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DECENTRALIZED DETECTION WITH CORRELATED GAUSSIAN OBSERVATIONS:  
PARALLEL AND TANDEM NETWORKS WITH TWO SENSORS

A Thesis  
presented in partial fulfillment of requirements  
for the degree of Master of Science  
in the Electrical Engineering  
The University of Mississippi

by  
Shailee Yagnik  
December, 2019

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## ABSTRACT

Signal detection in cognitive radio involves the determination of presence or absence of a primary user signal so that the secondary user may opportunistically gain access when the spectrum is unoccupied. In decentralized sensing scheme, two or more secondary users sense the spectrum, process individual observation and then pass the quantized data to a fusion center where a decision with regard to which hypothesis being true, that is, a signal being present or absent, is made.

In the first part of thesis, we study the error performance in a parallel network consisting of two sensors. In the parallel configuration, each sensor quantizes it's own observation into a single-bit and transmits them to the fusion center. At the fusion center, the performance of AND and OR rules are examined by assuming the observations at the two sensors are jointly Gaussian, with specific means, variances and correlation coefficient, under hypothesis  $H_1$ , whereas the observations under  $H_0$  are still Gaussian with specific means and variances but are statistically independent. The optimum quantizers at each sensor are found by minimizing the probability of error at the fusion center. We use a genetic algorithm (GA) to find a sub-optimal solution. It was observed that, when prior probabilities of hypotheses are equal, AND performs at least as well as OR.

In the second part of the thesis, we study Bayes error performance of two-sensor tandem network designed to detect the presence or absence of deterministic signals in correlated Gaussian noise. Hence, the correlation coefficient remains identical under both hypotheses. Specifically, we address the question of which sensor ought to serve as the fusion center for optimal detection performance. In the process of this query, we draw some inference parallel to the Good, Bad and Ugly signal regions formulated originally for the two-sensor one-bit-per-sensor parallel fusion network by Willet,et.al. In the tandem Good region, nu-

merical results conclusively show that the strategy of placing better sensor, i.e the sensor with higher signal to noise ratio, serving as the fusion center is preferred for better detection performance.

## ACKNOWLEDGEMENTS

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University of Mississippi

Shailee Yagnik

December 2019

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## CHAPTER 1

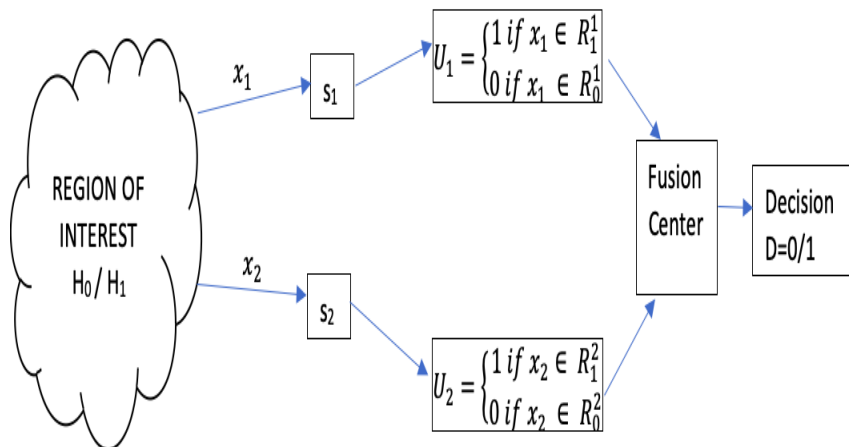
### INTRODUCTION

Recently, signal processing with distributed sensors has been gaining importance. The relatively low cost of sensors, the inherent redundancy possible with multiple sensors, the availability of high-speed communication networks, and increased computational capability have encouraged research in this topic [1]. In cognitive radio (CR) networks, spectrum detection is widely used to specify which frequency channel is being used by primary radio (PR) users, hence finding spectrum availability (a.k.a spectrum holes) is important [2]. Once a spectrum hole is detected, it is available for the secondary user to occupy the spectrum. There are two main types of methods used for making detection in the cooperative detection systems: Centralized Detection (CD) ([3]-[4]) and Decentralized Detection (DD) ([5]-[6])

DD makes processed data from each sensor available to a Fusion Center (FC) whereas in CD, all the sensors provide raw data to FC based on which a decision is delivered by the FC. Bandwidth limitations and high data costs prompted system designers to quantize the data at each sensor before it is relayed to the FC. This results in the degradation of overall performance of the system. Hence, it is important to understand the interplay of data compression, resource allocation, and the performance of distributed detection systems.

The works in [5] - [7] studied the case of sensors in a correlated sensing environment where the sensor sends a 1-bit decision to the FC (See Fig 1.1). By using a person-by-person optimization technique, a sub-optimal quantization rule at each sensor is arrived at by minimizing the probability of error ( $P_e$ ). The works [5] and [6] use AND, OR and XOR rules at the fusion center, whereas [7] used the majority rule (for more than two sensors) at FC. For optimal detection performance, a likelihood ratio test (LRT) based on

Figure 1.1. Parallel network with two sensors



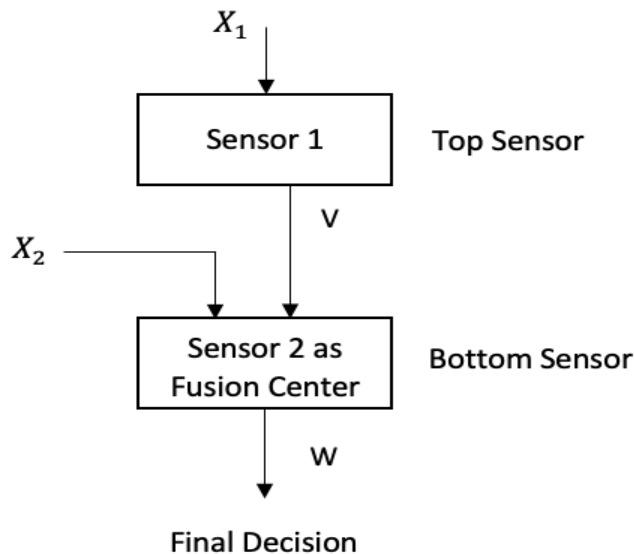
received information is optimal at the FC. In [7], the authors addressed the problem of having correlated sensing data at the sensors under the assumption that the marginal distribution of observations at the sensors are identical. It has been accepted that correlation degrades system performance, but more recently, [8] shows how strong correlation can be helpful in a centralized detection system for a large class of signal sets.

In a study of DD of testing of one of the two hypotheses, say  $H_0$  and  $H_1$ , where the noise corrupted signals at two sensors are bi-variate Gaussian conditioned on hypothesis, [5] explained the unexpected performance behavior of fusion of one-bit decisions from each sensor formulated as a Boolean logic. For this signal model, they identified three regions on the two-D signal plane, each axis denoting the signal level received by that particular sensor, where the behavior of the system performance can be characterized as good, bad, or ugly. A good region is where, for an optimized AND (or OR) Boolean fusion rule, each sensor employs a LRT and gives a decision favoring one of the two hypotheses by comparing the ratio of the likelihoods of two hypotheses with a threshold value that is optimized for that particular sensor. For the model in [5], likelihood ratio (LR) based on a sensor observation is

monotonic in observation. Hence, quantization of LR is equivalent to quantizing the sensor observations. In the bad region, either one of the sensors is ignored while the other sensor has a semi-infinite quantization interval for deciding  $H_1$ , or at least one of the sensors decides  $H_1$  based on its received observation falling in one of two or more unconnected intervals of observation. In the first situation, an optimal system would never ignore any information available to it, even though it is optimal for the given (AND or OR) Boolean logic fusion rule. In the second situation, when the sensor has two or more unconnected quantization intervals, complexity increases in finding the best-unconnected intervals. In either case, this region is termed a bad region. As numerical results have suggested, any decision rule, AND, OR or XOR (exclusive OR) can be optimal depending on the correlation coefficient and signal levels [5]. For optimal results, the sensors would employ multiple unconnected quantization intervals. In the ugly region, XOR is the best rule in several cases involving signal level, correlation coefficient and prior probability. In the sequel, when needed, we will simply refer to this paper as the GBU paper [5]. If the observations at the sensors are conditionally independent, then XOR will be a non-monotonic fusion rule and, thus, will never be optimal [1]. However, for dependence case and unequal signal levels at the sensors, for a distributed sensing system with correlated observation and a one-bit hard decision at each sensor,  $P_e \rightarrow 0$  as  $\rho \rightarrow 1$ , when the decision rule at the FC is XOR [9].

For the serial configuration of sensors, the first sensor passes its one-bit decision  $V$  to the second sensor and the second sensor makes a decision  $W$  based on the decision it received, and its own observation (see Fig 1.2). [10] shows that with two sensors, the serial configuration outperforms the two sensor parallel configuration, when the observations are conditionally independent on the hypothesis. This result holds true even if the observations are conditionally dependent. Papastrav and Athens [11] examined the two sensor serial configuration, the tandem network, and concluded that it is not always true to have the better quality sensor at the bottom of hierarchy when the observations at the sensors are conditionally independent under both hypotheses. Here, the better sensor is defined as the

Figure 1.2. Tandem network with two sensors



one receiving the stronger of the two available signals. However, in a practical scenario, independence between the sensors may not exist and sensor observations may be correlated.

Akofer and Chen established that for sensors in additive Gaussian noise, we get a better performance when the better sensor serves as FC for weak signal conditions [12]. For conditionally independent case, Akofer and Chen considered a modified form of tandem network, termed as interactive fusion [13]. In this form, the bottom sensor feeds back its decision to the top sensor, which then delivers the final decision using this decision and its own observation. The authors found that in general, feedback improves performance. However, for an asymptotically large number of independent samples drawn at the two sensors, the feedback shows no improvement in performance under Neyman-Pearson criterion [14]. In this thesis, we do not address interactive feedback.

## 1.1 THESIS WORK

As mentioned above, an earlier work looked at two sensors parallel network for the detection of deterministic signals in correlated Gaussian noise [5]. In this thesis, we consider the case where the signals at two sensors are independent under  $H_0$  but not under  $H_1$  for a

parallel network and numerically study the performance of AND and OR decision rules at the FC. Another contribution of this thesis is the partition of the signal plane into different regions for a tandem network for the case of GBU signal model. The goal is to identify good signal regions and design decision rules at the top and bottom sensors in good region, which will guarantee optimum performance. We find through numerical study that for the good region it is best to put the better quality sensor at the FC. For one, this configuration provides slightly better performance as compared to the reverse configuration. Second, the optimum performance is sensitive to the threshold of the top sensor if the better quality sensor were placed at the top, but not when the weaker quality sensor is placed at the top.

## 1.2 ORGANIZATION OF THESIS

The rest of the thesis is organized as follows. In chapter 2, we present the system model and concepts that are essential in understanding quantizer design and an analysis of the numerical results for parallel network of two sensors. This numerical study is for an extension of the model of [5] to the case of independent sensor observations under no signal hypothesis. Results are obtained through numerical investigation. In chapter 3, we present the system model of tandem network. We present the optimal decision rules at each sensor and identify the good and possible bad regions in signal plane. Numerical results are then analyzed in chapter 3. In chapter 4, we summarize and discuss the contributions of this work. Throughout the thesis, we consider only the minimization of Bayes error as the optimization criterion and did not consider the alternative Neyman-Pearson criterion. Recommendations for further research are also included in chapter 4.



## CHAPTER 2

### PARALLEL NETWORK PERFORMANCE ANALYSIS

Consider a set of two sensors sensing the presence of a PR in a frequency band by using a parallel network configuration given in Fig (1.1). Each sensor qauntizes it's information to single-bit and sends it to the FC. The physical channels between the sensors and the PR user, called sensing channels, are assumed to be additive noise channels, that are assumed to be independent of the signals present.

