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Computation of Coupon Values

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By Arthur S. Little

During the sound money campaign of 1896 there were distributed by the Republican national committee and other organizations, countless tons of educational literature; most of which was written or inspired by the brightest and ablest financiers and economists of the country. Among the numerous excellent tracts and pamphlets of this nature that came to my notice that which impressed me the most deeply, and I considered the best of all, was one with an artistically executed colored border of appropriately emblematical serpents, wherein the author urged the early and permanent retirement of the $346,000,000 "greenbacks," that were then and are still outstanding, and most ably defended the proposition that they had already cost the country many times what they had saved in interest on a funded loan, and that the fundamental idea upon which they were based, viz, that monetary wealth can be created with printing presses and a government's fiat, was the real cause of all the troubles from which we were then suffering and the evils with which we were menaced.

In a somewhat similar manner I have often thought, and do still think, that nearly all, if not all, the ignorance and misunderstanding that exist, with the resultant mischievously defective accountancy of transactions in and ownership of the ordinary type of investment—bonds of solvent municipalities and corporations—are due entirely to the fallacious belief that the coupons attached to such bonds are or represent or at any rate have something to do with interest on the capital that was borrowed by the issuer and invested by the purchaser and are therefore in an entirely different category from what is commonly designated as the par or face value, which in reality is merely coupon 41, 81 or 101, as the case may be, and is usually for from 50 to 33⅓ times as large an amount as all the others that preceded it, and payable on the same day as 40, 80 or 100. (This assertion refers solely to abstract accounting principles underlying actuarial work in transactions in or the distribution of earnings from prompt-paying

investment securities. I am well aware that, in the present state of enlightenment at least, a sharp distinction may exist under unusual conditions, e. g. bond calls, foreclosures, enforcement of guarantees, action by protective committees, etc.)

My views upon this subject are well covered in the article I contributed to this magazine in September, 1915, *The Tyranny of the Engraver* and on the present occasion I propose to assail merely the widely entertained belief that interest accrues on a bond between coupon dates on the par of the bond at a rate per annum obtained by dividing the sum of any two interest coupons attached thereto by said par—impulse having been afforded by the following:

".....interest accrued on bonds purchased is not a part of the cost of the bonds, but is a temporary advance to be reimbursed from the current coupons when they become due. The entry, therefore, is a charge to investments at whatever is paid for the bonds themselves, and to interest for the amount of the accrued interest paid."—THE JOURNAL OF ACCOUNTANCY, October, 1918, p. 318.

To begin, let us try a simple case wherein we assume that the interest accrued on bonds purchased is a part of the cost of the bonds—in other words, that the investor invests what he pays out—and see how it works. A bargain is struck whereby $1,000,000 7s, with 1¾ years to run, are sold upon terms to net the investor exactly 2½% semi-annual interest for the entire time. As no suitable bond tables happen to be available, the proper amount to be paid is ascertained by the same method that would have been employed on a coupon date, viz, by finding the present worth of the post-due negotiable instruments that constitute the bond, as follows:

$$\frac{\$1{,}000{,}000}{(1.0125)^{3.5}} + \frac{\$35{,}000}{(1.0125)^{3.5}} + \frac{\$35{,}000}{(1.0125)^{2.5}} + \frac{\$35{,}000}{(1.0125)^{1.5}} + \frac{\$35{,}000}{(1.0125)^{.5}}$$

$$= \$1{,}094{,}030.55.$$

which sum the investor pays for the bonds and puts on his books as the amount of the investment.....	$1,094,030.55
Three months later he charges the investment and credits his interest account with three months' semi-annual interest at 2½% (.623059%).....	6,816.46
	$1,100,847.01

The same day the investment is credited and cash charged with the proceeds of a coupon that is collected.....	35,000.00
New investment value	$1,065,847.01
Six months later a half-year's interest is charged (1.25% of balance)	13,323.09
	$1,079,170.10
Less coupon	35,000.00
	$1,044,170.10
	13,052.13
	$1,057,222.23
	35,000.00
	$1,022,222.23
	12,777.78
Investment value on day bond and last coupon are due.....	$1,035,000.01

It looks very much, therefore, as though this investor invested what he paid out, viz, $1,094,030.55;

that it was all repaid to him within 1¾ years;

and that during that time he earned 2½% semi-annual interest on every penny that he invested.

