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Computation of Commissions and Federal Taxes: Where Commissions Are Based on Profits After Deducting Taxes

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WHERE COMMISSIONS ARE BASED UPON PROFITS AFTER DEDUCTING TAXES

By A. van Oss

The computation of commissions and federal taxes is in several instances complicated by the necessity of computing the commission upon profits after deducting the federal tax. T asmuch as the commission itself is a proper deduction from the gross income, the tax can not be computed without determining the commission, and, vice versa, the commission can not be computed without knowing the amount of the tax.

In the succeeding pages an attempt is made to explain how a procedure can be established and to show the method of dealing with several concrete cases under the 1920 tax, as follows:

Problem   I—Commission payable after deducting the entire amount of both excess-profits and income taxes.

"   II—Commission payable after deducting the entire amount of excess-profits tax but before deducting the normal income tax, or vice versa.

"   III—Commission payable after deducting a portion of the federal tax.

"   IV—Commission payable on a sliding scale after deducting the entire amount of both excess-profits and income taxes.

"   V—Commission payable after deducting taxes but considering the commission itself as a deductible expense.

In the discussion of the above problems the same hypothetical case has been used throughout, viz.:

Invested capital	$100,000.00
Income before deducting taxes and commission...	70,000.00
Rate of commission, to be applied to profits after deducting federal taxes	20%

Wherever necessary the amount of the income and the rate of commission have been modified to cover the particularities of the individual case.

In the treatment of the above problems the principal object has been to present a simple solution by means of ordinary arithmetic, in each case followed by a proof of its correctness. The first problem has been approached from different angles and has been treated somewhat elaborately to show the methods by which certain fundamental rules and formulæ that have been applied throughout can be constructed. Inasmuch as in actual practice it is not always evident under which bracket the excess-profits tax has to be computed, a special effort has been made to establish a procedure whereby this can be definitely ascertained in each case by arithmetical processes.

In problem IV, where the commission is payable on a sliding scale, this procedure becomes rather complicated and recourse has been taken to a very simple algebraic demonstration; also in the treatment of problem V, giving a solution where the commission is considered a deductible expense rather than a distribution of profit, a simple equation of the first degree has been used. The problem has been introduced not so much because it will be frequently met in actual practice as to dispel any notion that such a consideration will materially complicate the solution.

PROBLEM I

COMMISSION PAYABLE AFTER DEDUCTING THE ENTIRE AMOUNT OF BOTH EXCESS-PROFITS AND INCOME TAXES

First method of solution:

The method of calculating followed herein will be (a) first to determine the taxes on the basis of the profits before deducting commission; (b) then to calculate a preliminary amount of commission payable on that basis; and (c) thereupon to ascertain the correct amounts of both taxes and commission.

Computation of Commissions and Federal Taxes

(a) *Preliminary computation of the tax*

		Income	Tax
Excess-profits credit:			
8% of invested capital ($100,000).....	$ 8,000.00		
Exemption	3,000.00	$11,000.00	
Portion of income taxable at 20%:			
20% of invested capital ($100,000)....	$20,000.00		
Less excess profits credit.............	11,000.00	9,000.00	$ 1,800.00
Portion of income taxable at 40%.......		50,000.00	20,000.00
Preliminary amount of excess-profits			
tax			$21,800.00
Income tax—10% of:			
Taxable income		$70,000.00	
Less excess-profits tax...............	$21,800.00		
Exemption	2,000.00	23,800.00	
		$46,200.00	4,620.00
Preliminary amount of federal tax.....			$26,420.00

(b) *Preliminary computation of commission*

Income before deducting commission........................ $70,000.00
Deduct preliminary amount of federal tax.................... 26,420.00

Amount on which preliminary commission is to be computed.... $43,580.00
Preliminary commission of 20%............................ 8,716.00

(c) *Calculation of correct amount of taxes and commission*

(1) It is evident that the commission actually payable will be higher than $8,716.00; on the other hand, the tax will be less than $26,420.00.

