Auditor's Approach to Statistical Sampling, Volume 3. (Supplementary Section) Stratified Random Sampling

American Institute of Certified Public Accountants. Professional Development Division. Individual Study Program

(Supplementary Section) **3**

AN AUDITOR'S APPROACH TO STATISTICAL SAMPLING

STRATIFIED RANDOM SAMPLING

Individual Study Program
Professional Development Division
American Institute of Certified Public Accountants

NOTICE TO READERS

This programed learning text is a publication of the staff of the American Institute of Certified Public Accountants and is not to be regarded as an official pronouncement of the Institute. It was prepared by David Monroe Miller, programing consultant; Robert E. Healy, CPA, and Morton J. Rossman, CPA, consultants; and Thomas R. Hanley, CPA, Manager, Special Projects. The members of the Committee on Statistical Sampling assisted in an advisory capacity.

STRATIFIED RANDOM SAMPLING

SUPPLEMENTARY SECTION

Programed for the
American Institute of Certified Public Accountants
by
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CONTENTS OF SUPPLEMENTARY SECTION

HOW TO USE THE EXHIBITS

Most of the Exhibits are worksheets used in working out the two illustrative sampling problems, "MNO Tool Company" and "JKL Component Corporation" (usually referred to in abbreviated form.) Answers are provided directly in back of those worksheets that contain difficult or unusual exercises. For the simpler worksheets, answers are in the usual place in the programed text – that is, directly after the frame that gives the exercise. In every case, the text will indicate where the answers are to be found.

Exhibits 1 through 4 are general in nature, and are used in Chapters 1 and 2. Exhibits 5 through 17 are used in conjunction with the MNO Tool Company Problem in Chapters 1 through 5. Exhibits 18 through 31 are used in conjunction with the JKL Corporation problem in Chapters 6 and 7. Finally, Exhibit 32 is a table of reliability coefficients, used in many chapters.

If you are working out the JKL Corporation problem and wish to look back at the corresponding worksheet for the MNO Tool Company problem, you can look up the Table of Contents to find the Exhibit with the same title.

Opposite each worksheet is a page of instructions indicating how it is to be used. Directions are also built into the worksheets themselves. Most of the worksheets are designed to be gone through all at once, but some are supposed to be interrupted by a frame in the programed texts. Specific instructions are given each time, but if in doubt, always refer to the frame in the text that originally directed you to the supplementary material.

SUMMARY OF STRATIFIED SAMPLING PROCEDURES

The steps below are based on the assumption that the decision to use stratified rather than unrestricted sampling has already been made. Factors influencing this decision are discussed in Chapter 1.

The asterisks indicate steps that have been performed for you in this teaching volume but which must be done by the auditor in an actual problem.

Chapter 6, Dollar-Value Stratification and 100% Inspection of a Stratum, and Chapter 7, Conclusion and Review, cover all steps listed below and are therefore not itemized in this summary.

CHAPTER 1. PRINCIPLES OF STRATIFICATION

1. Establish the desired precision and reliability for the estimate of the overall population value.
2. Designate the strata unambiguously, so that every element in the population is known, in advance, to belong to one and only one stratum.
3. Decide which of the strata, if any, is to be sampled on a 100% basis.

CHAPTER 2. PRELIMINARY SAMPLE SELECTION

*4. Use the regular random number table (not the two-page illustrative table) to select a preliminary sample of at least 30 items from each stratum.

*5. For each stratum, list the sample values and compute the squares of the values. (Suggested format for tabulating sample data appears in Volume 1, Exhibit 5.)

CHAPTER 3. SAMPLE SIZE ALLOCATION

6. Estimate the total population value based on the preliminary sample (Exhibits 7 and 21).

*7. Compute the estimated standard deviation of each stratum (Worksheet 1 in Volume 1).

8. Determine the proportion of the total sample that will be allocated to each of the strata (Exhibits 8 and 22).

CHAPTER 4. DETERMINATION OF SAMPLE SIZE

9. Solve the sample-size equation for the overall n (Exhibits 9 and 23).

10. Compute the sample size required from each stratum, and add 10% as a precaution (Exhibits 11 and 25).

*11. Select additional sample elements and record the individual values, as in steps 4 and 5.

CHAPTER 5. EVALUATION OF RESULTS

*12. Compute the new sample mean and estimated S^2_i of each stratum (Exhibits 13 and 27).

13. Estimate the total population value (Exhibits 7 and 21). The 100% strata, if any, are included in this estimate.
14. Compute the precision of this estimate at the desired reliability level (Exhibits 14 and 28).

