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Auditor's Approach to Statistical Sampling, Volume 5. Ratio and Difference Estimation

American Institute of Certified Public Accountants. Professional Development Division. Individual Study Program

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AN AUDITOR'S APPROACH TO STATISTICAL SAMPLING

RATIO AND DIFFERENCE ESTIMATION

Individual Study Program
Professional Development Division
American Institute of Certified Public Accountants

NOTICE TO READERS

This programed learning text is a publication of the staff of the American Institute of Certified Public Accountants, and is not to be regarded as an official pronouncement of the Institute. It was prepared under the general supervision of the members of the Committee on Statistical Sampling, with primary advisory responsibilities assigned to a special subcommittee composed of James W. Kelley, CPA, John R. Coker, CPA, John R. Rogers, CPA, and Dr. Robert Taylor, ÇPA. Dr. Donald M. Roberts acted as a statistical consultant in preparing this text for the Committee. Stephen J. Gallopo, CPA, Assistant Manager, Special Projects coordinated the activities as AICPA staff aide.

AN AUDITOR'S APPROACH TO STATISTICAL SAMPLING

Volume 5

RATIO AND DIFFERENCE ESTIMATION

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PREFACE

The Committee on Statistical Sampling of the American Institute of Certified Public Accountants has undertaken the development of a series of programed texts on statistical sampling techniques in auditing to broaden education in this area. *Ratio and Difference Estimation* is the fifth volume in this series.

The earlier volumes dealt with the basic principles of statistical sampling. This volume will not teach you basic principles. Rather, it will acquaint you with additional techniques which are useful in certain situations.

The techniques of ratio and difference estimation are concerned with the estimation of variables (total dollar amount). They can be used in connection with either unrestricted sampling as in Volume 1, or stratified sampling as in Volume 3. Those volumes are prerequisites to the study of this volume. Volumes 2 and 4 are basic texts with which the auditor with an interest in statistical sampling should be familiar, but are not required as prerequisites to this volume.

Volumes 1 and 3 discuss the technique of making an estimate based on a mean value computed from a random sample. Given the variance around the mean and the size of the sample, the auditor can compute the precision of the estimate at the reliability level he specifies. The basic technique of projecting from a sample mean ("mean estimation") can be used with both unrestricted random sampling and stratified random sampling.

The technique of mean estimation can, in theory, be used for any variables estimate; however, it is not always the best tool for the job. In some cases, the variability of the population will be so large that the precision and reliability requirements could be met only with a very large sample size. In other cases, a mean estimate would still be feasible, but a ratio or difference estimate would enable the auditor to use a smaller sample size and achieve the same precision and reliability, or alternatively use the same sample size and achieve tighter precision or greater reliability.

Ratio and difference estimation are used primarily as tools to help the auditor gain satisfaction that a client's book value is fairly stated. The audit use of ratio and difference estimation is illustrated in an accounts receivable and an inventory problem. The audit judgments made in these cases are necessary to illustrate the use of statistical techniques. In applications, you will have to decide the appropriate precision and reliability factors based solely on your judgment.

A second use of ratio estimation is to determine the total value of a certain category within a larger category of accounting data; for example, estimating the total overdue receivables within total accounts receivable. If total receivables are known, it is possible to compute the ratio of overdue amounts to total due on a sample of accounts, and then estimate the overdue amounts total using the ratio method.

Although the teaching examples in this volume are all concerned with estimation of a total dollar amount, an intermediate step involves the computation of a ratio which in itself can be of interest to the auditor. For example, using the techniques in this volume, the auditor could estimate the ratio of total service charges to total credit sales as a percentage. The next step—computing the actual dollar amount of service charges—may be of less interest than the ratio itself.

This booklet is being made avaliable to the Institute membership as part of the continuing education program of the Professional Development Division.

JOE R. FRITZEMEYER, CPA
Director, Auditing & Reporting

June, 1972

INTRODUCTION

This volume is similar in format to the four previous volumes in this series. Completing the programed text, the reader turns the page after each frame and checks his answer in the left-hand box. The responses called for may be choices, fill-ins or problems to work out. Some frames are marked "No answer required." These frames may contain important information and should be read as thoroughly as the others. As in previous volumes, maximum educational value will be obtained by writing answers directly in the spaces provided, and then changing them if they prove to be incorrect.

This volume is approximately the same length and of the same order of complexity as Volumes 1 and 3. The basic theory presented in these volumes applies here. Some of the formulas used in ratio estimation involve more numerical terms than those used previously. You are therefore advised to have access to a calculating machine when reading Chapters 2 through 6.

The Supplementary Section contains worksheets and other reference material. The text will always direct you to the appropriate exhibit in the teaching sequence. You can also refer to the Supplementary Section at any time to review work you have already done or to get a preview of what is to come.

The programed text begins on the following page.

CHAPTER 1 BASIC CONCEPTS

1-1. Ratio estimation consists of estimating an amount X by projecting a known amount Y by an estimate of the ratio of X to Y.

For example, if Y is $100, and the ratio of X to Y is 5:1 (5 to 1), then of course X is $______.

Note: Capital letters refer to population values throughout this volume, while lower case letters refer to sample values.

TURN THE PAGE

$500	1-2. We now show how this basic arithmetic can be used in auditing situations. Suppose that a population of 1,500 accounts payable has a book value, according to the control ledger, of $2,000,000. Assume if all accounts were audited (by direct confirmation or other means), the ratio of audited values to book values would be 1.07 to 1. <u>If</u> we knew this, we could conclude by simple arithmetic that the total audited book value would be $____________.

$2,140,000

($2,000,000 x 1.07)

REMINDER

1. As you go through this book you will be turning the page each time, rather than going down the page.

2. If your answer proves to be incorrect cross it out and substitute the correct answer.

3. The frames marked "No answer required" contain as much information as those which call for an answer.

TURN THE PAGE AND GO ON TO 1-3

1-38. We desire to estimate the total cost (X) of a given population of items. From our sample we have an estimated cost-to-retail ratio of .61.

To solve for an estimate of X, we simply multiply the B value times the ratio. Thus, the basic ratio-estimation formula is:

$$\hat{X} = \hat{R}B$$

In this formula, there is a caret over the X as well as the R because the total will be the (ACTUAL/ESTIMATED) total retail value.

3-2. According to the inventory records, the total book value of the inventory (B) is $2,500,000. To test this amount for reasonableness, the auditor first selects a preliminary sample. From this sample he computes an estimate of the total inventory value that would be obtained if every item in the inventory were audited. Therefore, we speak of the total audited value. Using either the formula or your own words, indicate how we estimate this total, once we have a sample ratio. ____________

____

____

No answer required.

1-3. In practice, of course, we would not know the ratio of audited to book values. However, we can estimate this ratio from the results of a random sample. If the total audited amount is $80,000 and the total book amount is $75,000, the sample ratio of audited values to book values is $80,000 to $75,000 or ________ to 1.

(Note that the lower-case 'a' will refer to the sum of the audited values in the sample while lower-case 'b' will refer to the sum of the book values in the sample.)

---

estimated

1-39. This procedure was the same one used in frame 1-1, although without the notation. B was $100, $\hat{R}$ was 5.00, and $\hat{X}$ was computed to be $500. To conclude our arithmetic and notational review, write the basic ratio estimation formula and, given the sample ratio in frame 1-37 and the known total in frame 1-34, use the space below to solve for $\hat{X}$.

Multiply B by $\hat{R}$ where $\hat{R}$ is the sample ratio of

$\sum a_i$ to $\sum b_i$

or

$\hat{X} = \hat{R} \cdot B$

3-3. From Exhibit 5, we can compute $\hat{R}$ to be 1.02. With that figure and the basic ratio-estimation formula, you can make a preliminary estimate of the total inventory value. Do this now, and compute the difference between $\hat{X}$ and B.

1.07 $\frac{a}{b} = \frac{80}{75}$ = 1.0667 rounded to 1.07	1-4. Let us summarize the procedure so far. 1. We want to estimate an unknown total - the value of 1,500 accounts payable when audited. We call the unknown total X. (Note that we use X instead of A, since A refers to precision.) 2. If we decide to use a ratio estimate, we have to relate the unknown value to a known value, B, which in this case would be the total book value of the same accounts. 3. Since B is known from the control ledger, we could easily compute X if we knew the ratio of X to B. 4. We can estimate the ratio of X to B by taking a random ______ of the accounts.
$\hat{X} = \hat{R}B$ $\hat{X}$ = .61 ($9,000) $\hat{X}$ = $5,490	1-40. This cannot be considered the final solution, since it is based on only a preliminary sample and does not contain any measure of precision and reliability. The topics of selection of sample size and computation of precision and reliability are covered in later chapters. (No answer required)
$\hat{X}$ = (1.02) (2,500,000) = 2,550,000 $\hat{X}$ - B = $50,000	3-4. The estimated $50,000 between the book value and the auditor's estimated value is not considered material. We must determine, however, if our sample is large enough to support the position that the actual difference is immaterial. In sampling terms, we need to know the __________ of the estimate for a particular reliability level.

sample	1-5. Although the four points in frame 1-4 accurately summarize the ratio estimation concept, several questions can be raised: 1. How can we measure the difference between the estimated ratio and the true ratio? 2. How large a sample size is necessary? 3. Why not take a sample of accounts to audit and proceed as in earlier volumes? We will begin with the last question. (No answer required)
No answer required.	1-41. As was the case with all other techniques discussed in this series, the decision to use ratio estimation must rest primarily on the auditor's judgment, but there are certain criteria available for determining when ratio estimation would be effective. The first criterion requires two sets of values which can be compared with one another. This is not only a criterion, but is virtually a definition of a ratio problem, since a ratio is composed of two numerical values. (No answer required)
precision	3-5. The achieved precision based on a sample of 50 is about twice the desired precision ($70,000) at the specified reliability of 95%. If an estimate based on a preliminary sample achieves the desired precision, we could stop. In this example, however, we must recognize that our sample is too small to satisfy our requirements. (No answer required)

No answer required.	1-6. Ratio estimation can be more efficient. To demonstrate the efficiency of ratio estimation, let us consider an alternative - estimation based on a sample <u>mean</u> rather than on a ratio. Assume that a random sample of 40 items has a total dollar value of $3,000, yielding a sample mean of $75. If there are 10,000 items in the population, an estimate of the total dollar value of the population would be _________ ($N\bar{x}$).
No answer required.	1-42. The second criterion provides that the two sets of values should be approximately ____________ to each other.
No answer required.	3-6. The next step is to determine the sample size required. For this purpose, turn to Exhibit 9. This exhibit is arranged so that through row 7, you are following the same procedure as you did in Volume 1, Supplementary Section,Worksheet 4, S-20. At row 7, you have computed the sample size when sampling (WITH REPLACEMENT/WITHOUT REPLACEMENT). Steps 8-10 are necessary when sampling is done (WITH REPLACEMENT/ WITHOUT REPLACEMENT.)