(2) Inasmuch as the commission is payable out of that part of the profit which is taxable at 46%, the tax will be $46 less than $26,420.00, for every hundred dollars of commission payable, namely:

On account of excess-profits tax under the second bracket.......... $40.00
On account of income tax of 10% on the remainder................ 6.00

Total $46.00

(3) For the same reason the commission will be $9.20 (20% of $46) higher than $8,716.00 for every $46 of tax thus overprovided.

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(4) It follows from (2) and (3) that for every $100 commission actually payable the tax of $26,420 will be reduced by $46 and the commission determined ($8,716) increased by $9.20, so that

(5) The amount of commission ($8,716) determined in the above preliminary calculation is 90.8% of the correct amount, which will therefore be $\dfrac{\$8716}{90.8} \times 100 = \$9,599.12$ and

(6) the correct amount of the tax will be $\dfrac{46}{100} \times \$9,599.12 =$

$4,415.60 less than $26,420, or $22,004.40.

Second method of solution:

The problem may also be approached by (a) determining as the first step the commissions on the basis of the profits before deducting taxes (instead of first determining the taxes on the basis of the profits before deducting commission as above), (b) calculating the preliminary amount of taxes on the amount of net profit so computed, and (c) thereupon computing the correct amounts of commission and taxes—as follows:

(a) *Preliminary computation of commission*

Income before deducting taxes and commission.............	$70,000.00
Commission at 20%.....................................	14,000.00
Balance	$56,000.00

(b) *Computation of the tax on the basis of $56,000.00 taxable income*

	Income	Tax
Excess profits credit..............	$11,000.00	
Portion of income taxable at 20%..	9,000.00	$ 1,800.00
Portion of income taxable at 40%..	36,000.00	14,400.00
Preliminary amount of excess profits tax		$16,200.00
Income tax—10% of:		
Taxable income	$56,000.00	
Less—excess-profits tax........ $16,200.00		
Exemption 2,000.00	18,200.00	
	$37,800.00	3,780.00
Preliminary amount of federal tax.		$19,980.00

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(c) *Calculation of correct amount of taxes and commission:*

In this case the following should be observed:

(1) The commission actually payable will be less than $14,000.00, whereas the tax will be more than $19,980.00.

(2) For every one hundred dollars of taxes payable the commission will be $20 less than $14,000.00.

(3) For every $20 less of commissions paid the tax will be $9.20 more, namely:

On account of the excess-profits tax under the second bracket, 40%, or $8.00
On account of the income tax, 10% of the balance of $12.00, or.... 1.20

   Total $9.20

(4) It follows from (2) and (3) that for every $100 of taxes actually payable the commission of $14,000.00 will be reduced by $20 and the tax computed ($19,980) increased by $9.20, so that

(5) the amount of the tax ($19,980) in the above preliminary calculation is 90.8% of the correct amount, which will therefore be $\dfrac{\$19,980}{90.8}$ x 100=$22,004.40, and

(6) the correct amount of commission will be 20% of $22,004.40 =$4,400.88 less than $14,000.00, or $9,599.12.

Proof of correctness of solutions:

The following statement will prove the above results:

Income before deducting taxes and
   commission $70,000.00
Deduct—commission payable........   9,599.12

   Taxable income $60,400.88

Computation of tax

	Income	Tax
Excess-profits credit	$11,000.00	
Portion of income taxable at 20%...	9,000.00	$ 1,800.00
Portion of income taxable at 40%...	40,400.88	16,160.35
Total excess-profits tax........		$17,960.35

Income tax—10% of:

Taxable income		$60,400.88
Less—excess-profits tax........	$17,960.35	
Exemption	2,000.00	19,960.35
		$40,440.53	4,044.05

Total federal tax $22,004.40

Computation of commission

Income before deducting taxes............................. $70,000.00
Deduct—federal tax payable................................   22,004.40

Profit on which 20% is payable............................ $47,995.60
Commission @ 20% (as above).............................   9,599.12

General rules and formulæ:

It will be noted that the same percentage (90.8%) is applied to the preliminary results of each method and is computed by (1) applying the rate of commission (20%) to the rate of tax (46%), (the above example illustrating the procedure in cases falling under the second bracket), and (2) thereupon deducting the resulting figure (9.2) from 100, in this case leaving 90.8.

