Acoustic Radiation Force on a Fluid to Fluid Boundary by Phase Plate Focused Ultrasound

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ACOUSTIC RADIATION FORCE ON A FLUID TO FLUID BOUNDARY BY PHASE PLATE FOCUSED ULTRASOUND

A Dissertation presented in partial fulfillment of requirements for the degree of Doctor of Philosophy in the Department of Physics and Astronomy The University of Mississippi

by
ROBERT LIRETTE
May 2019
ABSTRACT

In this study, the physics and applications of the ultrasonic radiation force at the interface between two immiscible fluids were investigated. These studies were performed using low-profile discrete-stepped lenses to modify the phase of the incident radiation generating multiple field morphologies. In its first application to acoustics, a fraxicon lens was developed to approximate the field generated by an axicon. This type of lens creates a minimally diffractive Bessel beam and long depth of focus that is useful in ultrasonic imaging, therapy, and non-destructive evaluation techniques. Fields modified by fraxicon, Fresnel, and axicon lenses were characterized experimentally by scanning hydrophone measurements and by numerical simulations. The results showed good agreement with each lens type. A theoretical framework for the focusing efficiency of fraxicon and Fresnel phase plate lens was also developed which compared well to the numerical simulations. Ultrasound focused by these lenses was then used to deform the boundary between two sets of fluids. First a water to carbon tetrachloride (CCl₄) boundary was explored. A central finding was that the near field features of the fraxicon provided the stability to extract a droplet from a fluid interface, a novel effect, allowing it to be transported over relatively large distances. Similar phenomena of acoustic trapping and transport typically requires arrays of multiple transducers or standing waves. Here a single sided transducer with a passive lens was used. This phenomenon has applications in chemical engineering, microfluidics, and advanced testing techniques. The trapping force was calculated from theory using simulations of the field and the results compared well to observations. Next the boundary between canola oil and water was investigated. Due to the impedance mismatch at the boundary significant heating occurred when it was insonified causing a decrease in the surface tension over time. When the surface tension is low enough, jetting of the oil into the water occurs. The findings in this work could lead to a broad range of applications involving fluid transport and manipulation.
DEDICATION

This dissertation work is dedicated to my mother Linda Lirette and my grandmother Jackie Malbrough whose love and constant support throughout kept me motivated to finish.
LIST OF ABBREVIATIONS AND SYMBOLS

Al aluminum
ASM angular spectrum method
CPS clear polystyrene
DOF depth of focus
FEM finite element method
FFT fast Fourier transform
MDZ minimal diffraction zone
PLA poly-lactic acid
PVC polyvinyl chloride
RF radio frequency

a acoustic attenuation coefficient
c sound propagation speed
c_1 longitudinal sound speed of fluid 1
c_2 longitudinal sound speed of fluid 2
c_{lens} longitudinal sound speed in the lens material
D_n direct distance from the n^{th} step of the lens to the focusing point
d_m the thickness of the m^{th} step within a phase group
d_{min} the minimum thickness of the lens where k = 1
E_I average incident energy density
F acoustic radiation force
f the focal length
g the acceleration due to gravity
G focal power fraction
\( h(r) \) the height profile of the fluid boundary deformation

\( I_{av} \) time averaged acoustic intensity

\( k \) acoustic wave number magnitude

\( k_{lens} \) acoustic wave number in the lens material

\( k_r \) the radial component of the wave vector

\( k_z \) the z component of the wave vector

\( L \) the depth of focus

\( M \) number of steps within one phasing group

\( \mathbf{n} \) unit vector normal to the fluid interface upward

\( p \) complex acoustic pressure

\( P_0 \) peak acoustic pressure

\( P_I \) incident pressure amplitude

\( R \) radius

\( r_{1,2} \) reflection coefficient

\( r_I \) intensity reflection coefficient

\( R_n \) radial distance to the \( n^{th} \) phasing step from the center of the lens

\( T \) maximum axicon thickness

\( t_I \) intensity transmission coefficient

\( u \) complex acoustic velocity

\( W \) total acoustic power

\( W_F \) power delivered to the 6 dB bounds of the focus

\( Z_i \) specific acoustic impedance of the \( i^{th} \) fluid

\( \alpha \) angle that defines the axicon thickness

\( \beta \) angle that defines the depth of focus
\( \eta \)  
diffraction efficiency

\( \theta_i \)  
angle of incidence

\( \theta_m \)  
the angular phase at the \( m^{th} \) step within a phase group

\( \theta_r \)  
angle of refraction

\( \lambda \)  
the ultrasonic wavelength in water

\( \lambda_{\text{lens}} \)  
the ultrasonic wavelength in the lens material

\( \rho \)  
density

\( \rho_1 \)  
density of fluid 1

\( \rho_2 \)  
density of fluid 2

\( \sigma \)  
surface tension

\( \psi \)  
complex velocity potential

\( \omega \)  
angular frequency
ACKNOWLEDGEMENTS

First I’d like to thank my research advisor, Dr. Joel Mobley, as well as the other researchers in the ultrasonics group at the National Center for Physical Acoustics (NCPA) Dr. Cecille Labuda, Dr. Likun Zhang, and Dr. Charles Church. I’d also like to thank the researchers at NCPA whom I’ve worked with in the past in other groups, Dr. Roger Waxler and Dr. Jim Sabatier, for their guidance along the way. Further, I am grateful to my graduate colleagues Dr. Ukesh Koju and Kevin Yi-Wei Lin for their technical assistance with my research. Lastly I wish to extend my thanks to everyone else at NCPA and the department of physics at the University of Mississippi whom I’ve worked with during my time here.
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CHAPTER 1

INTRODUCTION

Ultrasonic radiation that is focused on a boundary between two immiscible fluids can impart a radiation pressure on that boundary which is either in the direction of propagation or backwards toward the source. The extreme example of this is the ultrasonic fountain where ultrasound is focused at a water to air boundary creating a water fountain effect. This is a well known phenomenon and has applications in microfluidics as well as other commercial uses. Previous studies have been constrained to the deformations resulting from low incident amplitudes and traditional spherically focused sources.\textsuperscript{1,2} The goal of this work is to investigate the deformation and other phenomenon such as droplet capture and jetting using higher source amplitudes and non-conventional beam types. To that end, ultrasonic transducers with integrated low profile phase plate lenses were developed and used as sources. Phase plate lenses are much thinner than traditional refracting lenses and thus have less bulk attenuation and dispersion. They also allow for easier access to the near field than their refractive counterparts. One type of lens constructed was a Fresnel lens which approximates the focusing of a traditional spherical lens. A novel type developed in this work was a fraxicon phase plate lens which is designed to produce a finite Bessel beam. This approximates the characteristics of the cone shaped axicon lens.

Chapter 2 outlines the theoretical framework for the lens design and for the acoustic radiation force on a fluid boundary. Next in Chapters 2 through 5, the development, testing, and numerical simulations of the lenses is detailed. In Chapters 6 and 7, two sets of experiments are presented demonstrating both positive and negative acoustic radiation pressure on a fluid boundary.

The phase plate lenses were designed to alter the phase of the incident ultrasonic radiation in discrete steps so that constructive interference occurs at a focus in the case of the Fresnel and
along a depth of focus (DOF) in the case of the fraxicon. The Fresnel lens approximates a spherical lens whereas the fraxicon approximates an axicon which is a bulk refractive lens. Fresnel phase plate lenses have been previously developed and studied for both ultrasonic and optical use.\textsuperscript{3,4} The field generated by the fraxicon and axicon however is of interest because it creates a Bessel beam which is minimally diffractive along a long DOF.\textsuperscript{5,6}

In the initial stages of development, the lenses were manufactured out of polymer materials. They were then mounted to a ring which was spaced about 1 cm away from a flat piezoelectric crystal disk source and coupled to it using water. This had the advantage of being able to use multiple different lenses with the same transducer. In the later design that was used the lenses were machined out of aluminum and were coupled directly to the crystal with epoxy. This design allowed for at much higher powers and for longer durations without over heating. Each transducer was designed using a commercially available 50 mm diameter piezoelectric crystal with a resonant frequency of 1.2 MHz.

The fields modified by the various phase plate lens transducers were characterized by direct measurement in a water tank using a hydrophone. The polymer lens transducers were scanned using a low power pulsed source which allowed for the characterization of the broadband behavior of the different lens types. The aluminum lens transducers were scanned using a sinusoidal burst source to better represent their fields when a continuous wave was used. Next a study on the focusing and electrical efficiency of the aluminum lens transducers is presented. The hydrophone measurements are then compared in the frequency domain to simulations using the angular spectrum method (ASM). The angular spectrum simulation is a Fourier transform method which takes the field on a plane and propagates it in space by breaking down the field into a plane wave spectrum.\textsuperscript{7,8}

In the remaining chapters of this work, two experimental setups are detailed that demonstrate various phenomena related to the acoustic radiation pressure experienced by a fluid interface. In the first set of experiments a fluid combination of water and carbon tetrachloride was used to examine the phenomenon of a negative radiation pressure which deforms the fluid interface towards the source. For this setup the source was located in the water and the propagation direction was
downward into the carbon tetrachloride. These two fluids were chosen because they are impedance matched which results in minimal reflection and heating at the boundary. With this configuration and using the near-field radiation from the fraxicon phase plate transducer, a single droplet of carbon tetrachloride was separated, trapped, and transported through the water by the field. Typically a standing wave$^9$ or the field from an array of transducers$^{10}$ is required for acoustic trapping, however this effect is achieved using the field from a single transducer modified by a passive lens. Finite-element simulations of the incident and scattered fields and a theoretical prediction of the acoustic radiation force on the droplet are also presented.

Next a study of the positive radiation pressure on a fluid interface is presented. For this experimental setup canola oil and water were chosen as the fluid combination with the source in the canola oil with the field directed downward into the water. The phenomenon of thermal breakdown of surface tension at a fluid boundary, which results in the jetting of the canola oil into the water, is then presented and discussed.
2.1 FRESNEL AND FRAXICON PHASE PLATE LENSES

Phase plate lenses provide a custom modification of acoustic fields and are important in applications such as ultrasonic imaging, therapy and for non-destructive evaluation of materials. These lenses provide an inexpensive and low profile alternative to refractive lens types. Their low profile allows for a deeper focus and less attenuation through the bulk lens material. Traditional lenses work by the refraction of waves at smooth boundaries, such as the curved surface of an ordinary focusing lens. Fresnel lenses work by providing the curvature in a piecewise fashion in a low profile quasi-planar configuration. Phase plate Fresnel lenses take this further by approximating the curvature with a series of discrete phasing steps. Previous studies have demonstrated the focal properties and efficiencies of this type of lens in both optics and acoustics applications. An axicon lens is a bulk type with a continuous profile that refracts incoming waves to a line of focus producing a finite non-diffracting Bessel beam along a depth of focus. A fraxicon lens is a low profile stepped approximation to the axicon producing a Bessel beam of similar character. A related design based on refraction rather than phasing was first proposed for use in optics. Also as previously shown by Li et al. the phase of a reflected sound wave can be modified using acoustic metamaterials to produce a finite Bessel beam.