$750,000 (Sample mean of $75 multiplied by 10,000)	1-7. The technique of estimation based on a sample mean ("mean estimation") can be used either with unresticted sampling as in Volume 1, or with stratified sampling as in Volume 3. One potential disadvantage of this technique is that if the variability of the population is great, a large sample size will be required. Such a situation may occur if the range of values is (NARROW/WIDE).
proportional Note: The lack of proportionality does not invalidate the ratio technique but reduces its efficiency.	1-43. Another requirement for using a ratio estimate - one not mentioned earlier - is that there should be some individual differences between 'b' and 'a' values. Sometimes, in an auditing situation, each a_i in the sample is equal to its corresponding b_i value. But if no differences are observed in the sample, then $\sum b_i$ will equal $\sum a_i$ and the sample ratio will, of course, be equal to ___.
WITH REPLACEMENT WITHOUT REPLACEMENT	3-7. While the form is identical to the corresponding exhibit in Volume 1, the meaning of the terms is different. Specifically describe the difference between S_{R_j} and S_{X_j}. ______________________________

WIDE	1-8. This problem may be solved in part by stratifying the population, as discussed in Volume 3. Often stratification will not completely solve the problem, and in those cases where it does, it may be costly and difficult to carry out. (No answer required)
1	1-44. This, in turn, means that the estimate of the total, $\hat{X}$, is identical to the given value, B. In applying ratio techniques this creates a problem since when no differences are observed between a_i and b_i we cannot estimate the ratio population variance. This makes it impossible to determine precision and reliability. (No answer required)
S_{R_j} : The standard deviation of the population of ratios. S_{X_j} : The standard deviation of the population of dollar values.	3-8. Application of the sample-size formula in this volume, as in the other volumes in this series, is simply a question of following the worksheet instructions. In Exhibit 9 as in the other worksheets, the formula and the first column can be studied by those who wish to relate their computations to the formula. The second column gives the necessary explanation of the symbols, and column 3 provides the instruction for making the computations. Do all of Exhibit 9, checking occasionally with the answers in Exhibit 10.

No answer required.

I	II
$1,200	$ 801
72	473
10	372
1,050	550
800	438
40	540
302	280
750	770
$4,224	$4,224

$$\frac{\$4224}{8} = \$528$$

1-9. To review the concept of variability, consider the two populations at the left. Each has a mean of $528. You can see from inspecting the figures, however, that population (I/II) has a much higher standard deviation.

No answer required.

1-45. The way to handle this occurrence depends on the auditing situation. If the auditor has good reason to believe that there are few or no discrepancies, the ratio estimation technique should not be used.

(No answer required)

Answers are in Exhibit 10.

3-9. By comparing rows 7 and 10, we can see the reduction in sample size caused by using the without replacement method rather than with replacement. Furthermore, by looking at steps 8 and 9, we observe that the savings in terms of observations will not amount to much unless row 8 exceeds .05.

As a rule of thumb, lines 8-10 can be omitted unless the sample size calculated in row 7 is more than 5% of the population size. We carried through the steps here for illustrative purposes.

(No answer required)

I

Book Values b_i	Audited Values a_i	Ratio of a_i to b_i
$1,000	$1,200	1.20
80	72	.90
5	10	2.00
1,100	1,050	.95
600	800	1.33
40	40	1.00
320	302	.94
715	750	1.05
$3,860	$4,224	

$\frac{a}{b} = 1.09$

1-10. The high-variability data from page 10 appears in the middle left column. Assume that these eight elements represent a sample of audited accounts payable whose total we wish to estimate, while the figures in the left-hand column represent book values. The figures at the right represent the ratio of each a_i element (audited value) to the corresponding b_i element (book value). Notice that we use a subscript, i, to refer to an individual account - thus the values for a_1, a_2,...are 1,200, 72,...respectively, for b_1, b_2,...1,000, 80,...respectively, and a_1/b_1 would equal 1.20; whereas a and b without subscripts refer to the sample totals - thus a = $4,224 and b = $3,860.

(No answer required)

No answer required.

1-46. The final criterion for ratio estimation is that the B total must be known in advance to be the sum of the detail, so that it can be multiplied by the ratio to give our estimate of the true value. In the cost-to-retail problem, the B total is considered to be the correctly footed retail total. Similarly, in the problem of estimating the value of accounts receivable the B total is known but (IS/IS NOT) necessarily correct.

No answer required.

3-10. The safety factor of 10% added in line 11 is not scientifically based, but emphasizes that planning is based on a relatively small sample. The number in line 10 is only a good approximation. Thus the 10% addition should be viewed as:

__1. A practical hint which builds in some extra conservatism.

__2. A "must" in statistical sampling.

No answer required.

b_i	a_i	Ratio of a_i to b_i
$1,000	$1,200	1.20
80	72	.90
5	10	2.00
1,100	1,050	.95
600	800	1.33
40	40	1.00
320	302	.94
715	750	1.05
$3,860	$4,224	

$\frac{a}{b} = 1.09$

1-11. Each of the ratios of a_i to b_i is obtained by dividing the individual audited amount by the corresponding book amount. The overall ratio for this sample is 1.09. This ratio is not computed by averaging the individual ratios, but rather by ______________________________.

IS NOT (If the B total from the control ledger were known to be correct, there would be no need to make a statistical estimate in the first place.)

1-47. While the total value B need not be correct, it is important that it correspond to the addition of all b_i values in the population. The auditor must be aware of footing errors.

(No answer required)

1. A practical hint which builds in some extra conservatism.

3-11. Ratio estimation does not alter the procedure for computing the required sample size. The formula used is of the same form you worked with previously. The formula used depends on whether the sample was taken with or without replacement and whether the population was stratified. The problems of stratification will be considered in Chapter 5.

(No answer required)

dividing the sum of the audited values, $a = \sum_{1}^{8} a_i$, by the sum of the book values, $b = \sum_{1}^{8} b_i$, or $\frac{4224}{3860} = 1.09$	1-12. The sample ratio of $\frac{a}{b}$ is symbolized as $\hat{R}$. R stands for the population ratio, and the caret over R indicates that the sample ratio is considered to be the (TRUE/ ESTIMATED) ratio of X to B.
No answer required.	1-48. In review, here are (somewhat condensed the criteria for using a ratio estimate: 1. For each item in the population there are two numerical values (a_i, b_i); these two values should be approximately proportional. 2. There should be some observed differences between the a_i values and b_i values to permit calculation of precision and reliability. 3. The total population value B must be known, and correspond to the addition of all b_i values in the population. (No answer required)
No answer required.	3-12. The required sample size of 211 (including the extra 10% for safety) may seem somewhat large even though the precision requirement of $70,000 represents 2.8% of the total book amount. One standard of comparison would be the sample size required to achieve the same precision and reliability using the method of mean estimation. Therefore, you probably expect mean estimation to require a sample size which is (ABOUT THE SAME/LARGER) than 211.

ESTIMATED

b_i	a_i	Ratio of a_i to b_i
$1,000	$1,200	1.20
80	72	.90
5	10	2.00
1,100	1,050	.95
600	800	1.33
40	40	1.00
320	302	.94
715	750	1.05
$3,860	$4,224	

$\hat{R} = 1.09$

1-13. The table repeated at the left suggests that the standard deviation of the ratios may be considerably smaller than the standard deviation of audited values. We have already seen that the audited values seem to have a high standard deviation while the individual ratios appear more clustered.

(No answer required)

No answer required.

1-49. To summarize this discussion, the use of a ratio estimate can be considered when there are two corresponding sets of nearly proportional values, with other factors as set forth in frame 1-48.

In such circumstances, an estimate based on the sample ratio will result in greater sampling efficiency -- that is, lower sample size, tighter precision limits, or greater reliability, provided the data are nearly proportional. In cases where the relationship is not one of near proportionality, difference estimates could be considere This will be discussed further in Chapter 6.

LARGER

3-13. Larger is the obvious answer. To see how much larger, we would use the data from the preliminary sample of 50 to compute S_{X_j} which represents ____________________

____________________.

Using this and worksheet 4 in Volume 1, we could compute the required sample size. However, as we remarked earlier, Exhibit 9 uses the same formula (in rows 1-7) except that S_{R_j} is used instead of S_{X_j}.

No answer required.

1-14. There is one item in the sample -- the third one -- that does not conform to the pattern. The ratio is 2.00. But we are not averaging ratios; rather, we are dividing $\sum a_i$ by $\sum b_i$ to determine our sample ratio. Thus, the actual dollar amounts, $10 and $5, are so small that the effect of this item on the overall ratio of 1.09 is (LARGE/SMALL). Note that if a large dollar account had a ratio of 2.00, the effect would be much greater. Thus, if the amounts were $500 and $1,000 instead of $5 and $10 the ratio would be (LARGER/SMALLER).

END OF CHAPTER 1

an estimate of the standard deviation of the population of audited values corresponding to the a_i column

3-14. Using the data from the preliminary sample, $S^2_{X_j}$ may be computed to be about 270,000, while $S^2_{R_j}$ was computed in row 10, Exhibit 7, to be about 10,000. This is a ratio of nearly __________, and hence comparing the corresponding sample sizes on a with replacement basis (row 7, Exhibit 9), we would obtain a sample size of _________.

SMALL

LARGER

(1.19 instead of 1.09)

$$\frac{4224 - 10 + 1000}{3860 - 5 + 500} = \frac{5214}{4355}$$

= 1.19

1-15. This demonstrates two points. First, the effect of the $10 account on the sample mean is significant. Without it, the mean would be $602 rather than $528, and the large amount of variability contributed by this very low item would be reduced. Second, the ratio is responsive to differences in large dollar accounts corresponding to the auditor's concern with potential material error.

(No answer required)

CHAPTER 2 PRELIMINARY SAMPLE:
SELECTION AND ANALYSIS

2-1. In this chapter we will solve a ratio estimation problem which takes us through all of the procedures and most of the computations necessary in a field problem. Read Exhibit 1 in the Supplementary Section for the facts of the problem. This exhibit will be referred to at various points in the next 3 chapters.