Simple as this is, very few seem to appreciate the importance and indispensability of sound accountancy, through which this interest was earned, obtained, secured, collected, received, etc. If this investor had followed the usual custom of relying exclusively upon a pair of scissors for the collection of the income specified, he would not have earned interest in any sense of the word. He would simply have made a profit of $45,969.45 resulting from paying $1,094,030.55 for the bond and coupons and later retailing them for $1,140,000. But by correct accounting practice the interest for, say, the first three months, was collected and made available for investing a balance-sheet with gratifying qualities, being spent, handed over to living heirs, etc., through an exceedingly important journal entry whereby interest account was credited and this investment charged with the previously ascertained correct amount of $6,816.46. The immediate effect, it should be noted, is a material inflation of the book value of the investment, and, as far as the question of the soundness of such

inflation is conecrned, it is totally immaterial how soon it will be wholly or partly undone by cash deposits made by this investment for its credit. For example, in the case of bonds with annual coupons when the interest is taken at the usual semi-annual interval, the inflated value is carried for six months. The important point is that every one of these charges made to this investment will in due course be paid in full in cash, together with compound interest on balances; hence the asset represented by the swollen account is as good as it could possibly be and is comparable to a well secured broker's account, with this favorable distinction: that definite foreknowledge exists regarding amounts and dates of the deposits that are eventually to extinguish the account.

I am still decidedly of the opinion that the foregoing is the only correct aspect of the subject of income from bonds and similar securities, and that, until such doctrine is duly recognized and observed, very little progress will be made in the accountancy of investment of other people's money.

But, according to the dictum above quoted, $17,500 of the $1,094,030.55 is a temporary advance, to be tucked away for the next 90 days in the inanimate unproductive interest account, while whatever is paid for the bonds themselves—in this case presumably $1,094,030.55—$17,500=$1,076,530.55—is a charge to investments.

After getting started with such expert advice, therefore, the bank or trust officer who bought these bonds (very likely with the money of others) is hardly to be blamed for crediting interest account with $35,000 three months later, and thereupon consulting the ticker or awaiting the bullying edict of some politician at Harrisburg or Montpelier in order to ascertain how much of the book value he should write off.

Except in rare cases which have no bearing on the subject under consideration, bonds are bought only for the income they will yield. When the latter is what is agreed upon between the buyer and seller, what is given for the bonds is, as in the example just cited, an ascertained sum of money, which in turn is the present worth of the various negotiable instruments for fixed sums payable at specified dates that constitute the bond. When, as is frequently the case, the bonds are bought at a price—e. g. 102⅞

and interest, par and interest, etc.—a sum of money is paid for a collection of negotiable instruments amounting to what they do and maturing when they do, and the rate of interest that will be earned may be ascertained later to any degree of precision desired. The only difference, therefore, between bonds sold "on a basis" and "at a price" is merely that the unknown quantity has been shifted to another element of the equation; but in either case the important fact remains that the sum of money paid for the bonds is the present worth of the bonds and coupons at some fixed rate. Therefore, while there is nothing immoral or unsound in the fact that a dealer or holder sells and an investor buys $100,000 6s, with 49.7778 years to run, at 129¾ and accrued interest—the latter acting on the honest conviction that the bonds are a bargain at that price—nevertheless what he really does is to pay $131,083.33 for $400,000 non-interest-bearing notes maturing at various dates during the next 50 years, knowing from experience and knowledge acquired from consulting basis books that such a set of notes for that amount and maturing as they do, if bought today for a sum of money indicated by taking 129.75% of the face value of the largest and last maturing note, plus 80/180 of the first maturing note, would yield a rate of income that would be satisfactory to him, to wit, 4½%.