#### 2.1 SYSTEM MODEL

The hypothesis to be tested is the presence of the PR (hypothesis  $H_1$ ) over the channel versus the absence of the PR (hypothesis  $H_0$ ). Hence, the model under  $H_1$  can be defined as:

$$X_1 = Y_1 + V_1$$

$$X_2 = Y_2 + V_2$$

where  $X_1, X_2$  are the sensor observations at the two sensors,  $V_1, V_2$  are zero mean i.i.d Gaussian noise with variance  $\sigma_0^2 < 1$  and  $Y_1, Y_2$  are jointly Gaussian signals with means  $s_1$  and  $s_2$ , variances  $\sigma_{y_1}^2 = \sigma_{y_2}^2 = 1 - \sigma_0^2$ , and correlation coefficient  $\rho$ . Hence, the densities under each hypothesis can be defined as:

$$H_0 : X_1, X_2 \sim N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & 0 \\ 0 & \sigma_0^2 \end{bmatrix} \right)$$

$$H_1 : X_1, X_2 \sim N \left( \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right) \quad (2.1)$$

After the sensors sense the channel, each sensor quantizes its own observation to either 1 ( i.e PR is present:  $H_1$ ) or 0 ( i.e PR is absent :  $H_0$ ), according to the quantization rule  $U_i, i = 1, 2$ . The quantization rule decides 1 if  $x_i$  falls in the interval  $R_1^i$  and decides 0 when it falls in the interval  $R_0^i = \bar{R}_1^i$ . Hence  $U_i$  can be written as:

$$U_i = \begin{cases} 1, & \text{if } x_i \in R_1^i \\ 0, & \text{if } x_i \in R_0^i \end{cases} \quad (2.2)$$

Note that  $R_1^i \cap R_0^i = \phi$  and  $R_1^i \cup R_0^i = R$  the measure space of  $X_i$ . After a decision is made at the sensors, each sensor sends its decision to the FC on a reporting channel that is orthogonal and independent from other sensor's reporting channel. Based on the received decisions from every sensor, FC makes a global decision  $D = 1$  (i.e  $H_1$  is true) or  $D = 0$  (i.e  $H_0$  is true). The decision rule at the FC can be any Boolean logic rule, such as AND, OR or XOR.

## 2.2 MINIMUM PROBABILITY OF ERROR

In making a decision in any binary hypothesis testing problem, we can have the following four possibilities:

- (a)  $H_0$  is the true hypothesis, fusion center output is  $D = 0$
- (b)  $H_1$  is the true hypothesis, fusion center output is  $D = 1$
- (c)  $H_0$  is the true hypothesis, fusion center output is  $D = 1$
- (d)  $H_1$  is the true hypothesis, fusion center output is  $D = 0$

The first two correspond to correct decisions, whereas the last two indicate errors. In statistical literature, (c) is known as Type *I* error (or probability of false alarm) and (d) is known as Type *II* error (or probability of miss). The optimization problem is to design the decision

rules at sensor 1 and sensor 2 so that the overall Bayes cost  $C$  (or risk) is minimized. Let us consider the AND rule at the FC. We consider only uniform cost in this thesis, so the Bayes cost becomes the probability of error,  $P_e$ . Hence, the probability error of the decision system is given as following:

$$P_e = \pi_0 P(D = 1|H_0) + \pi_1 P(D = 0|H_1) \quad (2.3)$$

where

$$\begin{aligned} P(D = 1|H_0) &= \int_{R_1^2} \int_{R_1^1} p(x_1, x_2|H_0) dx_1 dx_2 \\ P(D = 0|H_1) &= \int_{R_0^2} \int_{R_0^1} p(x_1, x_2|H_1) dx_1 dx_2 \end{aligned} \quad (2.4)$$

Decision regions  $R_j^i$ , ( $i \in (1, 2), j \in (0, 1)$ ) could be semi-infinite region or a union of disjoint intervals. [5] showed the conditions for the good region, where the decision region would be semi-infinite region when the observations at the sensors are correlated with same correlation coefficient under both hypotheses. However, numerical results are obtained here for the model given by equation (2.1) where the correlation coefficient is zero under  $H_0$ .

As a reasonable solution to finding the optimum decision rules at the sensors, we use the Genetic Algorithm (GA) proposed in [9]. A GA in general yields locally optimum solution but initiations at several starting points could lead to a solution that is close to the optimum solution. Alternatively, using results in [12], we can observe that decide  $H_0$  regions for both sensors in AND fusion rule will be of the form: decide  $H_0$  if  $t_{i1} < x_i < t_{i2}$  ( $i = 1, 2$ ). However, for optimum result (i.e minimum  $P_e$ ), ( $t_{i1}, t_{i2}, i = 1, 2$ ) needs to be searched using a numerical procedure. See section (3.3) for a comment in a tandem case. The adopted GA has the following steps:

1. Generate a set  $K$  of solutions (i.e., a generation/set of chromosomes). In this work, we choose  $K = 12$ .

2. Evaluate the fitness function for each solution in  $K$  (i.e.,  $1-P_e$  of each solution).
3. Take a number of solutions that have the highest fitness function values from  $K$  and directly place them into the next generation of  $K$ . In this work, the chosen number of solutions is  $\frac{|K|}{2}$ .
4. Choose  $\frac{|K|}{4}$  pairs from the current  $K$  according to their fitness value by using the roulette wheel selection method. Then, perform  $c$  number of random crossovers between each pair to generate two new solutions. Doing this for all  $\frac{|K|}{4}$  chosen pairs produce  $\frac{|K|}{2}$  new solutions. These new solutions are used to fill the second half of the next generation (i.e., the  $K$  of the next iteration).
5. Perform mutation on the new solutions resulted from step 4 by flipping each bit with probability  $\frac{\epsilon}{100}$ . We used  $\epsilon = 5$ .
6. Repeat steps 2-5 for 6000 number of iterations.
7. Take the highest fitness valued solution from the resulting  $K$  as the final solution.

The author, Hadi Kasasbeh, generously shared his GA program with me to carry out this research. We present the results in the next section.

### 2.3 NUMERICAL RESULTS

In this section, we provide some of the numerical results obtained and study the performance of the system. In the GA optimization procedure, once the optimum decision regions at each sensors are found,  $P_e$  is computed numerically by solving the joint probability densities given in equation (2.4). In the tables shown below, I represents the sensor observation is ignored (i.e sensor always decides either  $H_1$  (in AND rule) or  $H_0$  (in OR rule), irrespective of the observation), S represents that particular sensor has a semi-infinite decision region and M represents the sensor decision region that has a union of multiple disjoint intervals.

Table 2.1. Performance comparison of two-sensor parallel configuration for AND and OR fusion rules, central LRT, better sensor as single sensor, when the correlation coefficient  $\rho = 0.3$ , noise variance  $\sigma_0^2 = 0.9$ ,  $\pi_0 = 0.5$

		AND Rule			Central LRT	Better Sensor as Single Sensor	OR Rule		
$s_0$	$s_1$	$t_1$	$t_2$	$P_e$	$P_e$	$P_e$	$P_e$	$t_1$	$t_2$
0.2	1.5	I	S	0.2207	0.2206	0.2206	0.2207	I	S
0.1	1	I	S	0.3039	0.303	0.3037	0.3039	I	S
1	3	I	S	0.0618	0.0582	0.0618	0.0618	I	S
1.5	2	S	S	0.1339	0.1131	0.1523	0.1482	S	S
3	2	S	S	0.0544	0.0409	0.0618	0.06	S	S
0.5	1	S	S	0.2976	0.2925	0.3037	0.3039	I	S
1	1.5	S	S	0.2072	0.1904	0.2206	0.2197	S	S
0.5	2	I	S	0.1524	0.1505	0.1523	0.1524	I	S
2	2	S	S	0.1068	0.0870	0.1523	0.1231	S	S
2	4	S	S	0.0192	0.0147	0.02	0.0201	I	S

Table 2.2. Performance comparison of two-sensor parallel configuration for AND and OR fusion rules, central LRT, better sensor as single sensor, when the correlation coefficient  $\rho = 0.5$ , noise variance  $\sigma_0^2 = 0.9$ ,  $\pi_0 = 0.5$

		AND Rule			Central LRT	Better Sensor as Single Sensor	OR Rule		
$s_1$	$s_2$	$t_1$	$t_2$	$P_e$	$P_e$	$P_e$	$P_e$	$t_1$	$t_2$
0.2	1.5	I	S	0.2207	0.2184	0.2206	0.2207	I	S
0.1	1	I	S	0.3039	0.299	0.3037	0.3039	I	S
1	3	I	S	0.0618	0.0603	0.0618	0.0618	I	S
1.5	2	S	S	0.1299	0.1182	0.1523	0.1524	I	S
2	2	S	S	0.1027	0.0929	0.1523	0.1364	S	S
3	2	S	S	0.0534	0.0466	0.0618	0.0618	S	I

**Table 2.2 continued from previous page**

		AND Rule			Central LRT	Better Sensor as Single Sensor	OR Rule		
0.5	1	S	S	0.2892	0.289	0.3037	0.3039	I	S
1	1.5	S	S	0.2011	0.1946	0.2206	0.2207	I	S
0.5	2	I	S	0.1524	0.1509	0.1523	0.1524	I	S
2	4	S	S	0.019	0.0165	0.02	0.0201	I	S

Table 2.3. Performance comparison of two-sensor parallel configuration for AND and OR fusion rules, central LRT, better sensor as single sensor, when the correlation coefficient  $\rho = 0.9$ , noise variance  $\sigma_0^2 = 0.9$ ,  $\pi_0 = 0.5$

		And Rule			Central LRT	Better sensor as Single Sensor	OR rule		
$s_0$	$s_1$	$t_1$	$t_2$	$P_e$	$P_e$	$P_e$	$P_e$	$t_1$	$t_2$
0.2	1.5	S	S	0.2106	0.156	0.2206	0.2207	I	S
0.1	1	S	S	0.2808	0.2002	0.3037	0.2889	S	S
1	3	S	S	0.0589	0.0513	0.0618	0.0618	I	S
1.5	2	S	S	0.1156	0.1055	0.1523	0.1524	I	S
3	2	S	S	0.0474	0.0439	0.0618	0.0618	S	I
0.5	1	S	S	0.258	0.2154	0.3037	0.3039	I	S
1	1.5	S	S	0.1778	0.1614	0.2206	0.2207	I	S
0.5	2	S	S	0.1444	0.1189	0.1523	0.1524	I	S
2	2	S	S	0.0884	0.0850	0.1523	0.1524	S	I
2	4	S	S	0.0178	0.0164	0.02	0.0201	I	S

Table 2.4. Performance comparison of two-sensor parallel configuration for AND and OR fusion rules, central LRT, better sensor as single sensor, when the correlation coefficient  $\rho = 0.7$ , noise variance  $\sigma_0^2 = 0.9$ ,  $\pi_0 = 0.9$

		AND Rule			Central LRT	Better Sensor as Single Sensor	OR rule		
$s_0$	$s_1$	$t_1$	$t_2$	$P_e$	$P_e$	$P_e$	$P_e$	$t_1$	$t_2$
0.2	1.5	M	S	0.0958	0.0793	0.0847	0.1	I	I
0.1	1	M	S	0.0995	0.0914	0.0971	0.1	I	I
1	3	S	S	0.0291	0.0287	0.0303	0.0317	I	S
1.5	2	S	S	0.0496	0.0474	0.0657	0.073	I	S
3	2	S	S	0.0228	0.021	0.0303	0.0317	S	I
0.5	1	M	I	0.1	0.0874	0.0971	0.1	I	I
1	1.5	S	S	0.0695	0.0689	0.0847	0.1	I	I
0.5	2	S	S	0.0636	0.0612	0.0657	0.073	I	S
2	2	S	S	0.0408	0.0354	0.0657	0.0730	S	I
2	4	S	S	0.009	0.0086	0.0105	0.0107	I	S
3	5	S	S	0.0021	0.0017	0.0028	0.0028	I	S
4	5	S	S	0.0017	0.000951	0.0028	0.0028	I	S

Table 2.5. Performance comparison of two-sensor parallel configuration for AND and OR fusion rules, central LRT, better sensor as single sensor, when the correlation coefficient  $\rho = 0.5$ , noise variance  $\sigma_0^2 = 0.9$ ,  $\pi_0 = 0.1$

		AND Rule			Central LRT	Better Sensor as Single Sensor	OR rule		
$s_1$	$s_2$	$t_1$	$t_2$	$P_e$	$P_e$	$P_e$	$P_e$	$t_1$	$t_2$
0.2	1.5	I	I	0.1	0.0801	0.0895	0.0858	S	S
0.1	1	I	I	0.1	0.0918	0.0994	0.0946	S	S
1	3	I	S	0.0334	0.0322	0.0324	0.0334	I	S
1.5	2	I	S	0.0768	0.0648	0.0702	0.0757	S	S

**Table 2.5 continued from previous page**

		AND Rule			Central LRT	Better Sensor as Single Sensor	OR rule		
3	2	S	I	0.0334	0.0271	0.0324	0.0334	S	S
0.5	1	I	I	0.1	0.0963	0.0994	0.1	M	I
1	1.5	I	I	0.1	0.0883	0.0895	0.0998	M	S
0.5	2	I	S	0.0768	0.0671	0.0702	0.0739	S	S
2	2	I	S	0.0768	0.0531	0.0702	0.0675	S	S
2	4	I	S	0.0113	0.0098	0.0112	0.0113	S	S
3	5	I	S	0.003	0.002	0.003	0.003	S	S
4	5	I	S	0.003	0.001	0.003	0.0027	S	S

In Tables 2.1-2.2, we notice that when the prior probabilities are equal, AND rule performs at-least as well as OR rule for all values of  $\rho$  and  $\sigma_0^2$ . From Tables 2.1-2.3, we can notice that with increasing  $\rho$ , for AND rule, the probability of error decreases somewhat, whereas for the OR rule, correlation has no effect on the system performance. In Tables 2.4-2.5, we observe that when prior probabilities are not equal, there are certain cases where information from both sensors is ignored at the FC for both the AND and OR rules. For example, looking at the signal pair (0.2, 1.5) in Table 2.4, under the OR fusion rule, the information from both the sensors is ignored, whereas for the AND rule at the FC, under the same conditions, the weaker sensor quantization has multiple interval regions and the stronger sensor implements a single threshold LRT. However, under the same conditions, the performance of the better sensor as a single sensor outperforms the performance of AND and OR as FC decision rules. Also, if we look at the signal pair (0.5, 2) in Table 2.5, the performances of AND and OR are worse as compared to the single sensor performance of the better sensor. This means that the GA ended up in a sub-optimal solution as the GA algorithm failed to pick the AND rule having the weaker sensor always deciding  $H_1$  and the better sensor implementing a single threshold LRT. Based on the above observations, we can



conclude the following:

1. For equal prior probabilities, the AND rule performs at-least as well as the OR rule.
2. For equal prior probabilities and AND rule, the probability of error decreases gradually as  $\rho \rightarrow 1$ . However, under the same conditions, the performance of the OR rule is about the same as the standalone performance of the better sensor as single sensor.

