also working forward advancing in scramjet propulsion technology [17]. The recent scramjet programs around the world are summarized Table 1-1. An in-depth knowledge about the history of scramjet technology can be obtained from [6] and [11].

1.4 Materials and Fuels

To determine the performance of scramjet a specific material and a specific fuel are required. It is difficult for the selection of materials and fuels with the increased heat loading and the requirement of high lower-heating value of the fuel. The performance of the engine are based on the selection of these two constituents. Hence, the present work is to select a material and a fuel that show potential for achieving good performance for the scramjet engine.

The aim of this work is to choose three different candidate combustor materials and three different candidate fuels, and then prepare a mathematical description of the performance parameters at various free stream Mach numbers to analyze how the selection of materials and fuels impact the performance of the scramjet.
Table 1-1. Recent important scramjet technology programs worldwide [12].

<table>
<thead>
<tr>
<th>Nation</th>
<th>Program</th>
<th>Emphasis</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia / US</td>
<td>HIFiRE</td>
<td>Flight test of a scramjet using a Terrier-Orion Sounding rocket to develop and validate scramjet technologies</td>
<td>Second HIFiRE hypersonic test flight was on March 22, 2010</td>
</tr>
<tr>
<td>Brazil</td>
<td>14X</td>
<td>Mach-6 hypersonic UAV propelled by H₂ scramjet engine. Intended for access to space.</td>
<td>Being tested in T3 Brazilian air force hypersonic wind tunnel.</td>
</tr>
<tr>
<td>France</td>
<td>LEA</td>
<td>Development of experimental vehicle propelled by dual-mode ram/scramjet engine to fly at Mach 10 - 12.</td>
<td>Scheduled to terminate in 2015 after four flight tests.</td>
</tr>
</tbody>
</table>
1.4.1 Materials

Many hypersonic research programs such as X-15, hyper-X, HySHOT, etc., have developed exotic materials to withstand the harsh environments associated with hypersonic flight. These materials are generally classified depending on the temperature range they can operate continuously. For temperatures ranging from 300 C to 980 C, alloys of titanium and metal matrix composites are preferred due to their low weight and ease of manufacture. Temperatures in excess of 1000 C require the use of ultra-high temperature ceramics and carbon-carbon composites to operate.

Each of these groups has their own benefits, but the materials are chosen from both groups for this work. One of them is chosen from metal alloys and the other two are chosen from ultra-high temperature ceramics, which consists of: borides, carbides and nitrides of transition elements such as zirconium, hafnium and tantalum [18]. The benefits of choosing these materials include high operating temperatures, good chemical and thermal stability; these can be found in various forms like monolith (solid pieces of material), matrix (composites) and coatings [19]. The maximum service or operating temperatures [20] of various materials are shown in Figure 1-7. The materials chosen for this analysis and their maximum service temperatures are: zirconium oxide ceramic (2700 K), nickel-chromium alloy (1600 K) and carbon-carbon carbide (2300 K) (Draper et al 2007) according to their properties defined in [18], [19], [21].
Figure 1-7. Maximum service temperature of various materials [20].
1.4.2 Fuels

The selection of fuel for the performance of a scramjet is very important as the thrust produced depends on the lower-heating value of the fuel. The important considerations to be made while selecting the fuel are the lower-heating value, storage of the fuel, the cost and availability of the fuel and the density.

The most common hydrocarbon jet fuels in practical use have a small range of $h_{PR}$ (lower heating value) [22]. Recently, the aircraft Ion Tiger from the Navy showed an electrochemical fuel cell propulsion system which is fueled by liquid hydrogen, which showed four times higher efficiency compared to the internal combustion engine [23]. The following fuels are chosen for this study: hydrogen, JP-5 and natural gas. Their lower-heating values are 120000 kJ/kg, 42800 kJ/kg and 47100 kJ/kg respectively [24].

1.5 Parametric Thermodynamic Cycle Analysis

Cycle analysis models the thermodynamic changes of the working fluid (air and products of combustion) as it flows through the engine. Parametric cycle analysis determines the performance of engines at different flight conditions and values of design choice [4]. The main objective of parametric analysis is to relate the engine performance parameters to design choices, limitations and flight environment.

Parametric cycle analysis is used to develop mathematical expressions for specific thrust, thrust-specific fuel consumption, fuel-to-air ratio, propulsive, thermal and overall efficiencies and
thrust flux for the ideal scramjet engine. The performance parameters developed in this work are similar to those performance parameters developed for ramjet and scramjet in [4], [25], [26], [27]. This work presents a unique parametric analysis that has not done previously in literature.

1.6 This Work

The parametric analysis for ideal scramjet engine is fundamentally based on the Brayton cycle [4]; which is described in the temperature versus entropy (T-s) diagram. The performance of the engine can be estimated by modeling the seven performance parameters, thus determined in this work. These are mainly dependent on the service temperature of material and the lower-heating value of the fuel. The analysis is first done on an ideal mass flow rate scramjet engine and then on a non-ideal mass flow rate scramjet engine and then the two are compared.

The performance of the engine for the materials chosen is determined for a specific fuel and then the performance of the engine for the fuels is determined for a specific material. All the results are presented as a function of free stream Mach number. The performance of the scramjet engine is described by specific thrust \( \frac{F}{\dot{m}_o} \), thrust-specific fuel consumption \( S \), fuel-to-air ratio \( f \), thermal, propulsive, overall efficiencies \( \eta_T, \eta_P, \eta_O \) and the thrust flux \( \frac{F}{A_2} \). Each of these results are presented versus the combustion Mach number \( M_2 \) at various free stream Mach numbers \( M_o \).
In this section of the thesis the basic concepts of scramjet propulsion, historical developments and importance of the present work have been covered. In the following chapter the physical statement of problem will be defined.
CHAPTER 2

STATEMENT OF THE PROBLEM

2.1 Definition of the Problem

The main aim of the present work is to determine the performance of a constant velocity combustor scramjet engine by using parametric thermodynamic cycle analysis; and then to compare the ideal mass flow rate and non-ideal mass flow rate flight conditions of the scramjet engine. The performance of the constant velocity ideal scramjet combustor is then determined for three different candidate materials and three different candidate fuels by varying the maximum service temperature of the material and the lower-heating value of the fuel in the performance expressions. The performance of the scramjet is determined by using the seven performance parameters namely specific thrust \( (F/\dot{m}_0) \), thrust-specific fuel consumption \( (S) \), fuel-to-air ratio \( (f) \), thermal efficiency \( (\eta_t) \), propulsive efficiency \( (\eta_p) \), overall efficiency \( (\eta_o) \) and thrust flux \( (F/A_2) \).

The present work can be divided into two sections: developing the mathematical expressions for the ideal mass flow rate case and then developing the mathematical expression for the non-ideal mass flow rate constant velocity combustor scramjet. The results will be presented in Chapter 4 for each of the performance indices as a function of free stream Mach number \( (M_o) \) at various combustion Mach numbers \( (M_2) \). The analysis for the constant velocity combustor
scramjet is based on the Brayton cycle as described via the temperature versus entropy (T-s) diagram. The (T-s) diagram is a common and efficient way to visualize the thermodynamics of the power cycle. This also helps in choosing the candidate material and candidate fuel for the combustor.

2.2 T-s Diagram

The temperature versus entropy (T-s) diagram for a constant velocity combustor scramjet, based on the Brayton cycle is as shown in the Figure 2-1. The ideal Brayton cycle for the parametric description of the scramjet engine consists of an isentropic inlet compression process, a constant pressure combustion process, an isentropic nozzle expansion process and a constant pressure heat rejection process where the nozzle exit static pressure ($P_9$) is equal to the freestream flight ambient static pressure, $P_o$. The ideal description also considers the mass flow rate of the fuel to be negligible as compared to the mass flow rate of the air flowing through the combustor. The present work only considers the situations where the combustion Mach number is less than or equal to the freestream Mach number ($M_2 \leq M_o$). The situations where ($M_2 > M_o$) is not considered as the flow entering the inlet of the engine is not considered to expand isentropically i.e., the speed of the incoming air is not considered to be increased in the inlet of the engine.
Figure 2-1. T- s diagram comparison for ramjet (0-t2-4-9-0) and scramjet (0-2-4-9-0).
2.2.1 Situations for $V_2 \leq V_o (M_2 \leq M_o)$

The T-s diagrams depictions of the ideal ramjet engine (0-t2-4-9-0) and the ideal scramjet engine (0-2-4-9-0) are shown in Figure 2-1, based on the Brayton cycle. The description of the ideal scramjet operation will also be compared to the ideal ramjet as part of this discussion. Referring to Figure 2-1, freestream air approaches the engine inlet (station 0 in Figure 2-1) at $T_0$ (ambient altitude temperature), $P_0$ (ambient altitude pressure), and $M_o$ (freestream Mach number). In an ideal ramjet engine, the air is isentropically brought (compressed) nearly to rest at the $T_{t2}$ essentially stagnation conditions. Next a constant-total-pressure (and constant combustion Mach number, $M_2 \approx 0$ or constant velocity, $V_2 \approx 0$) heat addition (combustion) process then takes place in the combustion chamber to raise the total temperature to $T_{\text{max}}$. Note that the total pressure and the static pressure are essentially the same constant pressure during the ideal ramjet combustion process since $M_2 \approx 0$ or $V_2 \approx 0$. Isentropic expansion then follows through the exit nozzle along the “ramjet” vertical dotted line in Figure 2-1. Next the gas flow exits the ideal ramjet engine exit nozzle at a static pressure equal to the freestream flight ambient static pressure, $P_0$, and at $T_9$, the static temperature of the engine exit nozzle along the “ramjet” vertical line in Figure 2-1. Note that $T_{\text{max}}$ corresponds to the engine material temperature limit.

In an ideal scramjet engine, the combustion process takes place at supersonic speeds, and therefore the total (stagnation) temperature is not experienced by the gas. However, it should be noted from the first law of thermodynamics that the amount of heat addition in the combustion process is determined in terms of the total temperature differences across the burner. In Figure 2-1 for the ideal scramjet, the flow again approaches the engine inlet but is not brought (compressed)
nearly to rest as in ramjet, but is brought (slowed) to supersonic speed at $T_2$. Next combustion starts at temperature $T_2$, and as with the ramjet, heat addition is assumed to occur at a constant pressure (Brayton cycle) and constant combustion velocity, but here $V_2$ is supersonic. Heat is added until achieving a static gas temperature of $T_{\text{max}}$ at the exit of the ideal scramjet burner process. Because of the aforementioned dependence of heat addition on total temperature differences across the burner, the heat released in the combustion chamber is dependent on the difference between $T_{t4}$ and $T_{t2}$. Finally, the gas exits the burner and is isentropically expanded from $T_{\text{max}}$ to $T_9$ through the exit nozzle along the first “scramjet” vertical line in Fig. 1 to the ambient free-stream pressure, $P_o$. The same maximum material limit temperature, $T_{\text{max}}$, is experienced by both (ramjet and scramjet) engine materials, for ramjet $T_{t4} = T_{\text{max}}$ and for scramjet $T_4 = T_{\text{max}}$. The temperature increase during the constant total pressure heat addition depends on the lower-heating value of the fuel. For scramjets, there may be additional entropy losses associated with the mixing of the fuel and the air.

2.3 Materials and Fuel selection

As aforementioned, the maximum service temperature of a material and the lower-heating value of a fuel impacts the thrust generated by the engine; hence it becomes important to select a material with a desirable maximum service temperature. Thus for the scramjet combustor, the material with the maximum service temperature allows production of more thrust by the engine. Table 2-1 shows the candidate materials chosen for this work.
The combustion in scramjet takes place at a Mach number greater than one, this implies that there will be very little time for the fuel to properly mix with air and to be ignited. Thus, the lower-heating value of the fuel helps in selection of the fuel; fuels with higher lower-heating value ($h_{PR}$) are chosen for this work. Table 2-2 shows the different candidate fuels chosen for this work.

Table 2-1. Selected materials and corresponding maximum service temperatures ($T_{\text{max}}$).

<table>
<thead>
<tr>
<th>Material</th>
<th>Maximum Service Temperature ($T_{\text{max}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zirconium-oxide ceramic</td>
<td>2700 K</td>
</tr>
<tr>
<td>Carbon – carbon carbide</td>
<td>2300 K</td>
</tr>
<tr>
<td>Nickel-chromium alloy</td>
<td>1600 K</td>
</tr>
</tbody>
</table>

Table 2-2. Selected fuels and corresponding lower-heating values.

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Lower-heating value (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid hydrogen</td>
<td>120,000</td>
</tr>
<tr>
<td>JP-5</td>
<td>42,800</td>
</tr>
<tr>
<td>Natural gas</td>
<td>47,100</td>
</tr>
</tbody>
</table>
Having presented a physical description of the scramjet in the chapter (Chapter 2), a mathematical description of scramjet will be described in the next chapter (Chapter 3).
3.1 Assumptions of Cycle Analysis

The parametric analysis of a scramjet engine is fundamentally based on the Brayton cycle as described in terms of the temperature versus entropy (T-s) diagram. The mathematical analysis presented here follows the basic notation and cycle analysis presented in detail in [4]. The analysis presented can be divided into two sections:

a) Ideal mass flow rate scramjet

b) Non-ideal mass flow rate scramjet.

