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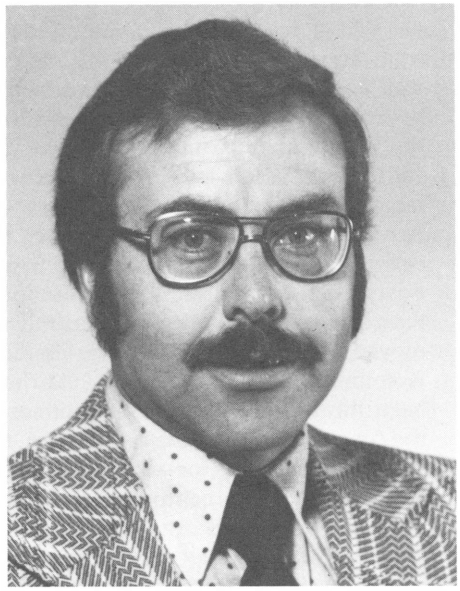

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The author explains the application of matrix theory to some common accounting problems.

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Research in accounting has revealed the benefits of utilizing interdisciplinary techniques in the formulation of specific theory.[1] Of increasing importance are the fields of mathematics, statistics, and computer science.[2] Although its use can in many cases produce "better results," the sophisticated level of mathematical analysis made available by these disciplines is, for the most part, ignored, discouraged, or not accepted.[3] Ironically, many of the reasons for this response seem to be the same as the advantages of using higher level mathematics. Mathematical notation facilitates formulation and recognition of relationships of variables; however, symbolisms and notations themselves are not easily understood or accepted. Through the use of mathematical notations, relationships can be established and evaluated, and a concentrated effort can be made on the problem itself rather than on the verbal terminology.[4]

This discussion will examine one area of accounting research involving mathematical analysis: matrix theory. Matrix theory can be applied directly to accounting and provides a method for analyzing particular accounting areas. In the simpler forms of matrix algebra, accounting theory finds both a theoretical approach and practical methodology. A more complex form of matrix theory is found in the concept of linear programming which requires the establishment of relationships of accounting data and results in a solution of values for an optimal relationship.

Matrix Models

Many authors have recognized the relationship between accounting data and matrix forms. Devine has pointed out that accountants have historically dealt with vectors in primitive forms of measurement in reporting financial information. His major criticism was that the information was not formulated to the extent necessary for utilizing effectively mathematical techniques.[5] Concentrating upon various data processing applications, Corcoran has done extensive analysis in matrix theory and accounting. Corcoran has concluded that there is a similar logic involved in the methods which can be described as an input-transformation-output procedure.[6]

Depreciation Models

Because of its common usage and diverse methods of valuation, depreciation is the first application to be examined. Following Corcoran's input-transformation-output method, depreciation can be viewed as the assets' bases (input) to be allocated, transformed by a rate over a period of time into a segmented schedule (output) showing the itemized annual expense. In matrix notation, this can be shown without elements as:

$$\underset{(N \times 1)}{\overset{A}{\begin{bmatrix} \quad \end{bmatrix}}} \bullet \underset{(1 \times M)}{\overset{x}{\begin{bmatrix} \quad \end{bmatrix}}} = \underset{(N \times M)}{\overset{B}{\begin{bmatrix} \quad \end{bmatrix}}}$$

The elements of A can be shown at any basis and can be as numerous as needed with the only limitation being that all elements must be depreciated by the same method and for the same time length. The rate vector, x, will have the same dimensions as the time length. The values of the elements of x are determined as fractions or percentages of annual allocations for whichever method is used. The product matrix B will have as elements each asset's yearly depreciation; i.e., b_{ij} refers to asset a_i and its depreciation in year x_j. Furthermore, $\sum_{j=1}^{N} b_{i1}$ will show the total depreciation for year 1 and $\sum_{j=1}^{M} b_{ij}$ will give the total depreciation allocation for the first asset shown in A.

As an example, assume 4 assets whose bases are $1,500, $3,000, $4,500, and $6,000 and all of which are to be depreciated over 5 years under a sum-of-the-years'-digits method.

$$\underset{(4 \times 1)}{\overset{A}{\begin{bmatrix} 1{,}500 \\ 3{,}000 \\ 4{,}500 \\ 6{,}000 \end{bmatrix}}} \bullet \underset{(1 \times 5)}{\overset{x}{\begin{bmatrix} 5/15, 4/15, 3/15, 2/15, 1/15 \end{bmatrix}}} =$$

$$B = \begin{bmatrix} 500 & 400 & 300 & 200 & 100 \\ 1{,}000 & 800 & 600 & 400 & 200 \\ 1{,}500 & 1{,}200 & 900 & 600 & 300 \\ 2{,}000 & 1{,}600 & 1{,}200 & 800 & 400 \end{bmatrix}$$