The foregoing quotation is found in the *Students' Department* and is part of the reply to an inquirer who has bought bonds at par and interest. No figures are given as to rates, amounts and dates; so let us assume, for example, that the purchase consisted of $100,000 annual 7s with nine months to run. The bill of the bond dealer of course reads:

$100,000 A., B. and C. 7s at par.....	$100,000.00
Accrued interest to date , 90 days at 7%.....	1,750.00
	$101,750.00

which is as correct as it can be as far as business methods are concerned. But in the name of sound accountancy I certainly protest against the further dissemination in the columns of this magazine of such puerile and pernicious doctrines as:

> that of the $101,750 that this investor laid out, $100,000 only was an investment; $1,750 thereof being a temporary advance to be charged up elsewhere.

This man gave $101,750 for $107,000 to be received nine months hence, whereby he earns 6.87907% simple interest or 6.8217628% if reduced to standard interest with semi-annual compounding.

The amount of unity at 6% compounded semi-annually is:

Years	*Amount*
0	$1.00
½	1.03
1	1.0609
1½	1.092727, etc.

Upon attempting to find, by interpolation, to what this $1.00 has grown at the end of three months we obtain $1.01488916, which, if correct, should be $(1.03)^{\frac{1}{2}}$. It is correct—the square root of 1.03 to 12 places being 1.01488915651. Similar success is obtained in attempting to find the present worth of unity for ¼, ¾, 1¼, etc. years, from which we are warranted in concluding that we may safely depend upon interpolation to take care of semi-annual compounding for periods that are not multiples of the half-year.

If the values for, say, ½ to 50 years shown in the tables for this 7% bond to yield 2½%, be investigated, either mathematically or mechanically (by proper plotting on profile paper), it will be found that they constitute a series, suggesting that interpolation may be employed. The tables, which all parties (except the misguided members of the reinvestment school) acknowledge to be correct, show 106.58 for 1½ years; 108.73 for 2 years; 110.84 for 2½, etc.; hence it would seem that the value for 1¾ years must lie between those for 1½ and 2 years. Now, while this 2-year value of 108.73 is the present worth at 1¼% per half-year, compound discount, of the parent bond and four coupons of $35,000 each, the value for 1¾ years is also the present worth of the same bond and four coupons; and as they are now all payable 3 months sooner, their present worth is of course greater. In fact, as we have already noted, it is $1,094,030.55. But by the application of interpolation we obtain $1,076,584.90 as a bond value for 1¾ years, differing from that obtained by the present-worth process, which stood the test of being multiplied down to maturity, to the extent of $17,445.65. As the bond at 1¾ and at 2 years has four coupons thereon and only three at 1½ years,

this discrepancy is undoubtedly connected with the $35,000 coupon that disappears when the 1½ year stage is reached; moreover it is suggestively close to the $17,500 accrued interest that is supposed to be on the bond mid-way between coupon dates. But this $17,445.65 is not 3 months' growth on $1,000,000 @ 7%, as that, with semi-annual compounding, would be only $17,349.50. Still less is it the present worth of the maturing coupon at either 2½% or 7%; nor is it 3 months' interest at either of these rates on the 2-year value. As this $17,445.65 does not appear to be "interest" obtainable through any plausible elements at hand, let us leave it for the time being and investigate at another period when the bond had longer to run. At 19¾ years the present worth of the bond and 40 coupons is $1,715,478.26 and the interpolated value between 19½ and 20 years is $1,698,032.61; yielding the same discrepancy as before, notwithstanding that the premium is about 62 points greater, and, what is still more significant, the first difference employed in interpolating is decidedly less. Further experiments at ¼, 9¼, 15¾, etc., years yield this same constant discrepancy—hence we are justified in concluding that it is not a fortuitous odd amount, but on the contrary a constant quantity, dependent upon some fixed law and possessing interesting and important significance.

We now experiment at another income rate, say 3%, and find that the accrued interest has shrunk to $17,434.85.

At 3½% it is $17,424.10
at 4%, $17.413.36
and at 4½%, $17.402.65.