Fresnel phase plate lenses approximate spherical lensing by modifying the phase of the incident wave using a series of steps so that maximal constructive interference occurs at the focus \( f \). Figure 2.1 demonstrates the discrete geometry required for focusing. In order for waves passing through the \( n \)th step to constructively interfere at the focus with those passing through the center of
the lens, the distance $D_n$ from that step to the focus at $f$ is

$$D_n = f + \frac{n\lambda}{M}$$  \hspace{1cm} (2.1)

where $\lambda$ is the wavelength in water. The radial distance to the $n^{th}$ step is then given by

$$R_n = \sqrt{D_n^2 - f^2}.$$  \hspace{1cm} (2.2)

The phase arriving at the focus is shifted radially outward in phase-wrapped groups by changing the thickness of the steps. Each group consists of $M$ steps that shift the phase by an integer fraction of $2\pi$ in total across the group (Fig. 2.1). For this work, the lens is designed to be used under water where the speed of sound $c$ is less than that of the lens material $c_{\text{lens}}$. The phase difference between a water only propagation path and that of $m^{th}$ segment of lens material $d_m$ within water and over the same distance as the water only path $d$ is

$$\theta_m = kd - [k_{\text{lens}}d_m + k(d - d_m)] = (k - k_{\text{lens}})d_m,$$  \hspace{1cm} (2.3)

where the wave number in water is $k = 2\pi/\lambda$ and in the lens material is $k_{\text{lens}} = 2\pi/\lambda_{\text{lens}}$. Enforcing the requirement that the phase shifts by an integer fraction of $2\pi$,

$$\theta_m = \frac{m - 1}{M}2\pi.$$  \hspace{1cm} (2.4)

Combining Eq. (2.3) and Eq. (2.4) and then adding the thickness of the supporting base $d_{\text{min}}$, the thickness of each step is then given by

$$d_m = \frac{m - 1}{M} \left( \frac{1}{\lambda} - \frac{1}{\lambda_{\text{lens}}} \right)^{-1} + d_{\text{min}}.$$  \hspace{1cm} (2.5)

The base thickness $d_{\text{min}}$ was designed as $3\lambda_{\text{lens}}/4$ in order to minimize standing waves from occurring.
The refractive axicon was initially designed for use in optics. It has an inverted cone shape that generates a finite Bessel beam. The two parameters $\alpha$ and $\beta$ are derived from Snell’s law. They determine the total thickness of a lens of radius $R$ and depth of focus (DOF) $L$ (Fig. 2.2). For an acoustic wave incident to the conical surface at an angle $\theta_i$ and refracting at an angle $\theta_r$ both with respect to normal to the cone,

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{\lambda_{\text{lens}}}{\lambda} = \frac{c_{\text{lens}}}{c}.$$  \hspace{1cm} (2.6)

Substituting $\theta_i = \alpha$ and $\theta_r = \alpha - \beta$ (Fig. 2.2 demonstrates the geometry)

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{\sin \alpha}{\sin(\alpha - \beta)} = \frac{\sin \alpha}{\sin \alpha \cos \beta - \cos \alpha \sin \beta} = \frac{1}{\cos \beta - (\sin \beta / \tan \alpha)}.$$  \hspace{1cm} (2.7)

Using this result and Eq. (2.6) the relationship between $\alpha$ and $\beta$ is then

$$\tan \alpha = \frac{\sin \beta}{\cos \beta - \frac{c}{c_{\text{lens}}}}.$$  \hspace{1cm} (2.8)

The depth of focus can then be set using $\tan \beta = R/L$ and the maximum thickness of the cone $T$ needed is determined by $\tan \alpha = T/R$. 
Bessel beam solutions to the wave equation were first shown in optics and later in acoustics.\textsuperscript{6,18} The complex acoustic pressure $p(r, z)$ of a Bessel beam propagating in the $z$ direction has the form

$$p(r, z) = P_0 \exp(i(k_z z - \omega t)) J_0(k_r r)$$

(2.9)

where $P_0$ is the peak acoustic pressure, $k_r$ and $k_z$ are the radial and $z$ components of the wave vector respectively, and $J_0(k_r r)$ is the zero-order Bessel function of the first kind. It is non-diffracting in the sense that the time averaged intensity $I_{av}$ is independent of the axial distance $z$. If $k_r$ and $k_z$ are real the average acoustic intensity in a fluid of density $\rho$ and sound speed $c$ is given by\textsuperscript{19}

$$I_{av} = \text{Re} \left( \frac{P(r, z)^* P(r, z)}{2 \rho c} \right) = \frac{P_0^2 J_0^2(k_r r)}{2 \rho c}.$$  

(2.10)

Using the design principle of the stepped Fresnel lens, the stepped fraxicon creates a line of focus using a series of steps approximating the linear grade of the axicon (Fig. 2.3). The phase is modified in segments that are integer fractions of the wavelength as in the Fresnel design. Therefore the equation for the thickness of each step is the same as before (Eq. (2.5)). The equation for the radial location of each step which determines the resulting beam profile however is different.

The incident wave is phased by the lens so that there are alternating positive and negative constructive interference points along a central line out to a DOF $L$. For this to occur the direct
distance $D_n$ from the $n^{th}$ step to a point that is an axial distance $l_n$ away from the lens which keeps the axicon angle $\alpha$ constant is

$$D_n = l_n + \frac{n\lambda}{2M}. \quad (2.11)$$

Unlike in the Fresnel design, the set of $D_n$ must be half integer fractions of a wavelength from $l_n$ so that the peak pressure alternates between positive and negative along the DOF. The line from the radial point at $R_n$ to the axial point at $l_n$ forms a like-triangle with the line from the outer radius $R$ to the DOF at $L$ (Fig. 2.3). So in terms of the radial point $R_n$ the axial length corresponding to the $n^{th}$ step is

$$l_n = \frac{D}{R} R_n \quad (2.12)$$

and by the Pythagorean Theorem,

$$R_n^2 + l_n^2 = \left(l_n + \frac{n\lambda}{2M}\right)^2. \quad (2.13)$$

Plugging Eq. (2.12) into Eq. (2.13) and solving for $R_n$, the radius of the $n^{th}$ phasing step is then given by

$$R_n = \frac{n\lambda}{2MR} \left( L + \sqrt{L^2 + R^2}\right). \quad (2.14)$$

![Figure 2.3: Radial profile of the fraxicon phase plate lens.](image)
2.2 ACOUSTIC RADIATION PRESSURE AT A FLUID TO FLUID BOUNDARY

It has been previously demonstrated that an acoustic beam focused at the interface of two fluids will deform that interface either in the direction of the incident radiation or in the opposite direction depending on the relative sound speeds and densities of the two fluids.\textsuperscript{1,20,2} This phenomenon is completely non-linear. The deformation increases with acoustic power up to a point where step-like features due to the waveguide properties of the deformed interface occurs.\textsuperscript{2,21} A further increase in power yields jetting of one fluid into the other.

The for small-amplitude sources this deformation has been demonstrated to agree well with the Langevin description of radiation pressure.\textsuperscript{22,2} The Langevin radiation pressure is given by the time averaged acoustic energy density $E$ in the medium. For a beam that is normally incident to an interface between two fluids, the energy density on either side of the interface can be found by considering the intensity $I = cE$ (where $c$ is the speed of sound) in each medium. The fluid containing the source is designated fluid 1 and the remote one is fluid 2. Assuming normal incidence, the net radiation pressure acting on the interface is then the difference in Langevin radiation pressure on either side of the boundary\textsuperscript{1} given by

$$P_{\text{net}} = \frac{I_I}{c_1} + r_I \frac{I_I}{c_1} - t_I \frac{I_I}{c_2}$$

(2.15)

where $I_I$ is the incident intensity, $r_I$ is the intensity reflection coefficient, $t_I = 1 - r_I$ is the intensity transmission coefficient, and $c_1$ and $c_2$ are the sound speeds of fluid 1 and fluid 2 respectively. Rewriting Eq. (2.15) in terms of the incident energy density $E_I$ and the reflection coefficient $r_{1,2}$ (using $r_I = r_{1,2}^2$) gives

$$P_{\text{net}} = E_I \left[ 1 - \frac{c_1}{c_2} + r_{1,2}^2 \left( 1 + \frac{c_1}{c_2} \right) \right]$$

(2.16)

and the reflection coefficient $r_{1,2}$ is given by

$$r_{1,2} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad Z_i = \rho_i c_i.$$

(2.17)
$Z_i$ is the characteristic acoustic impedance in the $i^{th}$ medium, $\rho_i$ is the density, and $c_i$ is the sound speed.\textsuperscript{23}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure2.4}
\caption{Density and sound speed dependence on the directionality of the acoustic radiation force at a boundary.}
\end{figure}

The solid black line in Fig. 2.4 represents when $P_{\text{net}} = 0$ in Eq. (2.16). The boundary between fluid combinations to the left of this line will experience a positive radiation force in the direction of the sound propagation and those to the right of it will experience a negative radiation force in the opposite direction. The dashed green line in this figure represents where the fluid densities are equal. Above this fluid 1 will sink and will be below fluid 2. Conversely for fluid combinations below this line fluid 1 will be above fluid 2 which is preferred for this study since the experimental setup is for top – down propagation. The dot-dash orange curve represents where the two fluids have equal acoustic impedances. Fluid combinations occurring on this line will have no reflections at the boundary. This is preferred to isolate heating effects due to reflections and absorption at the viscous boundary layer\textsuperscript{19} from the acoustic phenomena. The two fluid 1 to fluid 2 combinations used in this study are plotted here as well.
Previous studies have verified the Langevin theory using a spherically focused source at low intensities.\textsuperscript{2} So that gravitational effects can be taken into account, a more general and direction dependent form of Eq.(2.16) is used

\begin{equation}
P_{\text{net}} = A P_i^2(r) \quad A = \frac{\varepsilon Z_i^2 + Z_j^2 - 2Z_i\rho_j c_i}{Z_i c_i (Z_i + Z_j)^2},
\end{equation}

where $\varepsilon = 1$, $i = 1$, and $j = 2$ for upward propagation of the incident beam and $\varepsilon = -1$, $i = 2$, and $j = 1$ for downward propagation. Index 1 indicates the fluid on the bottom and index 2 indicates the upper fluid. $P_i(r)$ is the radial profile of the incident pressure amplitude in cylindrical coordinates at the interface of the two fluids.

The diffraction pattern for a spherically focused acoustic field in the focal plane is given by\textsuperscript{2,24}

\begin{equation}
P_i(r) = P_0 \left| J_1(\pi r/\lambda) \right| \left| \pi r/2\lambda \right|
\end{equation}

where $P_0$ is the peak pressure amplitude. For a Bessel beam in a plane that is located along the depth of focus perpendicular to the propagation axis the diffraction pattern is given by\textsuperscript{5}

\begin{equation}
P_i(r) = P_0 |J_0(\pi r/\lambda)|
\end{equation}

In each $J_i$ is the $i^{th}$ order Bessel function of the first kind.