In the ideal mass flow rate scramjet it is assumed that the mass of the combustion exhaust is equal only to the mass of the air entering the combustor and the mass of the fuel added is neglected (\( \dot{m}_o \approx \dot{m}_a + \dot{m}_f \)). In the non-ideal mass flow rate scramjet the mass of the exhaust is assumed to be composed of both the masses of air and fuel entering the combustor (\( \dot{m}_o \neq \dot{m}_a + \dot{m}_f \)).
3.2 Parametric Analysis for Ideal Mass Flow Rate Scramjet

A simple geometry of the burner for the ideal mass flow rate scramjet is as shown in Figure 3-1. Application of the steady-state energy equation (first law of thermodynamics) to a control volume across the combustion chamber for the ideal scramjet yields

\[ \dot{m}_0 c_p T_{t2} + \dot{m}_f h_{PR} = \dot{m}_0 c_p T_{t4} \]  
\[ (1) \]

or

\[ T_{t4} - T_{t2} = \frac{\dot{m}_f h_{PR}}{\dot{m}_0 c_p} \]  
\[ (2) \]

Now Euler’s equation is \( dP = -\rho V dV \), and since for the combustor \( dP = 0 \), then \( dV = 0 \) which implies

\[ V_2 = V_4 \]  
\[ (3) \]

hence Eq. (2) in terms of static temperatures becomes

\[ T_2 + \frac{\dot{m}_f h_{PR}}{\dot{m}_0 c_p} = T_4 \]  
\[ (4) \]

It is known that
\[ T_{t2} = T_2 \left[ 1 + \frac{\gamma - 1}{2} M_2^2 \right] \]  \hspace{1cm} (5) 

and

\[ T_{t4} = T_4 \left[ 1 + \frac{\gamma - 1}{2} M_4^2 \right] \]  \hspace{1cm} (6) 

substituting for \( T_2 \) and \( T_4 \) from Eq. (5) and Eq. (6) into Eq. (4) and substituting for

\( \frac{m_\ell}{m_o} )/(h_{PR}/c_p) \) from Eq. (2) into Eq. (4) yields

\[ \frac{1}{H} + \tau - 1 = \frac{\tau}{B} \]  \hspace{1cm} (7) 

where

\[ H = 1 + \frac{\gamma - 1}{2} M_2^2 \]  \hspace{1cm} (8a) 

\[ B = 1 + \frac{\gamma - 1}{2} M_4^2 \]  \hspace{1cm} (8b) 

\[ \tau = \frac{T_{t4}}{T_{t2}} \]  \hspace{1cm} (8c) 

rearranging Eq. (7) and solving for \( B \) yields

\[ B = \frac{\tau H}{1 + \tau H - H} \]  \hspace{1cm} (9) 

substituting back into Eq. (9) for \( H \) and \( B \) from Eq. (8a) and Eq. (8b), respectively, yields \( M_4 \) as
\[ M_4 = \frac{M_2}{\sqrt{\tau \left[ 1 + \frac{(\gamma - 1)}{2} M_2^2 \right] - \frac{(\gamma - 1)}{2} M_2^2}} \] \hspace{1cm} (10)

now from Fig. 1, it is seen that \( T_4 = T_{\text{max}} \). Therefore from Eq. (6)

\[ T_{t4} = T_{\text{max}} \left[ 1 + \frac{(\gamma - 1)}{2} M_4^2 \right] \] \hspace{1cm} (11a)

and

\[ \tau = \frac{T_{t4}}{T_{t2}} = \frac{T_{\text{max}}}{T_{t2}} \left[ 1 + \frac{(\gamma - 1)}{2} M_4^2 \right] = \frac{T_{\text{max}} B}{T_{t2}} \] \hspace{1cm} (11b)

let

\[ C = T_{\text{max}}/T_{t2} \text{ thus } \tau = CB \] \hspace{1cm} (12)

now substituting for \( \tau \) from Eq. (12) into Eq. (9) and solving for \( B \) yields

\[ B = 1 + \frac{(H - 1)}{CH} \] \hspace{1cm} (13)

Next from Eqs. (8a), (8b) and (12) insert the expressions for \( H \) and \( B \), and \( C \) into Eq. (13), and then solving for \( M_4 \) yields

\[ M_4 = \frac{M_2}{\sqrt{T_{\text{max}}/T_{t2} \left[ 1 + \frac{(\gamma - 1)}{2} M_2^2 \right]}} \] \hspace{1cm} (14)

Next insert Eq. (14) for \( M_4 \) into Eq. (11b), and solving for \( \tau \) yields
\[ \tau = \frac{T_{\text{max}}}{T_{t2}} + \frac{(y - 1)M_2^2}{2\left(1 + \frac{(y - 1)M_2^2}{2}\right)} \quad (15) \]

using the definitions from [4] it is seen that

\[ \frac{T_{\text{max}}}{T_{t2}} = \frac{T_{\text{max}}/T_o}{T_{t2}/T_o} = \frac{T_{\text{max}}/T_o}{T_{t2}/T_o} = \tau_{\lambda}/\tau_r \quad (16) \]

since \( T_{t2} = T_{t0} \) and by definition

\[ \tau_{\lambda} = \frac{T_{\text{max}}}{T_o} \quad (17) \]

and

\[ \tau_r = \frac{T_{t0}}{T_o} = 1 + \frac{(y - 1)M_0^2}{2} = \frac{T_{t2}}{T_o} \quad (18) \]

Thus Eq. (14) becomes

\[ M_4 = \frac{M_2}{\sqrt{\frac{\tau_{\lambda}}{\tau_r} \left[1 + \frac{(y - 1)M_2^2}{2}\right]}} \quad (19) \]

and Eq. (15) becomes

\[ \tau = \frac{\tau_{\lambda}}{\tau_r} + \frac{(y - 1)M_2^2}{2\left(1 + \frac{(y - 1)M_2^2}{2}\right)} \quad (20) \]

Equations (19) and (20) are necessary to develop the scramjet engine performance expressions for the constant velocity combustor process of the Brayton cycle. Equation (19) provides the
combustor exit Mach number, $M_4$, in terms of the combustor entrance Mach number, $M_2$, the freestream Mach number, $M_0(\tau_1)$, and the material temperature limit, $T_{\text{max}}(\tau_\lambda)$.

Next the engine performance expressions are presented in terms of $M_4$ and $\tau$ from Eqs. (19) and (20), respectively.

### 3.2 Engine Performance Expressions

It is known that, at station 2 and station 4 from Figure 2-1.

\[ P_{t4} = P_4 [1 + \frac{(\gamma - 1)}{2} M_4^2]^{\frac{\gamma}{\gamma - 1}} \tag{21} \]

and

\[ P_{t2} = P_2 [1 + \frac{(\gamma - 1)}{2} M_2^2]^{\frac{\gamma}{\gamma - 1}} \tag{22} \]

For the Brayton cycle $P_4 = P_2$. Hence, ratioing Eqs. (21) and (22) and substituting for $M_4$ from Eq. (10) yields, after some algebra

\[ \frac{P_{t4}}{P_0} = \frac{P_{t2}/P_0}{\left[\left\{1 + \frac{(\gamma - 1)}{2} M_2^2\right\} - \frac{1}{\tau}\left\{\frac{(\gamma - 1)}{2} M_2^2\right\}\right]^{\frac{\gamma}{\gamma - 1}}} \tag{23} \]

where

\[ \frac{P_{t2}}{P_0} = \frac{P_{t0}}{P_0} = \left[1 + \frac{(\gamma - 1)}{2} M_0^2\right]^{\frac{\gamma}{(\gamma - 1)}} \tag{24} \]

since $P_{t4} = P_{t9}$, it can be written that
\[
\frac{P_{t4}}{P_0} = \frac{P_{t9}}{P_0} = \left[ 1 + \frac{(\gamma - 1)}{2} M_9^2 \right]^\frac{\gamma}{(\gamma - 1)}
\]

(25)

and solving for \(M_9\) yields

\[
M_9 = \sqrt{\left[ \frac{\left( \frac{(\gamma - 1)}{2} \right)}{\left( \frac{P_{t4}}{P_0} \right)^{\frac{\gamma}{(\gamma - 1)}} - 1} \right] \frac{\gamma}{(\gamma - 1)}}
\]

(26)

now it can be written that

\[
\frac{V_9}{V_0} = \frac{M_9 a_9}{M_0 a_0} = \frac{M_9}{M_0} \sqrt{\frac{T_9}{T_0}}
\]

(27)

from Eq. (11b) it is seen that

\[
T_{t4} = T_{t9} = \tau T_{t2} = T_9 \left[ 1 + \frac{(\gamma - 1)}{2} M_9^2 \right]
\]

(28)

or

\[
\frac{T_9}{T_0} = \frac{\tau \left( \frac{T_{t2}}{T_0} \right)}{1 + \frac{(\gamma - 1)}{2} M_9^2} = \frac{\tau \tau_r}{1 + \frac{(\gamma - 1)}{2} M_9^2}
\]

(29)

The expression for specific thrust is given by [4] as

\[
\frac{F}{m_0} = \frac{M_0 a_0}{g_c} \left[ \frac{V_9}{V_0} - 1 \right]
\]

(30)
now using Eq. (27) for \((V_\phi/V_o)\), Eq. (29) for \((T_\phi/T_o)\), Eq. (26) for \(M_9\), Eq. (23) for \((P_{t4}/P_o)\), Eq. (24) for \((P_{t2}/P_o)\), and Eq. (20) for \(\tau\) along with Eq. (30) facilitates the computation of the specific thrust.

Next the fuel-to-air ratio, \(f = \dot{m}_f/\dot{m}_o\), is found from Eq. (2) as

\[
f = \frac{c_p}{h_{PR}} T_{t2} \left[\frac{T_{t4}}{T_{t2}} - 1\right] \tag{31}\]

or

\[
f = \frac{c_p}{h_{PR}} \tau T_o [\tau - 1] \tag{32}\]

where \(\tau\) is given by Eq. (20), and \(\tau_f\) is given by Eq. (18).

The next performance parameter is the thrust-specific fuel consumption, \(S\), which is given by [4]

\[
S = \frac{f}{(F/\dot{m}_o)} \tag{33}\]

where Eqs. (32) and (30) are employed to compute \(S\) from Eq. (33).

The thermal efficiency, \(\eta_T\), is given as [4]

\[
\eta_T = 1 - \frac{(T_\phi - T_o)}{(T_{t4} - T_{t2})} \tag{34}\]

or
\[ \eta_T = 1 - \frac{1}{\left( \frac{T_{t2}}{T_0} \right) \left( \frac{T_{t4}}{T_{t2}} - 1 \right)} \]  
(35)

or

\[ \eta_T = 1 - \frac{1}{\tau_r} \left[ \frac{(T_9/T_o) - 1}{\tau - 1} \right] \]  
(36)

where \( \tau_r \) is from Eq. (18), \( \tau \) is from Eq. (20), and \( (T_9/T_o) \) is given in Eq. (29).

Next the propulsive efficiency, \( \eta_p \), is given as [4]

\[ \eta_p = \frac{2}{V_9/V_o + 1} \]  
(37)

where \( (V_9/V_o) \) is given in Eq. (27). Next the overall efficiency, \( \eta_o \), is given as [4]

\[ \eta_o = \eta_T \eta_p \]  
(38)

where \( \eta_T \) is given by Eq. (36) and \( \eta_p \) is given by Eq. (37).

The thrust flux, \( F/A_2 \), is given as [4]

\[ \frac{F}{A_2} = \left( \frac{F}{m_o} \right) \left( \frac{m_o}{A_2} \right) \]  
(39)

where [4]

\[ \frac{m_o}{A_2} = g(\gamma, R) \left( \frac{A^*}{A} \right)^{\frac{1}{2}} \frac{P_o}{\sqrt{T_o}} \tau_r^3 \]  
(40)
with

\[
\left(\frac{A^*}{A}\right) = \left\{ \frac{1}{M^2} \left[ \frac{2}{(\gamma - 1)} \left( 1 + \frac{(\gamma - 1)}{2} M^2 \right) \right] \right\}^{-\frac{1}{2}} \tag{41}
\]

and

\[
g(\gamma, R) = \sqrt{\frac{\gamma}{R} \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{\gamma - 1}}} \tag{42}
\]

where \( M \) in Eq. (41) is evaluated at station 2 \((M = M_2)\).

Lastly the area ratio \(A_4/A_2\) across the burner can be determined from the continuity equation, since the mass flow rates at stations 2 and 4 are equal, namely

\[
m_{i_2} = m_{i_4} \tag{43}
\]

where \( m \) is given by [29]

\[
m = \frac{P_\gamma}{\sqrt{\gamma RT}} \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}} \left( \frac{A^*}{A} \right) A \tag{44}
\]

Employing Eq. (44) at stations 2 and 4 with Eq. (43) yields

\[
\frac{A_4}{A_2} = \left( \frac{A^*/A}{A} \right)_2 \frac{P_{i_2}^{1/2}}{P_{i_4}^{1/2}} \frac{T_{i_4}}{T_{i_2}} \tag{45}
\]

where Eq. (41), evaluated at stations 4 and 2, is used in Eq. (45), Eq. (20) is used for \( \tau = \frac{T_{i_4}}{T_{i_2}} \), where Eq.(23) is rearranged to determine \( P_{i_2}/P_{i_4} \). Equation (45) gives the expression for the
area ratio across the combustor that corresponds to constant pressure and constant velocity combustion.