## CHAPTER 3

### TANDEM NETWORK PERFORMANCE ANALYSIS

Consider a set of two sensors in tandem network monitoring a region of interest to determine the presence or absence of a signal. The first sensor quantizes its observation to a signal bit and passes it to the second sensor. The second sensor makes a final decision regarding which one of the two hypotheses is true, based on its own observation and the signal bit it had received.

#### 3.1 SYSTEM MODEL

Let  $H_0$  be the null hypothesis (no signal is present) and  $H_1$  be the alternative hypothesis (signal is present) with prior probability,  $\pi_0$  and  $\pi_1$ , respectively. The signals are assumed to be deterministic and are received in additive Gaussian noise. The received signal under both hypotheses are shown below

$$\begin{aligned} H_1 : \mathbf{X} &= \mathbf{S} + \mathbf{K} \\ H_0 : \mathbf{X} &= \mathbf{K} \end{aligned} \tag{3.1}$$

where  $\mathbf{X}^T = [X_1, X_2]$ ,  $\mathbf{S}^T = [s_1, s_2]$ . Noise  $\mathbf{K}$  is assumed to be a bivariate Gaussian with zero means, unit variances and correlation coefficient  $\rho$ . Since  $s_1, s_2$  can take either positive or negative values, without any loss of generality, we can restrict  $\rho$  to be between  $[0, 1]$  and also assume variances to be unity. Hence, the probability density functions under each hypothesis is given as:

$$\begin{aligned} H_1 : \mathbf{X} &\sim N(s_1, s_2, 1, 1, \rho) \\ H_0 : \mathbf{X} &\sim N(0, 0, 1, 1, \rho) \text{ where } 0 \leq \rho \leq 1 \end{aligned} \tag{3.2}$$

The tandem network configuration of these two sensors is shown in Fig 1.2. The top sensor, which receives the signal  $X_1$ , is denoted as sensor 1, whereas the bottom sensor, denoted as sensor 2, acts as the fusion center. Sensor 1 decision, denoted as  $V$ , is passed on to the sensor 2, which then combines  $V$  with its own observation  $X_2$  and delivers the final decision  $W$  regarding which one of the two hypothesis is true. We assume both the sensors use a single bit to represent their own decisions [6]. When sensor 1 (sensor 2) decides in favor of  $H_1$ ,  $V = 1$  ( $W = 1$ ). Similarly,  $V = 0$  ( $W = 0$ ) denotes sensor 1 (sensor 2) deciding in favor of  $H_0$ .

### 3.1.1 MINIMUM PROBABILITY OF ERROR

The optimization problem is to design the decision rules at sensor 1 and sensor 2 (serving as a fusion center) so that the overall Bayes cost  $C$  (or risk) is minimized. In this thesis, we consider the minimum probability of error criterion, which is the minimization of Bayes cost with 0-1 cost function. It is assumed that signal levels at the sensors, namely  $s_1, s_2$ , prior probabilities of each hypothesis  $(\pi_0, \pi_1)$  and  $\rho$  are known. The probability of error, denoted as  $C(W)$  in order to show its explicit dependence on the final decision  $W$ , can be written as :

$$C(W) = \pi_0 P(W = 1|H_0) + \pi_1 P(W = 0|H_1) \tag{3.3}$$

### 3.1.2 OPTIMUM DECISION RULE AND IDENTIFICATION OF GOOD REGION

In this section, we derive the nature of the decision rules at both the sensors so that the probability of error at sensor 2 ( $C(W)$ ) is minimized. Similar decision regions were derived for the random signals when the sensor noises were considered to be independent under both hypotheses [12]. However, no such derivations exist for deterministic signals in a correlated Gaussian noise environment. Furthermore, by studying the nature of those rules, we arrive at the good region for the two-sensor configuration, where decision rules that decide  $H_1$  are semi-infinite intervals on the real line, i.e., sensor 1 decides  $V = 1$  if  $X_1$  falls within a semi-infinite interval and sensor 2 decides  $W = 1$  when its observation falls over one semi-infinite

region, when  $V = 1$  and another semi-infinite region, when  $V = 0$ . For simplicity, let us use the following convention and notations for both the sensors: decision region favoring hypothesis  $H_1$  is denoted as  $R$  with a subscript denoting whether it is the decision region of sensor 1 or sensor 2. The complement of this region  $\Omega - R$  belongs to the decision region deciding  $H_0$  for each sensor. For sensor 2, an additional subscript on  $R$  will indicate if sensor 1 variable  $V$  is either 1 or 0. Let  $P(V = 1|H_0) = p_f$  (probability of false alarm of top sensor),  $P(V = 1|H_1) = p_d$  (probability of detection of top sensor). Probability density notations are  $p(x)$  for a marginal density,  $p(x, y)$  for a joint density, and  $p(x|y)$  for a conditional joint. Since  $C(W)$  is a function of both  $V$  and  $W$ , by optimizing  $C(W)$  with respect to both  $V$  and  $W$ , given the other variable is fixed, we obtain necessary conditions that yield the minimum error probability  $C(W)$ , as shown in the Appendix A. The results from the analysis provide the following decision rules. These rules are person-by-person optimal solutions [5].

$$R_{W|V=1} = \left\{ x_2 : L(x_2) \equiv \frac{\int_{R_{V=1}} p(x_1|x_2, H_1)p(x_2|H_1)dx_1}{\int_{R_{V=1}} p(x_1|x_2, H_0)p(x_2|H_0)dx_1} \geq \frac{\pi_0}{\pi_1} \right. \quad (3.4)$$

$$R_{W|V=0} = \left\{ x_2 : L(x_2) \equiv \frac{\int_{R_{V=0}} p(x_1|x_2, H_1)p(x_2|H_1)dx_1}{\int_{R_{V=0}} p(x_1|x_2, H_0)p(x_2|H_0)dx_1} \geq \frac{\pi_0}{\pi_1} \right. \quad (3.5)$$

$$R_V = \left\{ x_1 : L(x_1) \equiv \frac{\int_{R_{W|V=1}} p(x_2|x_1, H_1)p(x_1|H_1)dx_2 - \int_{R_{W|V=0}} p(x_2|x_1, H_1)p(x_1|H_1)dx_2}{\int_{R_{W|V=1}} p(x_2|x_1, H_0)p(x_1|H_0)dx_2 - \int_{R_{W|V=0}} p(x_2|x_1, H_0)p(x_1|H_0)dx_2} \geq \frac{\pi_0}{\pi_1} \right. \quad (3.6)$$

Equations (3.4) and (3.5) show the decision rules for sensor 2 and (3.6) shows the decision rule for sensor 1.

### 3.2 GOOD SIGNAL REGION

As mentioned earlier, in the good region, each decide  $H_1$  region for the two sensors specified by equations (3.4) through (3.6) become equivalent to semi-infinite interval on the real lines, i.e partitions of  $x_2$  for (3.4) and (3.5) and a partition of  $x_1$  for (3.6).

Let us first consider the decision region (3.4) of sensor 2. The likelihood ratio  $L(x_2)$  is similar to the likelihood ratio  $L(x_1)$  defined in equation (12) of the GBU paper. Following

GBU, we can easily show that

$$\frac{\partial L(x_2)}{\partial x_2} = \frac{L(x_2) [s_2 - \rho s_1 + \rho(\mu_1(x_2|H_1) - \mu_0(x_2|H_0))]}{1 - \rho^2} \quad (3.7)$$

where for  $j = 0, 1$ ,

$$\mu_j(x_2|H_j) = \frac{\int_{R_{V=1}} x_1 p(x_1|x_2, H_j) dx_1}{\int_{R_{V=1}} p(x_1|x_2, H_j) dx_1}$$

Irrespective of the nature of  $R_V$  region, i.e whether it is a semi-infinite interval or a union of multiple disjoint intervals, the partial derivative  $L(x_2)$  is non-negative, if both,  $s_2 \geq \rho s_1$  and  $s_1 \geq \rho s_2$  are satisfied. Note that this is a sufficient but not necessary condition as there can be certain sets of parameters for which the partial derivative could be non-negative as well. In general, it is difficult to identify those parameters. The sufficiency condition is identical to the sufficiency condition for the AND rule in [5]. Therefore, when this sufficiency condition, for the good region for the tandem network is met, the test in (3.4) is equivalent to a test of the form  $R_{W|V=1} : x_2 \geq t_{w1}$  when  $(s_1, s_2)$  lie in the first quadrant and it is of the form  $R_{W|V=1} : x_2 \leq t_{w1}$  when  $(s_1, s_2)$  lie in the third quadrant. Similarly, for (3.5), the same conditions on signal points  $(s_1, s_2)$  lead to semi-infinite intervals for  $R_{W|V=0}$  (replacing thresholds  $t_{w1}$  with  $t_{w0}$ ). To show monotonicity of  $L(x_1)$  with respect to  $x_1$ , we can rewrite this function as

$$L(x_1) = \frac{p(x_1|H_1) \left( \int_{R_{W|V=1}} p(x_2|x_1, H_1) dx_2 - \int_{R_{W|V=0}} p(x_2|x_1, H_1) dx_2 \right)}{p(x_1|H_0) \left( \int_{R_{W|V=1}} p(x_2|x_1, H_0) dx_2 - \int_{R_{W|V=0}} p(x_2|x_1, H_0) dx_2 \right)} \quad (3.8)$$

We note here that,  $X_2$  conditioned on  $X_1 = x_1$  is Gaussian distributed with mean  $\rho x_1$  and variance  $(1 - \rho^2)$  under  $H_0$  and under  $H_1$ , the variance remains the same while the mean is  $(s_2 + \rho(x_1 - s_1))$ . Also the ranges of the integrals are semi-infinite intervals in the good

region of signals  $(s_1, s_2)$  in the first quadrant. Hence, we can further simplify (3.8) as :

$$L(x_1) = \frac{p(x_1|H_1)}{p(x_1|H_0)} \frac{Q\left(\frac{t_{w1}-s_2-\rho(x_1-s_1)}{\sqrt{1-\rho^2}}\right) - Q\left(\frac{t_{w0}-s_2-\rho(x_1-s_1)}{\sqrt{1-\rho^2}}\right)}{Q\left(\frac{t_{w1}-\rho x_1}{\sqrt{1-\rho^2}}\right) - Q\left(\frac{t_{w0}-\rho x_1}{\sqrt{1-\rho^2}}\right)} \quad (3.9)$$

where  $Q(\cdot)$  represents the upper tail of standard Gaussian distribution. This equation is of the same form as equation (11) in [5]. Hence,  $L(x_1)$  is monotonically increasing with  $x_1$  as long as  $\rho s_1 \leq s_2 \leq \frac{s_1}{\rho}$ . Similarly, a good region in the third quadrant exists if  $\frac{s_1}{\rho} \leq s_2 \leq \rho s_1$ .