(No answer required)

---

27 to 1

5,400
(27 X 200)

3-15. This answer is correct only because we used a formula based on sampling with replacement. To obtain a more meaningful comparison, we should continue as in Exhibit 9, rows 8, 9 and 10 to adjust for sampling without replacement. This would yield a sample size of ______________ compared to 192. These additional steps take into account the fact that we are sampling ______________________.

No answer required.

1-16. We have not demonstrated mathematically that the standard deviation of ratios is smaller than the standard deviation of audited values. However, we can show why this should be the case in some circumstances Individual values often vary more than ratios This occurs when the two sets of data are approximately linearly related; that is, proportional.

State in your own words why there is often a linear relationship between the audited value of an account and the book value of the same account. ______________________
____
____
____
____

No answer required.

2-2. In this problem, what is the auditor attempting to do by means of statistical sampling? (Check answer)

__.1 Estimate the total value of the inventory.

__.2 Estimate the ratio between the audited inventory value and the book inventory value.

2596

(row 8: $\frac{5400}{5000} = 1.08$

row 9: $1 + 1.08 = 2.08$

row 10: $\frac{5400}{2.08} = 2596$)

without replacement

3-16. Once the sample size is selected by whatever means, the procedures are almost identical to those used in connection with unrestricted and stratified sampling.

In the following frames, we review the major concepts in ratio estimation. The reader is advised to look over Appendix 3, Chapters 1, 2 and 3. Then, to maximize the effectiveness of these review questions, do not refer to the programed text or the supplementary section.

(No answer required)

Answer to 1-16 appears in frame 1-17.

1-17. Although there are several reasons, you might have mentioned the following:

The size of a transaction affecting the account may be about proportional to the size of the account balance, and hence mistakes in processing transactions for large accounts will be larger than those affecting small accounts. When this is not so difference estimation may prove more efficient.

(No answer required)

1. Estimate the total value of the inventory.

2-3. The objective of the statistical procedure is to estimate the total value of the inventory. The procedure used to achieve this objective will involve computation of the estimated ratio between the ____________________ value and the book value. In Volume 1, a similar problem would have had the same objective but used different procedures.

No answer required.

3-17. Next to each of the symbols, give the meaning in your own words. The purpose is not simply to review the notation, but to make sure that the concepts are clear. Your answer should therefore be as complete as you can make it.

B: ____

____

$\hat{X}$: ____

____

$\hat{R}$: ____

____

____

No answer required.

1-18. Not all audited values can be expected to be nearly proportional to the corresponding book amount.

In Chapter 6 we shall present a procedure for analyzing the case when such a relationship does not exist.

Check the pairs that you might expect would have a linear relationship:

__ 1. Cost prices to retail prices.
__ 2. Audited inventory amount to amount shown on a priced perpetual record.
__ 3. Dollar amount of accounts more than 60 days past due to total receivables.

audited

2-4. As in any statistical estimation problem, the first step is to define precisely the quantity to be estimated. Turn to Exhibit 2 and keep it easily accessible.

In the top row, B has been filled in. There is no caret over B because it is known and hence it is not an estimate. In this example it is the total value of the client-priced inventory records.

In row 2, we have $\hat{X}$ to indicate that this quantity is an estimate of X. You are to fill in the description of $\hat{X}$ at this time.

B: Total value of the population according to existing records.

$\hat{X}$: Estimated value of the true population total.

$\hat{R}$: Estimated ratio of X to B, based on a sample (or: the actual ratio of the sample totals).

3-18. To make a ratio estimation practical, which of the following are necessary in regard to the a_i and b_i values?

__ 1. They should be approximately proportional.

__ 2. There should not be many differences between a_i and b_i.

__ 3. Differences must all be in the same direction.

__ 4. There should be at least a few differences between a_i and b_i.

1. Cost prices to retail prices.
2. Audited inventory amount to amount shown on a priced perpetual record.
3. Dollar amount of accounts more than 60 days past due to total receivables.

1-19. Although the foregoing may indicate several areas for applying ratio estimation, the objective is to illustrate the concept of linear relationship, - specifically, one in which the 'a' values in a population tend to be proportional to the 'b' values. In such a situation, no matter what the magnitude of the 'a' values, and how widely they are dispersed, the ratios of a_i to b_i will generally have a (SMALL/LARGE) standard deviation.

$\hat{X}$ is the estimated total inventory value based on audited prices and extensions.

2-5. Since we shall be using a ratio to estimate X, the remainder of this chapter discusses the steps involved in selecting and analyzing a preliminary sample.

The remaining rows in Exhibit 2 are essentially the same as "Data Sheets" in previous volumes. They can be completed by referring to the facts given in Exhibit 1.

Complete rows 3-6 in Exhibit 2, checking your answers in the next frame.

1. They should be approximately proportional.

4. There should be at least a few differences between a_i and b_i.

3-19. Which of the following should be true about the known (B) total? (Check your selection.)

__ 1. It must be given.

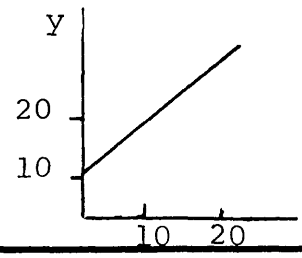
__ 2. It should be accurate in the sense of being free from footing errors.

__ 3. It should be accurate in the sense of closely approximating the true value.

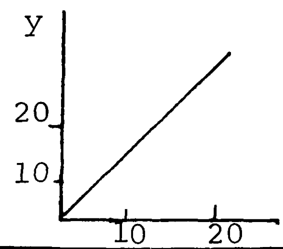
SMALL

1-20. Proportionality is a special type of linear relationship which can be expressed by the equation y=kx. This equality states that the value of y may be obtained by multiplying the x value by the constant factor k. This relationship can also be expressed graphically. Which of the following linear relationships represents a proportional relationship?

(A)



(B)



N: 5000

A: 70,000

R: 95%

U_R: 1.96

2-6. Since you can establish a true or audited value for each item in the inventory, and this audited value can be compared to the book value, the necessary condition for applying ratio estimation is present. Is it necessary for you to test the footing of the inventory records? (YES/NO) Why? ______________________________
____
____
____________________.

1. It must be given.

2. It should be accurate in the sense of being free from errors of arithmetic.

(The third choice is not a prerequisite of ratio estimation; rather it is something which the auditor is trying to ascertain.)

3-20. Why are many auditing problems well suited to ratio estimation?

____
____
____
____
____.

1-21. Choice (A) is incorrect. The relationship is linear, but not proportional. It fails to be proportional since y = 10 when x = 0, and it is consequently impossible to find a constant 'k' so that 10 = k(0). Look again at the graphs and then proceed to 1-22.

(No answer required)

If you choose (A), go to 1-21; if (B), go to 1-22.

2-7. Assuming the audited values are nearly proportional to book values, we would plan to use the ratio technique provided that ______________________________
____
____
______________________________.

YES

The ratio technique depends upon the total $2,500,000 being the sum of all the individual records.

3-21. Does the use of a ratio estimate, as opposed to a mean estimate, change any of the procedures we have learned concerning correspondence, route, and starting point in the random number table? (YES/NO)

1. There is usually a book value to which the auditor can compare his audited value.
2. The book and audited values are often approximately proportional.

No answer required	1-22. Choice (B) is correct. A proportional relationship can always be represented graphically by a straight line passing through the origin (where x = y = 0). In auditing, we do not expect an exact proportional relationship to exist. For example, cost-to-retail ratios might range between .50 and .75 to 1. Will there occasionally be a cost-to-retail value of .25 to 1? (YES/NO) Could there be a cost-to-retail ratio of more than 1 to 1? (YES/NO) Can we maintain that in general, retail price is approximately proportional to cost? (YES/NO)
there are a sufficient number of differences to permit estimation based on a reasonable sample size	2-8. You will often know if differences between book and audited values exist. They are quite common in inventories but may be rare in demand deposit bank accounts. Where differences are common, you can use a preliminary sample. Which of the following is the primary purpose of a preliminary sample? (Check your selection.) __ 1. To make a preliminary estimate of the total population value. __ 2. To estimate the variability associated with a particular method of estimation. __ 3. To determine the method of estimation to use.
NO	3-22. Suppose we used a preliminary sample to make a <u>mean</u> estimate instead of a ratio estimate. What would probably be the result? (Check your selection.) __ 1. The mean estimate would be identical to the ratio estimate. __ 2. We would do fewer computations, but probably require a larger sample size. __ 3. We would do more computations, but probably require a smaller sample size.

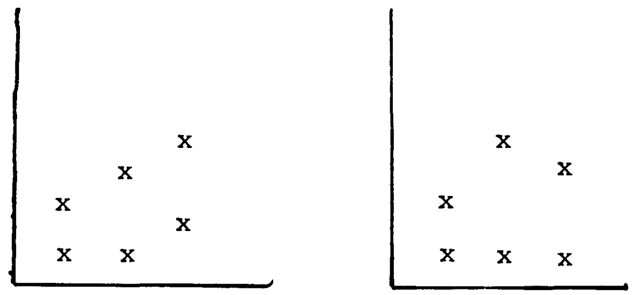
YES

YES (heavy markdowns)

YES

1-23. This suggests that if we plot cost vs. retail on a graph, the points will not all lie on a straight line going through the ______, but a line going through the origin will represent the data fairly closely. In which of the following two graphs would you expect a line through the origin to represent the data?

(A) (B)



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2. To estimate the variability associated with a particular method of estimation.

2-9. While (2) is the primary objective both (1) and (3) are important. We shall now show how to use the results of a preliminary sample in planning. In this text, using a preliminary sample means selecting a random sample of inventory items and establishing the audited value for each sample item.

For now we suppose that a preliminary sample is feasible, and that an unrestricted random sample without replacement is desirable.

NOTE: Volume 1 discussed "with replacement" to simplify the formulas, but in most applications "without replacement" is more useful; Volume 3 uses the more practical "without replacement" method.

(No answer required)

---

2. We would do fewer computations, but probably require a larger sample size.

3-23. Other factors being equal, ratio estimation tends to require a smaller sample size than mean estimation when

____

____.

Origin

(A)

1-24. We have now discussed our first question: Why use ratio estimation rather than mean estimation? Answer this question in your own words; then compare with the summary given in the next frame.__________

____.

No answer required.

2-10. To use a random number table to select our preliminary sample, we need to establish correspondence. One way this can be accomplished is ____.

the audited values are approximately proportional to the book values

END OF CHAPTER 3

See frame 1-25.