The height profile of the fluid interface $h(r)$ in the steady state is found by equating the pressure due to the weight of the displaced fluid and the Laplace pressure to the radiation pressure

\begin{equation}
P_{\text{net}} = (\rho_1 - \rho_2)g h(r) - \sigma \nabla \cdot \mathbf{n},
\end{equation}

where $g$ is the acceleration due to gravity, $\sigma$ is the surface tension of the interface, and $\nabla \cdot \mathbf{n}$ is the curvature of the interface ($\mathbf{n}$ is the surface normal upward from the interface). Here $P_{\text{net}}$ is defined negative if it causes an upward pull and positive if it pushes downward. Also it should be noted
for the densities in Eq. (2.21) index 2 indicates the upper fluid and index 1 the lower fluid. For small displacements \( n \) is approximately \( \nabla_r h(r) \) and Eq.(2.21) can be solved by taking the Hankel transform into wave number \( k \) space of both sides and using the identity \( \mathbf{H}[\nabla^2_r h(r)] = -k^2 \mathbf{H}[h(r)] \) where \( \mathbf{H}[h(r)] \) is the Hankel transform of \( h(r) \). Let \( \tilde{\pi}(k) \) be the Hankel transform of \( P_{20}^0(r) \), then the height profile as a function of the radial coordinate \( r \) is given by the inverse transform

\[
h(r) = A \int_0^{2\pi/\lambda} \frac{\tilde{\pi}(k)}{(\rho_1 - \rho_2)g + \sigma k^2} J_0(kr) k \, dk. \tag{2.22}
\]

Eq. (2.22) is shown to agree well with observation by Issenmann et al.\(^2\) for lower incident pressures. With higher incident power levels this model ceases to be valid.

It has been previously reported\(^21\) that at higher amplitudes step-like features in the deformation become apparent. This is due to acoustic waveguide effects, and each step can be represented as a cylindrical mode of the overall field. As the incident amplitude increases, more step-like modes appear until the surface tension breaks and jetting of one fluid into the other occurs. At non-impedance matched boundaries there can be significant heating due to reflections.\(^19\) As the boundary is heated, the surface tension decreases which can cause jetting to occur at lower power levels than what would be required for impedance matched boundaries. At even higher amplitudes the jetting liquid can atomize creating a mist and an acoustic fountain. This effect at a liquid to air boundary has been thoroughly investigated (for example by Fox and Griffing\(^25\), Lang\(^26\), Simon et al.\(^27\) and others) and is used in commercial humidifiers and other devices.

### 2.3 Acoustic Radiation Force on a Fluid Droplet

In part of this work the negative radiation pressure as described in the previous section is used to pull a fluid interface into the near field of a transducer using a fraxicon lens. Further it is shown that the near field of this type of transducer and lens has the ability to support and capture droplets of this fluid.
The theoretical foundation for the acoustic radiation force on a spherically symmetric object was first calculated by King\textsuperscript{28} in 1934. Yosioka and Kawasima\textsuperscript{29} in 1955 devised a model for a bubble suspended in a stationary acoustic field. Gor’kov\textsuperscript{30} later demonstrated in 1962 a formulation using the second-order time-averaged kinetic and potential energies for small compressible particles in a plane stationary field. Then in 1971 Crum\textsuperscript{9} derived the acoustic radiation force on a fluid sphere.

Another adaptation first described by Westervelt\textsuperscript{31} uses the acoustic radiation stress tensor $\Sigma_{i,j}$ for a sound scattering object in an ideal fluid.\textsuperscript{31,32,33,34} The acoustic radiation stress tensor components represent the forces exerted per unit area in the $i^{th}$ direction ($i = 1, 2, 3$) on the surface of an object which is outward normal in the $j^{th}$ direction ($j = 1, 2, 3$).\textsuperscript{19} Its divergence can be integrated over the volume surrounding a sound scattering object to calculate the net acoustic radiation force acting on that object. By the divergence theorem this can be represented as a surface integral over a fixed surface $S$ enclosing that volume. The acoustic radiation force on a fluid droplet can then be written as\textsuperscript{33,34}

$$ F = \int_S \left( \frac{\rho}{2} \langle |u|^2 \rangle - \frac{1}{2\rho c^2} \langle p^2 \rangle \right) dA - \int_S \rho \langle uu \rangle \cdot dA \tag{2.23} $$

where $u$ and $p$ are the time-varying acoustic fluid velocity and pressure respectively, $\rho$ and $c$ are the respective density and sound speed of the medium surrounding the droplet, and $\langle \rangle$ indicates the time average. The incident and scattered pressure and velocity fields can be calculated by numerical simulation and then Eq. (2.23) can be used to determine the net force a droplet would experience in that field. A detailed calculation of this force is given in Chapter 7.
3.1 LENS MATERIALS AND CONSTRUCTION

In this work, the properties of three lenses were investigated: the axicon, its stepped approximation, the fraxicon, and a stepped Fresnel lens. In the first part of the study plastic materials were used for the lenses and data were collected using a pulsed source (outlined in Section 3.2) to examine the dynamics of wave packets generated using the lenses. In the next part aluminum lenses were constructed and characterized using tone bursts to approximate the continuous response at higher power levels (section 3.3).

The transducer used with each lens was a 50 mm ceramic piezoelectric disk mounted inside a polyvinyl chloride (PVC) enclosure. Each of the low profile lenses were designed with 4 phasing steps in each phase group ($M = 4$ as described in Chapter 2) to maximize the efficiency of its focusing power. The plastic lenses were designed to be mounted using a threaded PVC ring which was fastened to the lens using silicone sealant (Fig. 3.1). This allowed them to be acoustically coupled to the transducer using water. The aluminum lenses however were designed to be directly coupled to the piezoelectric disk using epoxy (Fig. 3.2). This allowed for the use of much higher power sources and avoided any problems with the lenses melting or cavitation build up in the coupling medium.

Table 3.1 lists the relevant ultrasonic properties for various lens materials considered for this study. The in-house measured values in this table were taken using a pulse-echo technique. A contact transducer driven by an Olympus 5077PR pulser-receiver was attached to a sample of each material. The output of the pulser-receiver was monitored on an oscilloscope where echo delay time and attenuation was measured. From this the sound speed $c_m$ and attenuation coefficient
Figure 3.1: Transducer used in the pulse source experiments: (a) with and (b) without a lens mounted.

Figure 3.2: Interior of the transducer used in burst source experiments. This image was taken before the transducer was sealed with marine epoxy. It shows the rear side of the crystal and lens assembly.

\( a \) were obtained for each.* The densities of each was obtained using volume displacement and buoyancy methods.

For the pulsed source measurements, a 10 cm depth of focus (DOF) axicon lens was first constructed from polylactic acid (PLA) using a MakerGear M3 3D printer (Fig. 3.3 (a)). A 7 cm focal length stepped Fresnel (Fig. 3.3 (b)) and a 7 cm DOF fraxicon lens (Fig. 3.3 (c)) were also made using clear polystyrene (CPS) and machined to shape. PLA has a desirable sound speed for the lens material but is highly absorptive. CPS is a more desirable material due to its lower

*The attenuation coefficient of aluminum was not measurable using the pule-echo technique for a reasonably sized sample and is assumed to be much lower than that of the other materials listed.
Table 3.1: Measured ultrasonic properties for various lens materials considered for this study. PLA and polystyrene values were measured in-house and others* were previously published.35

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho ) (g/cm(^3))</th>
<th>( c_m ) (mm/( \mu )s)</th>
<th>( a ) (Np/cm @ 5 MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delrin*</td>
<td>1.42</td>
<td>2.43</td>
<td>3.49</td>
</tr>
<tr>
<td>Clear Polycarbonate*</td>
<td>1.18</td>
<td>2.27</td>
<td>2.87</td>
</tr>
<tr>
<td>PLA</td>
<td>1.24</td>
<td>2.10</td>
<td>7.12</td>
</tr>
<tr>
<td>White Polystyrene</td>
<td>1.09</td>
<td>2.05</td>
<td>1.92</td>
</tr>
<tr>
<td>Clear Polystyrene</td>
<td>1.18</td>
<td>2.68</td>
<td>1.17</td>
</tr>
<tr>
<td>Aluminum*</td>
<td>2.70</td>
<td>6.42</td>
<td>–</td>
</tr>
</tbody>
</table>

sound absorption. The attenuation in CPS is approximately 16% of that in PLA, 33% of that in Delrin, and 40% of that in polycarbonate.35 Attenuation is a critical property as it limits the level of acoustic power that can be used before thermal damage to the lens.

Figure 3.3: Lenses used for the pulse source measurements: (a) PLA 10 cm DOF axicon, (b) CPS 4-step 7 cm focus Fresnel, and (c) CPS 4-step 7 cm DOF fraxicon.

For the burst source measurements, a 5 cm focal length stepped Fresnel and a 10 cm DOF fraxicon lens were machined out of aluminum (Fig. 3.4). Each one was designed with a small lip around its bottom edge to center the crystal when it was epoxied to the lens (see Fig. 3.2). The crystal used with each lens has a wrap around contact so that they can be flush mounted. They were epoxied directly to the rear surface of the lenses using 3M DP270 epoxy. A PVC cap containing the electrical connector and a mounting screw was fastened to the back of each lens using marine epoxy which provided them with a water-proof seal.
3.2 PULSE SOURCE MEASUREMENTS

For the three plastic lenses described in Section 3.1, broadband pressure fields were measured in both the axial and transverse planes in an 8 gal tank of distilled water using a hydrophone controlled by a 3-axis positioning system (Fig. 3.5). The hydrophone is a Müller needle type with a diameter of 0.5 mm. The point-by-point scans were made using a 3-axis Velmex motorized positioning system. The system has an accuracy of 0.076 mm over 25 cm length of travel in all three directions. The axial scans were 7 cm in depth and 3 cm wide with a step size of 0.25 mm in each direction for a total of 280x120 steps. For each step, the transducer was excited by a broadband stimulus from an Olympus 5077PR. The signals received by the hydrophone were routed through a Perkin-Elmer 5185 preamp to a Tektronix TBS2102 100 MHz digital oscilloscope. The data were downloaded to a computer for storage and further processing.

The scanning and acquisition were automated using custom MATLAB code. The traverse scans were 2 cm by 2 cm with a step size of 0.25 mm for a total of 80x80 points. For each lens, the depth of the transverse scan plane was chosen by identifying the point in the axial scan with the maximum peak to peak amplitude (i.e., the broadband focal depth). To determine the frequency resolved fields, the fast Fourier transform (FFT) of each captured waveform was taken and the 1.2 MHz component was extracted. The spectrally resolved data is presented in Chapter 5 for comparison to angular spectrum simulations.
**Figure 3.5:** Hydrophone scan tank setup using pulse and burst sources. Pulse source measurements were done in an 8 gal water tank and burst measurements were conducted in a 45 gal tank.

The axicon lens produces an X-beam\textsuperscript{36,37,38} which refers to the shape of crossing wavefronts in the beam in a plane through the central axis. Broadly speaking, the conical shape of the axicon can be considered to be a source of angled wavefronts that converge on and cross the central axis. At the intersection of these wavefronts is the axially propagating wave packet. The long DOF of the lens and supersonic speed of the pulse is a consequence. The low profile fraxicon lens mimics the phase shifts of the axicon lens which is what endows the fraxicon with its DOF properties. However, due to the planar nature of the lens, the fraxicon does not produce an X-beam and thus the pulses do not propagate faster than the nominal sound speed in the water.