Equation (45) also applies for the constant pressure and constant Mach number combustor as well, except that in this case \( (A^*/A)_2 = (A^*/A)_4 \) since \( M_2 = M_4 \) and \( P_{t2} = P_{t4} \). Thus Eq. (45) becomes

\[
\frac{A_4}{A_2} = \sqrt{\frac{T_{t4}}{T_{t2}}} = \sqrt{\frac{T_{t4}/T_0}{T_{t2}/T_0}} \tag{46}
\]

or

\[
\frac{A_4}{A_2} = \left\{ \frac{\tau_v}{\tau_r} \left[ 1 + \frac{(\gamma - 1)}{2} M_2^2 \right] \right\}^{1/2} \tag{47}
\]

for the constant pressure and constant Mach number combustor model [7].

3.3 Parametric Analysis for Non-Ideal Mass Flow Rate Scramjet

The mathematical analysis presented here follows the basic notation and cycle analysis procedure presented in detail in [4], but is for the non-ideal mass flow rate situation (\( \dot{m}_o + \dot{m}_r \neq \dot{m}_o \)) and hence yields different mathematical expressions from those for the ideal mass flow rate model. Simple geometry of the burner for the non-ideal mass flow rate scramjet is as shown in Figure 3-2. Application of the first law of thermodynamics across the burner yields
\[ \dot{m}_o c_p T_{t2} + \dot{m}_f h_{PR} = (\dot{m}_f + \dot{m}_o) c_p T_{t4} \] (48)

or

\[ (T_{t4} - T_{t2}) + f T_{t4} = f \frac{h_{PR}}{c_p} \] (49)

Now Euler’s equation is \( dP = -\rho V dV \) and since for the combustor \( dP = 0 \), hence \( dV = 0 \) which implies

\[ V_2 = V_4 \] (50)

Hence Eq. (49) in terms of static temperatures becomes

\[ T_2 + f \frac{h_{PR}}{c_p} = (1 + f)T_4 + f \frac{V_4^2}{2c_p} \] (51)

It is known that

\[ T_{t2} = T_2 \left[ 1 + \frac{(\gamma - 1)}{2} M_2^2 \right] \] (52)

\[ T_{t4} = T_4 \left[ 1 + \frac{(\gamma - 1)}{2} M_4^2 \right] \] (53)
since \( V_4^2 = V_2^2 \) and since

\[
T_{t2} = T_2 + \frac{V_2^2}{2c_p}
\]  

(54)

combining Eq. (54) and Eq. (52) yields

\[
\frac{V_2^2}{2c_p T_{t2}} = \frac{V_4^2}{2c_p T_{t2}} = \frac{\left(\gamma - 1\right)/2}{[1 + \left(\gamma - 1\right)/2]M_2^2}
\]  

(55)

substituting for \( T_2 \) and \( T_4 \) from Eq. (52) and Eq. (53) into Eq. (51) and substituting for \( f h_{PR}/c_p \) from Eq. (49) into Eq. (51) and substituting Eq. (55) into Eq. (51) yields

\[
\frac{1}{H} + \tau - 1 + ft = \frac{(1 + f)\tau}{B} + \frac{f(H - 1)}{H}
\]  

(56)

where

\[
H = 1 + \frac{\left(\gamma - 1\right)}{2}M_2^2
\]  

(57a)

\[
B = 1 + \frac{\left(\gamma - 1\right)}{2}M_4^2
\]  

(57b)

\[
\tau = \frac{T_{t4}}{T_{t2}}
\]  

(57c)

Rearranging Eq. (56) and solving for \( B \) yields

\[
B = \frac{(1 + f)\tau H}{[1 + \tau H - H + ftH - f(H - 1)]}
\]  

(58)
Substituting back into Eq. (58) for H and B from Eq. (57a) and Eq. (57b) respectively yields $M_4$ as

$$M_4 = \frac{M_2}{\sqrt{\tau \left[ 1 + \frac{(\gamma - 1)}{2} M_2^2 \right] - \frac{(\gamma - 1)}{2} M_2^2}}$$  \hfill (59)$$

now from Figure 2-1 it is seen that $T_4 = T_{\text{max}}$, therefore from Eq. (63)

$$T_{t4} = T_{\text{max}} \left[ 1 + \frac{(\gamma - 1)}{2} M_4^2 \right]$$  \hfill (60a)$$

and

$$\tau = \frac{T_{t4}}{T_{t2}} = \frac{T_{\text{max}}}{T_{t2}} \left[ 1 + \frac{(\gamma - 1)}{2} M_4^2 \right] = \frac{T_{\text{max}}}{T_{t2}} B$$  \hfill (60b)$$

now let

$$C = \frac{T_{\text{max}}}{T_{t2}} \text{ thus } \tau = CB$$  \hfill (61)$$

next substituting for $\tau$ from Eq. (61) into Eq. (58) and solving for $B$ yields

$$B = 1 + \frac{(H - 1)}{CH}$$  \hfill (62)$$

next from Eqs. (57a) and (57b) insert the expressions for H and B into Eq. (62), and then solving for $M_4$ yields
\[ M_4 = \frac{M_2}{\sqrt{\frac{T_{\text{max}}}{T_{t2}} \left[ 1 + \frac{(y - 1)}{2} M_0^2 \right]}} \]  
\quad (63)

Next insert Eq. (63) for \( M_4 \) into Eq. (60b) and solving for \( \tau \) yields

\[ \tau = \frac{T_{\text{max}}}{T_{t2}} + \frac{(y - 1)}{2} \frac{M_2^2}{1 + \frac{(y - 1)}{2} M_0^2} \]  
\quad (64)

Using the definitions from [4] it is seen that

\[ \frac{T_{\text{max}}}{T_{t2}} = \frac{T_{\text{max}}}{T_{t2}/T_0} = \frac{T_{\text{max}}}{T_{t0}/T_0} = \frac{\tau_\lambda}{\tau_r} \]  
\quad (65)

Since \( T_{t2} = T_{t0} \) and by definition

\[ \tau_\lambda = \frac{T_{\text{max}}}{T_0} \]  
\quad (66)

and

\[ \tau_r = \frac{T_{t0}}{T_0} = 1 + \frac{(y - 1)}{2} M_0^2 = \frac{T_{t2}}{T_0} \]  
\quad (67)

Thus Eq. (63) becomes

\[ M_4 = \frac{M_2}{\sqrt{\frac{\tau_\lambda}{\tau_r} \left[ 1 + \frac{(y - 1)}{2} M_0^2 \right]}} \]  
\quad (68)

And Eq. (64) becomes
\[
\tau = \tau_\lambda + \frac{(\gamma - 1)}{2} M_2^2 \frac{1 + \frac{(\gamma - 1)}{2} M_2^2}{1 + \frac{(\gamma - 1)}{2} M_2^2} \tag{69}
\]

Although the mathematical development is completely different, the final expressions for \(M_4\) and \(\tau\) in Eqs. (68) and (69) for the non-ideal mass flow rate turn out to be identically the same as for the ideal mass flow rate case. Equation (68) provides the combustor exit Mach number, \(M_4\), in terms of the combustor entrance Mach number, \(M_2\), the freestream Mach number, \(M_0(\tau_r)\), and the material temperature limit, \(T_{\max}(\tau_\lambda)\). Next the engine performance parameters will be developed in terms of \(M_4\) and \(\tau\) from Eqs. (68) and (69) respectively.

3.3 A. Engine Performance Expressions

It is known that at station 2 and station 4 from Figure 2-1 that

\[
P_{t4} = P_4[1 + \frac{(\gamma - 1)}{2} M_4^2 \frac{\gamma}{\gamma - 1}] \tag{70}
\]

and

\[
P_{t2} = P_2[1 + \frac{(\gamma - 1)}{2} M_2^2 \frac{\gamma}{\gamma - 1}] \tag{71}
\]

For the Brayton cycle \(P_4 = P_2\); hence, ratioing Eqs. (70) and (71) and substituting for \(M_4\) from Eq. (59) yields after some algebra
\[
\frac{p_{t4}}{p_o} = \frac{p_{t2}/p_o}{\left[\left(1 + \frac{(y-1)}{2} M_o^2\right) - \frac{1}{\tau} \frac{(y-1)}{2} M_o^2\right]^{(\gamma-1)}}
\]  
(72)

where

\[
\frac{p_{t2}}{p_o} = \frac{p_{t9}}{p_o} = \left[1 + \frac{(y-1)}{2} M_o^2\right]^{\frac{\gamma}{(\gamma-1)}}
\]  
(73)

Since \(p_{t4} = p_{t9}\), it can be written that

\[
\frac{p_{t4}}{p_o} = \frac{p_{t9}}{p_o} = \left[1 + \frac{(y-1)}{2} M_o^2\right]^{\frac{\gamma}{(\gamma-1)}}
\]  
(74)

or solving for \(M_o\) yields

\[
M_o = \sqrt{\left[\left(\frac{p_{t4}}{p_o}\right)^{\frac{(y-1)}{\gamma}} - 1\right] \frac{2}{(\gamma-1)}}
\]  
(75)

Now it can be written that

\[
\frac{V_9}{V_o} = \frac{M_9 a_9}{M_o a_o} = \frac{M_9}{M_o} \sqrt{\frac{T_9}{T_o}}
\]  
(76)

From Eq. (60b) it is seen that

\[
T_{t4} = T_{t9} = \tau T_{t2} = T_9 \left[1 + \frac{(y-1)}{2} M_o^2\right]
\]  
(77)
Solving for $T_9/T_0$ yields

$$\frac{T_9}{T_0} = \frac{\tau \left( \frac{T_{t2}}{T_0} \right)}{\left[ 1 + \frac{(y - 1)}{2} M_9^2 \right]} = \frac{\tau \tau_r}{\left[ 1 + \frac{(y - 1)}{2} M_9^2 \right]}$$  \hspace{1cm} (78)

The expression for specific thrust is given by

$$\frac{F}{m_o} = \frac{M_o a_o}{g_c} \left[ \frac{(1 + f) V_9}{V_o} - 1 \right]$$  \hspace{1cm} (79)

now using Eq. (76) for $(V_9/V_o)$, Eq. (78) for $(T_9/T_0)$, Eq. (75) for $M_9$, Eq. (72) for $(P_{t4}/P_o)$, with Eq. (79) facilitates the computation of the specific thrust.

Next the fuel-to-air ratio, $f = \frac{\dot{m}_f}{m_o}$ is found from Eq. (49) as

$$f = \frac{T_{t4}/T_{t2} - 1}{\left( \frac{h_{PR}}{\frac{T_{t2} c_p}{T_{t4}}} - \frac{T_{t4}}{T_{t2}} \right)}$$  \hspace{1cm} (80)

and since $T_{t2} = \tau_r T_0$ then

$$f = \frac{[\tau - 1]}{\left( \frac{h_{PR}}{\frac{T_o \tau_r c_p}{\tau}} - \tau \right)}$$  \hspace{1cm} (81)

where $\tau$ is given by Eq. (69) and $\tau_r$ is given by Eq. (67).

The next engine performance parameter is the thrust-specific fuel consumption, $S$, which is given by [4] as
\[ S = \frac{f}{(F/\dot{m}_o)} \]  

where Eqs. (79) and (81) are employed into Eq. (82) to compute \( S \).

It can be shown that the thermal efficiency, \( \eta_T \), is given by

\[ \eta_T = 1 - \frac{1}{\tau_r} \left[ \frac{(1 + f) \left( \frac{T_9}{T_o} \right) - 1}{(1 + f) \tau - 1} \right] \]  

where \( \tau_r \) is from Eq. (67), \( \tau \) is from Eq. (69), and \( (T_9/T_o) \) is the same as explained just below Eq. (79).

Next the propulsive efficiency, \( \eta_p \), is given by

\[ \eta_p = \frac{2 \left[ (1 + f) \left( \frac{V_9}{V_o} \right) - 1 \right]}{\left[ (1 + f) \left( \frac{V_9}{V_o} \right)^2 - 1 \right]} \]  

where \( (V_9/V_o) \) is given by the explanation just below Eq. (79).

Next the overall efficiency, \( \eta_o \), is given from Eq. (83) and Eq. (84) as

\[ \eta_o = \eta_T \eta_p \]  

The thrust flux, \( F/A_2 \), is given [4] as

\[ \frac{F}{A_2} = \left( \frac{F}{\dot{m}_o} \right) \left( \frac{\dot{m}_o}{A_2} \right) \]  

where [4]
\[
\frac{\dot{m}_o}{A_2} = g(\gamma, R) \left( \frac{A^*}{A} \right) \frac{P_o}{\sqrt{T_o}} r^3
\]  

(87)

with

\[
\left( \frac{A^*}{A} \right) = \left( \frac{1}{M^2} \left[ \frac{2}{(\gamma - 1)} \left( 1 + \frac{\gamma - 1}{2} M^2 \right) \right] \right)^{-1/2}
\]  

(88)

and

\[
g(\gamma, R) = \sqrt{\frac{\gamma}{R} \left( \frac{2}{\gamma + 1} \right)^{\gamma+1} \gamma^{-1}}
\]  

(89)

where \( M \) in Eq. (88) is evaluated at station 2 (\( M = M_2 \)).