Further experiment is unnecessary, as it is now plainly apparent of what this discrepancy between the bond values obtained respectively by interpolation and the present-worth plan consists. It is an imaginary or theoretical coupon. All the tabular bond values that were employed in interpolating were for the occasion when the current coupon had just matured and been detached and paid—hence the same situation must exist with regard to a value for an intervening occasion when such value has been obtained in accordance with the law of the series of values at regular coupon dates, as given in the tables.

In order to view the subject from another standpoint, let us assume that this issue of 7s that we have been considering was

originally sold by a railroad company on a 7% basis (in other words, at par) and that during the last 30 years or so the bonds have fluctuated considerably in the market, due to changes in money rates and a material improvement in the road's credit, at times selling as high as a 2½% basis. Let us assume further that the various bondholders get together and for certain good reasons of their own request the railroad to take up the original bonds and issue in lieu thereof bonds similar in all respects except with quarterly instead of semi-annual coupons. The bondholders offer to bear the cost of engraving, and the railroad officers assent to the proposition on the condition that it be carried out in a strictly equitable manner in accordance with scientific actuarial principles—in other words, that the two quarterly coupons shall constitute a true equivalent for the original semi-annual coupon. As all existing conditions are to remain undisturbed except the frequency of the payment, it is hastily and not unnaturally concluded by all parties concerned that the proposed change is a very simple one—nevertheless it soon develops that the idea is altogether impracticable and will have to be abandoned. The railroad officers find that the loan is costing 7% semi-annually—hence the only quarterly coupon that spells equity to them is one for 1.73495%. In a similar manner, one of the bondholders who bought at 124 to net 4% requires a coupon for 1.741336%. Another bondholder who bought at 194 to yield 2½% can only accept a coupon for 1.744565%, etc.; in short, there would be required as many different sized quarterly coupons as there were income bases upon which the various bondholders had made their respective purchases.

This phase of bond value phenomena is somewhat similar to a rainbow that is observed by various scattered inhabitants of a large city. The essentials, consisting of one sun shining behind the observers and one shower in front of them, are common and constant; nevertheless each observer beholds his own special bow with its individual particular light rays, angular magnitude and position upon the celestial sphere.

In the article, *The Tyranny,* I laid down the doctrine that when bonds were held as an investment, the debtor owing to the investor was "this investment," personified, to which certain critics have objected, claiming that the only person or thing owing any-

thing to the investor was the individual, municipality or corporation that had issued the bonds and coupons he had bought and would pay them when they became due. A new line of thought is certainly afforded by the foregoing investigation. The seemingly anomalous situation, whereby the railroad owes $1,000. per bond, costing 7% interest and on the same day one individual owning one of the self-same bonds has $1,808.46 owing to him earning 2½% interest, is attributable to the indefinite amount of trading that has in the meanwhile taken place in the bonds of this issue. Some of the original purchasers have no doubt made handsome profits in addition to 7% interest on their investment, and other transient holders may have incurred heavy losses through having bought and sold on unwisely selected occasions, but this is of no concern whatever to the railroad, which at all times owed a uniform sum to this loan at 7% interest; and, although during the life of the bonds millions of dollars might be made and lost through change of ownership, the railroad company is not concerned in the slightest degree and would be no worse nor better off for being in total ignorance of the existence of all these changes in ownership and fluctuations in price. In a similar manner, an investor who buys at 180.84 when the bonds have 24 years to run is not concerned in the least with what the railroad originally received, or what has been made or lost by those who have owned the bonds prior to his purchase. All he knows or needs to know is that he has invested $1,808.46, $18,084.58, $180,845.83, etc., for 24 years, and that, attended by substantial and steadily increasing semi-annual payments on principal he will earn 2½% semi-annual interest on what he has invested—provided he keeps books and makes proper entries at least as frequently as whenever a coupon is collected.