We can simplify (3.4) and obtain the following equivalent decision regions, using the fact that a conditional distribution of two jointly Gaussian variable is Gaussian and region  $R_V$  for signals  $(s_1, s_2)$  in the good region of first quadrant is of the form  $R_V : x_1 \geq t_1$ ,

$$R_{W|V=1} = \left\{ x_2 : e^{-\frac{s_2^2}{2}} e^{x_2 s_2} \frac{Q\left(\frac{t_1-s_1-\rho x_2+\rho s_2}{\sqrt{1-\rho^2}}\right)}{Q\left(\frac{t_1-\rho x_2}{\sqrt{1-\rho^2}}\right)} \geq \frac{\pi_0}{\pi_1} \right\} \quad (3.10)$$

Similarly (3.5) simplifies as :

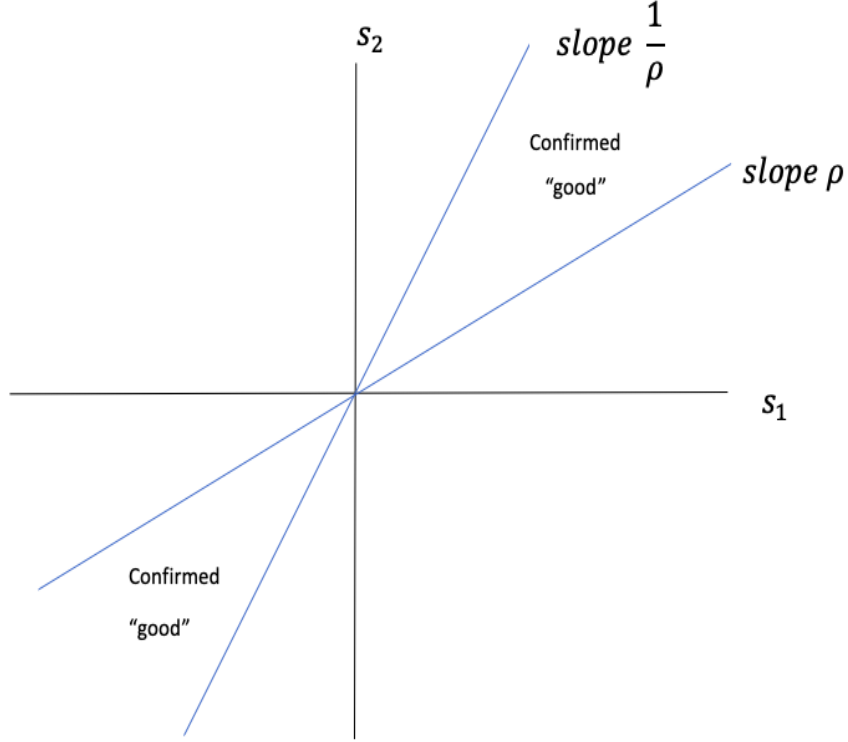
$$R_{W|V=0} = \left\{ x_2 : e^{-\frac{s_2^2}{2}} e^{x_2 s_2} \frac{1-Q\left(\frac{t_1-s_1-\rho x_2+\rho s_2}{\sqrt{1-\rho^2}}\right)}{1-Q\left(\frac{t_1-\rho x_2}{\sqrt{1-\rho^2}}\right)} \geq \frac{\pi_0}{\pi_1} \right\} \quad (3.11)$$

For the good region, using (3.10) and (3.11), it will be easy to find the required thresholds,  $t_{w1}$  and  $t_{w0}$ , using a simple numerical procedure since the expressions to the left of the inequalities are monotonic in  $x_2$ . Equations similar to (3.10) and (3.11) can be derived for signals in the third quadrant, namely  $\frac{s_1}{\rho} \leq s_2 \leq \rho s_1 \leq 0$ . The results can be described graphically as a division of the signal plane  $(s_1, s_2)$  into two regions: confirmed good and a portion possibly bad (see Fig 3.1)

### 3.3 IDENTIFICATION OF BAD REGION

The good region includes all the pairs of  $(s_1, s_2)$  such that convergence to a single-interval decision regions for both sensors in the optimal system is guaranteed. This can be pictured

Figure 3.1. Division of the signal plane  $(s_1, s_2)$  into good regions. Unmarked regions are possibly bad regions.



as all  $(s_1, s_2)$  where  $0 \leq \rho s_1 \leq s_2 \leq \frac{s_1}{\rho}$  in the first quadrant and  $\frac{s_1}{\rho} \leq s_2 \leq \rho s_1 \leq 0$  in the third quadrant. We define the bad region as the complement of the good region. Two possible behaviors were concluded for bad region in [5] for the parallel AND (or OR) rule.

1. One sensor has a single-interval quantization region and the other sensor is ignored.
2. At least one sensor has a non-single interval quantization region.

For the case of detection of a common Gaussian signal in independent noise, it was proved in [12] that the decision region for each sensor where they decide  $H_0$  respectively are of the general form  $\Omega - R_V = t_1 < x_1 < t_2$ ,  $\Omega - R_{W|V=1} = t_{w11} < x_2 < t_{w12}$ ,  $\Omega - R_{W|V=0} = t_{w01} < x_2 < t_{w02}$ . This was concluded by showing the convexity of the likelihood functions, similar to those in (3.4) through (3.6), for random signals. Unfortunately, convexity of (3.4) through (3.6) cannot be established for the case of deterministic signals in the correlated Gaussian model (3.1). Thus, by assuming an initial sensor rule for the top sensor, equations (3.4)

through (3.6) need to be solved iteratively to obtain a locally optimum solution. Then several initial sensor rules need to be considered in order to obtain a possible, best solution. Such a solution obtained is not guaranteed to be globally optimum, but only a reasonable good solution. Alternatively, a sub-optimal solution, such as those based on genetic algorithm, can be employed. Hence, we term all regions that are not in confirmed good as indeterminate although it is possible that some could be good. We defer the investigation of performance analysis in bad region to a future study.

### 3.4 PROCEDURE TO FIND OPTIMUM THRESHOLD IN GOOD REGION

Let us illustrate the procedure to find the optimum thresholds for each sensor in good region in the first quadrant. A similar procedure can be applied to find thresholds in good region in the third quadrant.

1. Consider a particular  $\pi_0$  value and a  $\rho$  value over  $0 \leq \rho < 1$ . Pick signal levels,  $s_1, s_2$  such that  $\rho s_1 \leq s_2 \leq \frac{s_1}{\rho}$ .
2. A minimization algorithm can search for minimum  $C(W)$  as a function of threshold  $t_1$ . Assume  $t_1$  over  $(-\infty, \infty)$ .
3. Given  $t_1$ , using (3.10), find the value of  $x_2 = t_{w1}$ , which will attain equality in (3.10). Similarly, use (3.11) for finding  $t_{w0}$ .
4. To calculate  $C(W)$  in (3.3), compute the conditional probabilities given below and then compute  $C(W)$ .

$$P(W = 1|H_0) = \int_{t_{w1}}^{\infty} \int_{t_1}^{\infty} p(x_1, x_2|H_0) dx_1 dx_2 + \int_{t_{w0}}^{\infty} \int_{-\infty}^{t_1} p(x_1, x_2|H_0) dx_1 dx_2$$

$$P(W = 0|H_1) = \int_{-\infty}^{t_{w1}} \int_{t_1}^{\infty} p(x_1, x_2|H_1) dx_1 dx_2 + \int_{-\infty}^{t_{w0}} \int_{-\infty}^{t_1} p(x_1, x_2|H_1) dx_1 dx_2$$

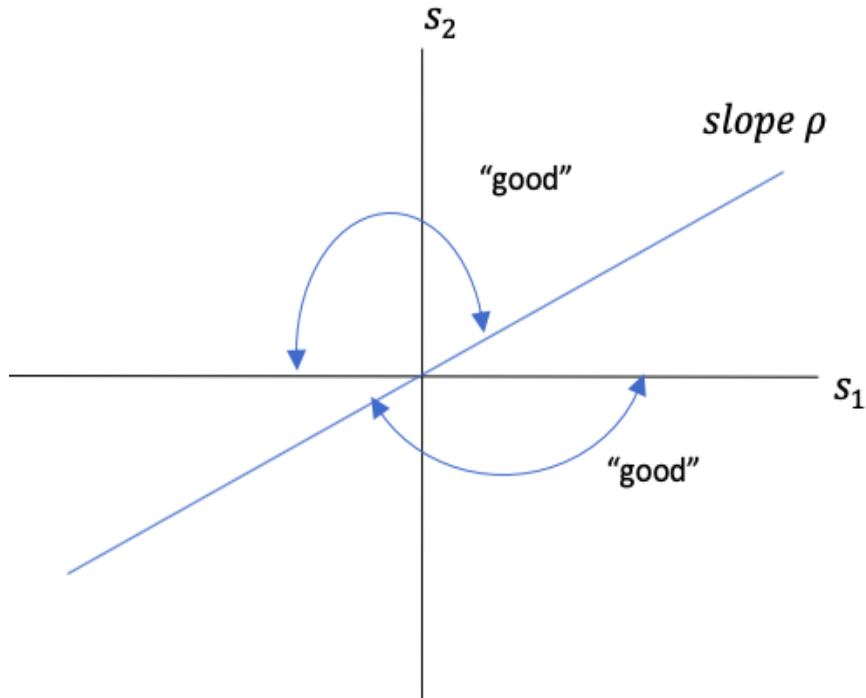


5. Minimization routine provides minimum  $C(W)$  and the corresponding  $t_1$ ,  $t_{w1}$  and  $t_{w0}$  for the chosen signal levels and correlation coefficient.
6. As needed, go to step 1 and choose different prior probabilities, signal levels and correlation coefficient.

### 3.5 SUB-OPTIMAL CASE

In this section, when the first sensor decision region where the sensor decides  $H_1$  is forced to be a single threshold region, we find the good regions for sensor 2. That is, within the restriction of sensor 1 decision to be a single threshold region, we identify good regions for sensor 2, so that minimization of  $P_e$  is possible with a simple numerical procedure. Detailed derivation is given in Appendix B. The results show the good region for sensor 2 as in Fig. 3.2.

Figure 3.2. Division of the signal plane into good regions for the bottom sensor when the top sensor decision is forced to be a single threshold LRT, unmarked regions correspond to possibly bad regions



### 3.6 SYSTEM PERFORMANCE AS $\rho \rightarrow 1$

We know from [8], for the central LRT and number of sensors equal to two, that for  $\rho$  exceeding a threshold value  $\rho^*$  given below, the probability of error decreases with increasing  $\rho$ . Here,

$$\rho^* = \begin{cases} \frac{s_1}{s_2}, & \text{if } \frac{s_1}{s_2} < 1 \\ \frac{s_2}{s_1}, & \text{otherwise} \end{cases} \quad (3.12)$$

For  $s_1 \neq s_2$ , if  $\rho > \rho^*$ ,  $P_e$  will decrease monotonically and approach 0, as  $\rho \rightarrow 1$  for the central LRT test.

For the tandem network, let us consider the following cases,

(a) The better sensor is at the FC

In this case,  $\frac{s_1}{s_2} < 1$  and  $\rho^* = \frac{s_1}{s_2}$  for a given  $(s_1, s_2)$  value. Hence, if  $\rho = \rho^* + h$ , where  $0 < h \leq 1 - \rho^*$ . Since  $s_2 = \frac{s_1}{\rho^*}$ , we have

$$\begin{aligned} s_2 &= \frac{s_1}{\rho - h} \\ &> \frac{s_1}{\rho} \end{aligned}$$

This implies, when the better sensor is at the bottom,  $\rho > \rho^*$ ,  $(s_1, s_2)$  will be in bad region.

(b) Better sensor at top

In this case,  $\frac{s_2}{s_1} < 1$  and  $\rho^* = \frac{s_2}{s_1}$  for a given  $(s_1, s_2)$  value. Hence if  $\rho = \rho^* + h$ , where  $0 < h \leq 1 - \rho^*$ . Since  $s_2 = \rho s_1$ , we have

$$\begin{aligned} s_2 &= s_1(\rho - h) \\ &< \rho s_1 \end{aligned}$$

In this case also,  $(s_1, s_2)$  will be in the bad region for  $\rho > \rho^*$ . The identification of good region allows for easy identification of globally optimal tests at the sensors. However, as we see in cases (a) and (b) presented above, for a given set of signals  $(s_1, s_2)$ , increasing correlation

$\rho > \rho^*$  puts the signal point in bad region. It can be expected that as  $\rho \rightarrow 1$ ,  $P_e \rightarrow 0$  for the optimal tandem test, based on inference from [9]. Hence, a study of probability of error performance for the signal in the bad region becomes important.

### 3.7 NUMERICAL RESULTS

Using the procedure stated in section 3.4, we studied the performance of the two-sensor tandem network in the good signal region of the first quadrant for the following cases:

- (i) Weaker sensor as the FC
- (ii) Better sensor as the FC

The performance of tandem network for cases (i) and (ii) listed above, and the performance of central LRT are listed in Tables 3.1-3.4. We can notice from Tables 3.1-3.4 that case (ii) outperforms case (i) when we compare the optimized (i.e minimum) probability of error achieved by choosing the corresponding optimum  $t_1$  values in each case. However, the difference between minimum errors in (i) and (ii) is minimal.