1-25. The discussion in frames 1-8 through 1-23 can be summarized as follows:

1. In mean estimation, as in Volumes 1 and 3, variability (standard deviation) is computed with reference to the dispersion of individual dollar values around the mean. If the range is wide the variability might require a sample size that is very large.

2. The advantage in using estimation is that ratios between 'a' and 'b' values will tend to vary less than the 'a' values themselves, provided that the 'a' and 'b' values are nearly ______________ to one another.

---

assigning a number to each item in the population

2-11. The necessity for observing some differences in the sample makes the selection of an appropriate size for a preliminary sample more involved than it was in Volume 1. Since the number of observed differences will depend both on the sample size and the incidence (rate) of differences in the population, your knowledge of the incidence of differences will play a key role in deciding whether to take a preliminary sample at all, and if so how large. As a rule of thumb, if you expect the incidence of differences to be smaller than 5%, using a preliminary sample may not be economical.

(No answer required)

CHAPTER 4. FINAL ESTIMATION

4-1. We are now ready to conclude the PQR inventory problem. Now that the required sample size has been determined, the remaining steps are similar to those techniques previously studied in this series. For an overview of these steps, read Appendix 3, chapter 4. Then locate Exhibit 11 and keep it easily accessible.

(No answer required)

proportional	1-26. In this context, what do we mean when we say that 'a' values and 'b' values are proportional? ________________________________.
No answer required.	2-12. If the incidence of differences is high (greater than 20%), then using a preliminary sample of 50 should produce several differences. If the incidence of differences equals 20%, attribute tables show that 95% of the time we will see at least 6 differences. (No answer required)
No answer required.	4-2. The process of combining the preliminary sample is common to all statistical sampling methods. The format in Exhibit 11 should therefore be familiar. The Worksheet already contains hypothetical results for the 161 additional elements, which added to the preliminary 50 will bring our sample size up to the required 211. Describe verbally, without actually doing the computations, how you would compute the ratio $\hat{R}$ based on the complete (not just the preliminary) sample. ________________________________.

There is an approximate linear relationship between 'a' and 'b' values represented by a = kb; that is, a straight line going through the origin.	1-27. Having gone over the rationale of ratio estimation, let us review the mathematics and the notation. X is the unknown total we wish to estimate. B is the known total with which X is to be compared by means of a ratio. When we say that B is known, does this mean that it is necessarily a correct total? (YES/NO) Explain your answer. ______________ ______________________________ ______________________________ ______________________________ ______________________________.
No answer required.	2-13. If the percentage of items showing differences lies somewhere between 5% and 20%, a preliminary sample of 50 is generally too small. Thus 80 to 100 may be necessary to obtain estimates of variability good enough for planning. In general then, the size of the preliminary sample is dependent upon ______________________ ______________________________ __________.
Divide the sum of 211 audited values by the sum of the 211 book values or Divide the sum of the differences by the sum of the book values and add the quotient to 1.0 (See Exhibit 11, row 16)	4-3. Exhibit 11 serves as a refresher in notation and in the mechanics of combining the preliminary and additional samples. The computations may be skipped, especially if you answered the previous question correctly. If you do the optional computations, check with the answers in Exhibit 12. If you do not complete this worksheet, glance at the answers and then go on to the next frame.

NO - for example, book value of accounts. If the total were known to be correct, there would be no need to make a statistical estimate in the first place. However, we must determine that the accounts foot to the total book value.

1-28. To estimate the ratio between X and B, we select a sample of the individual a_i and b_i elements. The required sample size and means of selection will be discussed in the next chapter. We know already that as in any statistical method of estimation, the statistical formulas are not valid unless it is a ___________ sample.

the incidence of differences in the population between book value and audited value

2-14. For illustrative purposes, the preliminary sample size used in this chapter is 50. For each sampling element selected - as, for example, "electric widgets, stock No. G-73215" - the auditor will determine a correct price and compute an extended value. The results in each of the 50 cases could be listed as in Exhibit 3. Note that the last column which is entitled "Difference" is computed by taking column __ minus column __.

Answers are in Exhibit 12.

4-4. The preceding worksheet has two purposes. First, we must compute the final ratio so that we can estimate the total population value. The computation could be done now but this will be more meaningful after we evaluate the sample results in terms of precision and reliability.

The second purpose of Exhibit 11 is to give the computational data necessary to make a revised estimate of the ratio population standard deviation. For this purpose, keep Exhibit 13 accessible.

(No answer required)

random	1-29. In our example, we listed not only the a_i and b_i values, but the individual ratios. This was done only to illustrate that ratios between a_i and b_i generally tend to vary less than the a_i values themselves. In practice, we need not compute the individual ratios. How, then, do we compute the sample ratio? ____________ ____ ______________.
9 6	2-15. Exhibit 4 contains the essential information for each of the 50 items in the preliminary sample. Notice that the first five entries are simply condensations of the data in Exhibit 3. The eleven observed differences could have been caused by: 1. ______________________. 2. ___________________________.
No answer required.	4-5. The formula for the estimated standard deviation which appears at the top of Exhibit 13 is the same formula used in connection with the preliminary sample, and Exhibit 13 is identical to Exhibit 7. For maximum teaching value these computations, unlike the earlier ones in this chapter, should not be skipped. The main distinguishing feature of ratio estimation, at least from a computational point of view, is the formula for the estimated standard deviation of the population of ratios. Now complete Exhibit 13, checking with the answers in Exhibit 14.

Divide the total of the a_i values by the total of the b_i values

1-30. The notation for the statement at the left would be:

$$\hat{R} = \frac{\sum a_i}{\sum b_i}$$

Why is there a caret over the R? ________
____.

What is an alternative way of expressing this formula? ________________________
____
______________________________________.

1. A difference in price.
2. A difference in extension.

NOTE: This test concerns only pricing and extensions.

2-16. While the auditor will be interested in the specific cause of any difference, for the stated statistical objective of estimating the variability of the population of ratios, it is (NECESSARY/UNNECESSARY) to specify causes.

Answers are in Exhibit 14.

4-6. As the result of the computations you have made in this chapter, the value of $\hat{R}$ is 1.02 and $S_{R_j} = 97.1$. The statistical objective is to estimate the total audited value of the population. This estimate is obtained in this example by multiplying ________ by _____________. To evaluate the agreement between this estimate and the true total audited value of the population, it is necessary to compute the _________ of our estimate at the desired ___________ level.

$\hat{R}$ is the estimated, not the true population ratio.

$\hat{R} = \frac{a}{b}$

$= \frac{\text{(total sample audited value)}}{\text{(total sample book value)}}$

1-31. Often in auditing it is simple to record exceptions - that is the amount of understatement or overstatement. In such circumstances, the information available consists of the book amount (b_i) and the difference between the book amount and the audited amount ($d_i = a_i - b_i$). Thus, if the book amount is \$250 while the audited amount is \$200 the difference is ________________.

The book value is larger and thus the difference corresponds to the amount of overstatement. If the audited amount had been \$300, the book amount would have been understated by \$50 and hence d_i = ____________.

UNNECESSARY, since the statistical procedures consider the magnitude of the differences regardless of cause.

2-17. To determine the sample size required to achieve a precision of \$70,000 at 95% reliability, using a ratio estimate, we need to compute an estimate of the standard deviation of the population of ______. This serves the same role as did (in Volume 1)

____

________________________________.

$\hat{R} = 1.02$

$B = 2,500,000$

$(\hat{X} = \hat{R} \cdot B = 2,550,000)$

precision

reliability

4-7. To compute the precision at the desired reliability level, turn to Exhibit 15. In this exhibit, rows 1-7 use the same formula as used in worksheet 3 of Volume 1. The formula for precision was

$$A = \frac{S_{X_j} \cdot U_R \cdot N}{\sqrt{n}}$$

. Here, the only change is that S_{R_j} takes the place of ____________

________________________________. The resulting formula appears in row 7 of Exhibit 15.

(50) or -50

50

Book Values (b_i)	Differences (d_i)
$1,000	$200
80	(8)
5	5
1,100	(50)
600	200
40	0
320	(18)
715	35
$3,860	$364

1-32. Note that in using the symbolic notation it is important to use the correct algebraic sign: "plus" corresponding to errors of understatement and "minus" corresponding to errors of overstatement. By adding the exceptions ($d = \sum d_i$), we can compute $\hat{R}$ by dividing the total exceptions by the total book values $\left(\frac{d}{b}\right)$, and adding this quotient to 1.0. In symbols, then, we can summarize this to say

$$\hat{R} = 1.0 + \frac{d}{b}$$

At the left, the data used earlier has been presented in this format. Using the formula above, what is the value of $\hat{R}$? ____________

---

ratios

the estimate of the standard deviation of the population of dollar values

2-18. The formula used in the exhibits is well suited to many auditing situations where most of the audited values agree with the book values. Turn to Exhibit 5 to see the required calculations. The first column indicates symbolically the operation to be performed. The second column describes the operation verbally Recall the definition of the symbol d_i (the observed difference in the i^{th} sample item) and state it symbolically.

d_i = __________

You should now complete Exhibit 5, checking your answers with Exhibit 6.

---

s_{x_j} (the standard deviation of dollar values)

4-8. In Volume 1 sampling was done with replacement, while in this volume we use sampling ____________________. This entails a modification of the formula for calculating the __________ of the ratio estimate. Rows 8, 9 and 10 of Exhibit 15 show how to calculate the required adjustment. In practice these steps can be skipped when the sample size is less than 5% of the population since the effect will be _____.

When $\frac{n}{N}$ = .05, for example, line 11 is 97% of line 7. For teaching purposes, however, you should complete all steps in Exhibit 15, checking your answer with Exhibit 16.