The method whereby the amount of an equivalent shorter coupon was found as a "discrepancy" is a clumsy, roundabout process; and it is now in order to devise a formula whereby direct calculation may be made for any frequency interval. Suppose a case where the coupons are to be increased from two to six. Denoting

Actual coupon by C
Short coupon by c
Semi-annual ratio by R

60-day ratio by r
Any semi-annual value by V
Value, ex-coupon, 2, 4 and 6 months thereafter by V′, V″ and V‴
then:

(1) $VR - C = V'''$
(2) $Vr - c = V'$
(3) $Vr^2 - rc - c = V''$
(4) $Vr^3 - rc^2 - rc - c = V'''$

As $r^3 = R$, we have from equations (1) and (4)

$C = r^2c + rc + c$

from which it will be found that $c = C(r - 1)$

$$\text{Odd coupon} = \frac{\text{(real semi-annual coupon) (odd rate)}}{\text{semi-annual rate}} \quad (R - 1) \text{ or}$$

Example: A trust company is the owner of a $100,000. 4% loan that was bought some years before at a price to yield 5%. The maker of the loan desires to take up the semi-annual "interest notes" originally uttered and to substitute a new set payable at 45-day intervals, to which the trust company assents. Required, the amount of the new coupons.

$$\frac{\$2{,}000. \times .00619225}{.025} = \$495.38.$$

PROOF

Value at, say, 17½ years	$88,427.42
Forty-five days' interest at 5%	547.56
Less coupon	*495.38*
	$88,479.60
	547.89
	495.38
	$88,532.11
	548.21
	495.38
	$88,584.94
	548.54
	495.38
Book value for 17 years, tallying with bond tables	$88,638.10

In view, therefore, of all the loose talk that has been and still is heard in various circles (as well as sundry standard tables and text-books) about "bonds with interest payable quarterly"; "bonds bearing interest at the rate of"; "total cash interest"; "surplus interest"; "deficient interest," etc., it is indeed refreshing to turn to an unpretentious little text-book, *Interest and Bond Values* published at Toronto in 1912 by M. A. Mackenzie of that city, and find nothing but such phrases as:

"the ability of the issuing corporation to earn the annual coupon payments."
"a bond due 20 years hence and bearing half-yearly coupons at 5%."
"had the bond borne coupons at 4½%."
"had the coupons been for $225. each ..... the coupons being only for $200. each."

One great difficulty with which conscientious school teachers have to contend is the disposition on the part of arithmetic pupils to ignore entirely the philosophical aspects of the various problems given them—working solely to get the answer, and, when that is accomplished, dismissing the matter from their minds as a closed incident. On account of the numerous coincidences that may and do arise in mathematics, it is quite probable that many a correct answer has been obtained by a dangerously incorrect process. As an example, a scholar called upon to find the area of a circle tries the experiment of multiplying the diameter into 3.14159 and taking half of the product. If the given diameter happens to be 2, the correct result that he obtains is no index to his understanding of that phase of mensuration.

This state of affairs is not confined entirely to school children—it is found occasionally among grown-ups. For instance, every single one of the thousands of semi-annual bond values published by one author are perfectly correct numerically, and yet the method by which they were calculated, as well as the philosophical interpretation and application, is as wrong as anything can be. Furthermore, that author's really (numerically) accurate values are, by his own admission, inaccurate, or at least in violation of what he considered correct principles, and were put out for commercial purposes to supply a popular demand which he honestly, even if erroneously, believed was founded upon error, which he appeared to be unable to stamp out. To be specific: if called upon to furnish a value for $1,000. twelve year 4s to yield 10% he would have

given the correct figures of $586.04, but his interpretation and application would have been substantially as follows:

In order to fulfill the conditions it is necessary:

(a) That you obtain $29.30 in actual cash every 6 months available for spending purposes.

(b) That your net capital 12 years hence when the bond matures be the same as at present, viz, $586.04.

Six months hence, at which time you are entitled to $29.30 in cash, you will obtain $20. from the coupon you collect. The remaining $9.30 you must borrow somewhere for 11½ years at 10% interest, compounded semi-annually. Twelve months hence you must borrow another $9.30 at the same rate for 11 years, etc. When the bond matures you will find that your total indebtedness on the 23 loans is $384.66, and after paying that, as well as paying yourself the last $29.30, you will have left your original capital of $586.04.

An hour later perhaps there might be another call for a value for some 40-year 1s to yield 2%, and the problem would be handled in a similar manner except that the figures given would contemplate that all this future borrowing would be done at the rate of 2%.