If the rate of commission is c and the tax rate is t, the general formula to ascertain the percentage X to be applied either to the preliminary amount of the commission obtained by the first method or the preliminary amount of the tax obtained by the second method will be

$$X = 100 - \frac{c\,t}{100}$$

In case the tax is to be calculated under the second bracket of the excess-profits tax, as in the example given, the above formula works out as follows:

$$X = 100 - \frac{20 \times 46}{100} = 100 - 9.2 = 90.8$$

If the tax is to be calculated under the first bracket,

$$X = 100 - \frac{20 \times 28}{100} = 100 - 5.6 = 94.4$$

If only the normal income tax is payable,

$$X = 100 - \frac{20 \times 10}{100} = 100 - 2 = 98$$

Inasmuch as there are many cases in which it is difficult to determine whether the tax is to be calculated under the first or the second bracket, it will be well to discuss a method whereby this can be readily determined.

Under the terms of the law, the 1920 excess-profits rate of the second bracket (40%) is applied to all profits over 20% of the invested capital. In other words, 20% of the invested capital is the high limit within which the 1920 rate of the first bracket (20%) will be applied, all income over and above that limit being taxable at 40%. It will also be seen that 20% of the invested capital would in such a case represent the earnings after deducting the *unknown* amount of the commission at the *known rate,* or the *taxable income.*

The problem is now reduced to the simple one of finding an amount from which a percentage is deducted when the percentage and the resulting balance are given, as for instance:

What is the amount (x) which leaves $16,580 after deducting 20% of x.

It will at once be seen that $16,580 represents 80% of x, which, therefore, is $20,725.

If we now apply this procedure to the adopted hypothetical case we find that 20% of the invested capital, or $20,000, represents the above-discussed taxable income. Upon this amount a federal tax of $3,420 is payable—$1,800 as excess profits and $1,620 as income tax—leaving $16,580 as the balance of profit after first deducting taxes and thereupon 20% commission.

The amount from which the commission is calculated is therefore $\frac{\$16,580}{.80} = \$20,725$, the commission $4,145, and the amount of income (before deducting either taxes or commission) beyond which the 40% rate will be applicable, $24,145.

In general terms, if the invested capital is C, the amount of the tax T and the rate of commission c, the amount which the earnings

(after deducting commission) can not exceed without becoming subject to the higher rate of excess-profits tax will be:

$$\frac{C}{5} - T \text{ divided by } \frac{100 - c}{100}$$

Again applying this test to the hypothetical case discussed in the foregoing pages, where the invested capital is $100,000.00 and the rate of commission 20%, we find:

$$\frac{\$20,000. - \$3,420}{.80} = \frac{\$16,580}{.80} = \$20,725$$

This is therefore the amount on which 20% commission, or $4,145, is to be paid, from which it follows that $24,145 is the limit beyond which, with an invested capital of $100,000 and the commission rate of 20%, the earnings before deducting taxes and commission can not rise without becoming taxable under the second bracket. The method or rule to follow in each case will therefore be:

(a) Take 20% of the invested capital........................ $20,000.00
(b) Compute the federal tax payable on this amount.......... 3,420.00
(c) The balance will represent the net profit after deducting
        taxes and commission............................... 16,580.00

(d) Divide this balance by $\dfrac{100 - c}{100}$ .... .80

(e) The quotient will represent the taxable income, viz........ 20,725.00
(f) Add the above amount of the tax....................... 3,420.00
(g) The sum will represent the limit of profit before deducting
        either taxes or commission......................... 24,145.00

PROBLEM II

COMMISSION PAYABLE AFTER DEDUCTING THE ENTIRE AMOUNT OF
EXCESS-PROFITS TAX, BUT BEFORE DEDUCTING THE NORMAL
INCOME TAX, OR VICE VERSA

Occasionally the commission is to be computed upon the earnings after deducting excess-profits taxes, but before deducting the normal income tax, or vice versa.