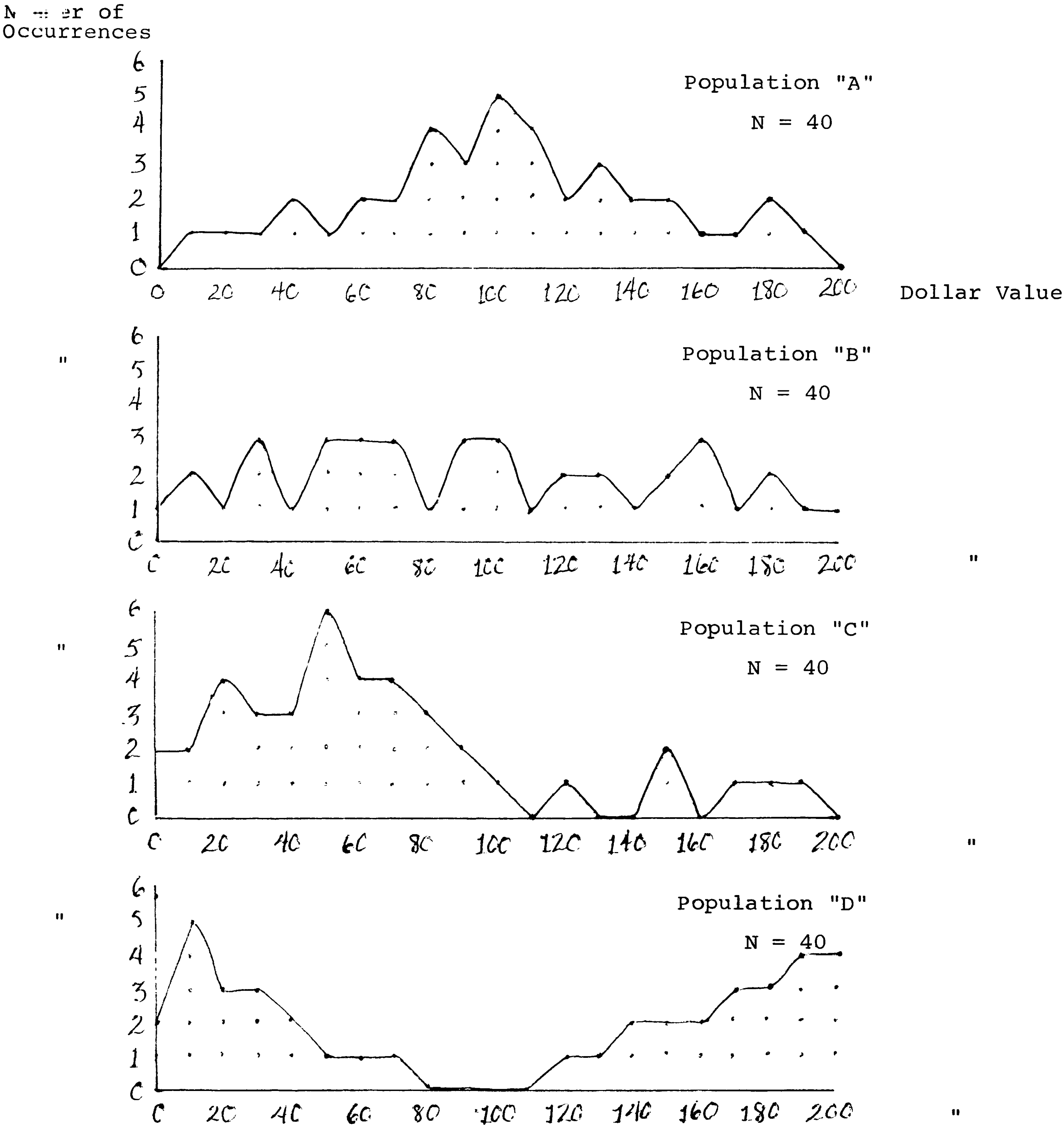
See third paragraph on preceding page.

Four Illustrative Populations

Population "A"	Population "B"	Population "C"	Population "D"
8	3	3	2
17	8	4	4
27	14	7	7
36	24	8	9
43	28	15	12
50	33	18	12
62	34	22	14
64	41	24	18
69	47	27	21
72	49	31	23
75	54	33	26
78	55	36	27
81	62	39	31
83	64	40	40
85	72	45	42
90	73	46	51
94	74	48	63
97	84	49	74
100	87	52	115
101	88	53	130
103	93	57	136
104	97	58	142
106	100	62	145
109	103	64	152
111	114	65	160
112	118	69	163
115	118	71	164
122	129	72	168
125	133	75	173
128	142	78	175
134	149	84	179
137	153	92	181
141	158	93	185
146	162	96	187
150	164	120	188
156	167	147	192
167	175	153	195
175	182	174	196
180	191	179	198
187	198	191	200
$\bar{X} = 101.0$	$\bar{X} = 96.0$	$\bar{X} = 65.0$	$\bar{X} = 105.0$

EXHIBIT 2

Graphic Representation of Four Illustrative Populations



Source: Exhibit 1 data, with all values shown to nearest $10

Using the Random Number Table

This exercise is to be done for the first stratum only (N = 1,500). In the spaces below, describe how you would establish correspondence between the inventory lots and the digits in the table, indicate a possible route, and describe how you would randomly select a starting point. Be as thorough and explicit as you can.

CORRESPONDENCE

ROUTE

STARTING POINT

Suggested answers are given on the next page.

Suggested Answers to Random Number Table Exercise

1. CORRESPONDENCE PLAN

Unless the lots happen to be consecutively numbered, they have to be numerically designated by the auditor. The most convenient way is to number them 0001 to 1500, so that each lot will have one corresponding four-digit number in the table.

2. ROUTE

You should specify in advance which digits in each row are to be used (1st through 4th, 2nd through 5th, etc.). You need not limit yourself to one row but could take, for example, the last two digits in one column and the first two in an adjoining column. Alternatively, and better, this could be determined randomly as suggested in paragraph #3 below.

You should also have specified the direction of the route (up or down), and what to do at the end of a column and page.

3. STARTING POINT

This could be determined simply by making a "blind stab" with a pencil. However, a more sophisticated method was suggested in Volume I. The "stab" is not itself the starting point, but rather gives the row numbers and column number. The next digit in the "stabbed" row could indicate the digit position. For example, the digits 53672 would indicate a starting point of Row 536, Column 7, beginning with the 2nd digit (and therefore using the 2nd through 5th digits).

CORRECT YOUR OWN ANSWERS, IF NECESSARY; THEN RETURN TO FRAME 2-18, PAGE 36.