It is to the credit of the reinvestmentists, however, that they were logical enough to see the absurdity into which they were thus led, and took the ground that tables intended for general use, in order to possess any utility, must provide for borrowing or reinvesting at some sane average rate. In fact, two authors have gone so far as to bring out a comprehensive set of tables of this character—both adopting the rate of 4%. For 12-year 4s to yield 10%, 63.799 is given, instead of the correct value of 58.604 as shown above.

All this is of course as absurd as it can be, being one result of the fallacy that there is no interest and that there can be no interest unless and until a cash payment in actual bills or coin takes place; in short, it is going the limit in refusing to acknowledge the efficacy or even the existence of accountancy, and possesses but one merit, to wit, it assumes that the investment is the sum of money which the investor pays for the bond.

Returning to the subject of obtaining correct results by incorrect, or at least philosophically unsound, methods, let us assume

that a corporation offers for sale an issue of $100,000 7s, to be dated May 1, 1920, due May 1, 1925, and finally accepts the best bid received, viz, a 7% basis. If delivery is made on May 1st the buyer pays $100,000 for his purchase, which is correct. But he, as well as many other people, are firmly of the opinion that he bought the bonds at par or their face value with no interest accrued, which is not correct. A closer scrutiny of what he bought reveals that it consisted of eleven different notes or bonds maturing serially from ½ to 5 years, with a total face value of $135,000 and an average life of 4.42 years. As all these notes (with one exception) have the same par value and (with one exception again) have different lengths of time to run, and yet are to yield the purchaser a uniform rate of interest, it is obvious that each one must have a different present worth, and the proper price for each note is found by a trustworthy true discount table to be:

Note	Years until due	Par value	Present worth	Discount
1	½	$ 3,500	$ 3,381.64	$ 118.36
2	1	3,500	3,267.29	232.71
3	1½	3,500	3,156.80	343.20
4	2	3,500	3,050.05	449.95
5	2½	3,500	2,946.91	553.09
6	3	3,500	2,847.25	652.75
7	3½	3,500	2,750.97	749.03
8	4	3,500	2,657.94	842.06
9	4½	3,500	2,568.06	931.94
10	5	3,500	2,481.22	1,018.78
11	5	100,000	70,891.88	29,108.12
		$135,000	$100,000.01	$34,999.99

The answer, $100,000, is the same in either case, but there is considerable (and decidedly important) philosophical distinction between the popular myth that one bond for $100,000, bearing interest at the rate of 7%, was bought at par and the actual fact that eleven bonds, aggregating $135,000, that do not, or at least need not so much as mention the word interest, were bought at an average discount of 25.93%.

Suppose that the company did not obtain an offer for the bonds until June 1st, when they were sold on the same terms, viz, a 7% basis. The company would demand, and no doubt obtain:

Par	$100,000.00
One month's accrued interest at 7%.....	583.33
	$100,583.33

The purchaser might object to the $583.33 on the grounds that it was obtained by monthly instead of semi-annual compounding. His claim, being manifestly just, is allowed, and the figures are amended to read $100,575.00, which are as incorrect as they can be except numerically. Five hundred and seventy-five dollars interest! How can there be any interest when nothing has as yet been borrowed or lent? During the last 30 days no change whatever has taken place in what is being bought and sold. It still consists of 11 notes aggregating $135.000 maturing $3,500. November 1, 1920; $3,500. May 1, 1921, etc., and the rate of interest to be earned is still 7% semi-annually. It is in the calendar that the change has occurred. The first note will mature and be paid in 5 months, the second in 11 months, etc.; hence the present worth of each is now greater, and by proper calculation is found to be:

Present worth	Discount
$ 3,401.08	$ 98.92
3,286.08	213.92
3,174.95	325.05
3,067.59	432.41
2,963.85	536.15
2,863.62	636.38
2,766.79	733.21
2,673.22	826.78
2,582.83	917.17
2,495.49	1,004.51
71,299.51	28,700.49
$100,575.01	$34,424.99