$\hat{R} = 1.09$ $= 1.0 + \frac{364}{3860}$	1-33. To obtain this value of $\hat{R}$, it was necessary to recognize that d = \$364, and hence $\frac{d}{b} = .09$. It follows that $\hat{R} = 1.0 + .09 = 1.09$. Remember, if the sample shows a net overstatement (d is negative), $\hat{R}$ must be______than 1.0, while if the sample shows a net understatement (d is positive) $\hat{R}$ must be_____than 1.0.
$a_i - b_i$ (audited value minus book value)	2-19. While the primary purpose of the preliminary sample is to obtain an estimate of the ___________________ of the population of ratios, the results also give us an estimate of the ratio. This estimate is contained in row 6 of Exhibit 5. Thus the observed ratio between audited value and book value is _________.
without replacement precision small	4-9. The calculated precision of \$64,313 (rounded to \$64,000) at a reliability level of 95% is smaller than the desired precision used in selecting the sample size. In some circumstances, the auditor may find that the precision achieved is larger than that desired - which indicates that the standard deviation was greater than anticipated. The auditor must then decide whether investigating possible causes or merely taking additional observations is more appropriate in the circumstances. (No answer required)

less greater	1-34. To review, suppose we wish to estimate the total cost of 1,000 items, the retail value of which is known to be \$9,000. This latter figure would be the (X/B) value.
standard deviation 1.02, $(\hat{R} - 1) = .02$	2-20. The data from Exhibit 5 provides the input to Exhibit 7 where we compute the estimated standard deviation of the population of ratios. As we noted before, this estimate plays the same role as ____ ______________________________ ______________________________ did in Volume 1. Again, Exhibit 8 is provided to enable you to check your computations in Exhibit 7. You should now complete Exhibit 7.
No answer required	4-10. The calculated precision of \$64,000 (rounded) at 95% reliability indicates that the chances are __ out of 100 that the <u>true</u> total value of the audited population differs from the estimated total value of the audited population by no more than ________.

B	1-35. A random sample of 80 items from the population has a total retail value of $800. The actual cost of the same 80 items totals $490. What is the estimated ratio for this population of cost to retail? ___
s_{X_j}: the estimated standard deviation of the population of dollar value	2-21. We have now completed all the preliminary steps to determine our required sample size for our given levels of precision and reliability. The formulas and computations were somewhat different, but the basic procedures are similar to those you encountered in previous volumes. (No answer required)
95 $64,000	4-11. As used here, the phrase "true total value of the audited population" refers to the total value you would obtain if you were to audit 100% of the items in the population. In this example, that would mean establishing a correct _____ and extension for each item in the inventory.

.61 If you were correct, the next frame may be skipped.	1-36. X, the unknown total, is total cost. The sample of 80 costed items, $\sum a_i$ = \$490. The corresponding 80 retail values, $\sum b_i$, turned out to be 800. The estimate of the population ratio, X to B, is the sample ratio, a to b. Dividing \$490 by \$800 and rounding to two decimals, the ratio is .61. (No answer required)
No answer required.	END OF CHAPTER 2
price (Note that inventory quantity is not being tested here.)	4-12. If the true and estimated totals ______ by no more than \$64,000 at 95% reliability, then we can also say that the interval beginning at the estimated total minus \$64,000, and extending to the estimated total _____ \$64,000, contains the true total with reliability of 95%. The lower value is called the lower precision limit and the higher value is called the upper precision limit. Turn now to Exhibit 17 and complete it, checking your answers with Exhibit 18.

No answer required

1-37. Although .61 may be a close estimate of the ratio of X (total cost prices) to B (total retail prices), it is of course subject to sampling variation. The true ratio might be less or more. In the following chapters, we discuss the variability of the sample ratio, and its effects on the precision and reliability of the final estimate. First, however, we discuss the mechanics of computing the estimate of total dollars.

(No answer required)

CHAPTER 3. COMPUTATIONS OF SAMPLE SIZE

3-1. In Chapter 2, we completed the preliminary sample computation and estimated the standard deviation of the population of ratios. Normally, the next step would be to compute the final sample size required to yield the $70,000 desired precision at 95% reliability. But for teaching purposes (and also in actual practice), a brief analysis of the preliminary results helps the auditor verify that he is on the right track. To make sure you have the necessary background facts, briefly reread Exhibit 1.

(No answer required)

differ

plus

Answers are in Exhibit 18

4-13. The calculated precision interval extending from $2,486,000 to $2,614,000 may fail to contain the true audited population total. However, the procedures used assure that if we were to repeatedly choose 211 items from this population, using unrestricted random sampling ________ replacement, 95% of the calculated precision intervals would contain the true value. Hence the chances that this one fails to contain the true value are ___.

No answer required.	NOW TURN BACK TO PAGE 3 AND BEGIN THE SECOND ROW
No answer required.	NOW TURN BACK TO PAGE 3 AND BEGIN THE THIRD ROW
without 5%	NOW TURN THE PAGE AND FLIP THE BOOK OVER SO THAT PAGE 40 IS RIGHT SIDE UP. CONTINUE WITH CHAPTER 4 EXACTLY AS BEFORE, BEGINNING WITH FRAME 4-14.

4-14. While the statistical objective was to estimate the total audited value of the inventory, based on audited __________ and extensions, the auditing objective was to decide whether the book value of $2,500,000 was fairly stated in terms of _________ and ____________. To relate these two objectives, you must choose a desired precision related to materiality and consider your actions under two contingencies: (1) the calculated precision interval contains the book value and (2) the calculated precision interval fails to contain the book value.

5-26. The ratio for each stratum is then extended by the total book value of the stratum, and these are added to produce the estimated total audited value of the population. If stratum 1 has a total book value of $600,000 and stratum 2 also has a total book value of $600,000, the estimated total audited value would be ______________.

6-5. Difference estimates are easily formed. For each item there is a book amount (b_i) and an audited amount (a_i), and consequently the difference ($d_i = a_i - b_i$). This difference will be zero when the book amount and audited amount agree, positive when the book amount is ______________, and negative when the book amount is _____________.

prices pricing extensions	4-15. If the book value is contained in the precision interval, we conclude that this test supports fair statement. This follows because the most the true value and book value could differ is the length of the precision interval at the stated reliability level and the length of the precision interval is smaller than an amount considered material. In our example, the lower precision limit is $14,000 below the book value, and the upper precision limit is $114,000 above the book value. This means we have 95% reliability that the book value is not overstated by more than $14,000, and not understated by more than $114,000. (No answer required)
$940,000 $[\frac{2}{3}(600{,}000) + \frac{9}{10}(600{,}000)]$	5-27. Note that in addition to the sample information we must know the total book value for each __________. This total book value is used with the sample ratio in each stratum to produce an estimated total audited value for each stratum. These separate estimates are then added together to produce the estimated ____________________ __________.
understated overstated	6-6. By examining each item in the population we establish the total difference, D. If we knew this total difference, the total audited value could be established by adding D to the total known book value, B. Difference estimation produces an estimate, $\hat{D}$, of the total difference, and based on this estimate we can estimate the total audited value by ____________________ $\hat{D}$ to the known total _______ value, B. This is expressed symbolically as $\hat{X} = B + \hat{D}$, where $\hat{X}$ stands for ____.

No answer required.	4-16. If the book value lies outside the precision interval, the statistical evidence does not support fair statement at the achieved precision and reliability. The lack of support of fair statement requires additional audit steps to determine whether there is a misstatement. We might say in these circumstances that at the specified precision and reliability, there is <u>potential</u> material misstatement. (No answer required)
stratum total audited value	5-28. The separate ratio estimate is useful when the ratios vary over the strata, but the sample size within each stratum must be large enough to obtain a separate estimate of the standard deviation of the ratio within each stratum. This means that some _______________ must be observed within each stratum. As a working rule, we suggest at least 30 observations per stratum.
adding book estimated total audited value	6-7. To estimate the total population difference, we can use either <u>unrestricted</u> random sampling or _______________ random sampling. If we use unrestricted random sampling, the average sample difference is multiplied by the number of items in the population to produce an estimate of the total population difference. In symbols, $\hat{D} = N\bar{d}$, where N is the number of items in the population and $\bar{d}$ is ____________________ _______________.

No answer required.	4-17. We have described a strategy to relate the sampling objective to the audit objective. This strategy calls for you to take different actions depending upon the outcome of the statistical test. If the precision interval contains the book value, you ____, while if the precision interval fails to contain the book value you ____.
differences	5-29. The other possible stratified ratio estimate is the combined ratio estimate. As the name implies, this estimate is calculated by combining the audited values over all strata and the book values over all strata to form a single ratio. To combine the audited values, proceed as in Volume 3; namely, compute the average audited value for a stratum, extend this by the number of items in the stratum, and add these extended values from each stratum. To do this with the data introduced in frame 5-25, we need to know ________________________________ and ____.
stratified the average sample difference	6-8. In Volume 1, mean estimation referred to the method of estimating the total population dollar value by multiplying the number of items in the population by the sample average. We could call the method described in frame 6-7 <u>mean estimation</u> applied to differences, because the methods are the same. Instead of the sample average of <u>dollar values</u>, we use the sample average of <u>differences</u>. Expressed in another way, in Volume 1 we were sampling from a population of dollar values; in this chapter we are sampling from a population of ________________.

accept the book value as fairly stated seek additional evidence	4-18. There are two ways this strategy could result in taking inappropriate action: the precision interval may fail to cover the book value when it should and the precision interval may cover the book value when it should not. You control the first type of error in selecting the reliability. Thus, if the reliability is set at 95%, you assume a ____ risk of not covering the book value when in fact it should.
number of items per stratum sample size per stratum	5-30. To complete the information, the sample size was 50 in each stratum; stratum 1 has 18,000 items while stratum 2 has 5,000 items. Recalling that stratum 1 yielded a total sample audited value of $1,000 while stratum 2 showed a total sample __________ value of $4,500, the combined estimated total audited value is ________________________.
differences	6-9. To illustrate difference estimation, we will analyze the problem of Chapter 2 from the standpoint of differences. The preliminary sample of 50 shown in Exhibit 4 had a total difference of 523. The average difference is _________ and N is 5,000. Thus the estimated total difference is ________________. The primary purpose of a preliminary sample is not this estimate but to estimate the _____________________ of the population of _________________.