The signals in this study had a -6 dB bandwidth of about 20\%, which allows for time resolved observation and analysis of the effects of the lenses. A general comparison of the fraxicon, axicon and Fresnel generated pulses is provided in Fig. 3.6. The top panels of the figure show the pressure fields at three instances of time. The central frame is in the region of maximal amplitude. The bottom panels show the energy profiles of the pulses. These were calculated as the square of the analytic signal envelopes of the signals, and are representative of the cycle-averaged RMS acoustic
energy density. The planar wavefronts in the central packet of the fraxicon and axicon fields are maintained with depth while the converging and diverging character of the focused Fresnel is apparent. The relative shape integrity of the axicon and fraxicon is also clearly apparent in Fig. 3.6.

The axicon pulse was remarkably stable, exhibiting very little dispersion over its depth of focus. The peak and signal velocities were nearly identical with a value near 1.53 mm/µs. This exceeds the nominal speed of sound in water (at 21.7 °C) of 1.49 mm/µs by 0.04 mm/µs. The temperature of the water would need to be raised by 18 °C to reach this nominal sound speed. In contrast, the fraxicon pulse exhibited significant dispersion over the range of propagation. The speed of the peak and the 5% signal edge averaged approximately 1.44 mm/µs over the range. At the same time the 1% signal velocity approached the nominal speed of sound in water at the acquisition temperature 22.5 °C. At closer inspection, the pulse was found to have behaviors tied to distinct zones along the path. In the stable zones, the envelope shape evolved slowly and the peak and 5% signal speeds were at or significantly above the nominal speed of sound in water. In between these zones there was a more rapid change in the shape with depth where the two speeds fell below nominal. These stable zones have a rough correspondence to the high amplitude regions shown in the frequency resolved beam patterns shown in Fig. 5.1. The spatial -6 dB widths of the axicon and fraxicon pulses vary less than 10% over the entire depth. The extended DOF’s of both are also apparent. The axicon pulse amplitude stays within -3 dB of its overall maximum over the entire 50 mm range, while the fraxicon stays within -3 dB for over 91% of this range. It should be noted that the axicon used in the pulsed measurements was designed to have a greater DOF (10 cm) than the fraxicon (7 cm). The Fresnel pulse varies greatly in amplitude and shape, which is expected since it is designed to have a point-like focus, yet it also never achieves the lateral or axial compactness of the other two fields. In contrast to the others, the Fresnel pulse only stays above -3 dB of the maximum amplitude for 44% of the range. Although the axicon packet exhibits nearly ideal behavior relative to the fraxicon, the fraxicon pulse does still exhibit a strong degree of shape integrity and maintains planar phase fronts over the observed depths. Thus the fraxicon can provide for many of the advantages of the axicon in pulse dynamics including later compactness,
planarity and morphologic stability. Since there are a wide variety of design modifications and parameter choices that can influence its broadband performance, further study of the fraxicon is warranted to better gauge its utility.

Figure 3.6: (Top) Acoustic pressure fields for a single wave packet for each lens type at three different instances in time. The lenses are aligned in the display region so that the focal zone each corresponds to 0 mm. (Bottom) The energy profiles for the same pulses as shown on the top row. The pulse amplitudes are self normalized. The propagation direction is upward.
3.3 SINUSOIDAL BURST MODE MEASUREMENTS

For the burst mode measurements the transducers with the epoxied Al lenses were driven by an Agilent 33250A arbitrary waveform generator through an E&I 2100L RF amplifier, with the scan performed inside a 45 gal tank of distilled water (Fig. 3.5). The larger tank was used to minimize standing waves. The source signal was a 1.2 MHz sinusoidal burst of 10 cycles for near measurements and 20 cycles for far measurements each at 1% duty cycle for further reduction of standing waves or reflection interference in the tank. The output power of the amplifier was monitored using an Agilent E4419B power meter with a Werlatone C5959 40 dB attenuated coupler. The positioning system used with the larger tank consists of a Centroid motion controller and Velmex tracks. MATLAB control and acquisition code similar to that used in the pulse source measurements of Section 3.2 with an Agilent 54622A 100 MHz digital oscilloscope was used to collect data.

Due to the wrap around contact on the crystal, the field generated is not entirely cylindrically symmetric. A 30 mm by 30 mm traverse plane scan (0.5 mm step resolution) was taken using a crystal without a lens attached to quantify the amount of asymmetry. Figure 3.7 shows the result of that scan. The results of the Al lens transducer scans are presented in Chapter 5 where they are compared to simulations. Despite the asymmetry, each transducer demonstrated the focal features they were designed for, as discussed in Chapter 5. The step size in each direction was 0.25 mm in the far field scans and 0.1 mm in the near field scans. The source amplitude was set to produce 10 W of electrical power at the output of the amplifier compensated for 1% duty.

To determine the focusing efficiency as described in Chapter 4, one dimensional scans were taken in the focal plane of the Fresnel lens and in the plane coinciding with half the depth of focus of the fraxicon (5 cm in each case) over 10 cm with a 0.1 mm resolution. The results of these are presented in Chapter 4. A near field scan of the fraxicon transducer was taken also with a resolution of 0.1 mm to determine the field features responsible for the droplet trapping phenomenon described in Chapters 6 and 7. The data from this scan is shown in Fig. 3.8.
**Figure 3.7:** Traverse plane scan data taken at an axial depth of 1 cm from a lensless piezoelectric crystal transducer. The color axis represents the peak-to-peak voltage at the received signal.

**Figure 3.8:** Near field axial plane of the fraxicon transducer. The color scale represents the amplitude of the 1.2 MHz spectral component of the waveform data normalized to the maximum value. The depth is measured from the front of the lens.
CHAPTER 4

FOCAL POWER FRACTION AND EFFICIENCY

4.1 FOCAL POWER FRACTION: BACKGROUND AND MOTIVATION

The focal power fraction $G$ is defined as the fractional amount of the total acoustic power that is delivered to an area in the focal plane of a transducer which is bounded by the -6 dB limits of the spatial peak intensity.\(^{40}\) By knowing $G$ for a particular transducer together with the total acoustic power $W$ output, the power delivered to the focus ($W_F = GW$) can be calculated without direct measurement, assuming $G$ does not depend on $W$. This is useful for determining the power delivered to the focus of each transducer used in the experiments described in Chapter 6 where access to the focal zone with a calibrated hydrophone is impractical.

From diffraction theory the focal power fraction of a shallow cylindrically symmetric radiator can be approximated by\(^{41}\)

$$G = \frac{1}{2} \int_0^{z_e} G_0^2(z) z \, dz; \quad z = kR \sin \theta$$

(4.1)

where $G_0$ is the diffraction pattern at the focal plane of the radiator, $\theta$ is measured from the center of the circular radiator to the central axis, $k$ is the wavenumber, $R$ is the outer radius of the spherical radiator, and $z_e$ is the dimensionless value for $z$ at the location of -6 dB of the maximum intensity. For a shallow spherical radiator, the diffraction pattern $G_0$ is approximately equal to that of a circular piston\(^{19}\)

$$G_0(z) = \frac{2J_1(z)}{z}.$$  \hspace{1cm} (4.2)

where $J_n$ is the $n^{th}$ order Bessel function of the first kind. Solving Eq. 4.2 numerically for $z_e$ when
\( G_0(z_e) = 0.5 \) (the -6 dB intensity location) gives \( z_e = 2.215 \). Next by plugging Eq. 4.2 into Eq. 4.1

\[
G = 2 \int_0^{z_e} \frac{J_2^2(z)}{z} \, dz; \quad z = kR \sin \theta
\] (4.3)

the focal power fraction for a shallow spherical radiator is \( G = 0.68 \).

The diffraction pattern in a plane perpendicular to the depth of focus of a shallow axicon is given by\(^{17}\)

\[
G_0(z) = J_0(z).
\] (4.4)

Using the same methods as with the spherical lens but using Eq. 4.4 instead of Eq. 4.2, \( z_e = 1.52 \) and \( G = 0.33 \) for a shallow axicon.

The focal power fraction for a stepped phase plate lens can be approximated by also considering the diffraction efficiency of this type of lens. The diffraction efficiency gives the amount of the source radiation which is diffracted and available at the focus.\(^{42,4,11}\) J. Swanson\(^{42}\) derived a theoretical model for the diffraction efficiency of a stepped phase plate by using the Fraunhofer approximation for the field modified by multilevel diffractive elements,

\[
P(x, y, z) = \frac{e^{ik_z z}}{i\lambda z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x_0, y_0, 0) \exp \left\{ \frac{ik_z}{2z} \left[ (x - x_0)^2 + (y - y_0)^2 \right] \right\} \, dx \, dy
\] (4.5)

where \( P(x_0, y_0, 0) \) is the initial source field which is then propagated in the \( z \) direction and \( k_z \) is the \( z \)-component of the wave vector. By spatially Fourier transforming \( P(x, y, z) \), the amplitudes of each diffractive order can be determined. In each dimension (\( x \) and \( y \)) and for each phasing step the source \( P(x_0, y_0, 0) \) is a rect function of a periodic width \( w_m \) centered about \( x_0 = (m - 1/2)w_m \) where \( m \) is an integer corresponding to a phasing step within a group up to \( m = M \).
Assuming a maximum phase delay of \( \phi \) and using the sinc property of the Fourier transform of a rect function, the normalized spectral component amplitude contributed by the \( m \)th phasing step is given by

\[
\hat{P}_m(k) = \frac{\sin(\pi w_m k/M)}{\pi w_m k/M} \exp\{-i2\pi(k x - (m - 1)\phi/M)\}.
\] (4.6)

The normalized total field then contributed by a full phase group of \( M \) steps is given by the summation

\[
\hat{P}(k) = \frac{1}{M} \sum_{m=1}^{M} \frac{\sin(\pi w_m k/M)}{\pi w_m k/M} \exp\{-i2\pi(k x_0 - (m - 1)\phi/M)\}.
\] (4.7)

For a large number of phasing groups, the \( l \)th diffractive order amplitude is non-zero when \( k = l/w_m \) and is given by

\[
A_l = \frac{1}{M} \sin(\pi l/M) \exp\{i\pi l/N\} \sum_{m=1}^{M} \exp\{-i2\pi(l - \phi)(m - 1)/M\}\}
\] (4.8)

The diffraction efficiency of the \( l \)th order is then

\[
\eta_l = A_l A_l^* = \left[ \frac{1}{M} \frac{\sin(\pi l/M)}{\pi l/M} \right] \left| \sum_{m=1}^{M} \exp\{-i2\pi(l - \phi)(m - 1)/M\} \right|^2
\] (4.9)

which evaluates to

\[
\eta_l = \left[ \frac{\sin(\pi(l - \phi))}{\pi l/M} \frac{\sin(\pi l/M)}{\sin(\pi(l - \phi))} \right]^2.
\] (4.10)

The maximum of the central \( l = 1 \) order occurs when \( \phi = 1 \). The diffraction efficiency \( \eta \) of an \( M \) level lens can then be approximated by

\[
\eta = \left[ \frac{\sin(\pi/M)}{\pi/M} \right]^2.
\] (4.11)

\[\text{It should be noted that Eq. 16 in J. Swanson}^{42} \text{ contains a typographical error which is corrected here.}\]
With $M = 4$ as with the phase plate lenses described in Chapter 3 the diffraction efficiency is $\eta = 0.81$. Assuming that a Fresnel phase plate lens is equivalent to a spherical lens with a focal power fraction $G = 0.68$ but is limited by a diffraction efficiency of 81%, the focal power fraction for the Fresnel lens is then $\eta G = 55\%$. Finally assuming that a fraxicon phase plate lens is equivalent to an axicon ($G = 0.33$) limited by this same diffraction efficiency, its focal power fraction is $\eta G = 26\%$.