Lastly the area ratio \( A_4/A_2 \) across the burner can be determined from the continuity equation; the mass balance across the burner yields

\[
\dot{m}_f + \dot{m}_2 = \dot{m}_4
\]  

(90)

or

\[
\left( \frac{\dot{m}_4}{\dot{m}_2} \right) = 1 + f
\]  

(91)

where [29] \( \dot{m} \) is given by

\[
\dot{m} = \frac{P_t \gamma}{\sqrt{\gamma RT_t}} \left( \frac{2}{\gamma + 1} \right)^{\gamma+1} \frac{1}{2(\gamma-1)} \left( \frac{A^*}{A} \right) A
\]  

(92)

Employing Eq. (92) at station 2 and 4 and combining with Eq. (91) yields
\[ \frac{A_4}{A_2} = \left( \frac{A^*}{A} \right)_2 \left( \frac{P_{t2}}{P_{t4}} \right) \frac{\sqrt{T_{t4}}}{T_{t2}} (1 + f) \]  

(93)

where Eq. (88) at stations 4 and 2 is used, Eq. (69) is used for \( \tau = T_{t4}/T_{t2} \) with Eq. (72) used to get \( (P_{t2}/P_{t4}) \). Equation (93) gives the expression for the area ratio across the combustor that corresponds to constant pressure and constant velocity combustion. The entropy can be determined by using the Eq. (94)

\[ \frac{\Delta s}{R} = \frac{\gamma}{\gamma - 1} \ln(\tau) - \ln \left( \frac{P_{t4}}{P_{t2}} \right) \]  

(94)

where \( \tau \) is given by Eq. (69) and Eq. (72) is rearranged to get \( (P_{t2}/P_{t4}) \).

In this chapter the mathematical expressions of performance parameters were derived for the parametric analysis of the scramjet (Chapter 3). In the following chapter the results for the performance analysis are presented for each performance parameter (Chapter 4).
CHAPTER 4

RESULTS

In this chapter the results for the performance parameters that are analytically developed in Chapter 3 are discussed. The results are discussed for the seven performance parameters: specific thrust ($F/\dot{m}_a$), fuel-to-air ratio ($f$), thrust-specific fuel consumption ($S$), thermal efficiency ($\eta_T$), propulsive efficiency ($\eta_P$), overall efficiency ($\eta_o$), thrust flux ($F/A_2$). Each of these results are presented versus the combustion Mach number ($M_2$) at various freestream Mach numbers ($M_o$). Whenever $M_2 = 0$, then the results correspond to the ramjet; for $M_2 = 0$ the results become identical to the ideal mass flow rate ramjet expressions in [4] and the non-ideal mass flow rate ramjet results in [9]. The values for $h_{PR} = 42,800$ kJ/kg, $T_o = 217$ K, $P_o = 19403$ Pa, $T_{max} = 1600$ K, $c_p = 1.004$ kJ/(kg K), and $\gamma = 1.4$ used here in the figures are the same as those employed in [4] for the ideal mass flow rate ramjet analysis in order to facilitate comparison with the results in [4]. Chapter 4 is sub-divided into four parts:

1) Performance parameters for constant velocity versus constant Mach number combustion

2) Performance parameters for ideal versus non-ideal mass flow rate combustion

3) Performance parameters for different combustor fuels

4) Performance parameters for different combustor materials
4.1 Performance parameters for constant velocity versus constant Mach number

Figures 4-1 to 4-11 illustrate the performance parameters for constant velocity versus constant Mach number combustion. The solid-line curves correspond to the situations of $V_2 = V_4$, constant velocity and constant pressure combustion. The dashed-line curves correspond to the situations of $M_2 = M_4$, constant Mach number and constant pressure combustion.

Shown in Fig. 4-1 is the specific thrust for the ideal scramjet ($M_2 \geq 1$) and ideal ramjet ($M_2 = 0$). The specific thrust of the ideal scramjet is plotted against freestream Mach number $M_o$ for various combustor entrance Mach numbers $M_2$. Ironically, it can be seen that the maximum (peak) ideal scramjet specific thrust increases with an increase in free stream Mach number in the case of $M_2 = M_4$, but decreases with an increase in free stream Mach number in the case $V_2 = V_4$. It is seen that the maximum (peak) ideal scramjet specific thrust increases and shifts to higher freestream Mach numbers as $M_2$ increases for $M_2 = M_4$; but the specific thrust peak decreases for $V_2 = V_4$ and shifts to higher freestream Mach numbers as $M_2$ increases. The specific thrust values for $V_2 = V_4$ are significantly lower than for $M_2 = M_4$. The line $M_2 = 0$ corresponds to the ramjet and it can be seen that it is same for both the cases. The ramjet ($M_2 = 0$) results are identical with the ramjet results in [4].

Shown in Fig. 4-2 is the fuel-to-air ratio for the ideal scramjet ($M_2 \geq 1$) and ideal ramjet ($M_2 = 0$). The fuel-to-air ratio, $f$, of the ideal scramjet is plotted against freestream Mach number, $M_o$, for various combustor entrance Mach numbers, $M_2$. It can be seen that, $f$, increases with an increase in the combustor inlet Mach number $M_2$ in the case of $M_2 = M_4$; but starts approximately at the same level even with increase in the combustor inlet Mach number $M_2$ in the case of $V_2 = V_4$. It can also be seen that $f$ increases as the combustor inlet Mach number ($M_2$) increases for a specific $M_o$. Conversely $f$ decreases as the flight Mach number increases for a specific $M_2$. The
fuel-to-air ratio is greater for the $M_2 = M_4$ case than for the $V_2 = V_4$ case. This corresponds to the higher specific thrust for $M_2 = M_4$. More fuel corresponds to more fuel energy being used for producing more specific thrust.

Depicted in Fig. 4-3 is the thrust-specific fuel consumption, $S$. The thrust-specific fuel consumption of the ideal scramjet is presented against freestream Mach number $M_o$ with combustor entrance Mach numbers $M_2$ as a parameter. It is seen that $S$ increases as $M_2$ increases and hence ideal scramjet engines will have higher thrust-specific fuel consumption than the ideal ramjet engine. More fuel consumption corresponds to more energy to produce greater specific thrust. The $M_2 = M_4$ cases have lower $S$ values than the $V_2 = V_4$ cases. The higher specific thrust for $M_2 = M_4$ cases more than compensates for the increase in the fuel-to-air ratio.

The thermal efficiency, $\eta_T$, for all the $M_2 = M_4$ cases is the same as for the ramjet case ($M_2 = 0$) as shown in Fig. 4-4; the $M_2 = M_4$ cases have higher $\eta_T$ values than the $V_2 = V_4$ cases. The thermal efficiency $\eta_T$ is a function of $\tau_r$ only (see Eq. (18)) for the $M_2 = M_4$ cases. The definition of $\tau_r$ is the ratio of ambient altitude total temperature ($T_{to}$) to the ambient altitude static temperature ($T_o$), and it is a function of freestream Mach number ($M_o$) only as derived in Chapter 3. The thermal efficiency is the same for both the ideal ramjet and $M_2 = M_4$ ideal scramjet, as is shown in Fig. 4-4. It is clear that both the ideal ramjet and the ideal scramjet exhibit higher thermal efficiency as the flight Mach number ($M_o$) increases. Since the ideal ramjet cannot exceed $M_o \approx 4$ then it cannot exceed a thermal efficiency of about 75%. Whereas since the ideal scramjet can operate at much higher $M_o$ values ($M_o \approx 8$), its thermal efficiency can exceed about 80% as shown in Fig. 4-4.
Figure 4-1. Specific Thrust for constant Mach number versus constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).

- $h_{PR} = 42,800 \text{ kJ/kg}$
- $T_0 = 217 \text{ K}$
- $T_{\text{max}} = 1600 \text{ K}$
- $c_p = 1.004 \text{ kJ/(kg.K)}$
- $\gamma = 1.4$
- $M_2 \leq M_o$
- $- M_2 = M_4$
- $- V_2 = V_4$
Figure 4-2. Fuel-to-air ratio for constant Mach number versus constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).
Figure 4-3. Thrust-specific fuel consumption for constant Mach number versus constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).

- $h_{PR} = 42,800 \text{ kJ/kg}$
- $T_o = 217 \text{ K}$
- $T_{\text{max}} = 1600 \text{ K}$
- $c_p = 1.004 \text{ kJ/(kg.K)}$
- $\gamma = 1.4$
- $M_2 \leq M_o$
- $M_2 = M_4$
- $V_2 = V_4$
Figure 4-4. Thermal efficiency for constant Mach number versus constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).

- $h_{PR} = 42,800 \text{ kJ/kg}$
- $T_o = 217 \text{ K}$
- $T_{max} = 1600 \text{ K}$
- $c_p = 1.004 \text{ kJ/(kg.K)}$
- $\gamma = 1.4$
- $M_2 \leq M_o$
- $M_2 = M_4$
- $V_2 = V_4$
Presented in Figs. 4-5 and 4-6 are the propulsive efficiency, $\eta_p$, and the overall efficiency, $\eta_o$, as a function of $M_o$ with $M_2$ as a parameter. In Fig. 4-5, the $M_2 = M_4$ cases have lower $\eta_p$ values than the $V_2 = V_4$ cases, also it can be seen that for the $V_2 = V_4$ case, for a particular combustor entrance Mach number the propulsive efficiency is high at lower freestream Mach numbers, then decreases and then again increases as the freestream Mach number increases. But, for $M_2 = M_4$ cases, the propulsive efficiency increases as the free stream Mach number increases for a particular combustor entrance Mach number. In Fig. 4-6, the $M_2 = M_4$ cases have higher $\eta_o$ values than the $V_2 = V_4$ cases.

Shown in Fig. 4-7 are the results for the thrust flux as a function of $M_o$ with $M_2$ held as a parameter for the conditions of $T_o = 217$ K, $P_o = 19403$ Pa (altitude = 12 km), and $T_{\text{max}} = 1600$ K. The thrust flux provides a better depiction [4] than the specific thrust of the freestream Mach number ($M_o$) at which the ideal ramjet/scramjet thrust reaches a maximum value as seen in Fig. 4-7. From Fig. 4-7 it is clear how rapidly the ideal scramjet thrust flux performance increases with increasing combustor inlet Mach number ($M_2$) and how the maximum thrust flux occurs at a considerably higher freestream Mach number than the maximum specific thrust (Fig. 4-1). Figure 4-7 shows that the performance behavior for both the $M_2 = M_4$ cases and the $V_2 = V_4$ cases are very similar except the magnitudes are significantly different. Both the $M_2 = M_4$ cases and the $V_2 = V_4$ cases show a peak thrust flux at about the same $M_o$ value for a specific $M_2$ value. Flight at $M_o \approx 8$ would require $M_2 \approx 3$. 
Figure 4-5. Propulsive efficiency for constant Mach number versus constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).
Figure 4-6. Overall efficiency for constant Mach number versus constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).
Figure 4-7. Thrust flux for constant Mach number versus constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).
Shown in Fig. 4-8 is the area ratio, $A_4/A_2$, across the combustor in order to achieve either constant velocity combustion (left-side y-axis) or constant Mach number combustion (right-side y-axis). To fly at $M_o = 6$ with a $M_2 = 2$, would require a divergent combustor area $A_4/A_2 \approx 1.20$ for the $M_2 = M_4$ case and a divergent combustor area $A_4/A_2 \approx 2.0$ for the $V_2 = V_4$ case. Figure 4-8 has significance for combustor design as to the proper value of $A_4/A_2$ for a specific $M_o$ and $M_2$.

Figure 4-9 illustrates (Eq. (19)) the constant velocity combustor exit Mach number, $M_4$, as a function of $M_o$ with $M_2$ taken as a parameter. Mostly $M_4 < M_2$ as expected, but in some cases, such as $M_o = 9$ and $M_2 = 3$, it is seen that the constant velocity combustion model predicts that $M_4 > M_2$. It is also seen that, in some cases even though $M_2 \geq 1$, $M_4$ is subsonic ($M_4 < 1$). Figure 4-10 shows how the total pressure ratio across the combustor varies for a constant velocity combustion process. The total pressure ratio is only constant and equal to one for the ramjet case ($M_2 = 0$). For all $M_2$ values the total pressure ratio approaches unity as $M_o$ increases to high values.
Figure 4-8. Combustor area ratio for constant Mach number versus constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).
Figure 4-9. Combustor exit Mach number for constant velocity combustion scramjet versus freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).
Figure 4-10. Combustor total pressure ratio for constant Mach number versus constant velocity combustion scramjet versus freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).