The solution of the first case is very simple and, applied to the standard problem used in this memorandum, will work out as follows:

Income before deducting commission or federal taxes..........	$70,000.00
Preliminary amount of excess-profits tax.....................	21,800.00
Amount on which preliminary commission is to be computed....	$48,200.00
Preliminary commission @ 20%............................	9,640.00

Inasmuch as for every $100 of commission actually payable the tax has been $40 overestimated, and for every $40 of taxes overestimated in the above calculation the commission will be $8 higher, it follows that the above preliminary amount of commission ($9,640) represents 92% of the correct amount, which is therefore $\dfrac{\$9,640}{.92}$ =$10.478.26. This will leave a taxable income of $59,-521.74 on which the excess-profits tax will be:

	Income	Tax
Excess-profits credit	$11,000.00	
Portion of income subject to 20% tax...........	9,000.00	$ 1,800.00
Portion of income subject to 40% tax...........	39,521.74	15,808.69
Total	$59,521.74	$17,608.69

Commission will therefore be payable on $70,000.00—$17,608.-69 = $52,391.31 and at the rate of 20% will amount to $10,478.26 as above.

It will also be seen that the formula, already given, to ascertain the percentage (X) to be applied to the preliminary amount of commission can be used also in this case, viz:

$$X = 100 - \frac{ct}{100} = 100 - \frac{20 \times 40}{100} = 100 - 8 = 92$$

Likewise the formula ($\frac{1}{5}$C — T divided by $\frac{100-c}{100}$) will provide the means to find the limit beyond which the taxable income will become subject to the 40% tax. The amount on which the commission would have to be paid would accordingly be

$$\frac{.80}{\$20,000 - \$1,800} = \$22,750.$$

The commission would be $4,550 and the high limit of income before deducting taxes and commission $24,550.

The solution of the alternate case, where the commission is payable on the net earnings after applying the normal income tax, but before deducting the excess-profits tax, is equally simple, viz:

Income before deducting commission or federal taxes..........	$70,000.00
Preliminary amount of income tax............................	4,620.00
Amount on which preliminary commission is to be computed....	$65,380.00
Preliminary amount of commission @ 20%....................	13,076.00

Inasmuch as for every $100 of commission actually payable the income tax proper will be decreased by $6.00 and for every $6.00 decrease in the tax the commission will be $1.20 higher, it follows that the above preliminary amount of commission ($13,076) represents 98.8% of the correct amount, which is therefore $\dfrac{\$13,076}{98.8}$

= $13,234.82. This will leave a taxable income of $56,765.18 on which the tax will be:

	Income	Tax
Excess profits credit.................	$11,000.00	
Portion of income subject to 20% tax.	9,000.00	$ 1,800.00
Portion of income subject to 40% tax.	36,765.18	14,706.07
Total excess-profits tax.........		$16,506.07
Income tax—10% of:		
Taxable income	$56,765.18	
Less—excess-profits tax........ $16,506.07		
Exemption 2,000.00	18,506.07	
	$38,259.11	3,825.91
Total federal taxes		$20,331.98

Commission will therefore be payable on $70,000 less $3,825.91 = $66,174.09 and at the rate of 20% will amount to $13,234.82, as above.

Also here the formula $X = 100 - \dfrac{ct}{100}$ can be applied, (t)

being in this instance 10% of the 60% remaining after the 40% excess-profits tax has been deducted from the taxable income, or

$$X = 100 - \frac{20 \times 6}{100} = 100 - 1.2 = 98.8$$

Likewise the limit beyond which the taxable income will become subject to the tax under the second bracket can be found with the formula previously used ($\frac{1}{5}$C—T divided by $\frac{100-c}{100}$). The amount on which the commission would have to be paid would accordingly be $\frac{\$20,000 - \$1,620}{.80} = \$22,975$. The commission would then be $4,595, and the high limit of income before deducting taxes and commission $24,595.