Description of Problem

The total dollar value (at cost) of finished-product inventory of the MNO Tool Company is to be estimated with a precision of ± $20,000 at 95% reliability. The inventory consists of hundreds of products ranging from low-value standard items (nuts and bolts, etc.) to moderately-priced hand tools. The inventory is divided into 2,000 lots. Many different kinds of items might be classified in a single lot, subject to the following restrictions:

1. Fifteen hundred (1,500) lots contain only the low-value standard items.

2. Five hundred (500) lots contain only hand tools.

The auditor has reason to believe that the individual lot values in the first group range from $40 to $200; in the second group, from $150 to $1,200. These dollar figures are, of course, only approximations based on previous records and experience.

The lots are designated with alphanumeric symbols in an irregular sequence (as in the XYZ Shoe Company example in Volume 1.)

EXHIBIT 6
MNO TOOL COMPANY

Data Sheet

A. AUDITOR'S DECISIONS

QUANTITY TO BE ESTIMATED	(X)	
DESIRED PRECISION	(A)	
DESIRED RELIABILITY	(R)	

B. POPULATION CONSTANTS

RELIABILITY COEFFICIENT	(U_R)	
SQUARE OF RELIABILITY COEFFICIENT	(U_R^2)	
SQUARE OF DESIRED PRECISION	(A^2)	

C.

STRATA	PRECISE DESCRIPTION	N_i	S_i	S_i^2	new S_i^2
STRATUM 1					
STRATUM 2					
STRATUM 3					
STRATUM 4					

INSTRUCTIONS FOR USING WORKSHEET (EXHIBIT 7)

1. In Column 1, enter the total of the sample values, and in Column 2, enter the preliminary sample size. This has already been done for the MNO Tool Co. preliminary estimate.

2. Compute the sample mean for each stratum by dividing Column 1 by Column 2.

3. In Column 4, enter the N for each stratum and total them. (This also has been done for the first MNO Tool Co. exercise.)

4. For each stratum, multiply the sample mean (Column 3) by the stratum N (Column 4). Enter the result in Column 5. This gives the estimated total value for each stratum.

5. To arrive at the estimated population total ($\hat{X}$), add up the Column 5 results.

6. When computing the <u>final</u> estimate of the population total -- that is, after the additional sample has been taken -- omit Columns 1 and 2. Instead, simply enter in Column 3 the combined sample mean that has been computed in Row 5 of Exhibit 13.

7. The <u>mean</u> of the population or of any of the strata is to be computed only if the particular auditing circumstances require it. For the teaching problems in this volume, the text gives instructions as to when and when not to use the last column.

EXHIBIT 7
MNO TOOL COMPANY

Estimate of Population Total

STRATA	(1) $\sum x_{ij}$	(2) n_i	(3) $\bar{x}_i$	(4) N_i	(5) $\bar{x}_i N_i$ $(=\hat{X}i)$	(6) $\hat{\bar{X}}$ (Optional)
Preliminary Estimate 1. Standard Items	$3,300	30		1,500		
2. Hand Tools	$21,000	30		500		
				N=2,000	$\hat{X}$=	$\hat{\bar{X}}$=
Final Estimate 1. Standard Items						
2. Hand Tools						
				N=	$\hat{X}$=	$\hat{\bar{X}}$=

INSTRUCTIONS FOR USING WORKSHEET (EXHIBIT 8)

1. In the first two columns, enter the N_i and estimated standard deviation (S_i) for each stratum. This has already been done for you in the MNO Tool Co. example.

2. For each stratum, multiply N_i by S_i and enter the results in the third column.

3. Add up the sums just computed, entering the total next to the notation $\Sigma N_i S_i$ =.

4. Divide each individual $N_i S_i$ figure by the total $N_i S_i$ figure. Compute to the nearest hundredth. Enter the result in the P_i Column.

5. As a check, add the P_i figures. The total should be 1.00 or 100%.

EXHIBIT 8
MNO TOOL COMPANY

Optimal Allocation of Stratum Sample Sizes

STRATUM	N_i	S_i	N_iS_i	P_i
1. Standard Items	1,500	\$36		
2. Hand Tools	500	\$205		

$\sum N_iS_i =$

INSTRUCTIONS FOR USING WORKSHEET (EXHIBIT 9)

1. All instructions for this Worksheet are in the second Column, headed "SOURCE." (The first column is for those who prefer to work from the formula, rather than from the instructions.) For each Row, 1 through 16, the auditor either enters a figure from previous Exhibits or performs the indicated computations.

2. Rows 1 through 7 are done for each stratum. Rows 8 through 16 apply to the population as a whole.

3. All rounding is done to the nearest whole number, except for U_R^2 (Row 9), which is never rounded.

EXHIBIT 9
MNO TOOL COMPANY

$$n = \frac{U_R^2}{A^2} \sum \frac{N_i S_i^2 (N_i - P_i n)}{P_i}$$

Sample Size Equation

	QUANTITY	SOURCE	STRATUM 1	STRATUM 2
1	N_i	Exhibit 6		
2	S_i^2	Exhibit 6		
3	$N_i S_i^2$	(1) x (2)		
4	P_i	Exhibit 8		
5	$\frac{N_i S_i^2}{P_i}$	(3) ÷ (4)		
6	$\frac{N_i S_i^2}{P_i} (N_i)$	(1) x (5)		
7	$\frac{N_i S_i^2}{P_i} (P_i n)$	(3) x (n)	n	n
8	$\sum \frac{N_i S_i^2}{P_i} (N_i)$	Total of Row 6 Figures		
9	U_R^2	Exhibit 6		
10	$U_R^2 \sum \frac{N_i S_i^2}{P_i} (N_i)$	(8) x (9)		
11	$\sum \frac{N_i S_i^2}{P_i} (P_i n)$	Total of Row 7 Figures		n
12	$U_R^2 \sum \frac{N_i S_i^2}{P_i} (P_i n)$	(9) x (11)		n
13	Desired A^2	Exhibit 6		
14	$A^2 n$	(13) x (n)		n
15	$A^2 n + \left[U_R^2 \sum \frac{N_i S_i^2 (P_i n)}{P_i} \right]$	(14) + (12)		n
16	n	(10) ÷ (15)		

$$n = \frac{U_R^2}{A^2} \sum \frac{N_i S_i^2 (N_i - P_i n)}{P_i}$$

EXHIBIT 10
MNO TOOL COMPANY

Sample Size Equation (ANSWERS)