5%	4-19. The second type of mistake is controlled by your choice of precision, which in turn is affected by your choice of ____________________. In other words, the smaller the length of the precision interval, the ___________ the risk of covering the book value when in fact the book value is not correct. Your chances of having the book value fall outside the precision interval increase as the magnitude of the difference between the book value and the actual value increases.
audited \$810,000 $[\frac{1000}{50}(18{,}000) + \frac{4500}{50}(5000) = 360{,}000 + 450{,}000]$ (If you obtained this answer, proceed to frame 5-32)	5-31. The procedure for obtaining the combined audited value is the same as used in Volume 3 to compute an estimated population value (see frame 5-14 and Exhibit 7 of Volume 3). The following table illustrates the procedure:

Total Audited Value	Sample Size	Sample Average	Stratum Size	Extended Value
\$1,000	50	20	18,000	\$360,000
\$4,500	50	90	5,000	450,000
				\$810,000

(No answer required)

\$10.46 \$52,300, (10.46 x 5000) standard deviation differences	6-10. The mechanics of calculating the estimated population standard deviation are exactly as in Volume 1. Exhibit 23 has been included in the Supplement to facilitate your computation. As you follow through the steps, compare the procedures to those you followed in Volume 1, work-sheet 1 (S-17). Notice that the differences d_j take the place of the dollar values x_j and the average $\bar{d}$ takes the place of the average $\bar{x}$. The resulting estimated standard deviation is symbolized as S_{D_j}: the estimated standard deviation of the population of ________________. You should now complete the steps in Exhibit 23, checking your answers with Exhibit 24.

sample size smaller	4-20. In fact, the chance that the precision interval contains the book value will be equal to $(\frac{100-R}{2})\%$ when the difference between the book value and actual value equals the length of the precision interval. In the PQR inventory example then, the risk would be ________________ of covering the book value when, in fact, the book value is understated by ________________. The same risk would apply to covering the book value when the book value is overstated by ____________________. Since this amount is smaller than the amount considered as material ($140,000), we have even smaller risk of missing a material error.
No answer required.	5-32. To obtain the combined estimated total book value, compute the average ____________ value for a stratum, extend this by ____________________________ and add the extended values of each stratum. The total sample book values were $1500 in stratum 1 and $5,000 in stratum 2. The average sample book values are _________ and __________. Extending these by the number of items in the stratum (18,000 in stratum 1 and 5,000 in stratum 2) yields $540,00 and $500,000, which added together yield $1,040,000.
differences	6-11. In Volume 1, the formula for the sample size is $$\sqrt{n} = \frac{S_{X_j} \cdot U_R \cdot N}{A}$$ A is the desired precision, U_R is the reliability factor appropriate for a reliability level of R%, N is the population size, and S_{X_j} is the estimated (dollar value) population standard deviation. To adapt this to difference estimation, only one change is required. What is it? __________________________.

2.5% \$128,000 \$128,000	4-21. This concludes the PQR Emporium example. You are advised to reread Appendix 3, Chapters 1 - 4 and to review the completed worksheets. In the following chapters, we will apply the technique or ratio estimation to an accounts receivable problem, in which the population will be stratified. It would be helpful to review the Summary and the worksheets in Volume 3. Even without the suggested review, you will find that the next example consists of concepts that are familiar either from the foregoing ratio estimation problem, or from Volume 3. (No answer required)
book number of items in the stratum \$30 \$100	5-33. The combined ratio is the quotient of the estimated total audited value divided by the estimated total book value. In the example this is _______________ divided by \$1,040,000 which yields .78. This combined ratio is multiplied by the total book value of \$1,200,000 to yield an estimated total audited value of ____________.
Change S_{X_j} to S_{D_j}	6-12. In addition to changing S_{X_j} to S_{D_j} we must also adjust because we are sampling without replacement. Turn to Exhibit 25. Compare this to worksheet 4 of Volume 1 (S-20) and note that column 9 is the same as row 7 except that we have used S_{D_j} rather than S_{X_j}. Also, note that Exhibit 25 is the same as Exhibit 9, except that S_{D_j} has replaced S_{R_j} in row 2. Complete Exhibit 25, checking your answers with Exhibit 26. (No answer required)

END OF CHAPTER 4

$810,000

$936,000, ($1,200,000 x .78)

5-34. In auditing, the combined ratio estimate is used more than the __________ ratio estimate. The chief reason is that it is not necessary to obtain differences in each stratum as it was in the case of the separate ratio estimate. Moreover, some strata may contain fewer than 30 observations provided the total sample size is reasonably large - about 50 as a working rule.

No answer required.

6-13. You have now used a preliminary sample to estimate the variability of the ________________ population and to compute the sample size necessary to achieve a _____________ of $70,000 at 95% __________. Note that the required sample size is about the same as that computed in Exhibit 11 using ratio estimation. To see why, examine Exhibit 33 where we have graphed the book and audited values of the preliminary sample. Points along the diagonal line are those whose book and audited values agree.

	CHAPTER 5 STRATIFIED RATIO ESTIMATION 5-1. In this chapter we discuss the steps necessary to adapt ratio estimation to stratified random sampling. The basic principles of stratified random sampling are discussed in Volume 3 of this series and are not repeated here. In Volume 3 we estimated a population total value using ________________. Here, of course, the objective is the same, namely estimating a population total, but we shall use the technique of ratio estimation.
separate	5-35. To determine how good any estimate is - that is, how close it is to the true value - we need to determine the _________ at a specified reliability. While it is possible to use formulas to calculate the precision of either a ____________ ratio estimate or a ____________ ratio estimate, the calculations involved are difficult to perform manually. Consequently, we discuss a simpler method which gives less exact results, but can be extremely useful in practice. The formulas are given in Appendix 1.
difference precision reliability	6-14. Those points above the diagonal line in Exhibit 33 represent errors of understatement (the audited value being larger than the book value). The points below the diagonal line represent errors of overstatement. In both, the vertical distance from the diagonal line measures the amount of over- or understatement. Examining the Exhibit, note that this vertical distance does not bear much relationship to the size of the book values. In other words, the differences do not appear to be ____________________ to the book values, and hence the condition favoring ratio estimation over difference estimation is not present.

mean estimation	5-2. Stratification is used with any method of estimation to obtain tighter ________________ for a given reliability and sample size, or smaller _____________ for a given precision and reliability. Stratified sampling plans with these characteristics are said to be more efficient than unrestricted random sampling.
precision separate combined	5-36. The method has a technical name - a replicated design - but is really quite simple. The basic idea of a __________ design is division of a sample into several smaller samples which we call subsamples. Estimates are calculated for each of the _______________. The dispersion among these subsamples allows us to measure the precision. To illustrate, suppose a sample of 200 is thought to be adequate to achieve a desired ________________ and reliability when an unstratified ratio estimate is used. You have already seen in Chapter 4 how to calculate the estimate of the total value and its precision.
proportional	6-15. Since the sample size of 205 required for difference estimation is nearly the same as the 211 previously used in the ratio estimation case, we shall use the larger sample size and suppose that the same sample items were chosen as in Exhibit 12. In other words, we shall evaluate these sample results using difference estimation. The first step is to compute the _________ _________________ of the population of differences. Exhibit 27 is included for this purpose. Use Exhibit 12 to obtain the values of sum of the differences and the sum of the squares of the differences. Now complete Exhibit 27 and check your answers with Exhibit 28.

precision sample size	5-3. A stratified sampling plan is more ___________ than unrestricted random sampling when the strata are selected so that the standard deviation within strata is much <u>LARGER/SMALLER</u> than the standard deviation of the whole population. Therefore, we desire to form strata with values within a stratum as <u>ALIKE/DIFFERENT</u> as possible.
replicated subsamples precision	5-37. Assume we drew the sample in 10 subsamples of 20 items each. In other words, we replicate or copy the sampling procedure 10 times to produce the required sample of 200 in 10 small samples of 20 items each. Labeling the sample items to identify to which subsample they belong, we compute a sample ratio for each subsample. Moreover, pooling all the subsamples, - that is, examining all 200 items - we produce the usual sample ratio. Since this is not obtained by averaging the subsample results, we shall use the term <u>pooled estimate</u> for this last estimate. (No answer required)
estimate of the standard deviation	6-16. The estimated standard deviation of the population of differences which you obtained in Exhibit 27 should be compared to S_{R_j} in Exhibit 14. Since the two numbers are close in value, what can you conclude about the population of differences in comparison to the population of ratios? ____ ____

efficient SMALLER ALIKE	5-4. In applications you must "designate the strata unambiguously, so that every element in the population is known, in advance, to belong to one and only one stratum." (This quotation is from the summary, page S-vii, of the Supplementary Section of Volume 3.) This limits the choice of a basis for identifying the strata to KNOWN/ESTIMATED characteristics of the population.
No answer required.	5-38. This results in 11 estimates: one for each of the _______ subsamples and the ______________________ estimate. We use superscripts to distinguish the subsample estimates, and the same notation as in earlier chapters for the estimate based on the whole sample. Thus, $\hat{R}$ refers to the pooled estimate since it does not have a _______________ while $\hat{R}^{(3)}$ refers to the estimate obtained from subsample number _______________. The eleven estimates referred to above can be written symbolically as: $\hat{R}$, $\hat{R}^{(1)}$, $\hat{R}^{(2)}$, $\hat{R}^{(3)}$, ..., $\hat{R}^{(10)}$.
The two populations have nearly the same variability	6-17. The computed precision of the ratio estimate was $64,000 (rounded to the nearest thousand). Do you think that the precision of the difference estimate will be (MUCH LARGER/MUCH SMALLER/ABOUT THE SAME)? Why? ______________________________ ______________________________

KNOWN

5-5. For example, in the MNO Tool Co. inventory in Volume 3, the basis for stratification was the type of lot - all standard item inventory lots in one stratum and all hand tool inventory lots in the second. Whether this stratification is worthwhile, i.e., is more ____________ than unrestricted random sampling, depends on how ____________ the dollar values of the lots are within each stratum.

10
pooled
superscript
3

Sub-sample No.	Book Value	Audited Value	Ratio
1	$1,000	$ 900	.90
2	1,100	1,000	.91
3	900	900	1.00
4	1,000	950	.95
5	800	900	1.12
6	1,100	1,050	.96
7	1,200	1,000	.83
8	900	850	.94
9	950	900	.95
10	1,150	1,050	.91
Pooled	$10,100	$9,500	.95

5-39. The table at the left illustrates this procedure numerically. Observe that each subsample provides an independent estimate of the population ratio, while the whole (pooled) sample is used in the ordinary way to provide the sample ratio to be used in estimating the total value.

In reading this table, $\hat{R}$ = ____________ while $\hat{R}^{(5)}$ = ____________.

ABOUT THE SAME

The estimated standard deviations are nearly equal.