- $h_{pr} = 42,800$ kJ/kg
- $T_o = 217$ K
- $T_{max} = 1600$ K
- $c_p = 1.004$ kJ/(kg.K)
- $\gamma = 1.4$
- $M_2 \leq M_o$
4.2 Performance parameters for ideal and non-ideal mass flow rate

In this section, the performance parameters for ideal and non-ideal mass flow rate cases for the constant velocity combustor scramjet are compared.

Presented in Figs. 4-11 to 4-20 are the results for $F/\dot{m}_o$, $f$, $S$, $\eta_f$, $\eta_p$, $\eta_o$, $F/A_2$, $A_4/A_2$, $M_2$, and $\tau$ expressed for the parametric equations developed in Chapter 3 for the non-ideal mass flow rate case. These performance parameters are shown with the combustor entrance Mach number, $M_2$, as a parameter. Whenever $M_2 = 0$ then the results correspond to the ramjet; for $M_2 = 0$ the results become identical to the ideal mass flow rate ramjet expressions in [4] and the non-ideal mass flow rate ramjet results in [9]. The values for $h_{PR} = 42,800$ kJ/kg, $T_o = 217$ K, $P_o = 19403$ Pa, $T_{max} = 1600$ K, $c_p = 1.004$ kJ/(kg K), and $\gamma = 1.4$ used here in the figures are the same as those employed in [4] for the ideal mass flow rate ramjet analysis in order to facilitate comparison with the results in [4]. The solid-line curves correspond to the non-ideal mass flow rate cases and the dashed-line curves correspond to the ideal mass flow rate cases for the situations of $M_2 \leq M_o$.

Shown in Fig. 4-11 is the specific thrust for the scramjet ($M_2 \geq 1$) and ramjet ($M_2 = 0$). It is clear from Fig. 4-11 that the scramjet specific thrust extends to much higher flight Mach numbers than the ramjet; at $M_o = 6$, ramjet would have no specific thrust whereas scramjet with $M_2 = 2$ or $M_2 = 3$ would have substantial specific thrust. It is seen that the maximum (peak) scramjet specific thrust decreases and shifts to higher freestream Mach numbers as $M_2$ increases. The non-ideal mass flow rate specific thrust cases are seen to be significantly higher (better) than the ideal mass flow rate cases for all $M_2$ values.
Figure 4-11. Specific thrust for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at free stream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).
Figure 4-12 illustrates the fuel-to-air ratio, \( f \), as a function of freestream Mach number, \( M_o \), with the combustor entrance Mach number, \( M_2 \), as a parameter. It is seen that \( f \) increases as the combustor inlet Mach number increases for a specific \( M_o \); however, \( f \) decreases as the flight Mach number increases for a specific \( M_2 \). Scramjet is seen to require a higher fuel-to-air ratio than ramjet (\( M_2 = 0 \)); the higher specific thrust in Fig. 4-11 requires a higher \( f \) value since more fuel corresponds to higher specific thrust. For all values of \( M_2 \geq 0 \), the non-ideal mass flow rate scramjet is seen to have a higher \( f \) value than the ideal mass flow rate cases.

Depicted in Fig. 4-13 is the thrust-specific fuel consumption, \( S \), as a function of \( M_o \) with \( M_2 \) as a parameter. It is seen that \( S \) increases as \( M_2 \) increases, thus scramjet engines will operate at higher thrust-specific fuel consumption than the ramjet engine. The non-ideal mass flow rate results are shown to have lower (better) thrust-specific fuel consumption for all cases as compared to the ideal mass flow rate cases.

The thermal efficiency, \( \eta_T \) as a function of \( M_o \) with \( M_2 \) as a parameter is shown in Fig. 4-14; the curve for ramjet is the same here as well as in [4, 9]; it is clear that both the ramjet and the scramjet exhibit higher thermal efficiency as the flight Mach number increases. Since the ramjet cannot exceed \( M_o \approx 5 \) then it cannot exceed a thermal efficiency of about 80%; whereas, since the scramjet can operate at much higher \( M_o \) values (\( M_o \approx 9 \)), its thermal efficiency can exceed about 80%. The thermal efficiency for the ideal mass flow rate and the non-ideal mass flow rate are nearly the same, with the non-ideal case being slightly higher.
Figure 4-12. Fuel-to-air ratio for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($\mathbf{M}_2 = 0, 1, 2, 3$).
Figure 4-13. Thrust-specific fuel consumption for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).
Figure 4-14. Thermal efficiency for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).

- $h_{PR} = 42,800$ kJ/kg
- $T_o = 217$ K
- $T_{max} = 1600$ K
- $c_p = 1.004$ kJ/(kg.K)
- $\gamma = 1.4$
- $M_2 \leq M_o$
- Non-ideal mass flow
- Ideal mass flow
Presented in Figs. 4-15 and 4-16 are the propulsive efficiency, $\eta_P$, and the overall efficiency, $\eta_o$, as a function of $M_o$ with $M_2$ as a parameter. The propulsive efficiency is seen to reach a minimum for scramjet ($M_2 \geq 1$) and these $M_o$ values should be avoided. The overall efficiency is seen to operate more efficiently as $M_o$ increases. Figure 4-15 shows that a ramjet ($M_2 = 0$) engine operating at $M_o = 3$ would have a lower propulsive efficiency than a scramjet engine ($M_2 = 2$) operating at $M_o = 6$; this demonstrates an advantage of scramjet. Similarly from Fig. 4-16, the overall efficiency for an ideal ramjet ($M_2 = 0$) engine operating at $M_o = 3$ would have a significantly lower overall efficiency than a scramjet engine ($M_2 = 2$) operating at $M_o = 6$. Figures 4-15 and 4-16 also show that the propulsive efficiency and the overall efficiency for the non-ideal mass flow rate cases are significantly higher (better) than the ideal mass flow rate cases and that the differences between the non-ideal mass flow rate cases and the ideal mass flow rate cases increase as $M_2$ increases.

Shown in Fig. 4-17 are the results for the thrust flux as a function of $M_o$ with $M_2$ held as a parameter for the conditions of $T_o = 217$ K, $P_o = 19403$ Pa (altitude = 12 km), and $T_{\text{max}} = 1600$ K. The thrust flux provides a better depiction [4] than the specific thrust of the freestream Mach number ($M_o$) at which the ramjet/scramjet thrust reaches a maximum value as illustrated in Fig. 4-17. From Fig. 4-17 it is clear how rapidly the scramjet thrust flux performance increases with increasing combustor inlet Mach number ($M_2$) and how the maximum thrust flux occurs at a considerably higher freestream Mach number than the maximum specific thrust (Fig. 4-11). The non-ideal mass flow rate cases are seen to be substantially higher (better) than the ideal mass flow rate cases and with scramjet being vastly better than ramjet ($M_2 = 0.5$).
Figure 4-15. Propulsive efficiency for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).
Figure 4-16. Overall efficiency for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).
Figure 4-17. Thrust flux for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers \((M_2 = 0, 1, 2, 3)\).
Figure 4-18 shows the area ratio (Eq. (46) and (93)) across the scramjet combustor in order to achieve constant velocity combustion. The values for the non-ideal and ideal mass flow rate cases are nearly the same. The information in Fig. 4-18 is valuable for scramjet combustor design.

Shown in Figs. 4-19 and 4-20 are $M_4$ (Eq. (21)) and $\tau$ (Eq. (22)) as a function of $M_0$ with $M_2$ as a parameter. These are the two important variables which are keys to the constant velocity combustor scramjet cycle analysis. These variables are identically the same for both the non-ideal and ideal mass flow rate cases. From Fig. 4-19 it is seen that even though $M_2 \geq 1$ that in some cases $M_4 < 1$; this means that the combustion at the combustor inlet is supersonic but becomes subsonic before reaching the combustor exit. It is also seen from Fig. 4-19 that for $M_2 = 2$ or $M_2 = 3$ and for high values of $M_0$ that $M_4$ becomes greater than $M_2$ ($M_4 > M_2$).

Figure 4-21 shows the entropy change (Eq. (94)) across the combustor which is the same for both the ideal and non-ideal mass flow rate cases. For a given $M_0$ value the entropy change across the combustor increases as $M_2$ increases; this is consistent with the T-s diagram depiction in Fig. 2-1.

Figure 4-22 shows the combustor velocity ratio as a function of $M_0$ with $M_2$ as a parameter. It can be seen that as the freestream Mach number increases for a particular combustor entrance Mach number the velocity ratio increases and then decreases. It can also be seen that as the combustor Mach number increases the maximum (peak) velocity ratio decreases.
Figure 4-18. Area ratio across the combustor for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).

- $h_{PR} = 42,800$ kJ/kg
- $T_o = 217$ K
- $T_{max} = 1600$ K
- $c_p = 1.004$ kJ/(kg.K)
- $\gamma = 1.4$
- $M_2 \leq M_o$

Non-ideal mass flow
Ideal mass flow

$A_1/A_2$ vs $M_o$
Figure 4-19. Combustor exit Mach number for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).

$h_{PR} = 42,800 \text{ kJ/kg}$
$T_o = 217 \text{ K}$
$T_{max} = 1600 \text{ K}$
$c_p = 1.004 \text{ kJ/(kg.K)}$  
$\gamma = 1.4$  
$M_2 \leq M_o$
Figure 4-20. Total temperature ratio for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).
Figure 4-21. Entropy change for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).
Figure 4-22. Combustor velocity ratio for ideal versus non-ideal mass flow rate constant velocity combustion scramjet at freestream Mach numbers for various combustion Mach numbers ($M_2 = 0, 1, 2, 3$).

- $h_{PR} = 42,800$ kJ/kg
- $T_o = 217$ K
- $T_{max} = 1600$ K
- $c_p = 1.004$ kJ/(kg.K)
- $\gamma = 1.4$
- $M_2 \leq M_o$
4.3 Performance parameters for different scramjet fuels

As described in the previous chapters, the selection of fuel for the performance of a scramjet is very important as the thrust produced depends on the lower-heating value, \( \text{h}_{\text{PR}} \), of the fuel. In this section the performance parameters: specific thrust \( (F/\dot{m}_o) \), fuel-to-air ratio \( (f) \), thrust-specific fuel consumption \( (S) \), thermal efficiency \( (\eta_T) \), propulsive efficiency \( (\eta_P) \), overall efficiency \( (\eta_o) \), thrust flux \( (F/A_2) \), are evaluated for three different candidate fuels and for a particular combustor material. Choosing a fuel for combustion significantly influences the parametric performance description of the scramjet; the lower heating value \( (\text{h}_{\text{PR}}) \) is an index for a fuel indicating the amount of heat released per unit mass of fuel. At the same time, the characteristics of hydrogen, such as the lower density and lower ignition temperature, are also unattractive for propulsion because of the handling difficulties. An assumption was made to only consider the amount of energy a fuel can release; the assumptions are made that storage constraints have been overcome for the liquid hydrogen fuel, and that proper handling has been achieved.

With these assumptions, the three candidate fuels chosen in this research are JP-5, natural gas, and liquid hydrogen with \( \text{h}_{\text{PR}} \) values of 42800 kJ/kg, 47100 kJ/kg, and 120000 kJ/kg respectively; these fuels are currently used in the propulsion field today. This section of Chapter 4 is used to illustrate and compare the different performance parameters for scramjet by employing different fuels. A specific maximum service temperature of the combustor material is chosen as the material basis for each figure to emphasize comparison with regard to fuel differences. Other than that, the same nominal performance properties are employed as in the previous sections in order to facilitate comparison with the results in [4]. In Section 4.4 these performance parameters are calculated for three different combustor materials zirconium oxide ceramic, nickel-chromium
alloy and carbon-carbon carbide and their maximum operating temperatures are 2700 K, 1600 K and 2300 K respectively.

The expressions derived for the non-ideal mass flow rate scramjet in Chapter 3 are employed for this evaluation as it is more realistic. The performance is evaluated by using these same expressions, but by changing the lower-heating value of combustor fuel and in Section 4.4 the maximum service temperature of combustor material.

Each performance parameter is presented versus the freestream Mach number with the combustor entrance Mach number, $M_2$, as a parameter. The combustor entrance Mach number for this performance evaluation is chosen to be Mach 2.5 as determined in [30]. The scramjet operates at about $M_o \approx 8$, so for a $M_o \approx 8$ from [30], it can see that the desirable combustor entrance Mach number is $M_2 \approx 2.5$. So, for this evaluation a combustor entrance Mach number of 2.5 is chosen in order to compare the performance of scramjet for different combustor fuels. All the results in this section contain 3 different performance curves, each corresponding to a particular combustor fuel for a particular combustor material and for $M_2 = 2.5$.