PROBLEM III

COMMISSION PAYABLE AFTER DEDUCTING A PORTION OF THE FEDERAL TAX

It may also happen that commission is to be computed after deduction of a portion of the excess-profits or income tax or both. The procedure is in principle the same as that followed in the previous examples. For instance, in case commission is to be computed on the net profit after deducting two-thirds of the entire amount of the federal tax the method will be as follows:

Income before deducting taxes and commission................ $70,000.00
Deduct two-thirds of preliminary amount of taxes, viz:

$\frac{2}{3} \times \$26,420 = $ 17,613.33

Amount on which preliminary commission is to be computed.... $52,386.67
Preliminary commission at 20%............................ 10,477.33

Inasmuch as for every $100 commission actually payable the tax will be decreased by $46 and for every $30.666 (or two-thirds of $46) of decrease in the tax, the preliminary commission will be increased by $9.20, it follows that for every $100 commission actually payable the preliminary commission will be increased by $\frac{2}{3}$ x $9.20, or 6.13\frac{1}{3}$. Consequently, the preliminary commission ($10,477.33) is 93.86$\frac{2}{3}$% of the correct amount and the latter amounts to $\frac{\$10,477.33}{.9386666} = \$11,161.93$. This will leave a taxable income of

$58,838.07 on which the federal tax will be

	Income	Tax
Excess-profits credit	$11,000.00	
Portion of income subject to 20% tax.	9,000.00	$ 1,800.00
Portion of income subject to 40% tax.	38,838.07	15,535.23
Total excess-profits tax		$17,335.23
Income tax—10% of:		
Total taxable income............	$58,838.07	
Less—excess-profits tax......... $17,335.23		
Exemption 2,000.00	19,335.23	
	$39,502.84 ·	3,950.28
Total federal tax		$21,285.51

Commission will therefore be payable on $70,000, less two thirds of $21,285.51 = $14,190.34, or on $55,809.66, and will, at the rate of 20%, amount to $11,161.93 as above.

The above percentage of 93.86⅔ can be obtained by the formula $x = 100 - \dfrac{ct}{100}$, in which t is two thirds of the tax rate (46%) or 30⅔%.

In the same manner will the formula $\dfrac{1/5\ C - \frac{2}{3}\ T}{\dfrac{100-c}{100}}$ lead to a quick determination as to whether the tax rate under the second bracket should be considered or not. The highest amount, leaving the tax within the first bracket, on which the commission would have to be paid would accordingly be

$$\frac{\$20,000 - \dfrac{2}{3} \times \$3,420}{.80} = \frac{\$17,720}{.80} = \$22,150$$

The commission thereon would be $4,430 and the high limit of income before deducting taxes and commission would be $24.430.

After the foregoing demonstrations there should be no difficulty in solving the majority of the problems arising when commissions are to be computed after deducting federal taxes in part or in total. It should now also be plain that their solution can be attained by comparatively simple arithmetical methods which lie within the grasp of average intelligence, and that no higher or lower algebra nor infinite series need enter into it.

PROBLEM IV

COMMISSION PAYABLE ON A SLIDING SCALE AFTER DEDUCTING THE
ENTIRE AMOUNT OF BOTH EXCESS-PROFITS AND INCOME TAXES

Sometimes the rate of commission payable is on a sliding scale,
for instance:

20% on the first..................................	$5,000
15% on the next..................................	5,000
10% on the next..................................	5,000
7½% on the next..................................	3,000
5% on the next..................................	3,000
2% on the balance.	

Also these cases can be solved by the general procedure used in
the solution of the preceding problems, viz.:

(a) *Preliminary computation of commission*

Income before deducting taxes and commission...............		$70,000.00
Commission:		
20% on $ 5,000..........................	$1,000	
15% on $ 5,000..........................	750	
10% on $ 5,000..........................	500	
7½% on $ 3,000..........................	225	
5% on $ 3,000..........................	150	
2% on $49,000..........................	980	
$70,000		$3,605.00
Balance		$66,395.00