	QUANTITY	SOURCE	STRATUM 1	STRATUM 2
1	N_i	Exhibit 6	1,500	500
2	S_i^2	Exhibit 6	1,296	42,025
3	$N_i S_i^2$	(1) x (2)	1,944,000	21,012,500
4	P_i	Exhibit 8	.35	.65
5	$\frac{N_i S_i^2}{P_i}$	(3) ÷ (4)	5,554,286	32,326,923
6	$\frac{N_i S_i^2}{P_i} (N_i)$	(1) x (5)	8,331,429,000	16,163,461,500
7	$\frac{N_i S_i^2}{P_i} (P_i n)$	(3) x (n)	1,944,000 n	21,012,500 n
8	$\sum \frac{N_i S_i^2}{P_i} (N_i)$	Total of Row 6 Figures	24,494,890,500	
9	U_R^2	Exhibit 6	3.84	
10	$U_R^2 \sum \frac{N_i S_i^2}{P_i} (N_i)$	(8) x (9)	94,060,379,520	
11	$\sum \frac{N_i S_i^2}{P_i} (P_i n)$	Total of Row 7 Figures	22,956,500 n	
12	$U_R^2 \sum \frac{N_i S_i^2}{P_i} (P_i n)$	(9) x (11)	88,152,960 n	
13	Desired A^2	Exhibit 6	400,000,000	
14	$A^2 n$	(13) x (n)	400,000,000 n	
15	$A^2 n + \left[U_R^2 \sum \frac{N_i S_i^2 (P_i n)}{P_i}\right]$	(14) + (12)	488,152,960 n	
16	n	(10) ÷ (15)	193	

INSTRUCTIONS FOR USING WORKSHEET (EXHIBIT 11)

1. As in Exhibit 9, the instructions come from the second column. The auditor either enters figures already known, or performs the indicated computations.

2. When beginning the worksheet, the letter "n" refers to the sample size computed in Exhibit 9. When the worksheet is completed, the letter "n" is henceforth used to denote the new and final figure in Row 8, not the original figure in Row 1.

3. Similarly, the n_i figures in Rows 3 and 6 are only temporary. They are multiplied by 1.1. (That is, 10% is added to them to obtain the actual n_i figures that will be used.) In these rows (3 and 6), do not round the n_i figure. Rather, simply multiply the result by 1.1.

4. Except for the intermediate n_i computations mentioned in Step 3, all figures are rounded to the nearest whole number.

Stratum Sample Sizes

	QUANTITY	SOURCE	RESULT
1	n derived by formula	Row 16, Exhibit 9	
2	P_1	Exhibit 8	
3	n_1	(1) x (2)	
4	n_1 + 10%	(3) x 1.1	
5	P_2	Exhibit 8	
6	n_2	(1) x (5)	
7	n_2 + 10%	(6) x 1.1	
8	Final n	(4) + (7)	

Stratum Sample Sizes (ANSWERS)

	QUANTITY	SOURCE	RESULT
1	n derived by formula	Row 16, Exhibit 9	193
2	P_1	Exhibit 8	.35
3	n_1	(1) x (2)	67.55
4	n_1 + 10%	(3) x 1.1	74
5	P_2	Exhibit 8	.65
6	n_2	(1) x (5)	125.45
7	n_2 + 10%	(6) x 1.1	138
8	Final n	(4) + (7)	212

Combined Sample Computations

	QUANTITY	SOURCE	STRATUM 1	STRATUM 2
1	Σx_i (preliminary)	Sample Data	$ 3,300	$ 21,000
2	Σx_i (additional)	" "	4,988	76,152
3	Σx_i (combined)	(1) + (2)	8,288	97,152
4	n_i (combined)	Rows 4 and 7 of Exhibit 11	74	138
5	$\bar{x}_i$	(3) ÷ (4)	112	704
6	$\bar{x}_i^2$	(5) x (5)	12,544	495,616
7	$\Sigma (x_i)^2$ (preliminary)	Sample Data	400,584	15,918,725
8	$\Sigma (x_i)^2$ (additional)	" "	633,084	58,066,431
9	$\Sigma (x_i)^2$ (combined)	(7) + (8)	1,033,668	73,985,156
10	$n\bar{x}_i^2$	(4) x (6)	928,256	68,395,008
11	$\Sigma (x_i)^2 - n\bar{x}_i^2$	(9) - (10)	105,412	5,590,148
12	s_i^2	(11) ÷ (n_i-1)	1,444	40,804

INSTRUCTIONS FOR USING WORKSHEET (EXHIBIT 14)

1. In this worksheet, like the others, instructions are given in the second column. The first column is for reference only, for those who prefer to work from the formula rather than from the instructions.

2. In Row 4, the n_i figure to be entered is the final n_i with 10% added. Do not use the preliminary n_i, which was 30 for each stratum, or the intermediate n_i, which was computed before the 10% was added.

3. In Row 10, the A^2 figure is the result of computations in this worksheet; not the desired A^2 which you have been working with up to now.