As noted from the performance parameter expressions derived in Chapter 3, the lower-heating value of fuel is directly related with the fuel-to-air ratio ($f$) and thrust-specific fuel consumption ($S$). It mainly effects $f$ and $S$ and thereby indirectly the other performance parameters also as can be seen in the results presented below.
4.3.1 Fuel-to-air ratio

Presented in Figs. 4-23 to 4-25 are the values of the fuel-to-air ratio, $f$, for the scramjet versus the freestream Mach number ($M_o$) at a combustor Mach number of 2.5 for different combustor fuels. Shown in Figs. 4-23 to 4-25 are for the non-ideal mass flow rate scramjet for $h_{PR}$ values of 42800 kJ/kg, 47100 kJ/kg, and 120000 kJ/kg respectively. Figure 4-23 corresponds to a maximum material operating temperature of 1600 K and Figs. 4-24 and 4-25 corresponds to a material operating temperatures of 2300 K and 2700 K respectively. The general behavior of fuel-to-air ratio ($f$) in this section is similar to that depicted in Section 4.2. These figures show that the higher the lower-heating value of combustor fuel ($h_{PR}$) the lower the fuel-to-air ratio. It can also be seen that the lower the maximum service temperature of combustor material the lower the fuel-to-air ratio as the lower heating value of fuel increases. Figures 4-23 to 4-25 show that the fuel-to-air ratio follows the same trend for all the different combustor fuels and for different combustor materials, but the difference is in the peak values of the fuel-to-air ratio. It can also be seen that the operating range of the scramjet varies with varying the combustor materials.
Figure 4-23. Fuel-to-air ratio for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.
Figure 4-24. Fuel-to-air ratio for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

\[ T_o = 217 \text{ K} \]
\[ T_{\text{max}} = 2300 \text{ K} \]
\[ c_p = 1.004 \text{ kJ/(kg.K)} \]
\[ \gamma = 1.4 \]
\[ M_2 = 2.5 \]

- 42,800 kJ/kg
- 47,100 kJ/kg
- 120,000 kJ/kg
Figure 4-25. Fuel-to-air ratio for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

- $T_o = 217$ K
- $T_{max} = 2700$ K
- $c_p = 1.004$ kJ/(kg.K)
- $\gamma = 1.4$
- $M_o = 2.5$

Fuel-to-air ratios:
- 42,800 kJ/kg
- 47,100 kJ/kg
- 120,000 kJ/kg
4.3.2 Specific thrust

Depicted in Figs. 4-26 to 4-28 are the values of specific thrust, $F/\dot{m}_o$, for the scramjet versus the freestream Mach number ($M_o$) at a combustor Mach number of 2.5 for different combustor fuels. Shown in Figs. 4-26 to 4-28 are for non-ideal mass flow rate scramjet for $h_{PR}$ values of 42800 kJ/kg, 47100 kJ/kg, and 120000 kJ/kg respectively. Figure 4-26 corresponds to a maximum material operating temperature of 1600 K and Figs. 4-27 and 4-28 corresponds to a material operating temperatures of 2300 K and 2700 K respectively. It is seen that the lower the fuel-to-air ratio, the lower is the specific thrust produced. The specific thrust follows the same general trend as discussed in the previous sections irrespective of the lower-heating value of fuel. It can also be seen that, with increase in the maximum service temperature of combustor material the time of flight to reach a specific destination decreases. It can be noted that the peak values of specific thrust occur at almost at the same flight Mach number irrespective of the fuel type; also there is not a big difference in specific thrust for the various fuels. Figures 4-26 to 4-28 show that the peak specific thrust is obtained at $M_o \approx 4.3$, 4.6 and 4.7 for the three candidate materials. It can also be seen that the peak specific thrust obtained is increasing significantly with change in the material operating temperature, but there is not much difference in the flight Mach number at which the peak thrust is obtained.
Figure 4-26. Specific thrust for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

\[ T_o = 217 \text{ K} \]
\[ T_{\text{max}} = 1600 \text{ K} \]
\[ c_p = 1.004 \text{ kJ/(kg.K)} \]
\[ \gamma = 1.4 \]
\[ M_2 = 2.5 \]

42,800 kJ/kg
47,100 kJ/kg
120,000 kJ/kg
Figure 4-27. Specific thrust for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

\[ T_0 = 217 \text{ K} \]
\[ T_{\text{max}} = 2300 \text{ K} \]
\[ c_p = 1.004 \text{ kJ/(kg.K)} \]
\[ \gamma = 1.4 \]
\[ M_2 = 2.5 \]

42,800 kJ/kg
47,100 kJ/kg
120,000 kJ/kg
Figure 4-28. Specific thrust for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

\[ F/\dot{m}_o \text{ [N/(kg/s)] } \]

- \( T_o = 217 \text{ K} \)
- \( T_{\text{max}} = 2700 \text{ K} \)
- \( c_p = 1.004 \text{ kJ/(kg.K)} \)
- \( \gamma = 1.4 \)
- \( M_o = 2.5 \)
- 42,800 kJ/kg
- 47,100 kJ/kg
- 120,000 kJ/kg
4.3.3 Thrust-specific fuel consumption

Shown in Figs. 4-29 to 4-31 are the values of thrust-specific fuel consumption, S, for the scramjet versus the freestream Mach number (M₀) at a combustor Mach number of 2.5 for different combustor fuels. Figures 4-29 to 4-31 are for the non-ideal mass flow rate scramjet with hPR values of 42800 kJ/kg, 47100 kJ/kg, and 120000 kJ/kg respectively. Figure 4-29 corresponds to a maximum material operating temperature of 1600 K and Figs. 4-30 and 4-31 corresponds to a material operating temperatures of 2300 K and 2700 K respectively. It can be noted that the minimum values of thrust-specific fuel consumption are obtained at about the same M₀ values where the maximum specific thrust is obtained. That is desired as it even simplifies the process of selection of fuel as the required maximum and minimum peak values of the performance parameters are obtained at the same flight Mach numbers. It can be seen that as the material operating temperature increases the minimum value of the thrust-specific fuel consumption decreases. Each curve has a defined length since it is also a function of specific thrust; however there is a termination of each curve because of no existing value of S when the specific thrust decreases to zero.
Figure 4-29. Thrust-specific fuel consumption for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

$T_o = 217$ K
$T_{max} = 1600$ K
$c_p = 1.004$ kJ/(kg.K)
$\gamma = 1.4$
$M_2 = 2.5$

---

47,100 kJ/kg
42,800 kJ/kg
120,000 kJ/kg
Figure 4-30. Thrust-specific fuel consumption for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.
Figure 4-31. Thrust-specific fuel consumption for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_o$</td>
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</tr>
<tr>
<td>$T_{\text{max}}$</td>
<td>2700 K</td>
</tr>
<tr>
<td>$c_p$</td>
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<tr>
<td>$\gamma$</td>
<td>1.4</td>
</tr>
<tr>
<td>$M_2$</td>
<td>2.5</td>
</tr>
</tbody>
</table>

47,100 kJ/kg  42,800 kJ/kg  120,000 kJ/kg
4.3.4 Propulsive and overall efficiencies

Presented in Figs. 4-32 to 4-37 are the propulsive efficiency, $\eta_P$, and the overall efficiency, $\eta_o$, for the scramjet versus the freestream Mach number ($M_o$) at a combustor Mach number of 2.5 for different combustor fuels. Figures 4-32 to 4-34 are the propulsive efficiencies whereas the Figs. 4-35 to 4-37 are the overall efficiencies. From these figures it can be noted that for a fuel of low lower-heating value the propulsive and overall efficiencies are having higher magnitudes than for a fuel of high lower-heating value. It can be seen that the minimum values of propulsive efficiency are obtained at a flight Mach number of about $M_o \approx 4$, just ahead of the $M_o$ where the maximum specific thrust is obtained. The propulsive and overall efficiencies approaches unity as the flight Mach number is increasing. It can even be noted that a fuel of high lower-heating value ($h_{PR}$) has the minimum propulsive efficiency. Both the efficiencies increases past about $M_o \approx 4$ as the flight Mach number increases.
Figure 4-32. Propulsive efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

\[ T_0 = 217 \text{ K} \]
\[ T_{\text{max}} = 1600 \text{ K} \]
\[ c_p = 1.004 \text{ kJ/(kg.K)} \]
\[ \gamma = 1.4 \]
\[ M_2 = 2.5 \]
Figure 4-33. Propulsive efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

- \( T_o = 217 \) K
- \( T_{max} = 2300 \) K
- \( c_p = 1.004 \) kJ/(kg.K)
- \( \gamma = 1.4 \)
- \( M_2 = 2.5 \)
Figure 4-34. Propulsive efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.
Figure 4-35. Overall efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

- $T_o = 217$ K
- $T_{max} = 1600$ K
- $c_p = 1.004$ kJ/(kg.K)
- $\gamma = 1.4$
- $M_2 = 2.5$

Efficiencies:
- 42,800 kJ/kg
- 47,100 kJ/kg
- 120,000 kJ/kg
Figure 4-36. Overall efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.
Figure 4-37. Overall efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.
4.3.5 Thrust flux

Shown in the Figs. 4-38 to 4-40 are the thrust flux, $F/A_2$, for the scramjet versus the freestream Mach number ($M_o$) at a combustor Mach number of 2.5 for different combustor fuels. Figures 4-38 to 4-40 show that for a fuel of low lower-heating value, the higher is the thrust flux produced. Figure 4-38 corresponds to a $h_{PR}$ value of 42,800 kJ/KG and the Figs. 4-39 and 4-40 relates to $h_{PR}$ values of 47,100 kJ/KG and 120,000 kJ/KG respectively. It is seen that the peak values of thrust flux for all the fuels are obtained at about the same flight Mach number of $M_o \approx 7.6$ for maximum service temperature of material, $T_{max} = 1600$ K and $M_o \approx 9.1$ and 9.9 for maximum service temperature of material, $T_{max} = 2300$ K and 2700 K respectively. The maximum values of thrust flux are obtained at a flight Mach number different from the maximum values obtained for the specific thrust. It can be seen that the maximum thrust flux shifts to higher $M_o$ values as the maximum material operating temperature increases. It can also be seen that the peak value of thrust flux increases as the maximum material operating temperature increases. Similar to the specific thrust, the fuel with a higher lower-heating value is having a lower thrust flux produced.
Figure 4-38. Thrust flux for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

$T_o = 217 \text{ K}$

$T_{max} = 1600 \text{ K}$

$c_p = 1.004 \text{ kJ/(kg.K)}$

$\gamma = 1.4$

$M_2 = 2.5$

$120,000 \text{ kJ/kg}$

$47,100 \text{ kJ/kg}$

$42,800 \text{ kJ/kg}$
Figure 4-39. Thrust flux for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

\[ T_o = 217 \text{ K} \]
\[ T_{max} = 2300 \text{ K} \]
\[ c_p = 1.004 \text{ kJ/(kg.K)} \]
\[ \gamma = 1.4 \]
\[ M_2 = 2.5 \]

- 42,800 kJ/kg
- 47,100 kJ/kg
- 120,000 kJ/kg
Figure 4-40. Thrust flux for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor fuels.

$T_o = 217 \text{ K}$
$T_{max} = 2700 \text{ K}$
$c_p = 1.004 \text{ kJ/(kg.K)}$
$\gamma = 1.4$
$M_2 = 2.5$

- 42,800 kJ/kg
- 47,100 kJ/kg
- 120,000 kJ/kg
4.3.6 Overall desirable fuel selection analysis

In order to select a fuel that shows the overall best performance among the three candidate fuels, an overall scoring system has been adopted. The overall scoring system is based on a given flight Mach number \( M_o \) and a weighted point scale for the seven performance parameters. For scramjet propulsion, a flight Mach number of \( M_o \approx 8 \) is selected from the X-43A scramjet project [12]. To compare the performance, \( M_o \approx 10 \) for scramjet is used as the basis for discussion. However, even the remaining performance parameters except thermal efficiency are discussed in this section; note that only the thrust-specific fuel consumption, \( S \), and the fuel to-to-air ratio, \( f \), are explicitly functions of \( h_{PR} \). However, specific thrust and thrust flux are also implicitly affected by the change of candidate fuel; but the change in magnitude is not great. For the weighted point scale, fuel-to-air ratio and thrust-specific fuel consumption are more heavily weighted since a lower amount of fuel consumed and the lower thrust-specific fuel consumption are considered most important for propulsion. Therefore, a graduated weight scale is used in this analysis; the scale for the parametric performances is assigned as 50 for \( S \), 50 for \( f \), and hence overall 100 points.

Since the parametric performance parameters for scramjet vary as a function of \( M_c \), then \( M_c \) needs to be chosen ahead for direct material comparison; \( M_c = 2.5 \) is chosen for comparison. In order for the scramjet engine to operate effectively with current fuel technology, a lower \( S \) and lowest \( f \) are desirable. As presented in the above results for \( M_c = 2.5 \), liquid hydrogen shows lowest \( S \) and \( f \) for the three candidate materials. For both thrust-specific fuel consumption and fuel-to-air ratio, the \( h_{PR} = 120000 \text{ kJ/kg} \) (liquid hydrogen) is recommended over the other two fuels as determined from the results. The results of this section indicate the preferred fuel to be for \( h_{PR} = 120000 \text{ kJ/kg} \) (hydrogen fuel).
Also considering the scramjet as not a commercial airliner but as a military and a surveillance jet, the fuel considerations like the availability, cost of the fuel and the handling and storage problems are not considered in the process of selection of fuel as the most important to obtain the optimum values of the performance parameters for the sustained flight of the scramjet.
4.4 Performance parameters for different combustor materials

This section of Chapter 4 is used to illustrate and compare the different parametric performance parameters among the scramjet engines by employing different materials. These figures show how the case of non-ideal mass flow rate impacts in the parametric performance parameters for the three material candidates. The parametric performance description of the scramjet engine with the non-ideal mass flow rate basis is significantly influenced by choosing an engine (combustion chamber) material. The non-ideal mass flow rate basis illustrates that there are greater increases in parametric performance for scramjet than the ideal mass flow rate basis as the material maximum service temperature, $T_{\text{max}}$, increases. Thus, the assumption of non-ideal mass flow rate also impacts more significantly the scramjet parametric performance parameters as the scramjet operates at both higher $M_c$ and higher $T_{\text{max}}$.