(b) *Computation of the tax on the basis of $66,395 taxable income*

	Income	Tax
Excess-profits credit	$11,000.00	
Portion of income taxable at 20%...	9,000.00	$ 1,800.00
Portion of income taxable at 40%...	46,395.00	18,558.00
Preliminary amount of excess-profits tax		$20,358.00
Income tax—10% of:		
Taxable income................	$66,395.00	
Less—excess-profits tax........ $20,358.00		
Exemption 2,000.00	22,358.00	
	$44,037.00	4,403.70
Preliminary amount of federal tax.....................		$24,761.70

(c) *Calculation of correct amount of taxes and commission*

Inasmuch as for every $100 taxes actually payable, the commission will evidently be $2.00 less, which in turn will decrease the tax by $.92, it follows that the above preliminary amount of taxes is 99.08% of the correct amount, which is therefore $\dfrac{\$24,761.70}{.9908}$ = $24,991.62. This will leave an amount of $45,008.38. on which commission is payable as follows:

On the first $21,000.00 on the basis of the sliding scale.......... $2,625.00
On the balance of $24,008.38 @ 2%............................ 480.17

$3,105.17

After deducting this commission from the $70,000 earned, the amount of $66,894.83 will represent the taxable income, on which the tax will be:

	Income	*Tax*
Excess-profits credit	$11,000.00	
Portion of income taxable at 20%...	9,000.00	$ 1,800.00
Portion of income taxable at 40%...	46,894.83	18,757.93
Amount of excess-profits tax....		$20,557.93
Income tax—10% of:		
Taxable income.................	$66,894.83	
Less—excess-profits tax........ $20,557.93		
Exemption 2,000.00	22,557.93	
	$44,336.90	4,433.69
Amount of federal tax (as above)		$24,991.62

General rules and formula:

It will be seen that the formula $x = 100 - \dfrac{ct}{100}$ can be used to obtain the above percentage of 99.08 if (c) signifies the rate of commission payable on the residuary amount of the profits, viz., 2%.

In case commission is payable on a sliding scale the value of (c) is not always self-evident, nor is the tax rate (t) always readily determined. The problem is to find the limits beyond which the earnings before deducting taxes and commission would become

subject to (a) a higher tax rate and (b) a lower rate of commission.

The first limit (a) may be determined by the method given hereunder.

In the example given herein, an income (after deducting commission) of $20,000 (being 20% of the invested capital of $100,-000) is the high limit within which the 20% excess-profits tax rate will govern, and the total federal tax payable on that amount is $3,420. The balance of profit after deducting both taxes and commission is therefore $16,580. On $18,000 the commission would be $2,475, being computed at rates decreasing from 20% to 7½%, and on $21,000 it would be $2,675, computed at rates decreasing from 20% to 5%. The problem to solve is to find the amount which after deducting therefrom commission on the above sliding scale will leave $20,000 subject to the federal tax.

It is evident that the commission will be higher than $2,475, for it has to be computed on an amount exceeding $22,475, inasmuch as, otherwise, the balance subject to the federal tax would become less than $20,000. On the other hand, it will be less than $2,625, for, otherwise, it would have to be computed on an amount higher than $23,625, which would bring the taxable income under the second bracket. It follows, therefore, that the lowest rate of commission to be considered in this instance is 5%, applicable to earnings (after deducting taxes) between $18,000 and $21,000.

If we now call the limit beyond which, under the above commission arrangement, the earnings before deducting taxes and commission will be subject to the excess-profits tax under the second bracket x and the commission actually payable y it will be seen

(1) $x - \$3,420 - y = \$16,580.$

Deducting from x the amount of $18,000 and from y the commission of $2,475 payable thereon, we obtain the following equation:

(2) $(x - \$3,420 - \$18,000) - (y - \$2,475) = \$16,580 - \$18,000$ plus $2,475, or

(3) $(x - \$21,420) - (y - \$2,475) = \$1,055.$

Inasmuch as $(y - \$2,475)$ represents 5% commission payable on $(x - \$21,420)$, we substitute as follows:

(4) $(x - \$21,420) - \dfrac{5}{100} (x - \$21,420) = \dfrac{95}{100} (x - \$21,420)$

$= \$1,055.$

Multiplying both sides of the equation by $\dfrac{100}{95}$ we find

(5) x — $21,420 = $1,110.53, and
(6) x = $22,530.53.
Substituting the value of x in (3) we find
(7) ($22,530.53 — $21,420) — (y — $2,475) = $1,055, or
(8) y = $22,530.53 — $21,420 —$1,055 plus $2,475 = $2,530.53.