4.2 DIRECT HYDROPHONE MEASUREMENT OF THE FOCAL POWER FRACTION

A method similar to what was outlined by Civale et al.\textsuperscript{40} was used to measure the focal power fraction directly. Pressure amplitudes were measured in two lines (horizontal and vertical) perpendicular to the propagation axis and at the location of the spatial peak intensity for each of the two aluminum phase plate transducers which were discussed in Chapter 3. Assuming cylindrical symmetry the horizontal and vertical scans were averaged together and the resulting one dimensional scan data was integrated over a half circle to determine the total power contained in the entire scan length. The locations of the -6 dB points on both sides of the focus were determined, and then the data set was integrated over a half circle again within this range to determine the power contained in the focus. The ratio of the power contained within the -6 dB margins of the focus to the total power emitted by the transducer.

The hydrophone scans were done using the 40 gal tank experimental setup described in Chapter 3. The source used for each scan was a 40 cycle burst of a 1.2 MHz sinusoid operating at 1 % duty cycle at a compensated net input power of 10.7 W with the Fresnel lens transducer and 9.42 W with the fraxicon lens transducer. The electrical power was monitored with a power meter. The location of the spatial peak intensity was found by first carefully aligning the hydrophone so that it was directly in the center of the transducer and scanning along the propagation axis in 0.1 mm steps to find the maximum pressure. For the Fresnel lens this was at 50.0 mm from the lens and for the fraxicon it was at a depth of 51.7 mm. The transverse scans were done over a range of 12 cm centered at the focus in steps of 0.1 mm in both the horizontal and vertical directions. Using this
range the amplitudes of the signals that were obtained at each extreme were around 40 dB less than those obtained at the center.

A propagation delay compensated gate of 5 cycles was applied to each of the resulting waveforms. The start of the gate was chosen near the center of the waveform captured at the center of the focus for each transducer. The FFT was taken for each waveform and the 1.2 MHz spectral amplitude was selected to isolate any noise and harmonic generation. Assuming circular symmetry, the horizontal and vertical data sets were then averaged together for each transducer. Next this data was sinc interpolated with a resample factor of 100 (Fig. 4.1).

The net power $W$ was determined by integration of square of the resulting normalized spectral amplitude $A(r)$ over a half-circle enclosing the entire range of the scan,

$$W = \frac{\pi}{\rho c} \int A(r)^2 r \, dr \quad (4.12)$$

where $\rho$ is the density and $c$ is the sound speed of water. Next the -6 dB positions on each side of the focus were located and Eq. 4.12 was used again over this range (the focal width) to determine the power contained within the focus $W_F$. The focal power fraction was then determined using $G = W_F/W$ for each transducer. The results of this gave a focal width of 2.12 mm and focal power fraction of 32.8% for the Fresnel transducer and a focal width of 2.07 mm and focal power fraction of 10.9% for the fraxicon.

---

$^\dagger$The normalized FFT amplitude was used in each calculation of $W$ and $W_F$ and not the pressure amplitude so the resulting units for power are not scaled to watts. However since the focal power fraction is a ratio of the two, this scaling factor is divided out.
**Figure 4.1:** 1.2 MHz spectral amplitudes from 1D scan data centered at the location of the spatial peak intensity for the (a) 5 cm focal point Al Fresnel transducer and (b) 10 cm DOF Al fraxicon transducer. In each the black dots represent the data and the blue curve is the sinc interpolation.
The results for the measured values of $G$ were about 40% lower than what is predicted by theory for the Fresnel and 58% than what was predicted by the fraxicon. The discrepancy could be due to manufacturing variances in the lenses or possibly resonance effects in the measuring tank.

The assumption of cylindrical symmetry was also checked by calculating the net power from a two dimensional scan of the Fresnel transducer and comparing that to the net power obtained by a one dimensional scan over the same range. The results of this demonstrated a slight over estimation from the one dimensional scan results of less than 10%. The theoretical values are again checked in Chapter 5 using an angular spectrum simulation which does show good agreement.

4.3 ELECTRIC CONVERSION EFFICIENCY

Using an acoustic force balance apparatus with a conical target similar to that which was described by Kossoff\textsuperscript{43} the net average acoustic output power was determined over a range of electric input powers for each of the two aluminum lens transducers. A lensless transducer with the same crystal as the two aluminum lens transducers, was also measured for comparison. The electric conversion efficiency, which is the efficiency of each transducer in converting electrical input power into acoustic energy (acoustic output divided by electrical input power), was then determined for each of these transducers. Using this value along with the focal power fraction and focal width measured in Section 4.2, the acoustic power contained in the focus and the incident energy density at the focus can be determined for each without measuring these values directly with a hydrophone. This method is important for the experiments outlined in Chapter 6 where the focal region is inaccessible to hydrophone measurements.

A relation governing the pressure exerted on a surface that is exposed to acoustic radiation was first derived by Lord Rayleigh in the early 1900’s.\textsuperscript{44,45} In 1952, F. E. Borgnis\textsuperscript{46} derived the result for the average acoustic radiation force $F_{av}$ on a reflecting cone of angle $\alpha$ due to an acoustic beam of cross sectional area $S$ and average incident energy density $E_I$,

\[
F_{av} = SE_I \left[1 - r_I \cos^2 \alpha \right] \tag{4.13}
\]
where \( r_I \) is the intensity reflection coefficient. The energy density is related to the acoustic intensity in the propagation direction by \( I = c E \) where \( c \) is the speed of sound. The net acoustic power radiated by a source is defined as \(^{19}\)

\[
W = \int_{S'} I dS
\]

(4.14)

where the surface element \( dS \) is perpendicular to the propagation direction and \( S' \) is a surface which encloses the source. Assuming a beam of constant surface area \( S \) and average intensity \( I_{av} \) where all of the source power is directed in one direction, the average power delivered by the beam is

\[
W_{av} = I_{av} S = c E I S.
\]

(4.15)

Substituting this into Eq. 4.13 gives the average acoustic power in terms of the average radiation force

\[
W_{av} = F_{av} \left( 1 - r_I \cos^2 \alpha \right)^{-1}.
\]

(4.16)

For a cone of angle \( \alpha = 90^\circ \) this reduces to

\[
W_{av} = c F_{av}.
\]

(4.17)

The force balance used in this study was an Ohmic UPM-DT100N. It has an 82 mm diameter aluminum reflecting cone with a conical angle \( \alpha = 90^\circ \) that is submerged under water inside a cylindrical 1.035 L tank which has 13 mm thick rubber absorbing walls. The reflecting cone is mechanically coupled to an Ohaus 4120 digital balance located under the supporting base of the tank (see Fig. 4.2). Distilled water at a temperature of 21° C was used for all measurements and thus a nominal sound speed \(^{23}\) of \( c = 1.49 \text{ mm/\mu s} \) was assumed. Using Eq. 4.17 and by converting weight measurements to mass with \( f = mg \ (g = 9.81 \text{ m/s}^2) \) a scaling factor of 14.65 W/g was calculated to convert mass readings in grams from the balance to acoustic power in watts. The source signal from a TC Power Conversion AG1006 RF generator was a 1.2 MHz continuous sinusoid. The source power was ramped up in steps of approximately 1 W starting with the 1 W setting up to
the 25 W setting on the AG1006. The average electric power delivered to the transducer was more accurately determined with an Agilent E4419B power meter coupled to the input signal with a Werlatone C5959 -40 dB coupler (the same as was used in Chapter 3). At each input power step the electric power transmitted to the transducer and the acoustic power measured on the force balance was recorded. This process was repeated 3 times each for the 5 cm focus Al Fresnel transducer, 10 cm DOF Al fraxicon transducer, and a lensless transducer.

The results of these measurements are shown in Fig. 4.3 as plots of the output acoustic power verses the input electrical power in watts. The error-bars in each of these figures represents the measurement uncertainty of the force balance specified by its manufacturer to be ±3.0% of the measurement plus 0.2 W. A linear fit was done with each data set, the slope of which is interpreted as the electric conversion efficiency. The Fresnel, fraxicon and lensless transducers had electric conversion efficiencies of 52%, 48%, and 78% respectively.

Using these values and the focal power fraction values from Section 4.2 the power delivered $W_F$ and the incident energy density in the focal region $E_I$ for each transducer can be determined by measuring the electric power delivered to it. For example: if 20.0 W of electric power is delivered to the 10 cm DOF Al fraxicon transducer which has an FPF of 10.9% then the net power delivered
to the 2.07 mm width focal zone is $20.0 \times 48\% \times 10.9\% = 1.05$ W. Assuming circular symmetry of the focus, the incident energy density at the focus can be calculated with Eq. 4.15 rearranged

$$E_I = \frac{W_F}{c \ S_F}$$

(4.18)

where $S_F = \pi r_F^2$ and $r_F$ is the focal zone half width. With the provided example: $W_F = 1.05$ W, $S_F = 3.36 \times 10^{-6}$ m$^2$, and $c = 1490$ m/s. The incident energy density is then 209 J/m$^3$. 
Figure 4.3: Acoustic power output verses electrical power input plots from force balance measurements for the (a) 5 cm Al Fresnel, (b) 10 cm Al DOF fraxicon, and (c) the lensless transducers.
5.1 OVERVIEW OF THE ANGULAR SPECTRUM METHOD

The acoustic fields generated by each of the lenses in this work were simulated using the angular spectrum method (ASM). It is a Fourier technique which transforms a phase screen into its plane wave spectrum that can be propagated outward in \( k \) space to reconstruct the field remote from the source.\(^7\)\(^8\)

The lenses were modeled by creating a phase aperture at the starting depth of the simulation. Assuming a plane wave is incident on the lens traveling in the \( z \) direction along the axis of symmetry of the lens, the resulting field after the sound has propagated through the lens is phase modified by \( e^{i\theta} \) with the shift given by

\[
\theta = 2\pi \left( \frac{d_{\text{lens}}}{\lambda_{\text{lens}}} + \frac{d_{\text{w}}}{\lambda} \right)
\]

where \( \lambda_{\text{lens}} \) is the wavelength within the lens material, \( \lambda \) is the wavelength in the surrounding water, \( d_{\text{lens}} \) is the propagation distance through the lens material, and \( d_{\text{w}} \) is the remaining path through the water. The thickness of the lens at each point \( d_{\text{lens}} \) was calculated using Eq. (2.5) for the fraxicon and Fresnel lenses and by Eq. (2.8) for the axicon. The water path \( d_{\text{w}} \) was then calculated by subtracting the maximum thickness of each lens from \( d_{\text{lens}} \). The frequency used was 1.2 MHz to match the center frequency of the transducer used in the experiments and material sound speeds for the various lenses that were modeled are given in Table (3.1).