6-18. Turn now to Exhibit 29. You now have the information required to compute the precision of the difference estimate. Observe that Exhibit 29 is nearly the same as Exhibit 15, used to compute the precision of the ratio estimate. As you can see, the only difference is that Exhibit 29 uses ______________________________ whereas Exhibit 15 uses ______________________________. Now complete Exhibit 29, checking your answeres with Exhibit 30.

efficient similar	5-6. In the inventory example the known characteristic is ______________ while the audited characteristic is ______________. When ratio estimation is used, the audited characteristic is the ratio of the audited amount to the book amount. To create efficient strata, we need to use a known characteristic which is related to the ______________.
.95 1.12	5-40. Note that the pooled estimate, $\hat{R}$ = .95, is calculated from the whole sample as before. It provides the basis for estimating the total population audited value as 95% of ______________. If the total book value is $100,000, we have, symbolically, $\hat{X} = \hat{R} \cdot B = \$95,000$ where $\hat{X}$ stands for ______________.
the estimated standard deviation of the population of differences the estimated standard deviation of the population of ratios	6-19. Since the estimate achieves the desired precision, we calculate the final estimate of the population total by using Exhibit 31. Note that in Row 3, the estimated total difference, $\hat{D}$, is required. To produce this estimate we multiply the ______________ by the ______________. Complete Exhibit 31 checking your answers with Exhibit 32.

type of lot dollar value ratio	5-7. It is not always easy to find a known characteristic of a population to serve as a basis for stratification. Some suggestions are: classes of accounts, inventory types, and book value. For a given choice of basis you would like the ______________________ of ratios to be small within each stratum.
total book value estimated total audited value	5-41. The purpose of the division of the sample into subsamples was to obtain an approximation of the precision. To do this, look for the highest ratio among the 10 subsamples, the lowest ratio among the 10 subsamples, and take the difference between them. You should observe that the highest ratio is 1.12 while the lowest is .83, so the ________________ is .29.
average sample difference ($\bar{d}$) number of items in the population (N)	6-20. Having an estimated total difference of $53,000 (rounded), the estimated total audited value is equal to $2,553,000 since $\hat{X} = B + \hat{D}$. The ratio estimate produced a value of $\hat{X}$ = $2,550,000. The closeness of these values in this example is unusually good. (No answer required)

standard deviation	5-8. The population of book values is a useful basis for stratification. Because of its practical importance, we use this basis in subsequent frames. After choosing the basis, you must choose the number of strata and the location of stratum boundaries. Computer programs provide sophisticated solutions to this problem. Here we present procedures found useful in manual operations. (No answer required)
difference	5-42. The difference between the highest and ____________ of the subsample ratios is then extended by the total population book value. Then that number is divided by the number of subsamples. For the example, we have .29 ( ) = \$29,000 and \$29,000/( ) = \$2,900. This number, \$2,900, represents the standard error of the estimate. In other words, this number takes the place of $NS_{R_j}/\sqrt{n}$ in the computation of precision (see Exhibit 15). To obtain the precision (with replacement, row 7), we need only multiply this number by U_R, the ____________.
No answer required.	6-21. The precision interval is from \$2,490,000 to \$2,616,000. Since this precision interval includes the book value, how much risk are you assuming if you accept the book value as fairly stated? ____________ ____________ ____________.

No answer required.	5-9. Generally three to five strata will produce a high degree of efficiency with the top stratum being sampled 100%. A frequently used procedure to locate the stratum boundaries is to divide the remaining book population in approximately equal dollar strata. If a population had a total book value of $2,000,000, and the top stratum consists of 20 items amounting to $500,000, the remaining $1,500,000 would be divided among the remaining strata; so, if we used 3 additional strata, each stratum would have approximately ________________.
lowest $100,000 10 reliability factor	5-43. We stated that $2,900 "takes the place of $NS_{R_j}/\sqrt{n}$" instead of "is equal to $NS_{R_j}/\sqrt{n}$." If we follow the formulas in Exhibits 13 and 15, we would not obtain 2900 as the value of $NS_{R_j}/\sqrt{n}$. However, we can expect the two numbers to be close, and rather obviously, the method we have just presented is easy. Therefore, it is recommended when ease of computation is important and approximate precision is sufficient. (No answer required)
5% with respect to the achieved precision of $63,000; less than 5% with respect to the desired precision of $70,000.	6-22. We have seen in this example that the method of difference estimation uses the differences in the same manner as the (audited) dollar values are used in Volume 1. The gain in efficiency arises because ________________________________ ________________________________. The analogy with mean estimation applies also when using stratified random sampling. Therefore, the techniques presented in Volume 3 concerning mean estimation of dollar values in stratified sampling apply to mean estimation of differences in stratified sampling.

$500,000	5-10. Once the stratum boundaries are selected, you must determine an appropriate sample size and allocate the sample to the strata. In Volume 3, the optimal allocation method was employed which takes into account the variability of each stratum as well as the number of elements. To use this method it is necessary to estimate the standard deviation in each stratum. How did Volume 3 suggest doing this? ________________________
No answer required.	5-44. It is necessary to keep track of the subsample to which the selected items belong. In using the selection techniques discussed in Volume 1 (modified so that sampling is without replacement), it is necessary only to divide the total sample - in the order in which it was selected - into the desired number of subsamples. Thus, for the example, in a total sample size of 200, and 10 subsamples, the first __ items selected would constitute subsample 1, the second __ items, subsample 2, etc.
the standard deviation of differences is smaller than the standard deviation of audited values.	6-23. In Chapter 5 we stressed that in planning to use a stratified random sample you must: 1. Choose the _____ for stratification. 2. Choose the ______ of strata and ________ of strata boundaries. 3. Choose the sample ____ and the __________ of this to the strata.

by using a preliminary sample of 30 from each stratum	5-11. To use the optimal allocation method in stratified ratio estimation, you must obtain an estimate of the standard deviation of ______ within each stratum. We apply the procedures discussed in Chapter 2 to each stratum separately. Which exhibit would you use to calculate the estimated standard deviation of ratios for each stratum from the preliminary sample? __________
20 20	5-45. Read the description of the problem concerning the auditing of the accounts receivable of the MNO Corporation. (See Exhibit 19.) It is often the case that you cannot use a preliminary sample for planning purposes. Rather, planning is done on the basis of previous experience, some knowledge of the population of book values, and some approximations. The important point is that the results of a statistical sample can always be evaluated. We run the risk, however, of not achieving the desired __________.
basis number, location size, allocation	6-24. In choosing the basis for stratification, you should choose a known characteristic of the population that is related to size of the ____________. The more effectively you can group items with similar differences within the same stratum, the smaller the sample size required to achieve a desired __________ at a specified ____________. Type of item, type of accounting control, location, and size of book value are frequently chosen as the basis.

ratios Exhibit 7 (If you did not obtain this answer review Chapter 2.)	5-12. After computing an estimate of the standard deviation of ratios within each stratum, you could then complete the appropriate exhibits in Volume 3 (Exhibits 8, 9, and 11) with your estimated standard deviation of ________ taking the place of S_i (the estimated standard deviation of audited dollar values).
precision	5-46. Exhibit 20 shows the sample data arranged by subsample within each of the two strata. Just as in the unstratified case, it is necessary to compute the estimated ratio from each subsample and the ______________ estimate. In addition, for the stratified case we must choose between a ______________ ratio estimate and a combined ratio estimate. Since computation would reveal little difference of the ratios between strata, we should choose the ______________ ratio estimate.
differences precision reliability	6-25. Using the book values as a basis for stratification can be very effective in many applications -- especially when there is an approximate linear relationship between the book and audited values. If book values are used as the ________, the number of strata will often be set at three to five when calculations are done manually. There are many ways to decide the location of stratum boundaries, but a frequently used procedure is to divide the dollars approximately equally among the strata (except the top stratum which is sampled 100%).

ratios	5-13. While the use of a preliminary sample to estimate the __________ of __________ is a sound method for deciding an appropriate sample size and allocation, there are many cases where selecting a preliminary sample would not be feasible. For example, using a preliminary sample would not be possible in many accounts receivable confirmation procedures. Even when feasible, the method has the disadvantage of requiring you to select enough observations from each stratum to observe some differences (see Chapter 2, frames 2-11 to 2-13).
pooled separate combined	5-47. For subsample 1 the estimated total audited value is computed as $13{,}000 \cdot \frac{\$53{,}900}{50} + 2000 \cdot \frac{\$172{,}380}{30} = \$25{,}506{,}000$. In this computation 13000 represents the number of items in stratum 1 while 50 represents __________. The estimated total book value from subsample 1 is $26,182,000. To obtain this, use the same formula with $55,900 taking the place of $53,900 and __________ taking the place of $172,380. The ratio of .97 is obtained by dividing $25,506.000 by __________.
basis	6-26. The choice of sample size and allocation to the strata can be done using procedures outlined in Volume 3 (see the Summary of Stratified Sampling Procedures page S-vii, in the Supplementary Section). These involve selecting a preliminary sample from each stratum to estimate the __________ of differences within each __________. As we pointed out in Chapter 2, the appropriate size of the preliminary sample depends upon the incidence of differences between book values and __________.

standard deviation ratios	5-14. In cases where preliminary samples are not feasible or desirable, we suggest two alternative approaches. The first uses the population of book values. You may take a preliminary sample of _______ values from each stratum, compute an estimate of the ____________________ of book values for each stratum, and use these estimates in Exhibits 8, 9, and 11 of Volume 3 with your estimated standard deviation of ______ values taking the place of S_i (the estimated standard deviation of audited dollar values).
sample size in stratum 1 $174,720 $26,182,000	5-48. Turn now to Exhibit 21. We have recorded the information for subsample 1. To make sure you can do the computations, you should complete the information for a few additional subsamples, checking your answers with Exhibit 22. In practice, it is unnecessary to extend each of the subsample ratios by the ____________________ of $25,000,000. However, this calculation is shown in Exhibit 22 to present the variation among the estimates.
standard deviation stratum audited values	6-27. It is not possible to give exact rules for the size of the preliminary sample, but as a practical approach we suggest selecting 50 items from each stratum, provided the incidence of _________ is 20% or more. In cases where this is not satisfied, either larger preliminary samples or other procedures should be used.

book standard deviation book	5-15. This alternative requires a __________ sample of book values only, and hence it is unnecessary to establish an ________ value for each item in the sample. Since you are using the population of book values instead of the population of ratios, you would expect this method to produce a sample size that is <u>SMALLER/LARGER</u> than that which would have been produced had you estimated the standard deviation of ratios.
total book value	5-49. As you observe from Exhibit 22, the lowest subsample ratio is ______ while the highest subsample ratio is 1.00. The difference is .04 which - extended by the total book value of $25,000,000 - yields $1,000,000. Dividing this by 10, the ________________________ gives $100,000 as the estimated __________________ of the pooled estimate.
differences	6-28. If a preliminary sample is feasible, and you have calculated an estimate of the _______________________ of differences in each stratum, you may use the same worksheets as in Volume 3 (Exhibits 8, 9 and 11) to calculate the sample size and allocation of the sample to the strata. In using these worksheets, you will substitute your calculated estimate of the standard deviation of _______________ in each stratum wherever the symbol S_i is used.