With these assumptions, three different combustor materials: zirconium oxide ceramic, carbon-carbon carbide and nickel-chromium alloy with maximum operating temperatures are 2700 K, 2300 K and 1600 K respectively were selected and the performance parameters evaluated. This section of Chapter 4 is used to illustrate and compare the different performance parameters among the scramjet by employing different fuels. The lower-heating value of fuel is chosen as the material basis for each figure to show comparison with regard to combustor materials. Other than that, the same nominal performance properties are employed as in the previous section for fuel selection in order to facilitate comparison with the results in [4]. These performance parameters are calculated for three candidate fuels: JP-5, natural gas, and liquid hydrogen with $h_{\text{PR}}$ values of 42,800 kJ/kg, 47,100 kJ/kg, and 120,000 kJ/kg respectively.

From the results obtained, we can notice that the combustor material has a significant impact on the range of flight. The combustor material with a high service temperature provides a
wide range of flight whereas the combustor material with low service temperature has a low range of flight conditions. All the graphs in this section contain three different performance curves each corresponding to a particular combustor material for a particular combustor fuel and for a particular combustor entrance Mach number. All the performance parameters are presented for three different candidate fuels to show how the change of fuel also effects the performance of the scramjet for different combustor materials.

The maximum service temperatures used in this analysis are 1600 K (nickel chromium alloy), 2300 K (carbon-carbon carbide) and 2700 K (zirconium oxide ceramic). The nominal properties for $T_o = 217$ K, $P_o = 19403$ Pa (altitude = 12 km), $c_p = 1.004$ kJ/(kg K), $a_o = 294.76$ m/s, and $\gamma = 1.4$ used in this section are the same as those employed in [4] for the ideal mass flow rate scramjet analysis in order to facilitate the comparison with the results in [4].
4.4.1 Fuel-to-air ratio

Figures 4-41 to 4-43 illustrate the fuel-to-air ratio, $f$, as a function of freestream Mach number $M_o$, with the combustion Mach number of $M_c = 2.5$ for different candidate materials. Shown in Fig. 4-41 is $f$ for maximum service temperature of 1600 K. It is seen that $f$ increases as the combustion Mach number increases for a specific $M_o$. Figure 4-41 demonstrates that for a scramjet engine operating at $M_o = 8$ and $M_c = 2.5$, for a lower-heating value of fuel, $h_{PR}$, value of 42,800 kJ/kg, for a material of $T_{max} = 1600$ K, $f$ is about $f \approx 0.075$; and for materials of $T_{max} = 2300$ K and 2700 K, fuel-to-air ratio is $f \approx 0.025$ and 0.035 respectively.

To compare the fuel-to-air ratios, $f$, of the three materials, they are presented in Figs. 4-41 to 4-43; the impact of varying the value of $T_{max}$ is shown along with varying the lower-heating values of fuels ($h_{PR}$). These figures show that $f$ increases as $T_{max}$ increases. Figures 4-42 and 4-43 show how the fuel-to-air ratio varies for different candidate materials for different fuels of $h_{PR} = 47,100$ kJ/kg and 120,000 kJ/kg respectively. All the results follow the same trend as explained in the previous sections. It can be seen from the Figs. 4-41 to 4-43, the fuel-to-air ratio is greater for a material of higher material operating temperature. It can also be seen that the fuel-to-air ratio, $f$, decreases for a particular candidate material as the lower-heating value of fuel increases.
Figure 4-41. Fuel-to-air ratio for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

\[ h_{pg} = 42,800 \text{ kJ/kg} \]
\[ T_o = 217 \text{ K} \]
\[ c_p = 1.004 \text{ kJ/(kg.K)} \]
\[ \gamma = 1.4 \]
\[ M_2 = 2.5 \]
Figure 4-42. Fuel-to-air ratio for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

\[ h_{pr} = 47,100 \text{ kJ/kg} \]
\[ T_{o} = 217 \text{ K} \]
\[ c_{p} = 1.004 \text{ kJ/(kg.K)} \]
\[ \gamma = 1.4 \]
\[ M_{2} = 2.5 \]
Figure 4-43. Fuel-to-air ratio for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.
4.4.2 Specific thrust

Depicted in Figs. 4-44 to 4-46 are the values of specific thrust, $F/\dot{m}_o$, for the scramjet versus the freestream Mach number ($M_o$) at a combustor Mach number of 2.5 for different combustor materials. Shown in Figs. 4-44 to 4-46 are for non-ideal mass flow rate scramjet for maximum material operating temperatures of $T_{\text{max}} = 1600$ K, 2300 K and 2700 K respectively. Figure 4-44 corresponds to a lower heating value of fuel, $h_{PR}$ value of 42,800 kJ/KG and Figs. 4-45 and 4-46 corresponds to a lower heating value of fuels, $h_{PR}$ values of 47,100 kJ/kg and 120,000 kJ/kg respectively. It is seen that the lower the fuel-to-air ratio, the lower is the specific thrust produced. Figure 4-44 shows that more specific thrust is produced for a material of high maximum operating temperature than for a material of low maximum operating temperature. The specific thrust follows the same general trend as discussed in the previous sections. It can also be seen that, with increase in the maximum service temperature of combustor material the time of flight to reach a specific destination decreases.

It can be noted that unlike in the fuel selection process, the peak values of specific thrust are not obtained at the same flight Mach number. From Fig. 4-44, for a fuel of $h_{PR} = 42,800$ kJ/kg and for a combustion Mach number of $M_c = 2.5$; for a material of $T_{\text{max}} = 1600$ K, the peak specific thrust is produced at $M_0 \approx 4.3$; and for $T_{\text{max}} = 2300$ K and 2700 K, the peak specific thrust is produced at $M_0 \approx 4.6$ and 4.7 respectively. Figures 4-45 and 4-46 also follow the similar trend but different peak specific thrust values are obtained. It can also be seen that the peak specific thrust obtained is increasing significantly with change in the material operating temperature but there is not much difference in the flight Mach number at which the peak thrust is obtained. It can also be seen that as the lower-heating value of fuel increases the peak specific thrust produced decreases.
Figure 4-44. Specific thrust for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.
Figure 4-45. Specific thrust for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.
Figure 4-46. Specific thrust for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

- $h_{PR} = 120,000$ kJ/kg
- $T_o = 217$ K
- $c_p = 1.004$ kJ/(kg.K)
- $\gamma = 1.4$
- $M_2 = 2.5$
4.4.3 Thrust-specific fuel consumption

Shown in Figs. 4-47 to 4-49 are the values of thrust-specific fuel consumption, $S$, for the scramjet versus the freestream Mach number ($M_o$) at a combustor Mach number of 2.5 for different combustor materials. Shown in Figs. 4-47 to 4-49 are for non-ideal mass flow rate scramjet for maximum material operating temperatures of $T_{max} = 1600$ K, 2300 K and 2700 K respectively. Figure 4-47 corresponds to a lower heating value of fuel, $h_{PR}$ value of 42,800 kJ/kg and Figs. 4-48 and 4-49 corresponds to a lower heating value of fuels, $h_{PR}$ values of 47,100 kJ/kg and 120,000 kJ/kg respectively. It can be seen that the lower the maximum service temperature of material, lower is the thrust-specific fuel consumption produced. It can be noted that the low values of thrust-specific fuel consumption are obtained at about the same $M_o$ values where the maximum specific thrust is obtained. Similar to fuel selection process each curve has a defined length since it is also a function of specific thrust; however there is a termination of each curve because of no existing value of $S$ when the specific thrust decreases to zero.

Figure 4-47 shows that, for a fuel of $h_{PR} = 42,800$ kJ/kg and for a combustion Mach number of $M_c = 2.5$; for a material of $T_{max} = 1600$ K, the minimum thrust-specific fuel consumption is produced at $M_0 \approx 4.3$; and for $T_{max} = 2300$ K and 2700 K, the minimum thrust-specific fuel consumption is produced at $M_0 \approx 4.6$ and 4.7 respectively. Figures 4-48 and 4-49 also follows a similar trend but different values of minimum thrust-specific fuel consumption are obtained at a flight Mach number higher than that obtained in the previous case.
Figure 4-47. Thrust-specific fuel consumption for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

- \( h_{PR} = 42,800 \text{ kJ/kg} \)
- \( T_0 = 217 \text{ K} \)
- \( c_p = 1.004 \text{ kJ/(kg.K)} \)
- \( \gamma = 1.4 \)
- \( M_2 = 2.5 \)
Figure 4-48. Thrust-specific fuel consumption for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

$h_{PR} = 47,100 \text{ kJ/kg}$
$T_0 = 217 \text{ K}$
$c_p = 1.004 \text{ kJ/(kg.K)}$
$\gamma = 1.4$
$M_2 = 2.5$
Figure 4-49. Thrust-specific fuel consumption for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

\[ S \text{ [mg/(N\cdot s)]} \]

- \( h_{PR} = 120,000 \text{ kJ/kg} \)
- \( T_0 = 217 \text{ K} \)
- \( c_p = 1.004 \text{ kJ/(kg.K)} \)
- \( \gamma = 1.4 \)
- \( M_2 = 2.5 \)

- 2700 K
- 2300 K
- 1600 K
4.4.4 Thermal efficiency

Shown in Figs. 4-50 to 4-52 are the values of thermal efficiency, $\eta_T$, for the scramjet versus the freestream Mach number ($M_o$) at a combustor Mach number of 2.5 for different combustor materials. Shown in Figs. 4-50 to 4-52 are for non-ideal mass flow rate scramjet for maximum material operating temperatures of $T_{\text{max}} = 1600$ K, 2300 K and 2700 K respectively. Figure 4-50 corresponds to a lower heating value of fuel, $h_{PR}$ value of 42,800 kJ/kg and Figs. 4-51 and 4-52 corresponds to a lower heating value of fuels, $h_{PR}$ values of 47,100 kJ/kg and 120,000 kJ/kg respectively. Figures 4-50 to 4-52 shows that the thermal efficiency is the same for all the candidate materials but varies very slightly as the lower-heating value of fuel used changes. This change in the thermal efficiency is not significant; this variation in the thermal efficiency is because it is an implicit function of the fuel-to-air ratio (see Eq. (83)). Because the scramjet can operate at much higher $M_o$ values ($M_o \approx 8$), its $\eta_T$ can reach about 90%. It also can be seen that $\eta_T$ rises rapidly for $0 < M_o < 5$ but rises less rapidly when $M_o > 5$. 
Figure 4-50. Thermal efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.
Figure 4-51. Thermal efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

- $h_{PR} = 47,100 \text{ kJ/kg}$
- $T_0 = 217 \text{ K}$
- $c_p = 1.004 \text{ kJ/(kg.K)}$
- $\gamma = 1.4$
- $M_2 = 2.5$

1600 K, 2300 K, 2700 K
Figure 4-52. Thermal efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

$\eta_T$ vs $M_o$

- $h_{PR} = 120.00$ kJ/kg
- $T_o = 217$ K
- $c_p = 1.004$ kJ/(kg.K)
- $\gamma = 1.4$
- $M_2 = 2.5$

1600 K, 2300 K, 2700 K
4.4.5 Propulsive and Overall efficiencies

Figures 4-53 to 4-58 are the values of propulsive and overall efficiencies, for the scramjet versus the freestream Mach number \( (M_o) \) at a combustor Mach number of 2.5 for different combustor materials. Shown in Figs. 4-53 to 4-58 are for non-ideal mass flow rate scramjet for maximum material operating temperatures of \( T_{\text{max}} = 1600 \, \text{K}, 2300 \, \text{K} \) and 2700 K respectively. Figures 4-53 and 4-56 corresponds to a lower heating value of fuel, \( h_{PR} \) value of 42,800 kJ/kg and Figs. 4-54, 4-57 and 4-55, 4-58 corresponds to a lower heating value of fuels, \( h_{PR} \) values of 47,100 kJ/kg and 120,000 kJ/kg respectively. It can be seen that the propulsive efficiency first decreases as the flight Mach number, \( M_o \) increases and then increases, whereas the overall efficiency increases as the flight Mach number, \( M_o \) increases. These figures present a clear comparison of how the propulsive and overall efficiencies vary by changing the candidate materials for a particular lower-heating value, \( h_{PR} \) of fuel. It can be seen that for a particular flight Mach number and for a particular fuel, the propulsive and overall efficiencies decreases as the maximum operating temperature of material increases.
Figure 4-53. Propulsive efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

- $h_{PR} = 42,800 \text{ kJ/kg}$
- $T_o = 217 \text{ K}$
- $c_p = 1.004 \text{ kJ/(kg.K)}$
- $\gamma = 1.4$
- $M_2 = 2.5$
Figure 4-54. Propulsive efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.
Figure 4-55. Propulsive efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