(4 x 5)

Further adjustments to the rate vector can be made to incorporate other information such as partial year allocation. For the previous example, if all the assets were purchased on May 1, the following rate vector would provide the proper annual depreciation.

$$x = \begin{bmatrix} 2/9, 13/45, 2/9, 7/45, 4/45, 1/45 \end{bmatrix}$$

(1 x 6)

Of all the depreciation methods available, double-declining balance would present the most problems for the matrix application. Using the declining-balance would require modification of the A matrix and a series of operations for each year or adjusting the x vector to reflect the different rates. A final depreciation schedule could be represented as a matrix whose elements are the separately computed matrices. For the above example, the procedure for adjusted matrix A is shown below:

$$.4\,A = B$$
$$.4\,(A - (B)) = C$$
$$.4\,(A - (B + C)) = D$$
$$.4\,(A - (B + C + D)) = E$$
$$.4\,(A - (B + C + D + E)) = F$$

The final depreciation schedule would be represented as a matrix composed of the vectors B, C, D, E, and F. The adjusted x vector which would accomplish the same results is shown below:

$$x = \begin{bmatrix} .4, .24, .144, .0864, .05184 \end{bmatrix}$$

The general form of the depreciation procedure can be changed to reflect only one year allocations by using the current year bases in the A matrix and corresponding percentages in the x vector.

Liquidation Models

A more complex matrix application can be made for indicating asset and creditor groups in preparation of a schedule of liquidation. Two matrices must be defined: one for realizable assets grouped by degree of pledged-security and one for the creditor groups and their respective claims upon each asset category. The input is the financial data of the asset categories; the transformation is the fractional creditors' claim on the asset categories; and the output is the monetary interest of each claimant in each asset category.

To use this procedure, the A matrix must contain realizable values and the owners' equity must be computed before values can be assigned to the B matrix. The A matrix is necessarily diagonalized for multiplication to occur. The B matrix will have a certain structure in that various creditor groups will have claims on related asset categories and none on other categories. For example, priority creditors will affect free assets and partially-secured creditors will not affect fully-pledged assets. The general form of the matrices is given below.[7]

Although the information listed in the C matrix is not itemized with respect to individual asset accounts, this is a minor point in view of the fact that money from the sale of the assets is distributed during liquidation, not the assets themselves. The schedule of the C matrix refers to this distribution of money by groups of liquidated assets which is the actual situation. The C matrix gives additional information that is not as obvious in the traditional Statement of Affairs.[8] The total interest of each creditor group can be found by summing the columns; i.e., $\sum b_{i}$. Likewise, each asset row indicates the amounts due each creditor group upon liquidation of the asset group.

A

Fully Secured	Partially Secured	Free Assets
a_{11}	0	0
0	a_{22}	0
0	0	a_{33}

(3 x 3)

B

Priority	Fully Secured	Partially Secured	Unsecured	Owners Equity
0	b_{21}	0	0	0
0	0	b_{23}	0	0
b	0	b_{33}	b_{34}	b_{35}

(3 x 5)

C

	Liabilities Having Priority	Fully Secured Liabilities	Partially Secured Liabilities	Unsecured Liabilities	Owners Equity
Assets: Pledged in Full Security					
Pledged in Partial Security					
Free					

Balance Sheet Models

Richards has written about a more general input-output matrix model. His model differs from those of Corcoran in that the inputs are the inflows and the outflows of the firm in the form of debits and credits. His model is divided into five basic areas of the balance sheet: all assets excluding long-term assets, long-term assets, equities, balance or catch-all accounts, and operations. The purpose of the model is to demonstrate the interrelationships of the accounts and to quantify that relationship.[9] The general formulas for the procedure are:

$$x - A\bullet = y \qquad \text{and } x = (I - A)^{-1} \bullet y$$

In the formulas, x is the vector of "account debits," A is the matrix formed by the five accounts indicating the inflow-outflow, and y is an unspecified vector of accounts in order of origination. The values of the y vector are the details of transactions. With this information, the effect on the related area and the size of that effect can be derived.

Due to the complexity of the form, Richards' input-output model will not be illustrated. It should be noted, however, that this model had a high correlation with the actual results of operations for Swift & Company during a six year period.[10]

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authors have listed practices that seem to them to account for the excellent management control they have found to exist in some nonprofit organizations.

This book could be very helpful for anyone involved in the operation of a nonprofit organization. Accountants are often asked to serve as board members for such organizations.

An active and concerned board of directors is a good point of beginning to improve the management control system within a nonprofit organization. An organization with a good control system can be assured that its goals are being reached in an efficient manner and that its service is of use to those it intends to serve.