4. In order to compute the precision (A) in the last Row, you will have to use Exhibit 16. When that worksheet is completed, you can enter the result in Row 11 at the bottom of Exhibit 14. However, in this teaching example (MNO Tool Co.), return to your place in the text before going on to Exhibit 16.

5. All computations are rounded to the nearest dollar.

$$A = U_R \sqrt{\sum \frac{N_i S_i^2 (N_i - n_i)}{n_i}}$$

EXHIBIT 14
MNO TOOL COMPANY

Evaluation of Results

	QUANTITY	SOURCE	STANDARD ITEMS	HAND TOOLS
1	N_i	Exhibit 6		
2	S_i^2 (new)	Exhibit 6		
3	$N_i S_i^2$	(1) x (2)		
4	n_i	Rows 4 and 7, Exhibit 11		
5	$N_i - n_i$	(1) - (4)		
6	$N_i S_i^2 (N_i - n_i)$	(3) x (5)		
7	$\frac{N_i S_i^2 (N_i - n_i)}{n_i}$	(6) ÷ (4)		
8	$\sum \frac{N_i S_i^2 (N_i - n_i)}{n_i}$	Total of Row 7 figures		
9	U_R^2	Exhibit 6		
10	A^2	(8) x (9)		
11	A	$\sqrt{}$ of Row 10. Use Exhibit 16.		

$$A = U_R \sqrt{\sum \frac{N_i S_i^2 (N_i - n_i)}{n_i}}$$

EXHIBIT 15
MNO TOOL COMPANY

Evaluation of Results (ANSWERS)

	QUANTITY	SOURCE	STANDARD ITEMS	HAND TOOLS
1	N_i	Exhibit 6	1,500	500
2	S_i^2 (new)	Exhibit 6	1,444	40,804
3	$N_i S_i^2$	(1) x (2)	2,166,000	20,402,000
4	n_i	Rows 4 and 7, Exhibit 11	74	138
5	$N_i - n_i$	(1) - (4)	1,426	362
6	$N_i S_i^2 (N_i - n_i)$	(3) x (5)	3,088,716,000	7,385,524,000
7	$\frac{N_i S_i^2 (N_i - n_i)}{n_i}$	(6) ÷ (4)	41,739,405	53,518,290
8	$\sum \frac{N_i S_i^2 (N_i - n_i)}{n_i}$	Total of Row 7 figures	95,257,695	
9	U_R^2	Exhibit 6	3.84	
10	A^2	(8) x (9)	365,789,549	
11	A	$\sqrt{\ }$ of Row 10. Use Exhibit 16.		

INSTRUCTIONS FOR WORKSHEET (EXHIBIT 16)

1. In Row 1, enter the square of the precision, as computed in Exhibit 14. (This has been done in the MNO Tool Company example.)

2. Enter any guess ("first approximation") of the square root in Row 2. The suggested procedure is to use the desired precision (A) as your first approximation.

3. From then on, perform the computations indicated in the second column. Each approximation brings you closer to the actual square root.

4. The procedure ends when you find yourself averaging two numbers that are only one dollar apart. This will usually happen in Row 8, but it may happen earlier, in which case the remaining rows are ignored.

5. Enter the final result in the bottom row of Exhibit 14.

6. In our two teaching examples, all rounding is done to the nearest dollar.

Computation of Precision by "Divide and Average" Method

	QUANTITY	SOURCE	RESULT
1	A^2	Row 10 of preceding worksheet	365,789,549
2	Approximate Square Root	Desired "A" (From Exhibit 6)	
3	1st Quotient	(1) ÷ (2)	
4	Second Approximation	Average of (2) and (3)	
5	2nd Quotient	(1) ÷ (4)	
6	Third Approximation	Average of (4) and (5)	
7	3rd Quotient	(1) ÷ (6)	
8	Fourth Approximation	Average of (6) and (7)	

Computation of Precision by "Divide and Average" Method (ANSWERS)

	QUANTITY	SOURCE	RESULT
1	A^2	Row 10 of preceding worksheet	365,789,549
2	Approximate Square Root	Desired "A" (From Exhibit 6)	20,000
3	1st Quotient	(1) ÷ (2)	18,289
4	Second Approximation	Average of (2) and (3)	19,145
5	2nd Quotient	(1) ÷ (4)	19,106
6	Third Approximation	Average of (4) and (5)	19,126
7	3rd Quotient	(1) ÷ (6)	19,125
8	Fourth Approximation	Average of (6) and (7)	19,126

EXHIBIT 18
JKL CORPORATION

Description of Problem (Part 1)

The JKL Component Corporation manufactures electronic hardware components which it sells to other manufacturing firms and to the Government. In this example, the problem is to estimate the total value of its 1,800 Government accounts receivable. The estimate will be based on a sample of the accounts, to be substantiated by auditing procedures other than confirmation. The sample size is to be large enough to enable the company to be 98% confident that the statistical estimate does not differ from the true value by more than ± $200,000.

Prior to this, a trial balance of $6,515,000 had been obtained for the value of the 1,800 accounts. At that time it was noted that 100 accounts had a balance of $10,000 or more, 500 had balances between $1,000 and $9,999, and the remaining 1,200 had balances of less than $1,000.

The auditor has already decided that all of the $10,000+ accounts will be substantiated, and that all of the accounts in the $1 - $999 category will not be substantiated, but that an appropriate number will be chosen by means of a random sample. The remaining questions, therefore, are as follows:

1. Given the precision and reliability requirements mentioned above, whether the middle 500 accounts should be statistically sampled or substantiated 100%;

2. How many of the low-value accounts to sample (and of the middle-value accounts, if sampling is decided upon for this category).