Deducting from $22,530.53 the tax of $3,420, there will be a balance of $19,110.53 on which commission is payable as follows:

On the first $18,000, as above......................	$2,475.00
On the balance of $1,110.53 @ 5%..................	55.53
Total as above	$2,530.53

Deducting this from $22,530.53 leaves a balance of $20,000, on which taxes are payable.

Therefore, if the earnings, before deducting commission on the above sliding scale, are higher than $22,530.53, they will become subject to taxation under both brackets of the excess-profits tax.

Instead of the above algebraic solution, the following general rule may be followed:

(a) Take 20% of the invested capital....................... $20,000
(b) Compute the tax payable on this amount............... 3,420
(c) The balance will represent the net profit after deducting
      taxes and commission 16,580
(d) Ascertain by inspection the portion of the earnings subject
      to the higher rate of commission................... 18,000
(e) Ascertain the commission payable thereon at these rates.. 2,475
(f) The rate of commission to be considered in subsequent
      calculations will be the next lower rate............ 5%
(g) Ascertain the portion of the income remaining after com-
      mission at that lower rate has been deducted there-,
      from, viz:
          (a plus e) — (b plus d), or (20,000 plus 2,475) —
          ($3,420 plus 18,000)............................ 1,055
(h) The amount will represent (100 — c) % of that portion.. 95%
(i) The portion of the income to which the lower rate of com-

      mission applies is therefore (g) divided by $\dfrac{100 - c}{100}$

          ($1,055 divided by .95)............................ 1,110.53
(j) The total amount subject to commission will therefore be
          (d) plus (i) $18,000 plus $1,110,53.................. 19,110.53
(k) Add the above amount of the tax (b).................. 3,420.00
(l) The sum will represent the limit of profit before deduct-
      ing either taxes or commission..................... 22,530.53

The limit beyond which the earnings, after deducting taxes, would become subject to a lower commission rate may be ascertained by inspection as already indicated in the particular case discussed in the preceding paragraph. In general, the amount beyond which the earnings will be subject to the higher excess-profits rate ($22,530.53 in the example used herein) should be ascertained first, so that the bracket under which the excess-profits tax is to be considered may be definitely established.

If the given amount of earnings before deducting commission and taxes is, say, $19,000 instead of $70,000, the tax thereon would be $3,140, namely $1,600 for excess-profits and $1,540 for income tax. The commission payable on the remainder of $15,860 would be $2,314.50, as follows:

20% on $5,000....................................	$1,000.00
15% on $5,000....................................	750.00
10% on $5,000....................................	500.00
7½% on $ 860....................................	64.50
$15,860	$2,314.50

It is evident that the taxes actually payable will be less than $3,140 and the commission more than $2,314.50, so that commission will be payable on more than $19,000 — $3,140 = $15,860, and on less than $18,000; the latter because more than $1,000 of taxes will be payable in the given circumstances. Consequently, the lowest rate of commission payable will be 7½% applicable to earnings between $18,000 and $15,000.

If the given amount of earnings before deducting commission and taxes should be, say, $23,000, the tax thereon would be $4,800, viz., $1,800 under the first and $1,200 under the second bracket of the excess-profits tax and $1,800 for the income tax. The commission payable on the remainder of $18,200 would be $2,485, viz., $2,475 on the first $18,000 plus 5%, or $10 on the remaining $200. Evidently the tax actually payable will be less than $4,800 and the commission more than $2,485, so that commission will be payable on more than $23,000 — $4,800, or $18,200, and on less than $21,000; the latter obviously because more than $2,000 of tax will be payable in the given circumstances. The lowest rate of commission payable will therefore be 5%, applicable to the earnings between $21,000 and $18,000.