Table 3.1. Performance comparison of tandem network configuration and central LRT for correlation coefficient  $\rho = 0.3$ ,  $\pi_0 = 0.5$

$s_1$	$s_2$	Tandem $P_e$ (Better Sensor as FC)	Tandem $P_e$ (Weaker sensor as FC)	Central LRT $P_e$
0.5	1	0.306	0.3084	0.3047
1	2	0.1552	0.1585	0.1535
1.5	4	0.0225	0.0228	0.0224
2	3	0.0578	0.0628	0.054
3	5	0.0049	0.0059	0.0044

Table 3.2. Performance comparison of tandem network configuration with central LRT for correlation coefficient  $\rho = 0.5$ ,  $\pi_0 = 0.5$

$s_1$	$s_2$	Tandem $P_e$ (Better Sensor as FC)	Tandem $P_e$ (Weaker sensor as FC)	Central LRT $P_e$
0.3	0.5	0.4009	0.4013	0.4007
1	1.5	0.2238	0.2262	0.2225
2	3	0.0644	0.0665	0.0633
3	5	0.006	0.0062	0.0059
4	5	0.0046	0.0053	0.0041

Table 3.3. Performance comparison of tandem network configuration with central LRT for correlation coefficient  $\rho = 0.3$ ,  $\pi_0 = 0.3$

$s_1$	$s_2$	Tandem $P_e$ (Better sensor as FC)	Tandem $P_e$ (Weaker sensor as FC)	Central LRT $P_e$
0.3	0.5	0.2951	0.2955	0.2948
1	1.5	0.1854	0.1899	0.1815
2	3	0.0517	0.0560	0.0484
3	4	0.0143	0.0164	0.0122
4	5	0.0029	0.0035	0.0022

Table 3.4. Performance comparison of tandem network configuration with central LRT for correlation coefficient  $\rho = 0.5$ ,  $\pi_0 = 0.3$

$s_1$	$s_2$	Tandem $P_e$ (Better Sensor as FC)	Tandem $P_e$ (Weaker sensor as FC)	Central LRT $P_e$
0.3	0.5	0.2958	0.2958	0.2957
1	1.5	0.1918	0.1936	0.1907

**Table 3.4 continued from previous page**

$s_1$	$s_2$	Tandem $P_e$ (Better Sensor as FC)	Tandem $P_e$ (Weaker sensor as FC)	Central LRT $P_e$
2	3	0.0576	0.0594	0.0566
3	5	0.0055	0.0056	0.0054
4	5	0.0042	0.0048	0.0037

In Figs. 3.3-3.8 we show the variation of probability of error  $C(W)$ , as a function of threshold of sensor 1,  $t_1$ , for the cases (i) and (ii). We can observe from Figs. 3.3-3.8 that the minimum probability of error is sensitive to threshold  $t_1$  in case (i), whereas when the better sensor is at FC, the minimum probability of error is only very slightly dependent on  $t_1$ , i.e. it is not sensitive to threshold  $t_1$  of first sensor. Also, in the Figs 3.9-3.11, we can see the error performances of single sensor, better sensor as FC, weaker sensor as FC and central LRT, as a function of  $\rho$ . We can observe that, at each  $\rho$ ,  $C(W)$  is minimum for central LRT followed by the case of better sensor as FC. At  $\rho = 0$ , that is when the observations of the sensors are statistically independent, we observe a significant difference between the probabilities of error. The probabilities of error increase gradually with increasing  $\rho$ . In Fig 3.10, we observe that, when the difference between  $s_1$  and  $s_2$  is quite large, the performance of tandem network when the weaker sensor is placed at the bottom is same as the standalone better sensor performance. As observed in Figs 3.9 -3.12, better sensor as the FC is the preferred choice for signals in the good region. This conclusion is arrived based on extensive numerical study, for various correlation coefficients, signal levels, and prior probabilities, even though only representative results are shown here. In Figs. 3.13 -3.15, we observe that, when the better sensor is placed at the bottom, the error becomes maximum at the break-point  $\rho^* = \frac{s_1}{s_2}$  and then starts to decrease gradually afterwards, as  $\rho$  increases further. Again, the trend observed in Figs. 3.9-3.15 seem to be valid for many signal levels and prior probabilities. In Figs. 3.13-3.15, curves for  $\rho > \rho^*$  are obtained for the sub-optimal case of

sensor 1 employing a single threshold LRT with the second sensor quantization optimized accordingly to the results in Appendix B.

Figure 3.3. Probability of error versus threshold  $t_1$ , better sensor as fusion center,  $s_1 = 2$ ,  $s_2 = 3$

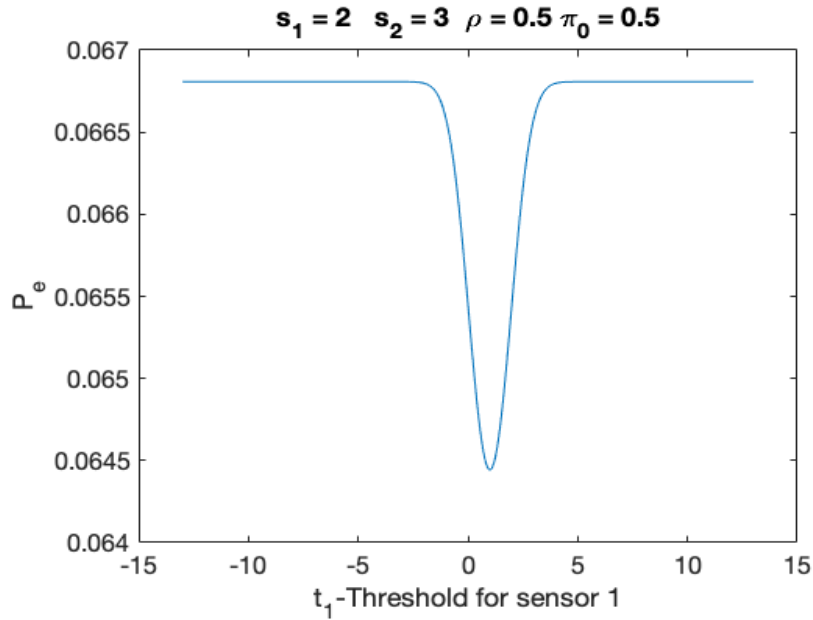


Figure 3.4. Probability of error versus threshold  $t_1$ , weaker sensor as fusion center,  $s_1 = 3$ ,  $s_2 = 2$

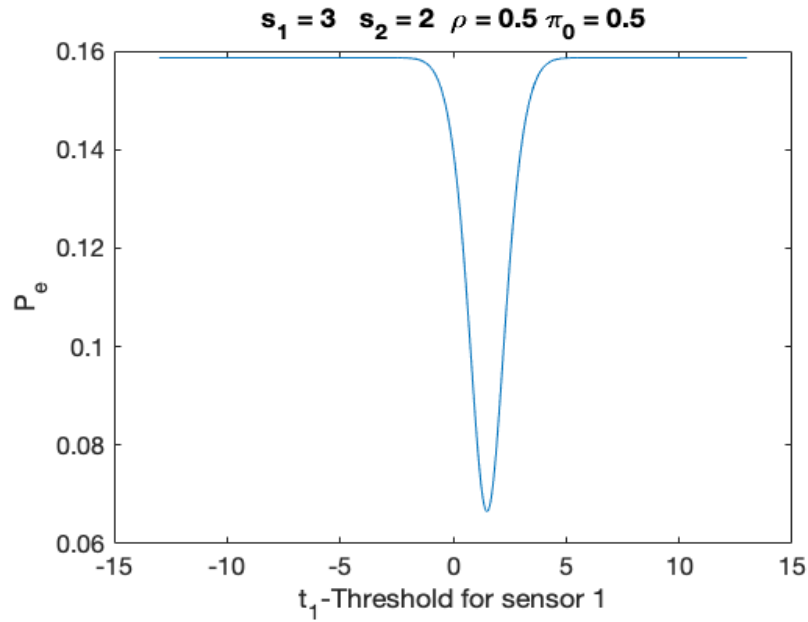


Figure 3.5. Probability of error versus threshold  $t_1$ , better sensor as fusion center,  $s_1 = 2$ ,  $s_2 = 4$

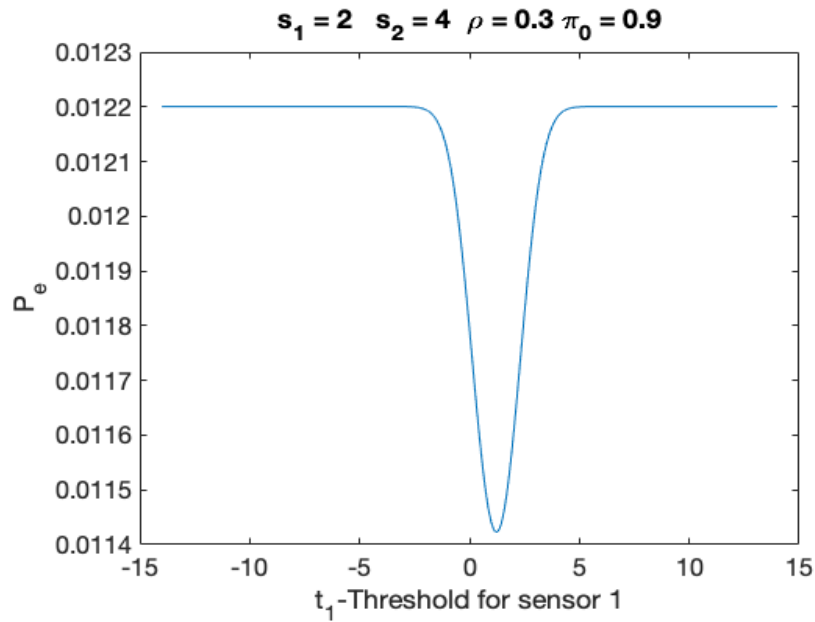


Figure 3.6. Probability of error versus threshold  $t_1$ , weaker sensor as fusion center,  $s_1 = 4$ ,  $s_2 = 2$

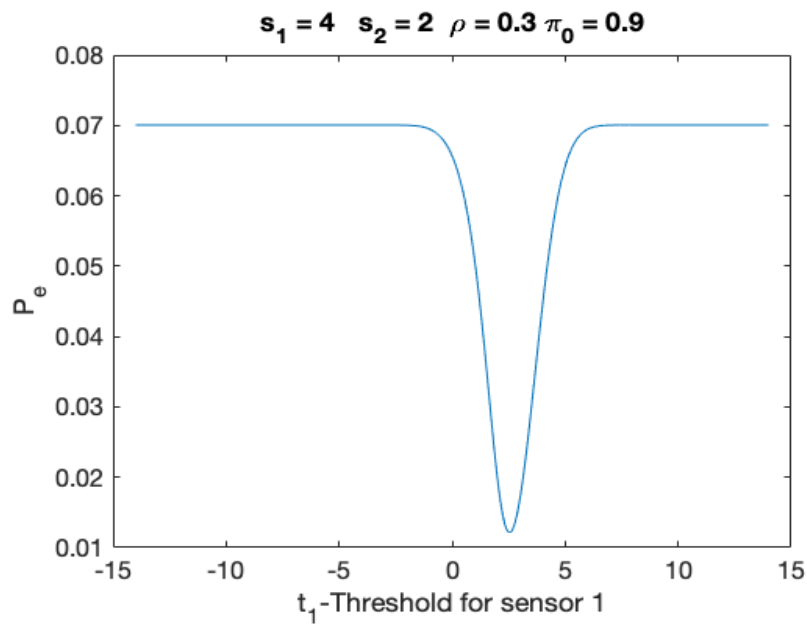


Figure 3.7. Probability of error versus threshold  $t_1$ , better sensor as fusion center,  $s_1 = 1$ ,  $s_2 = 3$