EXHIBIT 19
JKL CORPORATION

Description of Problem (Part 2)

For reasons stated in the text, the auditor has decided to take a stratified random sample of the 1,800 Government accounts, but to sample the high-value stratum on a 100% basis.

The following data will be relevant to this problem. Consult this chart whenever necessary. For the first two strata, the data are based on a preliminary sample of the 30 substantiated accounts in each category. (At this point, you are concerned only with the upper chart.)

	STRATUM	N_i	n_i	Sample Total	S_i	$(X_i)^2$
1.	$1-$999	1,200	30	$15,000	$ 240	$ 8,670,400
2.	$1,000-$9,999	500	30	$156,000	$1,200	$815,376,000
3.	$10,000 and over	100	100	$3,000,000	(not needed)	(not needed)

* * * * *

After the required additional elements were selected, those accounts were substantiated by means other than confirmation, like the 60 accounts in the preliminary sample, and the following data were obtained (for the additional elements):

	STRATUM	N_i	n_i	Sample Total	$(x_i)^2$
1.	$1-999	1,200	4	$3,020	$2,942,700
2.	$1,000-9,999	500	43	$245,500	$1,489,757,200

(Stratum 3 data not relevant in this part of the problem.)

Data Sheet

A. AUDITOR'S DECISIONS

QUANTITY TO BE ESTIMATED	(X)	
DESIRED PRECISION	(A)	
DESIRED RELIABILITY	(R)	

B. POPULATION CONSTANTS

RELIABILITY COEFFICIENT	(U_R)	
SQUARE OF RELIABILITY COEFFICIENT	(U_R^2)	
SQUARE OF DESIRED PRECISION	(A^2)	

C.

STRATA	PRECISE DESCRIPTION	N_i	S_i	S_i^2	new S_i^2
STRATUM 1					
STRATUM 2					
STRATUM 3					
STRATUM 4					

INSTRUCTIONS FOR USING WORKSHEET (EXHIBIT 21)

1. (Steps 1 through 3 need not be done for the stratum that has been sampled 100%.) In Column 1, enter the total of the sample values, and in Column 2, enter the preliminary sample size. For the JKL Corporation teaching problem, these figures are given in Exhibit 19.

2. Compute the sample mean for each stratum by dividing Column 1 by Column 2.

3. In Column 4, enter the N for each stratum. Multiply each stratum sample mean (Column 3) by the stratum N (Column 4). Enter the results in Column 5. This gives the estimated total value for each stratum.

4. The above steps are not necessary for the 100% stratum, because its total value is known rather than estimated. Enter the known total (from Exhibit 19) in Column 5.

5. To arrive at the estimated population total (X), add up the Column 5 results.

6. When computing the final estimate of the population total--that is, after the additional sample has been taken-- use the bottom half of the worksheet and omit Columns 1 and 2. Instead, simply enter in Column 3 the combined sample mean that has been computed in Row 5 of Exhibit 27. (As before, when working with the 100% stratum, go directly to Column 5.)

7. The mean of the population or of any of the strata is to be computed only if the particular auditing circumstances require it. For the teaching problems in this volume, the text gives instructions as to when and when not to use the last column.

EXHIBIT 21
JKL CORPORATION

Estimate of Population Total

STRATA	(1) Σx_{ij}	(2) n_i	(3) $\bar{x}_i$	(4) N_i	(5) $\bar{x}_i N_i$ $(=\hat{X}i)$	(6) $\hat{\bar{X}}$ (Optional)
Preliminary Estimate						
1. $1 - 999						
2. $1,000-9,999						
3. $10,000 or over						
				N=	$\hat{X}$=	$\hat{\bar{X}}$=
Final Estimate						
1. $1 - 999						
2. $1,000 - 9,999						
3. $10,000 or over						
				N=	$\hat{X}$=	$\hat{\bar{X}}$=

INSTRUCTIONS FOR USING WORKSHEET (EXHIBIT 22)

1. In the first two columns, enter the N_i and estimated standard deviation (S_i) for each stratum, except for the 100% stratum which is not included in these computations.

2. For each stratum, multiply N_i and S_i and enter the results in the third column.

3. Add up the sums just computed, entering the total next to the notation $\Sigma N_i S_i =$.

4. Divide each individual $N_i S_i$ figure by the total $N_i S_i$ figure. Compute to the nearest hundredth. Enter the result in the P_i column.

5. As a check, add the P_i figures. The total should be 1.00 or 100%.

EXHIBIT 22
JKL CORPORATION

Optimal Allocation of Stratum Sample Sizes

STRATUM	N_i	S_i	$N_i S_i$	P_i
1. $1 - 999				
2. $1,000 - 9,999				

$$\sum N_i S_i =$$

INSTRUCTIONS FOR USING WORKSHEET (EXHIBIT 23)

1. All instructions for this Worksheet are in the second Column, headed "SOURCE." (The first column is for those who prefer to work from the formula, rather than from the instructions.) For each Row, 1 through 16, the auditor either enters a figure from previous Exhibits or performs the indicated computations.

2. We do not, of course, compute the "sample size" for the stratum that has been sampled 100%. Rows 1 through 7 are therefore done for each of the other strata, and Rows 8 through 16 apply to the totals of those two strata.