The resulting phase plane was then transformed to \( k \) space by taking the FFT. The magnitude of the wave vector is \( k = 2\pi / \lambda \) where \( k = \sqrt{k_x^2 + k_y^2 + k_z^2} \). It was then propagated in the \( k_z \) direction by multiplying it with the propagation kernel \( e^{ik_z\Delta z} \) where the \( z \) component of the wave vector \( k_z \) is given by \( k_z = \sqrt{k^2 - k_x^2 - k_y^2} \) and \( \Delta z \) is the distance it was propagated in coordinate space. The
propagated field is then transformed back into coordinate space by the inverse FFT. This process of Fourier transforming a plane into $k$ space, applying the propagation kernel, and then calculating the inverse Fourier transform was then repeated for each depth of the simulations. The results of these simulations for the various lenses are shown and compared to the corresponding data in the following sections.

5.2 SIMULATION RESULTS AND COMPARISON TO DATA

The fields generated by each of the lenses discussed in Chapters 2 through 4 were simulated using the ASM. For each lens, the widths of the focal zones and other features predicted by the ASM simulations are in good agreement with the measurements, as shown in Figs. 5.1 – 5.7. The fields from the stepped Fresnel lenses are analogous to a spherically focused field as seen in Figs. 5.1 and 5.4. For the 7 cm focus CPS Fresnel, it converges to the focal zone which has a -3 dB width of approximately 4 mm and a -3 dB focal depth of 3 cm. The 5 cm focus Al Fresnel transducer has a slightly sharper focus with a -3 dB width of 2 mm and a -3 dB focal depth of about 1 cm. The Fresnel lenses were designed to focus a 1.2 MHz source at a depth of 5 cm and 7 cm for the Al and CPS lenses respectively. The transducers used each has a center frequency of 1.2 MHz so the lenses do create a strong focus at the designed focal points as shown in Figs. 5.1 and 5.4.

As seen in Figs. 5.2, 5.3, and 5.5 the axicon and fraxicon lenses demonstrate similar Bessel type fields throughout the scanned area. Both exhibit a minimally diffracting field that remains unchanged over a long depth and spreads only slightly as the end of the DOF is approached. The width of the minimal diffraction zone (MDZ) for the axicon (3 mm), CPS fraxicon (2 mm), and Al fraxicon (2 mm) are narrower relative to the spherical Fresnel lenses focal zone. The data shows a central MDZ line in the three lenses which has little diffraction over a broad range of frequencies. The spectra from the central waveforms in each had center frequencies near 1.2 MHz and -6 dB bandwidths of about 0.3 MHz (25% bandwidth). Each demonstrates a similarly narrow MDZ and long DOF.
Figure 5.1: Magnitudes for the 1.2 MHz spectral component of the fields produced with the CPS 7 cm focus Fresnel lens. Each pair of images has the measured field on the left and angular spectrum simulation on the right. Results shown in the (a) axial plane and (b) transverse plane at 7 cm.
Figure 5.2: Magnitudes for the 1.2 MHz spectral component of the fields produced with the PLA 10 cm DOF axicon. Each pair of images has the measured field on the left and angular spectrum simulation on the right. Results shown in the (a) axial plane and (b) transverse plane at 4 cm.
Figure 5.3: Magnitudes for the 1.2 MHz spectral component of the fields produced with the CPS 7 cm DOF fraxicon. Each pair of images has the measured field on the left and angular spectrum simulation on the right. Results shown in the (a) axial plane and (b) transverse plane at 4 cm.
Figure 5.4: Magnitudes for the 1.2 MHz spectral component of the fields produced with the Al 5 cm focus Fresnel. Each pair of images has the measured field on the left and angular spectrum simulation on the right. Results shown in the (a) axial plane and (b) transverse plane at 5 cm.
Figure 5.5: Magnitudes for the 1.2 MHz spectral component of the fields produced with the Al 10 cm depth of focus fraxicon. Each pair of images has the measured field on the left and angular spectrum simulation on the right. Results shown in the (a) axial plane and (b) transverse plane at 5.2 cm.
The measured widths of the central focus for each lens match well to ASM simulations as shown in Figs. 5.6 and 5.7. Much of the side lobe behavior is also simulated well by ASM. As seen in Figs. 5.1 – 5.5 the depth of the maximum amplitudes for each lens are also in good agreement with that predicted by ASM. Also the axial depth at which the focus is above -3 dB is in agreement with simulation. The fraxicon lenses demonstrate an MDZ, where the amplitude is above -6 dB, that is several centimeters longer than that of the Fresnel lenses.

![Graphs of data and simulations](image)

**Figure 5.6:** Comparison of the magnitude of the Fourier component of the data and the angular spectrum simulations at 1.2 MHz, taken through the center of focus for each lens. (a) CPS 7.0 cm focus Fresnel, (b) PLA 10.0 cm DOF axicon, (c) CPS 7.0 cm DOF fraxicon. In each case the amplitudes were unit normalized.
5.3 FPF ESTIMATION USING AN ANGULAR SPECTRUM SIMULATION

For the aluminum Fresnel and fraxicon lenses discussed in Chapters 3 and 4, ASM simulations provided another estimate of the focal power fraction (FPF) to compare with theory and measurements discussed in Chapter 4. The simulations covered 120 mm in the lateral directions and 35 mm in the axial direction to match the measurement planes of the experimental data. The simulated pressure fields are shown in Fig. 5.8. In these figures the color scale represents the peak pressure and each is normalized to 1. Only a small portion of the lateral dimension of the simulation is depicted in these figures so that the focal features of each are large enough to be seen.

For the two simulations, the depth of spatial peak intensity was determined to be 50.3 mm for the field from the Fresnel lens and 59.9 mm for the fraxicon lens. The FPF was then calculated at these depths for each using the same method outlined in Chapter 4. These simulated FPF values were calculated to be 51.7% for the Fresnel and 24.9% for the fraxicon. This compared well to the theoretical FPF values of 55% and 26% respectively. The measured values were however significantly different at 32.8% and 10.9% respectively. This was possibly due to the non-ideal nature of the lens and transducer construction as discussed in Chapter 4.
**Figure 5.8:** 1.2 MHz spectral component of the ASM for the Al lenses. (a) Fresnel axial plane and (c) traverse plane at the depth of peak intensity 50.3 mm. (b) Fraxicon axial plane and (d) traverse plane at the peak intensity depth 59.9 mm
CHAPTER 6

BOUNDARY DEFORMATION AND DROPLET EXTRACTION

6.1 EXPERIMENTAL SETUP AND OVERVIEW

The boundary between two sets of immiscible fluids was insonified from above inside a 2.4 L square base polycarbonate tank. The experimental setup is shown in Fig. 6.1. In the first set of experiments fluid 1 was water and fluid 2 was CCl\textsubscript{4}. In the second set fluid 1 was canola oil and fluid 2 was water. A TC Power Conversion AG1006 amplified signal generator provided a continuous wave signal at varying power levels to each of the different transducers used. The transducers characterized in chapters 2 and 3 were used with a source frequency of 1.2 MHz. A spherically focused Ultrasound Technologies (UST) transducer operating at 4.7 MHz was also used. The electrical power delivered to each transducer was monitored with an Agilent E4419B power meter. All observations were recorded using either an Edgertronic high speed video camera or a Canon EOS Rebel camera. With each image or video taken an initial image including a ruler was taken for scale reference.

![Experimental setup](image)

**Figure 6.1:** Experimental setup for insonifying the interface between two immiscible fluids. For each experiment the source transducer was located inside fluid 1 and the propagation direction was downward into fluid 2.
6.2 FLUID INTERFACE DEFORMATION HEIGHT AND SHAPE OBSERVATIONS

When incident ultrasound radiation was focused at the fluid boundary at acoustic power levels above approximately 1 W, stepped like features such as previously documented by Bertin et al.\textsuperscript{21} were observed (Fig. 6.2). The deformation height and number of steps depended on the amplitude of the signal as well as the width of the focal zone. These stepped features are a result of waveguide effects where each of the steps can be represented as a cylindrical waveguide.\textsuperscript{21}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.2.png}
\caption{Observed stepped features of the deformation of the interface between water and CCl$_4$. The interface was insonified by the 5 cm focus Al Fresnel transducer at a moderate power level (9 W signal source power or 1.3 W acoustic power at the focus using the conversion factors discussed in Chapter 3). (a) Ruler shown for scale and (b) the same image zoomed in.}
\end{figure}

Beyond a certain amplitude the fluid interface deformation becomes unsteady, and the stepped morphologies become masked with turbulence. The amplitude at which this occurs seems to depend on the width of the focus as sharper focused transducers require a lower amplitude to create this unsteady deformation. In Fig. 6.3, a UST 4.7 MHz transducer with a traditional spherical lens having a focal length of 4 cm was used. This lens has a much tighter focus than the others used in this study which caused the features of the interface became unstable at a much lower source amplitudes. In contrast much higher power levels were required to create unsteady deformations using the less tightly focused Al Fresnel transducer as seen in Fig. 6.4.
This unsteady state is similar to the acoustic fountain effect observed at a water to air interface as described in Section 2.2. Due to the viscous nature of the water medium however, ejected droplets do not have enough momentum to travel very far and instead fall to the sides of the deformation as can be seen in Figs. 6.3 and 6.4.

Figure 6.3: Water to CCl₄ fluid interface insonified from 4 cm above by a UST 4.7 MHz spherically focused transducer with the source signal power at (a) 1.84 W and (b) 4.55 W. The much tighter focus of this transducer produces an unsteady deformation at much lower power levels than with the others used in this study. Both are cropped with the same scale and the ruler is shown in (b) to indicate the height of deformation.

Figure 6.4: Unsteady deformation generated by the Al Fresnel lens transducer with its focal region located at the interface between water and CCl₄. The source power was 27 W which equates to 4.5 W of acoustic power with this transducer.
6.3 DROPLET CAPTURE AND TRANSPORT

The fraxicon transducer produces similar stepped morphologies in the fluid boundary as described in the previous section. This occurs when the fraxicon is placed at any location within its DOF above the interface. For the Al 10 cm DOF fraxicon, when it is placed between 12 mm to 18 mm from the boundary, the deformation is pulled up into the near field region of the transducer and a droplet is broken away from it (Fig. 6.5). The acoustic field in this region traps the droplet firmly, and it is then able to be transported to another location by moving the transducer (Figs. 6.6 and 6.7).