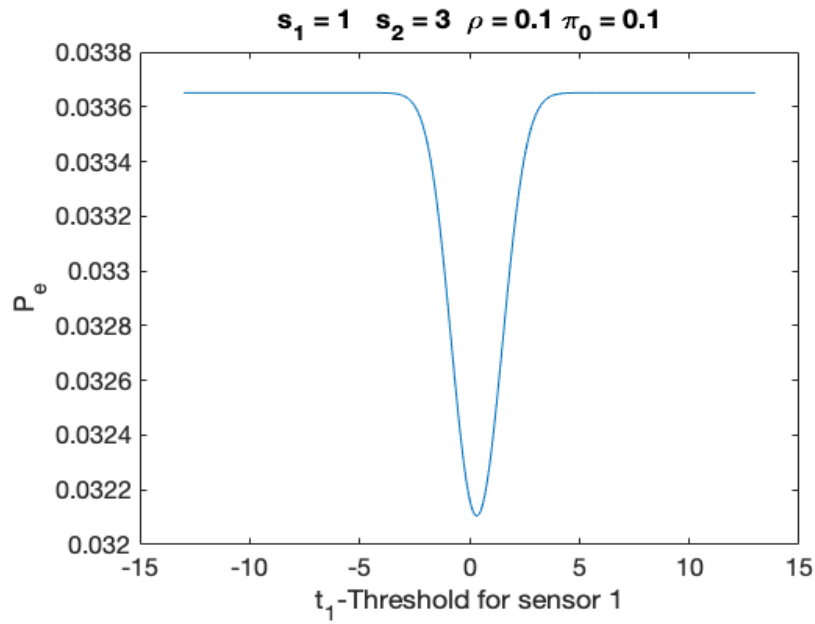


Figure 3.8. Probability of error versus threshold  $t_1$ , weaker sensor as fusion center,  $s_1 = 3$ ,  $s_2 = 1$

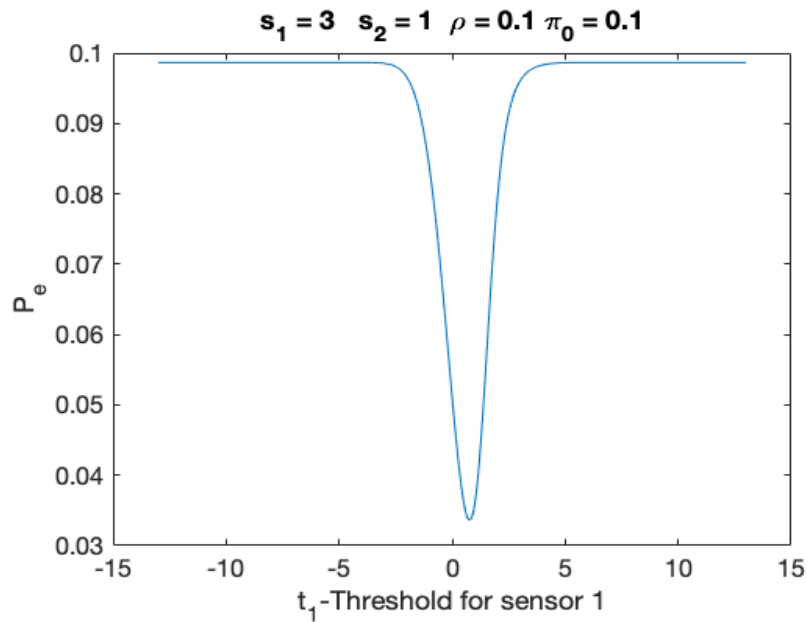




Figure 3.9. Probability of error versus correlation coefficient  $\rho$  for good region, received signals,  $\min(S) = 1.5$ ,  $\max(S) = 2$  prior probability  $\pi_0 = 0.5$

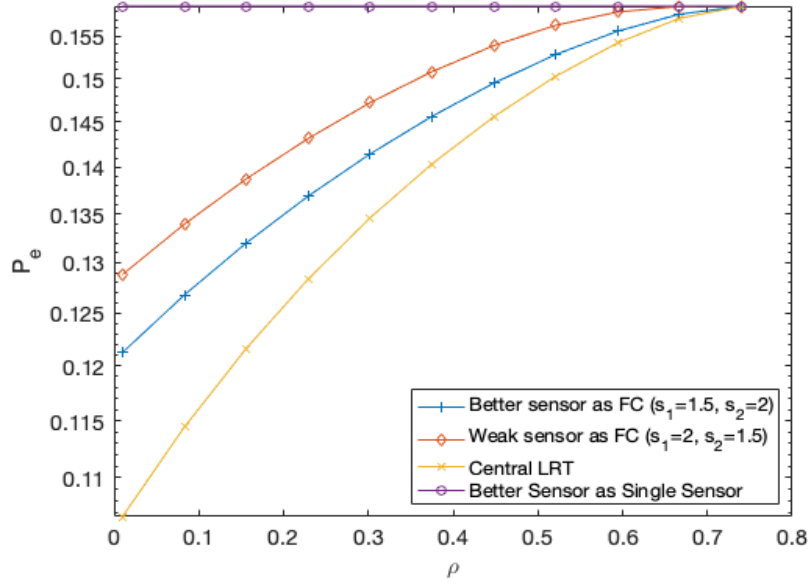


Figure 3.10. Probability of error versus correlation coefficient  $\rho$  for good region, received signals,  $\min(S) = 1$ ,  $\max(S) = 4$ , prior probability  $\pi_0 = 0.5$

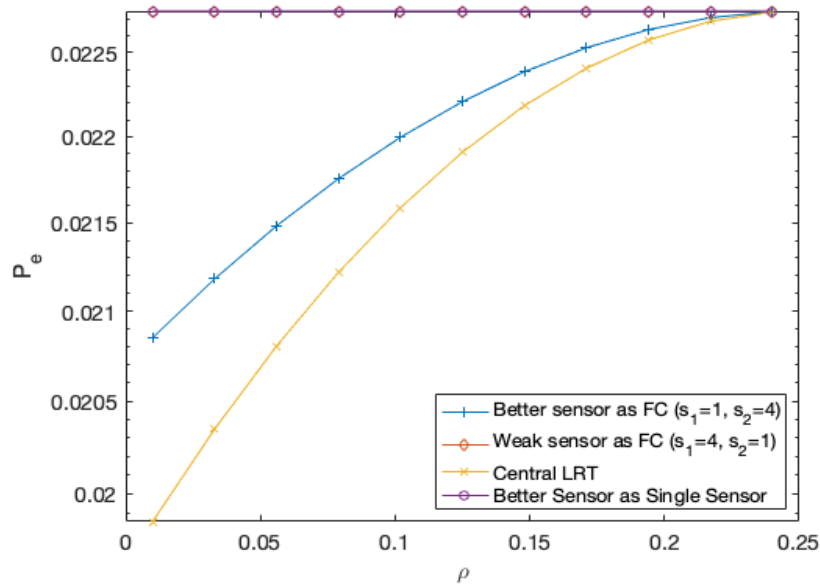


Figure 3.11. Probability of error versus correlation coefficient  $\rho$  for good region, received signals,  $\min(S) = 3$ ,  $\max(S) = 4$ , prior probability  $\pi_0 = 0.9$

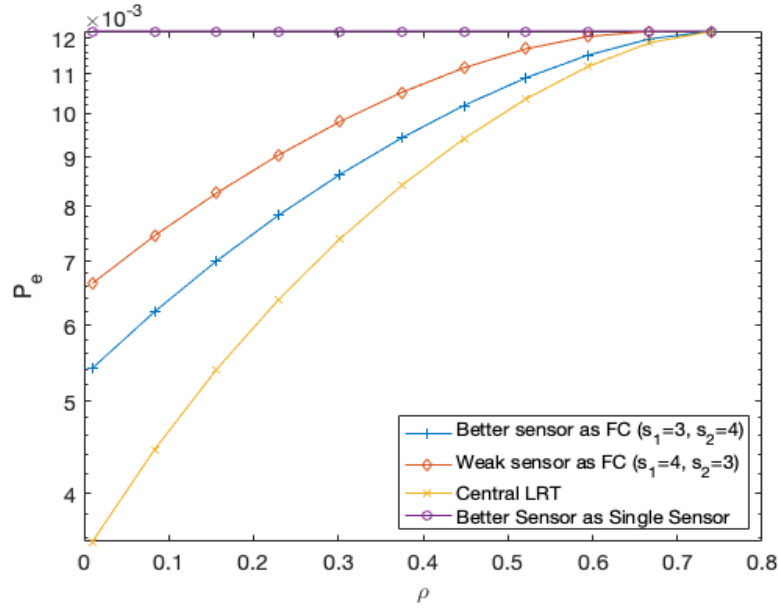


Figure 3.12. Probability of error versus correlation coefficient  $\rho$  for good region, received signals,  $\min(S) = 1.5$ ,  $\max(S) = 3$ , prior probability  $\pi_0 = 0.3$

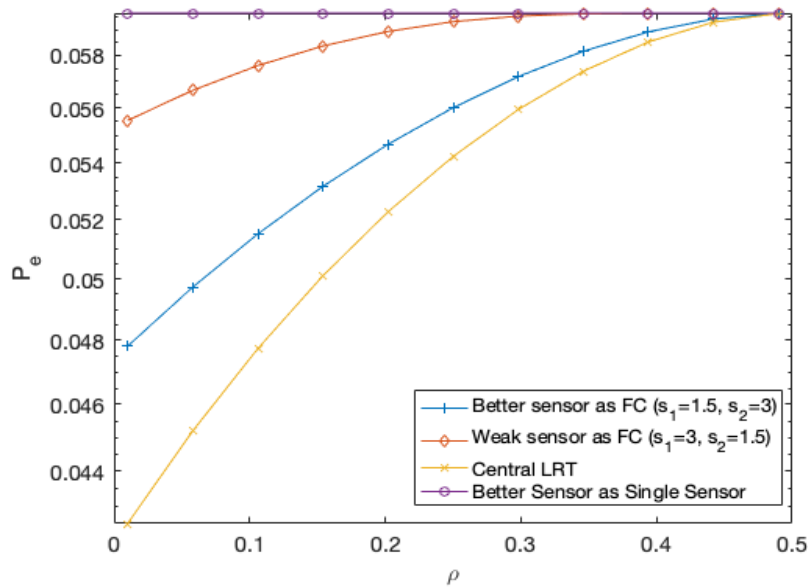
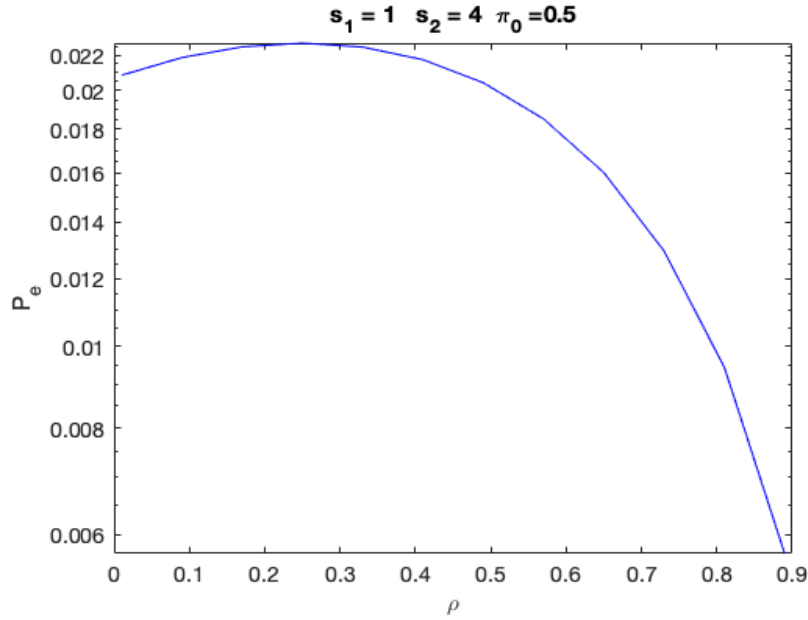
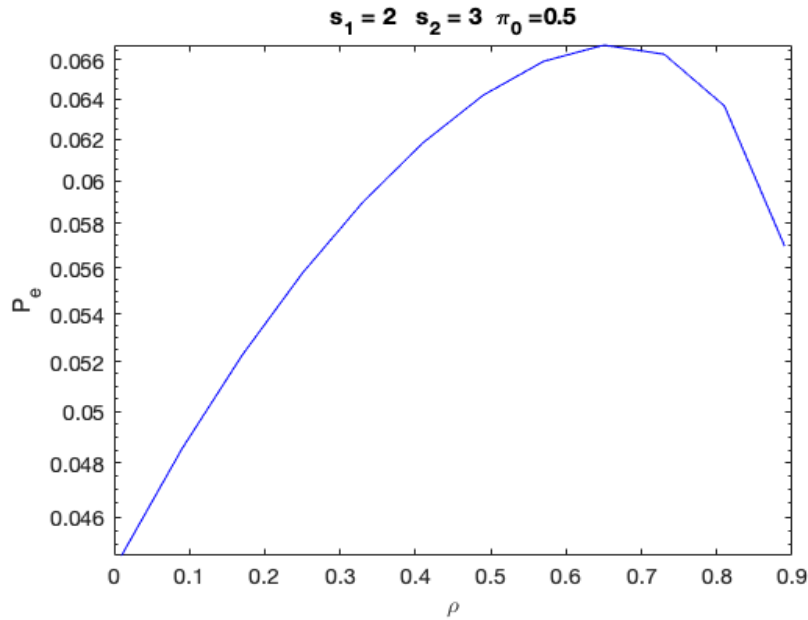