3. All rounding is done to the nearest whole number, except for U_R^2 (Row 9), which is never rounded.

$$n = \frac{U_R^2}{A^2} \sum \frac{N_i S_i^2 (N_i - P_i n)}{P_i}$$

EXHIBIT 23
JKL CORPORATION

Sample Size Equation

	QUANTITY	SOURCE	STRATUM 1	STRATUM 2
1	N_i	Exhibit 20		
2	S_i^2	Exhibit 20		
3	$N_i S_i^2$	(1) x (2)		
4	P_i	Exhibit 22		
5	$\frac{N_i S_i^2}{P_i}$	(3) ÷ (4)		
6	$\frac{N_i S_i^2}{P_i} (N_i)$	(1) x (5)		
7	$\frac{N_i S_i^2}{P_i} (P_i n)$	(3) x (n)	n	n
8	$\sum \frac{N_i S_i^2}{P_i} (N_i)$	Total of Row 6 Figures		
9	U_R^2	Exhibit 20		
10	$U_R^2 \sum \frac{N_i S_i^2}{P_i} (N_i)$	(8) x (9)		
11	$\sum \frac{N_i S_i^2}{P_i} (P_i n)$	Total of Row 7 Figures	n	
12	$U_R^2 \sum \frac{N_i S_i^2}{P_i} (P_i n)$	(9) x (11)	n	
13	Desired A^2	Exhibit 20		
14	$A^2 n$	(13) x (n)	n	
15	$A^2 n + \left[U_R^2 \sum \frac{N_i S_i^2 (P_i n)}{P_i}\right]$	(14) + (12)	n	
16	n	(10) ÷ (15)		

$$n = \frac{U_R^2}{A^2} \sum \frac{N_i S_i^2 (N_i - P_i n)}{P_i}$$

EXHIBIT 24
JKL CORPORATION

<u>Sample Size Equation</u> (Answers)

	QUANTITY	SOURCE	STRATUM 1	STRATUM 2
1	N_i	Exhibit 20	1,200	500
2	S_i^2	Exhibit 20	57,600	1,440,000
3	$N_i S_i^2$	(1) x (2)	69,120,000	720,000,000
4	P_i	Exhibit 22	.32	.68
5	$\frac{N_i S_i^2}{P_i}$	(3) ÷ (4)	216,000,000	1,058,823,529
6	$\frac{N_i S_i^2}{P_i} (N_i)$	(1) x (5)	259,200,000,000	529,411,764,500
7	$\frac{N_i S_i^2}{P_i} (P_i n)$	(3) x (n)	69,120,000n	720,000,000n
8	$\sum \frac{N_i S_i^2}{P_i} (N_i)$	Total of Row 6 Figures	788,611,764,500	
9	U_R^2	Exhibit 20	5.43	
10	$U_R^2 \sum \frac{N_i S_i^2}{P_i} (N_i)$	(8) x (9)	4,282,161,881,235	
11	$\sum \frac{N_i S_i^2}{P_i} (P_i n)$	Total of Row 7 Figures	789,120,000 n	
12	$U_R^2 \sum \frac{N_i S_i^2}{P_i} (P_i n)$	(9) x (11)	4,284,921,600 n	
13	Desired A^2	Exhibit 20	40,000,000,000	
14	$A^2 n$	(13) x (n)	40,000,000,000 n	
15	$A^2 n + \left[U_R^2 \sum \frac{N_i S_i^2 (P_i n)}{P_i}\right]$	(14) + (12)	44,284,921,600 n	
16	n	(10) ÷ (15)	97	

INSTRUCTIONS FOR USING WORKSHEET (EXHIBIT 25)

1. As in previous worksheets, the instructions come from the second column. The auditor either enters figures already known, or performs the indicated computations.

2. When beginning the worksheet, the letter "n" refers to the sample size computed in Exhibit 23. When the worksheet is completed, the letter "n" is henceforth used to denote the new and final figure in Row 8, not the original figure in Row 1.

3. Similarly, the n_i figures in Rows 3 and 6 are only temporary. They are multiplied by 1.1. (That is, 10% is added to them to obtain the actual n_i figures that will be used.) In these rows (3 and 6), do not round the n_i figure. Rather, simply multiply the result by 1.1.

4. Except for the intermediate n_i computations mentioned in Step 3, all figures are rounded to the nearest whole number.

Stratum Sample Sizes

	QUANTITY	SOURCE	RESULT
1	n derived by formula	Exhibit 23	
2	P_1	Exhibit 22	
3	n_1	(1) x (2)	
4	n_1 + 10%	(3) x 1.1	
5	P_2	Exhibit 22	
6	n_2	(1) x (5)	
7	n_2 + 10%	(6) x 1.1	
8	Final n	(4) + (7)	

Stratum Sample Sizes (ANSWERS)

	QUANTITY	SOURCE	RESULT
1	n derived by formula	Exhibit 23	97
2	P_1	Exhibit 22	.32
3	n_1	(1) x (2)	31.04
4	n_1 + 10%	(3) x 1.1	34
5	P_2	Exhibit 22	.68
6	n_2	(1) x (5)	65.96
7	n_2 + 10%	(6) x 1.1	73
8	Final n	(4) + (7)	107

Combined Sample Computations

	QUANTITY	SOURCE	STRATUM 1	STRATUM 2
1	Σx_i (preliminary)	Sample Data	$ 15,000	$ 156,000
2	Σx_i (additional)	" "	3,020	245,500
3	Σx_i (combined)	(1) + (2)	18,020	401,500
4	n_i (combined)	Rows 4 & 7 of Exhibit 25	34	73
5	$\bar{x}_i$	(3) ÷ (4)	530	5,500
6	$\bar{x}_i^2$	(5) x (5)	280,900	3,025,000
7	$\Sigma(x_i)^2$ (preliminary)	Sample Data	8,670,400	815,376,000
8	$\Sigma(x_i)^2$ (additional)	" "	2,942,700	1,489,757,200
9	$\Sigma(x_i)^2$ (combined)	(7) + (8)	11,613,100	2,305,133,200
10	$n\bar{x}_i^2$	(4) x (6)	9,550,600	2,208,250,000
11	$\Sigma(x_i)^2 - n\bar{x}_i^2$	(9) - (10)	2,062,500	96,883,200
12	s_i^2	(11) ÷ (n_i-1)	62,500	1,345,600

INSTRUCTIONS FOR USING WORKSHEET (EXHIBIT 28)

1. In this worksheet, like the others, instructions are given in the second column. The first column is for reference only, for those who perfer to work from the formula rather than from the instructions.