![Figure 6.5: Droplet capture by the Al fraxicon transducer. Here the front of the lens is located 18 mm from the fluid boundary and the droplet is trapped at a depth of 11.8 mm from the lens.](image)

The experiment was repeated for various source power levels, and all produced the same size droplet at the same distance from the lens. The initial height was also varied, and for trapping to occur the lens needed to be between 12 mm and 18 mm from the interface. Inside this range the fluid boundary was deformed far enough so as to place the droplet within the trapping region of the acoustic field. The source frequency was also varied within the operating range of the transducer. However due to the significant decrease in power outside of the transducers resonate frequency, its effect was not determined in this study.
Figure 6.6: Droplet capture and transport by the Al fraxicon transducer. Here the source power is 15 W. The front of the transducer is (a) 16 mm from the fluid boundary and (b) transported upwards by 12 mm with the droplet still trapped by the field 11.8 mm from the lens.

Figure 6.7: With the transducer set at an angle with respect to the fluid boundary trapping and transport of a droplet is still possible. The droplet shown here is trapped 11.8 mm from the front of the lens. The source power is 18 W. The center of the front of the lens is (a) 18 mm from the fluid boundary and then (b) translated upwards by 19 mm.
This effect was also observed using the polystyrene fraxicon lens (Fig. 6.8). The 7 cm DOF polystyrene lens (Fig. 3.3b) and transducer characterized in Chapter 2 was used to insonify the water – CCl₄ interface. Similar morphologies to those obtained with the Al lens transducers were observed. Due to the much lower efficiency of this lens, and thus lower power available at the focus, however, it was not able to trap the droplet for more than about a second. The amount of power available at the focus is much less than that of the Al lens transducer due to poor coupling between the transducer and lens. With high speed video it was observed that the droplet was momentarily trapped and held (Fig. 6.8). The droplet would then deform out of its trapped location and fall back into the fluid below about a second later.

Figure 6.8: Polystyrene fraxicon lens droplet capture attempt. Here the source power is 17 W. The droplet is at 12 mm from the front of the lens which is 20 mm from the fluid interface.
6.4 THERMAL INDUCED JETTING OF A FLUID INTERFACE

For the water – CCl₄ interface, when it is insonified the deformation is in the opposite direction of the propagation of the ultrasound. Using the same setup as shown in Fig. 6.1 the boundary between canola oil (fluid 1) and water (fluid 2) was insonified to demonstrate the opposite effect, a positive radiation force on a fluid boundary. This fluid combination has a large impedance mismatch so there is a large amount of reflection off of the interface. Consequently standing waves were also generated between the boundary and transducer, reflecting power back at the transducer. For this reason the fraxicon was not used with this interface due to its already low focal power efficiency and thus weak effect on the deformation of this boundary. The Al Fresnel transducer, with its much higher focal efficiency, was able to deform the surface however. Still, much higher powers were needed as a result to create similar deformation heights as with the water – CCl₄ boundary.

One phenomenon observed with this fluid combination was the jetting of the canola oil into the water after a few seconds of continuously insonifying the boundary. When the transducer height is adjusted so that its focus is at the fluid interface, there is significant heating of the fluid boundary layer due to the large impedance mismatch and resulting reflections.¹⁹ This heating decreases the surface tension between the two fluids over time which causes the jetting to occur (Fig. 6.9).

![Figure 6.9: Jetting effect at the interface between canola oil and water: (a) the initial deformation and (b) jetting after 2 seconds of continuous insonication. The Al Fresnel transducer was used here located 5 cm above the interface. Its source power was 41 W.](image)
The surface tension constitutes all of the attractive molecular forces at a fluid surface that keeps the surface together. Contact angle measurements are a qualitative method of measuring the surface tension. For a droplet of fluid placed on a smooth solid surface the angle which the edge of the droplet makes with that surface will be larger if its surface tension is larger. This is also a common method of measuring wettability or the hydrophobicity of a surface material by using water droplets. Contact angle measurements were done using a droplet of canola oil at various temperatures to confirm that its surface tension does decrease with temperature. The surface used in this measurement was a stainless steel plate so that the wettability with canola oil was negligible. The steel plate was placed on a heating element and pictures of the droplet were taken on its edge at temperatures ranging from room temperature to 100° C. As shown in Fig. 6.10, the contact angle of the droplet and thus the surface tension did decrease with increasing temperature. This supports the assumption that the delayed spontaneous jetting that occurs at the canola oil – water interface is due to heating and a breakdown of the surface tension at the focus of the transducer.

![Figure 6.10: Contact angle measurements of canola oil drop on a steel plate: (a) 21.3° at 30° C, (b) 17.6° at 70° C, and (c) 16.0° at 100° C. The angle the droplet makes with the surface is outlined in red.](image)
The vertical component of the acoustic radiation force on the droplet of CCl$_4$ trapped by the fraxicon transducer as described in section 6.3 was modeled using Eq. (2.23). The incident and scattered complex pressure was modeled using the finite element method (FEM) in COMSOL Multiphysics with the CCl$_4$ droplet at various depths away from the Al fraxicon lens. The finite element method used by COMSOL approximates the solution to the time independent acoustic wave equation (Helmholtz equation) at a specific frequency (1.2 MHz used here) on a discrete grid.

To shorten computation time cylindrical symmetry was assumed and each calculation was done over a 50 mm by 120 mm simulation domain. On the right side of the simulation domain a cylindrical radiation boundary condition was used. On the top and bottom, spherical wave radiation conditions were used. This was done to eliminate any standing waves in the simulation. The entire simulation domain was discretized by a triangular mesh having a maximum element size of one fifth of the wavelength in the near field and one third of the wavelength in the far field to minimize aliasing.

The model of the 10 cm DOF Al fraxicon lens was positioned so that the front side of it was located at $z = 0$ mm. A line source pressure amplitude of 1 Pa was used along the base of the lens. The droplet was modeled as a prolate spheroid using the material properties of CCl$_4$ and having a width and height equal to that of the observed droplet: 0.8 mm and 1.5 mm respectively.

Multiple simulations were done with the center of the droplet at various different depths covering a total range of 6 mm to 20 mm from the front of the lens. The results of each simulation of the complex pressure field were then exported to a text file containing the $r$ (radial) and $z$ (axial)
coordinates and the real and imaginary parts of the pressure field for later processing in Matlab. Figure 7.1 shows an example of the pressure amplitude fields from two of the simulations where the droplet was located at \( z = 11 \) mm and \( z = 18 \) mm. (It should be noted that the simulations shown by these figures were done only on the right half of each. Due to the axial symmetry this was then mirrored about the \( z \) axis in the final plot for a better physical representation of the field.) In Fig. 7.2 the result of the simulation with the droplet at a depth of \( z = 11.5 \) mm is shown on top of and with the same scale as the experimental observation from Fig. 6.5 for comparison.

![Figure 7.1: COMSOL simulation for the pressure field with the droplet at two different depths: (a) 11 mm and (b) 18 mm from the front of the lens. The color represents the complex amplitude of the pressure field normalized by the source amplitude. The lens is located at the bottom and the forward propagation direction shown here is upward.](image-url)
Figure 7.2: COMSOL simulation of the pressure magnitude with the droplet 11.5 mm from the lens overlaid on top of the experimental observation of the droplet capture. The color represents the pressure amplitude and the scale on the left indicates the distance from the front of the lens in mm.

7.2 SCALING THE SIMULATIONS

Since the simulations were done with a line source of an arbitrary value, another simulation was done without the droplet to compare to measured data. The pressure amplitude $P_{\text{sim}}$ was taken at the center of the depth of focus from this simulation ($z = 50$ mm) for the comparison. To get the measured pressure $P_{\text{meas}}$ at the center of the depth of focus, first the average energy density within the focal zone was calculated for the Al Fraxicon transducer using the method described in Chapter 4 with a source power of 20 W. This value was chosen because it was within the range required for the experimentally observed droplet capture. $P_{\text{meas}}$ was calculated by the relation $^{19}$

$$P_{\text{meas}} = 2c\sqrt{\rho E_l}$$  \hspace{1cm} (7.1)

A scaling factor was calculated by taking the ratio $P_{\text{meas}}/P_{\text{sim}}$ which came out to $9.70 \times 10^5$. This was then used as a multiplier for all of the simulated pressure data.
7.3 PARTICLE VELOCITY DETERMINATION

The FEM simulation returns the complex pressure, $p$. However, Eq. (2.23) also requires the particle velocity $u$. To calculate this, first the velocity potential $\psi$ was determined from the pressure using

$$p = -\rho \frac{\partial \psi}{\partial t}$$  \hspace{1cm} (7.2)

where $\rho$ is the density. By assuming a single frequency $\omega$, $\psi = \psi_0 e^{-i\omega t}$ where $\psi_0$ is the complex amplitude, and $p = p_0 e^{-i\omega t}$ with a complex amplitude $p_0$. Substituting these into Eq. (7.2), $\psi_0$ was calculated from

$$\psi_0 = -i \frac{\omega}{\rho} p_0.$$  \hspace{1cm} (7.3)

Because of the time averages in the force integrals (indicated by the brackets $\langle \rangle$ in Eq. (2.23)) the time dependence $e^{-i\omega t}$ averages to unity in each term and therefore can be ignored. Each of the vector components of $\mathbf{u}$ was then calculated using $\mathbf{u} = \nabla \psi$.

7.4 INTEGRATION OF THE ACOUSTIC RADIATION STRESS TENSOR

Because of the cylindrical symmetry, Eq. (2.23) was integrated over the surface of a cylinder which was centered at the droplet. The height $H$ of the cylinder was chosen to be three times the length of the droplet in the $z$ direction (4.5 mm) and its radius $R$ was chosen as three times the maximum radius of the droplet (1.2 mm). The surface integrals in Eq. (2.23) come from the divergence theorem by integrating the divergence of the acoustic radiation stress tensor. Therefore any size surface may be used as long as it is sufficiently larger in every dimension than $1/k$ which in this case is 0.198 mm. Figure 7.3 demonstrates this integration surface.

$$\mathbf{F} = \int_S \left( \frac{\rho}{2} \langle |\mathbf{u}|^2 \rangle - \frac{1}{2\rho c^2} \langle p^2 \rangle \right) d\mathbf{A} - \int_S \rho \langle \mathbf{uu} \rangle \cdot d\mathbf{A}$$  \hspace{1cm} (2.23)
Figure 7.3: Integration surface for the radiation force calculation. $S_1$ is the top surface, $S_2$ is the bottom surface, and $S_3$ is the surface of the cylindrical shell. It is centered about the location of the droplet.