Figure 3.13. Probability of error versus correlation coefficient  $\rho$ , when better sensor is at fusion center



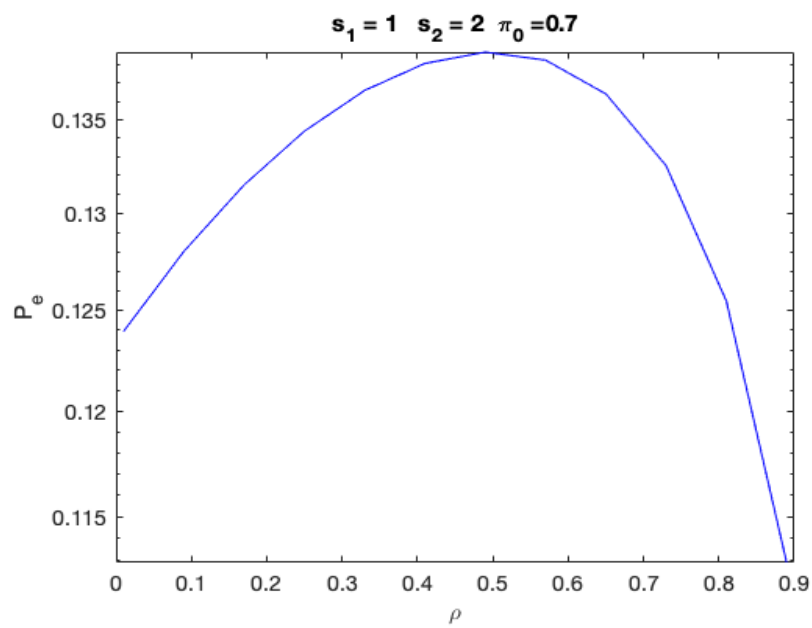
Note:  $\rho > \frac{s_1}{s_2}$  corresponds to sub-optimal single threshold LRT at sensor 1

Figure 3.14. Probability of error versus correlation coefficient  $\rho$ , when better sensor is at fusion center



Note:  $\rho > \frac{s_1}{s_2}$  corresponds to sub-optimal single threshold LRT at sensor 1

Figure 3.15. Probability of error versus correlation coefficient  $\rho$



Note:  $\rho > \frac{s_1}{s_2}$  corresponds to sub-optimal single threshold LRT at sensor 1

## CHAPTER 4

### CONCLUSION AND FUTURE RESEARCH

In this thesis, we studied error performance of a two-sensor decentralized detection system that detects the presence of signals in Gaussian noise. We first present an analysis of detection of Gaussian signals in Gaussian noise using parallel fusion algorithm employing an AND or OR fusion rule at the FC. Results show :

1. For equal prior probabilities, the AND rule performs at-least as well as OR rule.
2. For equal prior probabilities and AND rule, the probability of error decreases gradually as  $\rho \rightarrow 1$ . However, under same conditions, the performance of OR rule is about the same as the standalone performance of better sensor as single sensor.

We also studied Bayes error performance of two sensor tandem network for detection of deterministic signals in correlated Gaussian noise. Specifically, we identified the good region for signal points, where the optimum tests for both sensors are single threshold tests based on the observations at the sensors. Therefore, in good region, simple numerical computations can be used to find the optimum decision threshold levels at each sensor. Numerical results show that placing the sensor with better signal quality at the bottom gives a lower error performance as compared to the reverse configuration, for all signal points located in the good region. Notably, when the better sensor is at the top, the optimum performance is sensitive to the threshold of the top sensor, but not when the weaker signal sensor is at the top. We also notice that in the bad region, increasing noise correlation is expected to lead to decreasing error. Hence, it is important to study this region.

For the tandem network with the GBU model, we have to resort to a sub-optimal search algorithm, based on a genetic algorithm, in order to obtain quantization intervals in the bad

signal region. We defer this to a future study. Also, changing the optimization criterion to Neyman-Pearson may lead to different results than those obtained using the Bayes criterion. For example, for a certain range of probability of false alarm at FC, would weaker sensor as the FC might lead to higher probability of detection?

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## BIBLIOGRAPHY

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## APPENDICES

## APPENDIX A

### DERIVATION OF DECISION REGION RULES FOR EACH SENSOR IN TW0-SENSOR TANDEM NETWORK

In this appendix we show the derivations that lead to equations First, let us minimize  $C(W)$  by finding the decision region  $R_W$  at sensor 2 given the decision region at sensor 1 is already fixed. Let the whole sample space be denoted by  $\Omega$ . Using the Bayes rules, we can simplify the risk as follows:

$$\begin{aligned} C(W) &= \pi_0 \int_{R_W} p(x_2, V|H_0)dx_2 + \pi_1 \int_{\Omega-R_W} p(x_2, V|H_1)dx_2 \\ &= \pi_0 \int_{R_W} p(x_2, V|H_0)dx_2 + \pi_1 [1 - \int_{R_W} p(x_2, V|H_1)dx_2] \end{aligned} \quad (\text{A.1})$$

The above equation can be further simplified as :

$$\begin{aligned} C(W) &= \pi_1 + \int_{R_{W|V=1}} [\pi_0 p(x_2|V=1, H_0)p_f - \pi_1 p(x_2|V=1, H_1)p_d] dx_2 \\ &+ \int_{R_{W|V=0}} [\pi_0 p(x_2|V=0, H_0)(1-p_f) - \pi_1 p(x_2|V=0, H_1)(1-p_d)] dx_2. \end{aligned} \quad (\text{A.2})$$

To minimize  $C(W)$ , we need to minimize each integral in equation (A.2). First integral can be minimized by assigning all those points  $x_2$  that make the term present inside the square bracket non-positive to the region  $R_{W|V=1}$ . Writing this out explicitly yields the collection of all  $x_2$  points that satisfy

$$\frac{p(x_2|V=1, H_1)p_d}{p(x_2|V=1, H_0)p_f} \geq \frac{\pi_0}{\pi_1}$$

We can further simplify the numerator conditional density term on the left of the inequality as shown below:

$$\begin{aligned}
p(x_2|V = 1, H_1) &= \frac{P(x_2 < X_2 \leq x_2 + \delta x, V = 1|H_1)}{P(V = 1|H_1)} \\
&= \frac{\int_{R_{V=1}} p(x_2, x_1|H_1)dx_1}{p_d} \\
&= \frac{\int_{R_{V=1}} p(x_1|x_2, H_1)p(x_2|H_1)dx_1}{p_d}
\end{aligned}$$

Similarly, further simplification of the conditional probability density function in the denominator to the left of the inequality leads to equation (3.4) given earlier. Similarly, we can minimize the second integral by assigning all those points  $x_2$  that make the term present inside the square bracket non-positive to the region  $R_{W|V=0}$ . Writing this out explicitly yields the collection of all  $x_2$  points that satisfy

$$\frac{p(x_2|V = 0, H_1)(1 - p_d)}{p(x_2|V = 0, H_0)(1 - p_f)} \geq \frac{\pi_0}{\pi_1}$$

We can further simplify the numerator conditional density term on the left of the inequality as shown below:

$$\begin{aligned}
p(x_2|V = 0, H_1) &= \frac{P(x_2 < X_2 \leq x_2 + \delta x, V = 0|H_1)}{P(V = 0|H_1)} \\
&= \frac{\int_{R_{V=1}} p(x_2, x_1|H_1)dx_1}{1 - p_d} \\
&= \frac{\int_{R_{V=1}} p(x_1|x_2, H_1)p(x_2|H_1)dx_1}{1 - p_d}
\end{aligned}$$

Similarly, further simplification of the conditional probability density function in the denominator to the left of the inequality leads to equation (3.5) given earlier.

Next, let us minimize  $C(W)$  by finding the decision region  $R_V$  at sensor 1, given that the

decision rules at sensor 2 is already fixed. We can write the overall cost as follows:

$$\begin{aligned}
C(W) &= \pi_0[P(W = 1|V = 0, H_0)(1 - p_f) + P(W = 1|V = 1, H_0)p_f] \\
&\quad + \pi_1[P(W = 0|V = 0, H_1)(1 - p_d) + P(W = 0|V = 1, H_1)p_d].
\end{aligned} \tag{A.3}$$

The above equation can be simplified as

$$\begin{aligned}
C(W) &= \pi_0P(W = 1|V = 0, H_0) + \pi_1P(W = 0|V = 0, H_1) \\
&\quad + p_f\pi_0[P(W = 1|V = 1, H_0) - P(W = 1|V = 0, H_0)] \\
&\quad + p_d\pi_1[P(W = 0|V = 1, H_1) - P(W = 0|V = 0, H_1)].
\end{aligned} \tag{A.4}$$

The terms involving  $p_f$  and  $p_d$ , are the only terms that depend on the determination of  $R_V$  regions. Hence, the sum of these two terms in the above equation needs to be minimized by appropriately choosing the decision region  $R_V$ . This sum can be considered as the expected cost, given that the decision regions  $R_{W|V=1}$  and  $R_{W|V=0}$  are already determined. Hence, this sum is minimized by assigning all those points  $x_1$  to  $R_V$  that will make the integrand below non-positive. (Given decision regions of sensor 2 for  $V = 1$  and  $V = 0$  are fixed, the probabilities involving  $W$  are constants and hence can be taken inside the integral shown below)

$$\begin{aligned}
&= \int_{R_V} [\pi_0(P(W = 1|V = 1, H_0) - P(W = 1|V = 0, H_0))p(x_1|H_0) \\
&\quad - \pi_1(-P(W = 0|V = 1, H_1) + P(W = 0|V = 0, H_1))p(x_1|H_1)]dx_1
\end{aligned} \tag{A.5}$$

The above integrand can be further simplified using the results  $P(W = 0|V = 0, H_1) = 1 - P(W = 1|V = 0, H_1)$ ,  $P(W = 0|V = 1, H_1) = 1 - P(W = 1|V = 1, H_1)$ . Writing this

out explicitly yields the collection of all  $x_1$  points belonging to  $R_V$  region as

$$\begin{aligned} & \pi_0(P(W = 1|V = 1, H_0) - P(W = 1|V = 0, H_0))p(x_1|H_0) \\ & < \pi_1(P(W = 1|V = 1, H_1) - P(W = 1|V = 0, H_1))p(x_1|H_1) \end{aligned}$$

Writing out the above conditional probabilities in terms of integrals of conditional densities lead to the equation (3.6) given earlier. Designing sensors' decision rules as given by (3.5), (3.4) and (3.6) is a requirement for person-by-person optimal solution for minimizing  $C(W)$ . As mentioned in [5], this is a requirement for globally optimal solution as well. Although similar derivation of decision rules at the two sensors is provided in [12] for the detection of random signal in independent noise, the derivation provided here is much more straightforward and simpler.

## APPENDIX B

### DERIVATION OF GOOD REGION FOR SENSOR 2 IN TANDEM NETWORK FOR THE SUB-OPTIMAL CASE OF SENSOR 1 HAVING A SINGLE THRESHOLD LRT

In this appendix we derive sufficiency condition of good region in signal plane  $(s_1, s_2)$  for the sub-optimal case when sensor 1 is forced to be a single threshold LRT. We assume decision regions of sensor 1 to be semi-infinite intervals. In (3.5), let  $\mu_1(x_2) - \mu_0(x_2) = \delta$  (for simplicity, dependence on  $x_2$  is not shown explicitly) where  $\mu_1(x_2)$  is conditional expectation of  $G(s_1 - \rho s_2 + \rho x_2, 1 - \rho^2)$  (under  $H_1$ ) and  $\mu_0(x_2)$  is conditional expectation of  $G(\rho x_2, 1 - \rho^2)$  (under  $H_0$ ). Hence,

$$\delta = \frac{\int_A x_1 f(x_1 - \mu) dx_1}{\int_A f(x_1 - \mu) dx_1} - \frac{\int_A x_1 f(x_1) dx_1}{\int_A f(x_1) dx_1} \quad (\text{B.1})$$

where  $f$  denotes a Gaussian density with mean  $\rho x_2$  and variance  $1 - \rho^2$  and  $\mu = (s_1 - \rho s_2)$ . According to Lemma 1 of GBU [5], we have the following result

$$\delta > 0 \text{ if } \mu > 0$$

$$\delta = 0 \text{ if } \mu = 0$$

$$\delta < 0 \text{ if } \mu < 0 \quad (\text{B.2})$$

We now study different cases of signal  $(s_1, s_2)$  regions.