2. In Row 4, the n_i figure to be entered is the final n_i with 10% added. Do not use the preliminary n_i, which was 30 for each stratum, or the intermediate n_i, which was computed before the 10% was added.

3. In Row 10, the A^2 figure is the <u>result</u> of computations in this worksheet, not the desired A^2 which you have been working with up to now.

4. After computing A^2 in the next to last row, check your answer in Exhibit 29 and then go right on to Exhibit 30. After using that exhibit to compute the square root of A^2, enter it, for the record, in the last row of Exhibit 28.

$$A = U_R \sqrt{\sum \frac{N_i S_i^2 (N_i - n_i)}{n_i}}$$

EXHIBIT 28
JKL CORPORATION

Evaluation of Results

	QUANTITY	SOURCE	\$1 - 999	\$1,000 - 9,999
1	N_i	Exhibit 20		
2	S_i^2 (new)	Exhibit 20		
3	$N_i S_i^2$	(1) x (2)		
4	n_i	Rows 4 and 7, Exhibit 25		
5	$N_i - n_i$	(1) - (4)		
6	$N_i S_i^2 (N_i - n_i)$	(3) x (5)		
7	$\frac{N_i S_i^2 (N_i - n_i)}{n_i}$	(6) ÷ (4)		
8	$\sum \frac{N_i S_i^2 (N_i - n_i)}{n_i}$	Total of Row 7 figures		
9	U_R^2	Exhibit 20		
10	A^2	(8) x (9)		
11	A	$\sqrt{\ }$ of Row 10. Use Exhibit 30.		

$$A = U_R \sqrt{\sum \frac{N_i S_i^2 (N_i - n_i)}{n_i}}$$

EXHIBIT 29
JKL CORPORATION

Evaluation of Results (Answers)

	QUANTITY	SOURCE	\$1 - 999	\$1,000 - 9,999
1	N_i	Exhibit 20	1,200	500
2	S_i^2 (new)	Exhibit 20	62,500	1,345,600
3	$N_i S_i^2$	(1) x (2)	75,000,000	672,800,000
4	n_i	Rows 4 and 7, Exhibit 25	34	73
5	$N_i - n_i$	(1) - (4)	1,166	427
6	$N_i S_i^2 (N_i - n_i)$	(3) x (5)	87,450,000,000	287,285,600,000
7	$\frac{N_i S_i^2 (N_i - n_i)}{n_i}$	(6) ÷ (4)	2,572,058,824	3,935,419,178
8	$\sum \frac{N_i S_i^2 (N_i - n_i)}{n_i}$	Total of Row 7 figures	6,507,478,002	
9	U_R^2	Exhibit 20	5.43	
10	A^2	(8) x (9)	35,335,605,551	
11	A	$\sqrt{\ }$ of Row 10. Use Exhibit 30.		

INSTRUCTIONS FOR USING WORKSHEET (EXHIBIT 30)

1. In Row 1, enter the square of the precision, as computed in Exhibit 28.

2. Enter any guess ("first approximation") of the square root in Row 2. The suggested procedure is to use the desired precision (A) as your first approximation.

3. From then on, perform the computations indicated in the second column. Each approximation brings you closer to the actual square root.

4. The procedure ends when you find yourself averaging two numbers that are only one dollar apart. This will usually happen in Row 8, but it may happen earlier, in which case the remaining rows are ignored.

5. Enter the final result in the bottom row of Exhibit 28.

6. In our two teaching examples, all rounding is done to the nearest dollar. In practice, any degree of accuracy may be specified. However, there is no point in computing the precision to a greater degree of accuracy than has been used earlier in the problem.

Computation of Precision by "Divide and Average" Method

	QUANTITY	SOURCE	RESULT
1	A^2	Row 10 of preceding worksheet	
2	Approximate Square Root	Desired "A" (From Exhibit 20)	
3	1st Quotient	(1) ÷ (2)	
4	Second Approximation	Average of (2) and (3)	
5	2nd Quotient	(1) ÷ (4)	
6	Third Approximation	Average of (4) and (5)	
7	3rd Quotient	(1) ÷ (6)	
8	Fourth Approximation	Average of (6) and (7)	

Computation of Precision by "Divide and Average" Method

(Answers)

	QUANTITY	SOURCE	RESULT
1	A^2	Row 10 of preceding worksheet	35,335,605,551
2	Approximate Square Root	Desired "A" (From Exhibit 20)	200,000
3	1st Quotient	(1) ÷ (2)	176,678
4	Second Approximation	Average of (2) and (3)	188,339
5	2nd Quotient	(1) ÷ (4)	187,617
6	Third Approximation	Average of (4) and (5)	187,978
7	3rd Quotient	(1) ÷ (6)	187,977
8	Fourth Approximation	Average of (6) and (7)	187,978

EXHIBIT 32

Conversion of Reliability Percentages to U_R and U_R^2 Values

Desired R	U_R	U_R^2
75%	1.15	1.32
80%	1.28	1.64
85%	1.44	2.07
90%	1.64	2.69
95%	1.96	3.84
98%	2.33	5.43
99%	2.58	6.66