In cylindrical coordinates the elements of area $dA$ for each surface of the cylinder are

\[
dA_1 = r \, dr \, d\theta \, \hat{z}, \quad dA_2 = -r \, dr \, d\theta \, \hat{z}, \quad \text{and} \quad dA_3 = R \, dz \, d\theta \, \hat{r} \quad (7.4)
\]

where subscript 1 indicates the top, 2 indicates the bottom, and 3 indicates the side of the cylindrical surface. The dot product of the dyad $\langle uu \rangle$ with the incremental area $dA$ in the last term of Eq. (2.23) can be rearranged and evaluated using the associative identity for the dot product of a dyad with a vector as

\[
\langle uu \rangle \cdot dA = \langle u (u \cdot dA) \rangle. \quad (7.5)
\]

By using the incremental areas in Eq. (7.4) this can be expressed in cylindrical coordinates. Over $S_1$ this becomes

\[
\langle u (u \cdot dA_1) \rangle = \langle uu_z \rangle r \, dr \, d\theta, \quad (7.6)
\]

Over $S_2$ it is

\[
\langle u (u \cdot dA_2) \rangle = -\langle uu_z \rangle r \, dr \, d\theta, \quad (7.7)
\]

and on $S_3$

\[
\langle u (u \cdot dA_3) \rangle = \langle uu_r \rangle R \, dz \, d\theta. \quad (7.8)
\]
The components of Eq. (7.6) though (7.8) are determined by the particle velocity \( \mathbf{u} \) vector in each. The computation of these integrals was simplified by only considering the \( z \) component of the acoustic radiation force since it is the most relevant when considering the levitation of the droplet. A further simplification is due to the cylindrical symmetry of the pressure simulation as each integral over the angular coordinate \( \theta \) simply yields a factor of \( 2\pi \). The \( z \) component of the force in Eq. 2.23 can then be written as the sum of the contributions from each side of the integration cylinder \( F_z = F_1 + F_2 + F_3 \). By Eq. (2.23), (7.4), and (7.6) the contribution from the top surface is

\[
F_1 = \pi \int_{S_1} \left( \rho \left\langle |\mathbf{u}|^2 \right\rangle - \frac{1}{\rho c^2} \left\langle p^2 \right\rangle \right) r \, dr - 2\pi \int_{S_1} \rho \left\langle u_z^2 \right\rangle r \, dr. \tag{7.9}
\]

Similarly using Eq. (7.7) on the bottom of the cylinder the contribution is

\[
F_2 = -\pi \int_{S_2} \left( \rho \left\langle |\mathbf{u}|^2 \right\rangle - \frac{1}{\rho c^2} \left\langle p^2 \right\rangle \right) r \, dr + 2\pi \int_{S_2} \rho \left\langle u_z^2 \right\rangle r \, dr. \tag{7.10}
\]

Finally since \( d\mathbf{A}_3 \) doesn’t have a \( z \) component the contribution from the side surface only comes from the last term containing the dyad in Eq. (2.23) which is given by Eq. (7.8). This term then reduces to

\[
F_3 = -2\pi \int_{S_3} \rho \left\langle u_r u_z \right\rangle R \, dz. \tag{7.11}
\]

The time averages indicated by the brackets \( \langle \rangle \) in Eqs. (7.9) though (7.11) were calculated assuming \( e^{-i\omega t} \) time dependence. The time average of the product of two such complex quantities \( A \) and \( B \) is then given by \(^{19}\)

\[
\langle AB \rangle = \frac{1}{2} \text{Re} (AB^*). \tag{7.12}
\]

### 7.5 RESULTS OF THE CALCULATION

These integrals were carried out numerically in Matlab at various droplet depths using the trapezoid rule. The results of these calculations are summarized in Fig. 7.4. The dashed line in the figure represents the force that is required to balance the weight and buoyancy of the droplet \( F_g \).
This was calculated by assuming the droplet is a prolate spheroid with an equatorial radius $a$ and axial dimension $b$. The volume of this shape is given by

$$V = \frac{4}{3} \pi a^2 b.$$  \hspace{1cm} (7.13)

The sum of its weight and buoyancy can then be written as

$$F_g = (\rho_2 - \rho_1)Vg$$  \hspace{1cm} (7.14)

with downward as a positive force, $g$ is the acceleration due to gravity, $\rho_1$ is the density of water, and $\rho_2$ is the density of CC$_4$. Using these parameters the net weight was calculated to be $2.94 \mu N$.

**Figure 7.4:** Acoustic radiation force experienced by the simulated droplet verses the depth of the droplet with respect to the front of the lens. Each black dot represents the calculated acoustic radiation force experienced by the CCl$_4$ droplet when it is located at the depth given by the horizontal axis. The dashed line at the bottom represents the force required to balance the droplets weight and buoyancy.

As can be seen in Fig. 7.4 the calculated maximum negative radiation force was at a depth of 10.8 mm. At this depth the upward radiation force is greater than the net weight of the droplet. This agrees reasonably well with the experimental depth of 11.8 mm at which the droplet was observed to become trapped.
CHAPTER 8

CONCLUSIONS

The main goal of this work was to acoustically deform a fluid boundary layer from fields created by low profile phase-modifying lens-based transducers. Lenses generating two different beam types, spherically focused and finite Bessel beam, were investigated. In the first part of this work Fresnel (spherically focused) and fraxicon (finite Bessel beam) phase-modifying lenses were constructed. The fraxicon, designed as an analog to the axicon lens, is a unique type and this is the first known use of such a lens in acoustics. The fields modified by these lenses were characterized both experimentally and through simulations. The results compared well to the known characteristics of their respective refractive counterparts, the spherical lens and the axicon. In the second part of this work, the focused fields from these lenses were used to insonify the boundary between two immiscible fluids. Two fluid combinations were used, one demonstrating a negative radiation force and the other a positive radiation force. A significant finding in this study was the phenomenon of droplet capture using the fraxicon lens transducer on a water–CCl₄ interface which has not previously been reported. The delayed jetting due to heating within the focal zone at a canola oil–water boundary, another novel phenomenon, was observed.

The development of the phase plate lens transducers was detailed in Chapter 3, along with hydrophone measurements of the fields generated by each lens. The results of these measurements showed that these phase plate lenses were able to recreate many of the same field properties as their refractive counterparts. The two Fresnel lens transducers, which approximate spherical focusing, each demonstrated a strong focus at their designed focal lengths. The fraxicon transducers each created a long minimally diffracting zone over its designed depth of focus thus closely approximating this aspect of the field from an axicon. In Chapter 5 the experimental results were compared with
ASM simulations of the fields generated by the lenses. The numerical simulations all show good agreement with the measurements.

In Chapter 4 the efficacy of the aluminum lens transducers was tested by measuring their focal power fractions (FPFs) and electrical efficiencies. Theoretical values for the focal power fraction for each lens type are presented in Table 8.1 along side the results of direct measurements and values obtained from the ASM simulations outlined in Chapter 5. The measured results were significantly lower than that of theory and simulations. This was likely due to inefficiencies introduced by the asymmetry which was created by the wrap around contact of these transducers as discussed in Chapter 3 (see Fig. 3.7). The theoretical values were in good agreement with the ones obtained from numerical simulations. This is indicative that with a more ideal lens and transducer construction, the measurements may be closer to the predicted FPF values. The electrical efficiencies of the Al lens transducers were tested using a force balance technique. The results showed that in each about half of the electrical input power was converted into acoustic power. From these two measurements, the acoustic energy density contained within the focusing region of each was determined for a wide range of source levels.

<table>
<thead>
<tr>
<th>Lens type</th>
<th>Theory</th>
<th>Measurement</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fresnel</td>
<td>55%</td>
<td>32.8%</td>
<td>51.7%</td>
</tr>
<tr>
<td>fraxicon</td>
<td>26%</td>
<td>10.9%</td>
<td>24.9%</td>
</tr>
</tbody>
</table>

Table 8.1: FPF theoretical values and results from measurements and numerical simulations of the Al lens transducers.

As outlined in Chapter 6 the Al Fresnel and fraxicon transducers were used to insonify the interface between two fluid pairs, in one case, water–CCl₄, demonstrating a negative radiation force (pull) and in the other (canola oil–water) demonstrating a positive force (push). At lower signal amplitudes the results and deformation features observed were consistent with previously published observations. At higher amplitudes, unsteady deformations, droplet capturing, and jetting of one fluid into the other were observed.
With the Al fraxicon insonifying the water–CCl₄ interface at moderate drive levels of around 15 W, the acoustic radiation pressure was strong enough to pull the boundary layer into a region of the field capable of trapping a single droplet of CCl₄. This region where a droplet can be trapped was located in the near field, 11.8 mm from the front of the fraxicon lens. The trapping forces were strong enough that the droplet could then be transported through the water several centimeters away from the interface and once captured, the droplets were never observed to escape from the trap. These observations indicate that the trapping phenomenon had little to do with interactions of the field with the boundary since it was sustained when the transducer was moved several centimeters away from the boundary. In Chapter 7 the acoustic fields from the fraxicon transducer incident on and scattered by the droplet were numerically simulated and the radiation force was theoretically calculated. These calculations predicted that the droplet would experience a negative radiation force sufficient for trapping around 11 mm from the front of the fraxicon, consistent with the observations.

In the next set of experiments detailed in Chapter 6, a spherically focused Fresnel transducer insonified a canola oil–water boundary which resulted in a positive radiation pressure at the focus. With this setup, a delayed jetting of the canola oil into the water was observed. It was determined that this phenomenon was due to heating at the viscous boundary layer of the canola oil surface which breaks down the surface tension of the interface. This was supported by contact angle measurements of a canola oil drop at a range of temperatures.

Future work should focus on finding other fluid combinations where droplet capture is possible, to determine if this phenomenon is limited to fraxicon modified fields, and to investigate other beam types. Methods for improving the efficiency of these lensed transducers should also be explored, including the use of a more symmetric piezoelectric crystal, together with techniques for flush mounting the lenses.
LIST OF REFERENCES


VITA

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Education

2010 – M.Sc. in physics at the University of Memphis
2005 – B.Sc. in physics at the University of Louisiana at Lafayette

Research

2013-2019 – Research assistant under the advice of Dr. Joel Mobley at the National Center for Physical Acoustics located at the University of Mississippi. Research topics included acoustic lensing, radiation force, and material characterization.
2008 - 2010 – Research assistant under the advice of Dr. Firouzeh Sabri at the University of Memphis. Research topics included Martian dust capture mechanisms characterization and treatments of polymers including aerogels and polydimethylsiloxane (PDMS).
2004-2007 – Research assistant under the advice of Dr. Jim Sabatier at the National Center for Physical Acoustics located at the University of Mississippi. Research topics included acoustic detection of buried land-mines and methods for reducing false positives. Also assisted in developing and characterizing a 16 and a 256 beam laser Doppler vibrometer.

Academic Employment

2018 – Instructor, General Physics II, Dept. of Physics and Astronomy, University of Mississippi.

2011 - 2013 – Adjunct professor, General Physics I and II and associated lab classes,
Dept. of Natural Sciences, Southwest TN Community College.

2011 - 2012 – Adjunct professor, General Physics I and II and associated lab classes, Dept. of Physics, University of Memphis.

2008 - 2010 – Teaching assistant, Physics I and II labs, Dept. of Physics, University of Memphis.

Professional Activities

2018 – Physical Acoustics Summer School at the University of Mississippi. A week long lecture series on topics related to physical acoustics.

2003 – Research for Undergraduates Program, Texas A&M Institute for Quantum Studies. A month long summer program including classes and hands on workshops on quantum optics and laser physics at Texas A&M and Casper College, WY, ending with a joint Texas A&M, DARPA, and ONR conference on quantum optics in Jackson Hole, WY.