- $h_{PR} = 120,000 \text{ kJ/kg}$
- $T_o = 217 \text{ K}$
- $c_p = 1.004 \text{ kJ/(kg.K)}$
- $\gamma = 1.4$
- $M_2 = 2.5$
Figure 4-56. Overall efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

- $h_{PR} = 42,800 \text{ kJ/kg}$
- $T_o = 217 \text{ K}$
- $c_p = 1.004 \text{ kJ/(kg.K)}$
- $\gamma = 1.4$
- $M_2 = 2.5$
Figure 4-57. Overall efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

\[ h_{PR} = 47,100 \text{ kJ/kg} \]
\[ T_o = 217 \text{ K} \]
\[ c_p = 1.004 \text{ kJ/(kg.K)} \]
\[ \gamma = 1.4 \]
\[ M_2 = 2.5 \]
Figure 4-58. Overall efficiency for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.
4.4.6 Thrust flux

Shown in Figs. 4-59 to 4-61 are the values of thrust flux, F/A₂, for the scramjet versus the freestream Mach number (M₀) at a combustor Mach number of 2.5 for different combustor materials. Shown in Figs. 4-59 to 4-61 are for non-ideal mass flow rate scramjet for maximum material operating temperatures of T_{max} = 1600 K, 2300 K and 2700 K respectively. Figure 4-59 corresponds to a lower heating value of fuel, h_{PR} value of 42,800 kJ/kg and Figs. 4-60 and 4-61 corresponds to a lower heating value of fuels, h_{PR} values of 47,100 kJ/kg and 120,000 kJ/kg respectively. From Figs. 4-59 to 4-61, it is clear how rapidly the non-ideal scramjet thrust flux performance increases with increasing the maximum operating temperature, T_{max} of candidate material and how the maximum thrust flux occurs at a considerably higher freestream Mach number than the maximum specific thrust. It can be seen that the maximum thrust flux shifts along as the maximum material operating temperature increases, also the peak value of thrust flux increases as the material operating temperature increases.

Figure 4-59 shows that, for a fuel of h_{PR} = 42,800 kJ/kg and for a combustion Mach number of M_c = 2.5; for a material of T_{max} = 1600 K, the maximum thrust flux is produced at M₀ ≈ 7.6; and for T_{max} = 2300 K and 2700 K, the minimum thrust flux is produced at M₀ ≈ 9.1 and 9.9 respectively. Figures 4-60 and 4-61 also follows the same trend but different peak values of thrust flux are obtained at the same flight Mach numbers, M₀.
Figure 4-59. Thrust flux for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.

- $h_{PR} = 42,800 \text{ kJ/kg}$
- $T_o = 217 \text{ K}$
- $c_p = 1.004 \text{ kJ/(kg.K)}$
- $\gamma = 1.4$
- $M_2 = 2.5$

- 2700 K
- 2300 K
- 1600 K
Figure 4-60. Thrust flux for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.
Figure 4-61. Thrust flux for non-ideal mass flow rate constant velocity combustion scramjet versus freestream Mach numbers for different combustor materials.
4.4.7 Overall desirable material selection analysis

In order to select a material that shows the overall best performance among the three candidate materials, an overall scoring system has been adopted like in the case of the fuel selection. To compare the performance, \( M_o \approx 10 \) for scramjet is used as the basis for discussion. For the weighted point scale, thrust has been more heavily weighted since a higher amount of thrust is considered most important for propulsion. Especially thrust flux \((F/A_2)\) is most heavily weighted in the scoring system. Note that a lesser weight of performance is assigned for thrust-specific fuel consumption \((S)\) and fuel-to-air ratio \((f)\), and there is no weight for thermal efficiency \((\eta_T)\) since it turns out to be independent of material. For this work the weighted point scale for each parametric performance was chosen as 20 for specific thrust \((F/\dot{m}_o)\), 10 for \(S\), 10 for \(f\), zero for \(\eta_T\), 10 for propulsive efficiency \((\eta_P)\), 10 for overall efficiency \((\eta_o)\), and 40 for \(F/A_2\), and hence overall 100 points.

As for the fuel selection, \(M_c = 2.5\) is chosen for comparison. As seen in the results presented above only \(F/\dot{m}_o\) shows better performance at \(T_{\text{max}} = 2700\ K\); whereas, the other five performance parameters show better performance at \(T_{\text{max}} = 1600\ K\) except the case for thrust flux with \(T_{\text{max}} = 2700\ K\) as seen above.

From the results presented above, the \(T_{\text{max}} = 2700\ K\) (zirconium oxide ceramic) with \(M_c = 2.5\) is better for specific thrust than the other two materials because scramjet with the \(T_{\text{max}} = 2700\ K\) achieves the highest \(F/\dot{m}_o\) among three candidate materials. In order for the scramjet engine to operate effectively with contemporary fuel technology, a lower \(S\) and lower \(f\) are desirable. For thrust-specific fuel consumption, the \(T_{\text{max}} = 1600\ K\) (nickel chromium alloy) is recommended over the other two materials since this engine achieves the lowest value of \(S\). For fuel-to-air ratio, the \(T_{\text{max}} = 1600\ K\) (nickel chromium alloy) is more appropriate than the other two
candidate materials because of its lower values of \( f \). For propulsive efficiency and overall efficiency, \( T_{\text{max}} = 1600 \) K (nickel chromium alloy) is more desirable than the other two candidate materials due to the decreasing trend of \( \eta_p \) and \( \eta_o \) as \( T_{\text{max}} \) increasing. For thrust flux, \( T_{\text{max}} = 2700 \) K (zirconium oxide ceramic) shows much better performance than the other two candidate materials.

Though only specific thrust and thrust flux show better performance with \( T_{\text{max}} = 2700 \) K, the total overall performance for \( T_{\text{max}} = 2700 \) K is superior to the other two candidate materials. The results of this section indicate the preferred material to be for \( T_{\text{max}} = 2700 \) K (zirconium oxide ceramic).
CHAPTER 5
CONCLUSIONS

In this study, the seven parametric performance parameters of scramjet propulsion have been investigated and compared based on Brayton-cycle analysis. The results for specific thrust ($F/n_i$), thrust-specific fuel consumption ($S$), fuel-to-air ratio ($f$), thermal, propulsive, overall efficiencies ($\eta_T$, $\eta_P$, $\eta_o$), and thrust flux ($F/A_2$) and combustor area ratio, $A_4/A_2$ have been shown for parametric performance analysis for understanding scramjet engine application. These parameters were modeled using simple algebraic expressions similar to [4]. The results presented are also in close relation to [30]. The results presented in this study can be useful to predict the parametric performances of scramjet when employing different engine materials and fuels. The results demonstrate that the non-ideal mass flow rate analysis plays a significant role for scramjet propulsion especially at high combustion Mach numbers. Non-ideal mass flow rate analysis also provides guidance to determine the appropriate combustion Mach number to achieve better engine performance as the flight changes. The main modelling in this work which were specifically shown in detail in the Results chapter are as follows:

- Constant velocity versus constant Mach number combustion comparison
- Three different candidate materials and fuels comparison
- Ideal and non-ideal mass flow rate combustion cases comparison
The main conclusions of this work are summarized below.

- Both the constant velocity and constant Mach number combustion behave in the same general manner. The constant Mach number combustion has higher specific and thrust flux produced than the constant velocity combustion; whereas, the fuel-to-air ratio, thrust-specific fuel consumption and all the efficiencies are better in the case of constant velocity combustion than in the constant Mach number combustion.

- Among the three candidate engine materials for the non-ideal mass flow rate constant velocity combustor scramjet, zirconium oxide ceramic ($T_{\text{max}} = 2700$ K) was chosen as the best material. The higher $T_{\text{max}}$ value for an engine material is favorable for the achieving greater thrust for a scramjet engine, though there is a disadvantage for some other parametric performance parameters. This study employed a weighted point system, and thrust was more heavily weighted since a higher amount of thrust was considered most important for a surveillance aircraft. Among the three candidate fuels for the non-ideal mass flow rate constant velocity combustor scramjet, liquid hydrogen ($h_{\text{PR}} = 120,000$ kJ/kg) was selected as the best fuel. Only the two parametric expressions of thrust-specific fuel consumption and fuel-to-air ratio are given strongly influenced by the process of selection of fuel, and a much higher $h_{\text{PR}}$ value for liquid hydrogen helps to reduce the thrust-specific fuel consumption and fuel-to-air ratio significantly. It is assumed here that the storage and infrastructure constraints have been overcome for the liquid hydrogen fuel, and that proper handling has been achieved.

- The major advantage of modeling the non-ideal mass flow rate case is that it is more realistic and shows better performance for the scramjet engine especially at higher combustion Mach numbers with constant velocity combustion. For scramjet propulsion,
which consumes a significant amount of fuel, the mass of fuel cannot be ignored by assuming \( \dot{m}_o + \dot{m}_f \approx \dot{m}_o \). The performance parameters obtained for the non-ideal mass flow rate case are higher than those obtained for the ideal mass flow rate case. These quantitative results show the non-ideal mass flow rate to be superior and more realistic than the ideal mass flow rate performances.

This study basically focused on deriving parametric performance expressions for scramjet propulsion for various parameters; hence, now a useful model exists for understanding and predicting scramjet engine performance. The results presented here and in [31] are hoped to provide useful guidance to improve and predict the performance of a scramjet engine as well as determining an adequate material and fuel for scramjet propulsion.
LIST OF REFERENCES


(published online but will appear later in print)
VITA

OBJECTIVE
An entry-level engineering position requiring strong analytical and organizational skills

EDUCATION

Master of Science in Mechanical Engineering
University of Mississippi, University, MS, USA                                        GPA 3.46/4.0
August 2014 to May 2016

Bachelor of Technology in Mechanical Engineering
JNTUK Kakinada, AP, India                                GPA 7.0/10.0
August 2010 to May 2014

TECHNICAL SKILLS

COMPUTER : Auto CAD, Pro/ENGINEER, CATIA, MATLAB, C Programming, MS OFFICE, PTC Creo, Java, SQL.

EQUIPMENT : Lathe, Milling and drilling Machine, Welding, material testing

PERSONAL : Leadership, Adaptability, excellent communication and organizational skills, Problem solving, Teamwork

PROFESSIONAL EXPERIENCE

Graduate Teaching Assistant at The University of Mississippi, University, MS   August 2014 – Present

• Responsibilities include helping the professor in organizing classes, conducting tests and grading; and also assisting students in understanding the critical concepts of heat transfer, thermal engineering, thermodynamics and other engineering courses

Biological lab assistant at the National Center for Natural Products Research (NCNPR), University of Mississippi      January 2015 to January 2016

• Helping the research scientists in conducting the research experiments and documenting the results of the experiments in MS office.
Worked voluntarily as Assistant Production Engineer in Sri Madhavi Industries, India  
June 2013 to July 2014

- Handled fabrication assignments that involved installing, repairing and fabricating materials. Became adept at reading blueprints/drawings and fulfilling work orders.
- Demonstrated advanced skills in operating equipment and machines including CNC machines, cutting and drilling for the fabrication of mechanical components.

RESEARCH AND ACADEMIC EXPERIENCE

Graduate research assistant working Parametric Analysis of constant velocity combustion scramjet

- Objective is to determine performance of a constant velocity combustion scramjet engine by parametrically analyzing the engine parameters: specific thrust, fuel-to-air ratio, thrust specific fuel consumption, thermal efficiency, propulsive efficiency, overall efficiency and thrust flux and then to determine the desirable performance terms of the scramjet, by varying three different candidate fuels and three different candidate materials for the combustion chamber.

Thermal Heat transfer in a 2-Dimentional Plane

- Temperature distribution on the two-dimensional plate with known boundary conditions is visualized.
- The discrete partial differential equations at each node were solved using a TDMA (Tri Diagonal Matrix Algorithm) to find the temperature distribution at all the points on the plane

Fabrication of a Hexapod Robot using Mechanical linkages  
July 2013 to April 2014

- Locomotive robots are fast replacing the wheeled ones as they have distinct advantages like climbing heights, walking over uneven and unpredictable paths etc.
- The aim of the project is to build a six-legged walking robot that is capable of performing basic mobility tasks, such as walking forward and backward

Design and Fabrication of a Multi-Purpose Machine  
July 2013 to December 2013

- Objective is to design and fabricate a Multipurpose Machine which performs various machining operations like cutting, drilling, sawing and grinding.
- It can be used in small scale production like batch production or job production, it saves the material transportation time

PUBLICATIONS

**COURSE WORK**
• Finite Element Analysis, CAD/CAM, Machine Design, Engineering Graphics, material science
• Dynamics of Machinery, Vibrations, IC Engines, Automobile Engineering
• Thermodynamics, Thermal Engineering, Fluid Mechanics, Numerical Heat Transfer, Modern compressible flow, Heat Transfer

**ACCOMPLISHMENTS**
• Undergone training session on AutoCAD organized by AUTODESK and Certified as **AutoCAD Associate**
• Undergone FIRE & SAFETY training conducted by Kings Institute of Fire & Safety Engineers