Now that an investment has been made and an indebtedness incurred, matters are changed, and interest begins to accrue in earnest at the rate per $100,000. of $19.11 daily, $191.30 every 10 days, $287.09 every 15 days, etc. It is, however, accruing, not on the bonds, as is generally supposed, but on the $100,575.00 which the company borrowed and the investor lent. Assuming, for instance, that both parties see fit to make entries in interest account at monthly intervals, then their correctly kept books will show the

following amounts respectively owing to/owed by this loan investment:

July 1.....	$101,153.31
August 1.....	101,734.95
September 1.....	102,319.93
October 1.....	102,908.27
November 1.....	103,500.00

After the coupon is paid the balance is pulled down to $100,000, and during each ensuing six months throughout the life of the loan the balance will work up to $103,500. and then fall to $100,000. in exactly the same manner; but it is somewhat rash to assert or take for granted that one of the notes, as it matures, is interest on the original investment simply because it happens to be for the same sum of money that represents interest earned up to that time.

This will work very well so long as all the coupons are for the same amount, mature at regular intervals, and date from and finally mature with the bonds themselves, which is of course true in an overwhelming majority of cases, and yet does not need to be ever the case—at least as far as providing suitable and satisfactory loan investment at some fixed interest rate is concerned.

In 1878 a small issue of bonds was put out by the Brunswick & Chillicothe R. R. Co. (a part of the Wabash system) with coupons as follows: 2% for 6 years; 3% for 19 years and 4% for the remaining 3 years, which must have proven hard sledding for the par-worshippers of those days, to whom was yet unrevealed even the contemptible lazy-man's makeshift known as the pro-rata method and who, as far as I know, had nothing to turn to for help except a small pioneer edition of 4-place bond values designated by its author as "stock tables."

The normal logical starting point when the security is supposedly unquestioned is 0% basis. In a case of $1,000. 20-year 6s for instance this would be $2,200., which is the true par. If bought at this price there is no preponderance of the amount repaid over the amount invested; the ratio of increase for any epoch is 1.00; the interest $0.00 and time immaterial. If, however, the bond be bought for $2,199.99, interest amounting in the aggregate to 1 cent will be earned in the 20 years, and (at the expense of considerable labor) the average semi-annual rate may be ascertained

with the utmost precision; and, provided fractions of cents are duly accounted for, the investment can be carried along and made to stand at $1,000. at maturity in exactly the same manner and with the same accuracy as if the bond had been bought to yield say 5%. As the amount paid for the investment decreases the effective rate of interest increases and finally becomes infinitely large when the bond is bought for $0.00. Taking 1 cent as the minimum variation that may be made in a sum of money, it follows that there are 220,000 different income bases ranging from 0% to infinity% upon which this $1,000. bond may be purchased. For $10,000 worth there are 2,200,000; for $100,000 worth there are 22,000,000, etc. As might be expected from our familiarity with the laws of chance, it happens that a single one of these thousands or millions of income bases possesses special unique properties, just as .31622777 + is the reciprocal of 3.1622777 + or .137128857 is the log. of 1.37128857, but to seize on this "punctual interest basis" as an ideal standard and consider everything else between zero and infinity as distortions thereof through the operations of premiums and discounts, instead of regarding it as a special incidental case characterized by interesting and extraordinary properties, is in my opinion quite as bad as it would be for astronomers to consider the orbits of all the planets as circles of varying degrees of distortedness.

The professional accountants of the country should look upon the subject of investment accountancy with a somewhat broader vision than a pawnbroker or a drudging discount clerk in a commercial bank; awaken to a realization of the fact that something besides a pair of shears and a ticker is required to collect and distribute the income from the bonds owned by a trust estate or insurance company, or ascertain the interest charges, outstanding indebtedness and surplus of a modern corporation; learn that the effulgent halo that is supposed to encircle the sacred brow of par is no more existent than the pink elephants and green-tailed monkeys seen by inmates of inebriate wards—in short, not allow themselves to be influenced and guided entirely in their professional conduct by the customs, traditions and precepts of a Broad street curb specialist or some politician at Augusta or Lansing.