Management Control in Nonprofit Organizations should be required reading for all members of government, particularly those involved with such mammoths as the Department of Health, Education, and Welfare, and the Department of Defense.

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"THE MARKETING OF ACCOUNTING SERVICES," D. Larry Crumbley, Ray Barnhardt, and Robert J. Boewadt, THE CPA JOURNAL, Vol. XLV, No. 5, May 1975.

The main thrust expressed by the authors is that CPA's must become "client oriented." The first step is a determination of client's needs by a program of marketing research.

While informal client research may yield information as to a client's needs and wants, market research should provide data involving what client's want, what they do not like, and why they have changed accountants. Long-term client relations will benefit from an analysis of data of this nature.

In some cases clients may not realize a firm can perform the service it wants. Exchanging views with lawyers, bankers, and other accountants provides useful information as to what these services might be and whether or not they should be added.

Once an accountant has located in a community, a marketing orientation toward clientele should be adopted. The accountant should attempt to create a favorable image among both present and possible future clients. This may be accomplished by accepting speaking engagements before both accounting and non-accounting groups; writing accounting articles aimed at the public-at-large or specific business groups; and engaging in activities of civic, religious, or political organizations.

To enhance the marketing of accounting services, the accounting profession should continue to prepare films and pamphlets to inform the public about the services performed by CPA's.

It is the opinion of this reviewer that a marketing orientation toward clientele by the individual accountant backed by group public information and publicity programs by the accounting profession will result in not only a cultivation of new clients, but more longstanding relationships with present clients.

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Conclusions

The applications presented here are by no means singular. There is virtually no limit to the number of possibilities for application, except, of course, if one limits the number of processes in accounting practice. Multi-reciprocal consolidations, secondary overhead allocations, period budgeting, responsibility accounting, and variance analysis for costs are other suggested areas for matrix uses.

The advantages of the matrix models are basically two: (1) their compact and simple format, and (2) the ease of application. As long as there can be identified input and formulated transformations, any output report can be constructed through the use of matrices.

Notes

[1]John W. Buckley, Paul Kircher, and Russel L. Mathews, "Methodology in Accounting Theory," *Accounting Review* XLIII (April, 1968), 281.

[2]A. Wayne Corcoran, "Applied Mathematics and Accounting," *Management Accounting* LI (August, 1969), 29.

[3]*Ibid.*

[4]*Ibid.*, pp. 29-30.

[5]Carl T. Devine, "Some Conceptual Problems in Accounting Measurement," *Research in Accounting Measurement: American Accounting Association Collected Papers*, Edited by Robert K. Jaedicke, Yuji Ijiri, and Oswald Nielson (Menasha, Wisconsin: American Accounting Association, 1966), p. 20.

[6]A. Wayne Corcoran, *Mathematical Applications in Accounting* (New York: Harcourt, Brace & World, Inc., 1968), p. 133.

[7]*Ibid.*, p. 162.

[8]*Ibid.*, pp. 160-165.

[9]Allen B. Richards, "Input-Output Accounting for Business," *Accounting Review* XXXV (July, 1960), 429-437.

[10]*Ibid.*, p. 435.

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Frequently, however, the correction process involves a certain amount of manual checking, discussion with the operational department involved, or reconstruction of the data. In these instances processing the correct items is usually allowed to continue without waiting for correction of the error item. When the error is corrected, the corrected transaction can be introduced into a subsequent or the next processing cycle.

Failure to exercise control over the correction procedure can be a serious weakness in a program for data control. There should be formal, carefully prescribed procedures to insure that once errors in data are discovered corrections are effected and the corrected data is properly reintroduced into the processing activities. One of the observations the auditor should make when reviewing a client's system of controls is the attention paid to the error correction program and to the assignment of responsibility for its implementation.

Procedures aimed at determining the accuracy of data are equally applicable to batch systems and real-time systems. In both cases data relationships such as the internal consistency of fields within a transaction record, the consistency of the transaction record to the master record against which it is applied, and the existence or absence of required fields for particular types of transactions can be used to check validity, whether one is handling one transaction in a real-time environment or a batch of transactions in a batch-processing environment. The primary distinction between correction in the two environments occurs in the time frame during which errors are recognized and acted upon.

In determining how extensive the checking facilities should be in a given installation, its management should make a conscious effort to measure the expense to the installation of instituting certain controls as well as the expense to the installation if the errors that these controls are designed to prevent should occur. The optimum situation from an installation management point of view would be to provide controls to the point where the cost of the controls would equal the cost of failure that could have been prevented by those controls. This optimum control level may be further affected by legal requirements, but it does not excuse the installation management from recognizing the relative advantages of alternative operations.