preliminary audited LARGER	5-16. While the sample size will be larger than required to achieve a specified ______ and ____________ using ratio estimation, this method does have the advantage of providing a sample size large enough to be representative of the book value population. To illustrate, suppose a population shows a total book value of $5,000,000 and you specified precision to be $250,000 with a reliability of 95%. The estimated total book value computed from the selected sample should be between __________ and __________. 95% of the time.
.96 number of subsamples standard error	5-50. To obtain the precision of __________ ____________________________ at 95% reliability, multiply the estimated standard error of $100,000 by______, the U_R factor. From Exhibit 22, we see that the pooled estimate is $24,500,000 with a precision of $200,000 (rounded) at 95% reliability.
standard deviation differences	6-29. When preliminary samples are not feasible, one alternative is to select a sample of book values from each stratum and use the book values instead of the audited values in the appropriate worksheets of Volume 3 (Exhibits 8, 9, and 11). This alternative uses the estimated standard deviation of book values instead of the estimated standard deviation of __________ within each stratum. Since the standard deviation of ____________ will generally be larger than the ______________________ of differences this method will produce a sample size larger than required to meet the desired precision and reliability.

precision reliability 4,750,000 5,250,000 (If you obtained the correct answer skip to frame 5-18)	5-17. To see why the answer given for frame 5-16 is correct, observe that the sample is selected from a book population with a known total book value ($5,000,000). Since the sample size was computed to achieve a precision of $250,000 at a reliability level of 95%, the estimated total book value calculated from the sample should differ from the true total population value by no more than $250,000, 95 times out of 100. Knowing the book value is $5,000,000 permits the observation that the estimated total book value should be within $250,000 of $5,000,000 and hence the answer. (No answer required)
estimated total audited value 1.96	5-51. If the accounts in excess of $10,000 showed an audited value of $3,900,000 compared to a book amount of $4,000,000, the estimated total audited value of the accounts receivable population would be ____________ with a precision of ________ at 95% reliability.
differences book values standard deviation	6-30. Evaluating the results of stratified random samples using difference estimation requires the same calculations on the observed differences as were performed in Volume 3 using the observed _________ values. These calculations involve: computing the new sample mean of differences and standard deviations of ______________ within each stratum (Exhibit 13), estimating the total population difference (Exhibit 7), and computing the precision of this estimate at the desired reliability level (Exhibit 14).

No answer required.	5-18. A second alternative separates the problem of selecting an appropriate sample size and allocating this to the strata. To decide the total sample size, use the procedures described in Chapter 2. These would provide a sample size adequate to meet your desired precision and reliability without stratification. This permits you to achieve your desired ________________ and ________________ even when stratifying would not yield a smaller sample size than unrestricted random sampling.
\$28,400,000 \$200,000	5-52. This gives the precision interval of \$28,200,000 to \$28,600,000 at 95% reliability. However, while the statistical evidence suggests that the book value of the accounts receivable is not accurate (since with 95% reliability the precision interval contains the _________ value), you must refer back to the statement of the problem in order to judge the fairness.
audited differences	6-31. In comparing the statements in 6-30 with those in Volume 3 (pp. S-vii and S-viii, nos. 12, 13 and 14), observe that we have emphasized that you treat the differences in exactly the same manner as you treated the ____________ values in Volume 3. The result is an estimate of the total population difference -- signified symbolically by $\hat{D}$. To estimate the total audited value, we should ______ this estimate to the total book value, B. This is expressed in symbols by $\hat{X}$ = ________.

precision reliability	5-19. To allocate the sample to the strata, we suggest you make the stratum sample sizes proportional to the size of the book amount in the stratum. If you had three strata and stratum 1 contained \$400,000, stratum 2 contained \$600,000, and stratum 3 contained \$1,000,000, the suggested allocation scheme would give _____% of the observations to stratum 1, _______% of observations to stratum 2, and _____% of the observations to stratum 3.
true	5-53. Since the statement of the problem asked for \$600,000 precision, based on \$1,200,000 being material, it would seem that this is the figure to use in judging fairness. Using the \$600,000 as the precision, the reliability of the precision interval from \$27,800,000 to \$29,000,000 is nearly 100%. Hence the test does support the fairness of the book amount. (No answer required)
audited add $B + \hat{D}$	6-32. Alternatively, we can use the subsample technique described in Chapter 5 to obtain the approximate precision of the difference estimate. Look again at Exhibit 20 to observe the column containing the total difference within each subsample in each stratum. Dividing each of the 10 subsample total differences in stratum 1 by 50, and each of the 10 ______________________ in stratum 2 by 30 gives the average difference in each subsample in each stratum.

20% 30% 50%	5-20. This suggested allocation scheme is very effective when the standard deviation of the differences between audited amounts and book amounts is nearly proportional to the size of the book amount. This means that the variability of differences ____________ as the size of the book values increases. For example, if the standard deviation of differences were $1 among accounts showing a book amount of $100, then you might expect the standard deviation of differences to be ______ among accounts showing balances of $1,000.
No answer required.	5-54. This illustrates the idea that when achieved precision is much better than planned precision, the audit implications of the statistical test should be judged with respect to the _______________ precision since this is chosen in relation to _________
subsample total differences	6-33. To obtain the estimated total difference in stratum 1 for each of the 10 subsamples, we extend each subsample average difference by the number of items in stratum 1 (13,000). Likewise, for stratum 2 we __________ each subsample average difference by the _____________________ in stratum 2 (2,000). By adding these stratum estimates for each subsample, we obtain 10 estimates of the total ______________ in the population.

increases $10	5-21. While exact proportionality between the ________________________ of differences and _______ of book values will not occur, the fact that such a relationship exists approximately in many applications makes this a good allocation method. If you base your stratification on book amounts, and create strata containing approximately equal dollars, this method will make the sample sizes within the strata nearly __________.
planned materiality	END OF CHAPTER 5
extend number of items difference	6-34. The results of these calculations are shown in Exhibit 34. Examination of the resulting estimates shows the largest estimate of overstatement is $1,161,015 while the only estimate of understatement is $76,980, so that the <u>range</u> of the subsample estimates is $1,237,995. To compute the precision of the pooled estimate, we must divide this range by the number of subsamples (10) and extend this by the appropriate reliability factor. (No answer required)

standard deviation size equal	5-22. Thus far in this chapter we have discussed topics pertinent to planning; namely, choosing a _______ for stratification, choosing the number of _______ and the stratum boundaries, and choosing a total ________________ and an allocation of this to the strata. Chapter 5 of Appendix 3 summarizes this material.
	CHAPTER 6 DIFFERENCE ESTIMATION 6-1. Four conditions have been emphasized as necessary for ratio estimation. These are: 1. A book value exists for each item in the population. 2. There are some observed differences between audited values and book values. 3. The total book value is known and corresponds to the addition of the individual book values. 4. To be efficient, audited values should be nearly proportional to book values. (No answer required)
No answer required.	6-35. Using 95% reliability, the factor is 1.96 and consequently the precision of the pooled estimate is ________________. The pooled estimate of the total difference is shown in Exhibit 34 to be ______________ ____. Since the known book value is $25,000,000, the estimated total audited value is ______ ____ for accounts in stratum 1 and stratum 2.

basis

strata

sample size

5-23. Once the sample size and allocation are determined, the next step is to select the sample. The process of selection was described in Volumes 1 and 3; it is not discussed here except to remind you that the sampling is without replacement. The formulas presented assume that the sample is selected without replacement.

(No answer required)

No answer required.

6-2. Like ratio estimation, difference estimation requires the first three conditions specified in frame 6-1. Unlike ratio estimation, however, difference estimation is more appropriate when there is not a near proportional relationship between the book value of an item and its corresponding audited value. Nonproportional relationships can entail either the lack of any linear relationship between book and audited values, or linear relationships which do not pass through the ______________.

242,648 (or 240,000 rounded)

$$(427,900) = 13,000\left(\frac{8350}{50}\right) + 2000\left(\frac{31620}{30}\right)$$

$24,572,100 (or $24,600,000 rounded)

6-36. To this we must add the audited value of the 150 accounts with book values exceeding $10,000. This amounted to $3,900,000, and hence the estimated total audited amount for the entire population is ______________ (rounded). Since the precision of this estimate is $240,000 (rounded) at 95% reliability, the precision interval ranges between $28,260,000 and $28,740,000.

No answer required.	5-24. After selecting the sample, appropriate auditing procedures are employed to establish the __________ value for each item in the sample. For each sample item we then have the following: the stratum to which it belongs, its _____________ value, and its audited value. Using this sample information, we can estimate the total audited value of the population and compute the __________ of this estimate at a desired reliability level.
origin	6-3. If there is a proportional relationship between the book and audited values, there is also a proportional relationship between the book values and differences. In this circumstance, if the equation describing the proportionality is $a_i = .92b_i$, which indicates that each audited value is 92% of its ______ value, then simple algebra would lead to $d_i = a_i - b_i = -.08b_i$. This means the difference is 8% of the book value, and the negative sign indicates that the book value is (OVERSTATED/UNDERSTATED) by _____.
$28,500,000	6-37. As suggested in Chapter 5, while the statistical evidence does not support the accuracy of the book value of $29,000,000 the fairness of the book value should be judged on the basis of the planned precision of $600,000. Thus, the interval between $27,900,000 and $29,100,000 has a reliability of nearly ______, and since the book value falls within this interval, the fairness of the book value is supported by the evidence.

audited book precision	5-25. Two different stratified ratio estimates are possible: the separate ratio estimate and the combined ratio estimate. In using the separate ratio estimate, we first calculate a ratio for each stratum. This can be done by dividing the total sample ________ value by the total sample ________ value within each stratum. If the total sample audited value from stratum 1 is \$1,000, while the total sample book value is \$1,500, and from stratum 2 the total sample audited value is \$4,500 while the total sample book value is \$5,000, then the ratios are $\frac{10}{15}$ for stratum 1 and ________ for stratum 2.
book OVERSTATED 8%	6-4. Lack of a proportional relationship between book values and audited values can therefore be expressed in terms of no proportional relationship between __________ and book values. Difference estimation is most appropriate when either there is no _________ relationship or the line describing the linear relationship does not pass through the origin. For example, in a population of customer accounts of a record club offering similarly priced records to its members, the size of any difference between the audited value and the book amount might bear little relation to the book amount.
100%	END OF CHAPTER 6

audited book $\frac{9}{10}$ or .90	NOW TURN BACK TO PAGE 40 AND BEGIN THE SECOND ROW.
differences linear	NOW TURN BACK TO PAGE 40 AND BEGIN THE THIRD ROW.