Admittedly, the above method of ascertaining these rates by inspection is not scientific, but it seems hardly necessary to develop an algebraic method that will determine the exact limit beyond which the residuary earnings will become subject to lower commission rates. A close inspection will usually suffice to define a high and a low limit, and even if the wrong rates should be chosen, this can be rectified with less work than a proper determination of the exact limit will require.

PROBLEM V

COMMISSION PAYABLE AFTER DEDUCTING TAXES AND CONSIDERING THE COMMISSION ITSELF AS A DEDUCTIBLE EXPENSE

In the hypothetical case used throughout in problems I, II and III the consideration of the commission itself as first deductible from income would be equivalent to a reduction of the given percentage of commission (20%) to the actual (16-2/3%), for evidently 16-2/3% of every $100 of income before deducting commission ($16.66 2/3%) is equivalent to 20% of the remainder 83.33 1/3%). The solution of problem I in the circumstances would be as follows:

(a) *Preliminary computation of the tax*

By first method of solution of problem I....................... $26,420.00

(b) *Preliminary computation of commission*

Income before deducting commission.......................... $70,000.00
Deduct—preliminary amount of federal tax.................... 26,420.00

Amount on which preliminary commission is to be computed.... 43,580.00

Preliminary commission of 16-2/3%........................... 7,263.33

(c) *Calculation of correct amount of taxes and commission*

Reasoning on the same lines as in the first method of solution of problem I, it will be evident that:

(c-4) For every $100 of commission actually payable the tax of $26,420 will be reduced by $46 and the commission determined ($7,263.33) increased by 16-2/3% of $46, or $7.66 2/3 (instead of 20% of $46, or $9.20), so that

(c-5) The amount of commission ($7,263.33) obtained in the
above preliminary calculation is 92.33-1/3% of the
correct amount, which will therefore be

$$\frac{7{,}263.33}{92.33\text{-}1/3} \times 100 = 7{,}866.43,\text{ and}$$

(c-6) The amount of the tax will be

$$\frac{46}{100} \times 7{,}866.43 = 3{,}618.56,\text{ less than \$26,420, or \$22,801.44.}$$

Proof of correctness of above solution:

Out of profits amounting to....................		$70,000.00
there will be paid:		
Taxes	$22,801.44	
Commission	7,866.43	
		30,667.87
Leaving a balance of...........................		$39,332.13

20% of which equals the amount of commission.

In the above example the solution hinges upon the transforma-
tion of the given percentage (20) into the actual percentage
(16-2/3) that can be substituted in the solution given for prob-
lem I.

To establish a method by which this actual percentage (X)
can be directly determined from the given rate (c), it should be
observed

(a) That the given percentage applies to an amount that is X%
less than the amount of income before deducting com-
mission; therefore

(b) If the amount of income before deducting commission is 100,
the balance left after deducting commission at the
normal rate (X) would be 100 — X.

(c) This amount (100 — X) is the amount to which the given
rate (c) is to be applied, consequently

(d) The actual commission payable will be:

$$\frac{c}{100}(100 - X)$$

(e) This amount (d) added to the amount of (100 — X) will,
of course, equal 100, consequently

(f)

1. $\dfrac{c}{100}$ (100 — X) plus (100 — X) = 100.

2. c (100 — X) plus 100 (100 — X) = 10,000.

3. 100 c — c X plus 10,000 — 100 X = 10,000.

4. (100 plus ·c) X = 100 c.

5. $X = \dfrac{100\ c}{100\ \text{plus}\ c}$

The rule to follow in determining this actual percentage will therefore be:

To divide one hundred times the actual rate (2000) by one hundred plus the actual rate (120)

The quotient (16 2/3) will be the actual rate of commission to be used in the solution.

It may, therefore, be concluded that the methods used in any of the preceding problems will not be affected in the least by the provision that the commission itself is to be considered as a deductible expense. It will only be necessary to express the commission actually payable in a percentage of the earnings from which only the taxes have been deducted.