1.  $s_1 \geq 0, s_2 \geq 0$  (i.e first quadrant)

We can write the expression (3.10) when sensor 1 decision is 1 ( $V = 1$ ) as

$$x_2 : g_1(x_2) = e^{-\frac{s_2^2}{2}} e^{s_2 x_2} \frac{Q\left(\frac{t_1 - \rho x_2}{\sqrt{1 - \rho^2}} - \frac{s_1 - \rho s_2}{\sqrt{1 - \rho^2}}\right)}{Q\left(\frac{t_1 - \rho x_2}{\sqrt{1 - \rho^2}}\right)} \geq \frac{\pi_0}{\pi_1} \quad (\text{B.3})$$

Let  $\mu^* = \frac{s_1 - \rho s_2}{\sqrt{1 - \rho^2}}$ ,  $y_2 = \frac{t_1 - \rho x_2}{\sqrt{1 - \rho^2}}$  in equation (B.3), then

$$g_1(x_2) = \underbrace{e^{-\frac{s_2^2}{2}} e^{s_2 x_2}}_{h_1(x_2)} \underbrace{\frac{Q(y_2 - \mu^*)}{Q(y_2)}}_{h_2(y_2)} \quad (\text{B.4})$$

Note that  $h_1(x_2)$  is monotonically increasing w.r.t  $x_2$  if  $s_2 > 0$ .  $g_1(x_2)$  is monotone increasing w.r.t  $x_2$  when  $h_2(y_2)$  is monotonically increasing with  $x_2$ . Taking first derivative of  $h_2(y_2)$  in equation (B.4) w.r.t  $x_2$  we get

$$\frac{dh_2(y)}{dx_2} = \frac{d}{dy_2} \left( \frac{Q(y_2 - \mu^*)}{Q(y_2)} \right) \frac{dy_2}{dx_2} \quad (\text{B.5})$$

According to lemma 3 of GBU [5],  $\frac{Q(y_2 - \mu^*)}{Q(y_2)}$  increases monotonically with  $y_2$  if  $\mu^* > 0$  and decreases monotonically if  $\mu^* < 0$ . Also,  $\frac{dy_2}{dx_2} = \frac{-\rho}{\sqrt{1 - \rho^2}}$ . Hence,  $\frac{dg_1(x_2)}{dx_2} > 0$  when

$$\frac{d}{dy_2} \frac{Q(y_2 - \mu^*)}{Q(y_2)} < 0 \quad (\text{B.6})$$

The above condition is satisfied when  $\mu^* < 0$ . Hence,  $R_{W|V=1}$  is a single semi-infinite region of the form  $(t_{w1}, \infty)$  if  $\mu^* < 0$ , or we can say when

$$s_2 > \frac{s_1}{\rho} \quad (\text{B.7})$$

Similarly, for  $V = 0$ , we have,

$$\begin{aligned} x_2 : g_0(x_2) &= e^{-\frac{s_2^2}{2}} e^{s_2 x_2} \frac{\Phi\left(\frac{t_1 - \rho x_2}{\sqrt{1 - \rho^2}} - \frac{s_1 - \rho s_2}{\sqrt{1 - \rho^2}}\right)}{\Phi\left(\frac{t_1 - \rho x_2}{\sqrt{1 - \rho^2}}\right)} \leq \frac{\pi_0}{\pi_1} \\ &= e^{-\frac{s_2^2}{2}} e^{s_2 x_2} \frac{\Phi(y_2 - \mu^*)}{\Phi(y_2)} \\ &= \underbrace{e^{-\frac{s_2^2}{2}} e^{s_2 x_2}}_{h_1(x_2)} \underbrace{\frac{Q(-y_2 + \mu^*)}{Q(-y_2)}}_{h_2(y_2)} \end{aligned} \quad (\text{B.8})$$



For  $s_2 > 0$ ,  $h_1(x_2)$  is monotonic increasing with  $x_2$ .  $g_0(x_2)$  will be monotonic increasing w.r.t  $x_2$  when  $h_2(y_2)$  will be monotonically increasing with  $x_2$ . This is true when  $-\mu^* > 0$  or  $\mu^* < 0$ . The condition is same as in  $V = 1$  or  $s_2 > \frac{s_1}{\rho}$ . Now, let us look another way. Rewriting equation (3.7), we have the following

$$\frac{\partial L(x_2)}{\partial x_2} = \frac{L(x_2)[s_2 - \rho s_1 + \rho \delta]}{1 - \rho^2} \quad (\text{B.9})$$

If  $s_2 - \rho s_1 > 0$  and  $s_1 - \rho s_2 > 0$  (equation (B.2) shows  $\delta > 0$ ),

$$\frac{\partial L(x_2)}{\partial x_2} \geq 0 \quad (\text{B.10})$$

Hence,  $R_{W|V=1}$  and  $R_{W|V=0}$  are single semi-infinite regions and of the form  $(t_{w1}, \infty)$  and  $(t_{w0}, \infty)$  and will be optimal. Hence, sufficient condition becomes

$$s_2 > \rho s_1 \text{ and } s_1 > \rho s_2, \text{ or}$$

$$\rho s_1 < s_2 < \frac{s_1}{\rho} \quad (\text{B.11})$$

Combining (B.11) and (B.7) we observe that the sufficient condition for the good region in first quadrant is simply

$$s_2 > \rho s_1 > 0 \quad (\text{B.12})$$

Note : From  $\frac{\partial L(x_2)}{\partial x_2}$ , both  $s_2 - \rho s_1 < 0$  and  $s_1 - \rho s_2 < 0$  are not possible (note  $\rho < 1, s_1 > 0, s_2 > 0$ ). Hence,  $R_{W|V=1}$  and  $R_{W|V=0}$  cannot be of the form  $(-\infty, t_{w1})$  and  $(-\infty, t_{w0})$  respectively. So, if  $s_2 < \rho s_1$ , there is a possibility of  $R_{W|V=1}$  to be of the form  $(t_{w1}, \infty)$  or a union of multiple disjoint intervals and  $R_{W|V=0}$  to be of the form  $(t_{w0}, \infty)$  or a union of multiple disjoint intervals. This can be observed numerically when  $s_2 < \rho s_1$ .

2.  $s_1 < 0, s_2 > 0$  (second quadrant)

For  $s_1 < 0$ , the decision region  $R_V$  is of the form  $(-\infty, t_1)$ . So, when  $V = 0$ , we have

[since  $R_V = (-\infty, t_1)$ ,  $V = 1$  type region for  $s_1 > 0$  case becomes  $V = 0$  for  $s_1 < 0$  case and vice-versa],

$$x_2 : g_0(x_2) = \underbrace{e^{\frac{-s_2^2}{2}} e^{s_2 x_2}}_{h_1(x_2)} \frac{Q(y_2 - \mu^*)}{\underbrace{Q(y_2)}_{h_2(y_2)}} \geq \frac{\pi_0}{\pi_1} \quad (\text{B.13})$$

Since  $s_2 > 0$ , conclusion in case 1 still holds. That is, if  $s_2 > \frac{s_1}{\rho}$ ,  $R_{W|V=0} : x_2 \in (t_{w0}, \infty)$  and  $R_{W|V=1} : x_2 \in (t_{w1}, \infty)$ . Since  $s_2 > 0, s_1 < 0$ , this is satisfied for the whole of second quadrant.

3.  $s_1 < 0, s_2 < 0$  (third quadrant)

Since  $s_1 < 0$ ,  $R_V$  is of the form  $(-\infty, t_1)$ . For  $V = 0$ , we have,

$$x_2 : g_0(x_2) = \underbrace{e^{\frac{-s_2^2}{2}} e^{s_2 x_2}}_{h_1(x_2)} \frac{Q(y_2 - \mu^*)}{\underbrace{Q(y_2)}_{h_2(y_2)}} \geq \frac{\pi_0}{\pi_1} \quad (\text{B.14})$$

For  $s_2 < 0$ ,  $h_1(x_2)$  will be monotonically decreasing w.r.t  $x_2$ . To make  $g_0(x_2)$  monotonically decreasing w.r.t  $x_2$ , we require  $h_2(y_2)$  to be monotone decreasing w.r.t  $x_2$ . This happens when  $\mu^* > 0$ . Thus,  $R_{W|V=0}$  will be of the form  $(-\infty, t_{w0})$ , if

$$s_2 < \frac{s_1}{\rho} \quad (\text{B.15})$$

When,  $V = 1$ , we have

$$x_2 : g_1(x_2) = \underbrace{e^{\frac{-s_2^2}{2}} e^{s_2 x_2}}_{h_1(x_2)} \frac{Q(-y_2 - (-\mu^*))}{\underbrace{Q(-y_2)}_{h_2(y_2)}} \geq \frac{\pi_0}{\pi_1} \quad (\text{B.16})$$

For  $g_1(x_2)$  to be monotonically decreasing w.r.t  $x_2$ , we need  $h_2(y_2)$  to be monotonic decreasing w.r.t  $x_2$ . This happens when  $\mu^* > 0$  or when we have,

$$s_2 < \frac{s_1}{\rho} \quad (\text{B.17})$$

Alternatively, considering equation (B.9), for  $\frac{dL(x_2)}{\delta x_2} < 0$ , sufficiency condition becomes

$$s_1 - \rho s_2 < 0 \text{ and } s_2 - \rho s_1 < 0, \text{ or}$$

$$\frac{s_1}{\rho} < s_2 < \rho s_1 \quad (\text{B.18})$$

Putting (B.18) and (B.17) together, sufficiency condition becomes

$$s_2 < \rho s_1 \quad (\text{B.19})$$

4.  $s_1 > 0, s_2 < 0$  (fourth quadrant)

For  $s_1 > 0$ , decision region of  $R_V$  is of the form  $(t_1, \infty)$ . For  $V = 1$ , we have

$$x_2 : g_1(x_2) = \underbrace{e^{\frac{-s_2^2}{2}} e^{s_2 x_2}}_{h_1(x_2)} \frac{Q(y_2 - \mu^*)}{\underbrace{Q(y_2)}_{h_2(y_2)}} \geq \frac{\pi_0}{\pi_1} \quad (\text{B.20})$$

For  $g_1(x_2)$  to be monotonically decreasing w.r.t  $x_2$ , we require  $h_2(y_2)$  to decrease monotonically with  $x_2$  as  $h_1(x_2)$  is monotonic decreasing w.r.t  $x_2$ . This happens when  $\mu^* > 0$  or  $s_1 - \rho s_2 > 0$ . Hence, for  $R_{W|V=1}$  to be optimal with semi-infinite region of the form  $(-\infty, t_{w1})$ , signals must satisfy the following condition

$$s_2 < \frac{s_1}{\rho} \quad (\text{B.21})$$

The above condition will always be satisfied in the fourth quadrant as  $s_1 > 0, s_2 < 0$ .

Similarly, when  $V = 0$ , we have

$$x_2 : g_0(x_2) = \underbrace{e^{\frac{-s_2^2}{2}} e^{s_2 x_2}}_{h_1(x_2)} \frac{Q(-y_2 - (-\mu^*))}{\underbrace{Q(-y_2)}_{h_2(y_2)}} \geq \frac{\pi_0}{\pi_1} \quad (\text{B.22})$$

We require  $h_2(y_2)$  to be a monotonically decreasing w.r.t  $x_2$ , in order for  $g_0(x_2)$  to be

monotonically decreasing w.r.t  $x_2$ . If  $\mu^* > 0$ , then this condition will be satisfied and  $R_{W|V=0}$  will be an optimal semi-infinite region of the form  $(-\infty, t_{w0})$ . This is the same condition as what we get for  $R_{W|V=1}$  in fourth quadrant of signal plane. Hence, over the entire fourth quadrant,  $R_{W|V=1} : x_2 \in (-\infty, t_{w1})$  and  $R_{W|V=0} : x_2 \in (-\infty, t_{w0})$ , will be optimal.

The result here shows that for  $(s_1 > 0, s_2 > 0)$  case, if the first sensor is forced to be single threshold LRT, then the good region for sensor 2 will be valid as long as  $s_2 > \rho s_1$ . Hence, for a given  $(s_1, s_2)$  and  $\rho^* = \frac{\min(s_1, s_2)}{\max(s_1, s_2)}$ ,  $\rho > \rho^*$  will be in good region for sensor 2, for both cases of weaker sensor as the FC and the better sensor as the FC. However, notice that for globally optimum tandem network case,  $\rho > \rho^*$  puts the first sensor test in possibly bad region and hence, single threshold LRT at sensor 1 is not likely to be globally optimal for  $\rho > \rho^*$ . In fact, for a globally optimal tandem network,  $P_e \rightarrow 0$  as  $\rho \rightarrow 1$  (see comments in section 3.6). Graphs in 3.13-3.15 show for the sub-optimal case considered here in Appendix B,  $P_e$  decreases with  $\rho$  beyond  $\rho^*$ , but never approaches zero as  $\rho$  approaches one.

## VITA

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