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The Impact of Market Fragmentation on Stock Exchange Events and Thinly Traded Securities

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THE IMAPCT OF MARKET FRAGMENTATION ON STOCK EXCHANGE EVENTS AND THINILY TRADED SECURITIES

A DISSERTATION PRESENTED IN PARTIAL FULFILLMENT OF REQUIREMENTS FOR THE DEGREE OF DOCTORATE OF PHILOSOPHY IN BUSINESS ADMINISTRATION, DEPARTMENT OF FINANCE, UNIVERSITY OF MISSISSIPPI

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ABSTRACT

In Part 1, I analyze independent technological malfunctions that forced trading halts at various equity exchanges over the past decade. During each halt, all other exchanges remained open. The primary purpose of this study is to examine intraday trading activity before, during, and after a technological malfunction, which are events that are neither driven by an informational event nor an order imbalance. Of the events I record in this study (8 technological malfunctions), a majority document a reduction in liquidity and an increase in short term volatility during and immediately after a technological malfunction. Furthermore, these affects appear to be relatively short-term but, in a few events, I see abnormal trading as far out as 10 days. Additionally, I investigate what impact these events have on algorithmic trading activity and find that algorithmic trading activity increases intra- and post-suspension. In part 2, I examine the extent algorithmic trading and highly fragmented stock markets are related. In this study, I use multiple methods to determine the level of fragmentation and examine algorithmic trading activity. Additionally, I dissect this relation further to determine what influence trading fees have on algorithmic trading and the possible appeal that different fee venues provide algorithmic traders. I find evidence that suggest more fragmented stocks will have a more algorithmic trading activity, and that this activity will be concentrated on make-take venues where algorithmic traders are paid a rebate to provide liquidity. I also demonstrate using the number of daily stock venues recorded by the SEC's Midas dataset that there exists an inverted 'U' shape pattern between the number of daily venues a stock trades on and trading costs. In part 3, considering the

SEC's recent focus on addressing liquidity concerns for stocks with an average daily volume (ADV) below 100,000 shares, thinly traded securities, in this study I identify possible determinants of the poor liquidity of these types of securities. Among the commentary at a roundtable discussion held by the SEC in October of 2019, I identify there to be three prominent factors influencing daily liquidity in thinly traded stocks: (1) spatial fragmentation, (2) temporal fragmentation, and (3) market making activity. I find evidence that suggests that temporal fragmentation and market making activity appear to be more prominent factors contributing to the poor liquidity of thinly traded stocks. I also find that the market is capable of creating liquidity on its own without special advantages given to select exchanges and that spatial fragmentation doesn't appear to be severely impacting transactions costs in thinly traded stocks. I further make use of the SEC's Tick Size Pilot Program as a robustness check to confirm that temporal fragmentation and the lack of market makers are two driving factors influencing the differences in liquidity between thinly traded stocks and actively traded stocks.

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PART 1: EXAMINING THE IMPACT TECHNOLOGICAL MALFUNCTIONS CAN HAVE ON INTRADAY TRADING

I. INTRODUCTION

Technological innovations over the past couple of decades have vastly changed financial markets. While positive changes abound, a down-side to technological innovation is that technology-reliant systems require maintenance and sometimes malfunction. Malfunctions in financial markets have real consequences, ranging from a loss of investor confidence to a breakdown of the marketplace. For instance, in August 2019, Bob Pisani of CNBC reported a trade reporting "glitch" in the Securities Information Processor for NYSE-listed securities created, "considerable confusion in the last hour, and cast doubt on whether the prices disseminated for major indexes like the Dow Jones Industrial Average and the S&P 500 were accurate."[1] Another trading glitch occurred during the initial public offering of Facebook, resulting in mass trader confusion and Nasdaq receiving a $10 million dollar fine, the largest fine ever levied by the Securities and Exchange Commission (SEC) against an exchange, for poor systems and decision-making.[2]

The primary purpose of this study is to examine trading around technological malfunctions. Technological malfunctions are events that are not driven by an informational event nor an imbalance in orders. We seek to analyze trading before, during, and after trading malfunctions to determine whether these non-informational events create transitory or permanent effects. We use a sample of eight trading halts on various equity exchanges that occur due to a

[1] See https://www.cnbc.com/2019/08/13/stock-tape-glitch-means-its-still-not-exactly-clear-where-the-dow-sp-500-closed-on-monday.html

[2] See https://slate.com/news-and-politics/2013/08/nasdaq-trading-is-suspended-for-hours-due-to-technical-glitch.html

technical issue and result in suspended trading while the issue is mitigated. Analyzed technological malfunctions occur during various years, starting in 2013 and ending in 2019. As technological disruptions have not garnered a lot of academic attention, we rely on theoretical and empirical literature that explores trading halts brought about by excessive information asymmetries or trading imbalances. This approach allows us to indirectly compare the findings and predictions of halts driven by informational events with that of our sample, halts that result from technical errors.

Studies into the effect of traditional trading halts suggest these events produce a significant increase in quoted depths, number of trades, trading volume, and spreads, indicating a reduction in market liquidity (Jiang, McInish, and Upson, 2009). Jiang et al. not only find these effects on overall market quality for the individual halted stock, but also find significant liquidity impacts on informationally related securities. Other research suggests market deterioration around trading halts. For instance, increased trading activity is closely associated with increased volatility for both NYSE and Nasdaq stocks (Corwin and Lipson, 2000; Christie, Corwin, and Harris, 2002; and Lee, Ready, and Seguin, 1994).

While the aforementioned papers are instrumental in providing empirical evidence regarding trading halts, these studies' samples pre-date more technologically advanced markets. Extrapolating their results to today's mostly electronic markets dominated by algorithmic trading is problematic. Much of the prior literature analyzes trading halts resulting from excessive information asymmetries and low depth level (Edelen and Gervais, 2003). The study most closely related to ours is that of Clark-Joseph, Ye, and Zi (2017), who focus on the impact of Designated Market Maker (DMM) participation on trading around technology-driven halts. Clark-Joseph et al. study two technological glitches that result in two separate trading halts, one at the NYSE and the other at EDGX. Their analysis is the first to look at halts resulting from technological malfunctions rather than resulting from information asymmetries or low depth levels. Clark-Joseph et al.'s analysis does not expand beyond their DMM focus. Our contribution to the literature is that we examine trading around technological malfunctions and document market quality before, during, and after these halts and across many exchanges as well as over a reasonably long period of time. Additionally, we extend current research on the behaviors of algorithmic traders during stressful market conditions by observing algorithmic trading, both during and after these technology-driven trading halts.

II. HYPOTHESIS DEVELOPMENT

TRADING HALTS

The SEC identifies two types of trading halts—regulatory and non-regulatory. Regulatory trading halts, the most common type, occur when pending news significantly impacts a security's price or when there is uncertainty about a security's ability to continue to meet a market's listing standards. All U.S. equity markets honor regulatory trading halts imposed by a security's primary market.[3] No trading takes place in a halted security on any exchange during a regulatory halt. Non-regulatory halts occur when an exchange experiences significant imbalance in a security's pending buy and sell orders. Non-regulatory halts, however, don't preclude other markets from trading the halted security.[4]

In general, the underlying factor driving a regulatory or non-regulatory trading halt, as defined by the SEC, is new fundamental information to be released by a firm or information garnered by security analysts prompting an order imbalance. We study trading halts that differ from regulatory and non-regulatory halts in two aspects:

1. Technological malfunction halts are not driven by an informational event nor an imbalance in orders, but are the direct result of a technological malfunction or glitch, and

[3] See https://www.sec.gov/fast-answers/answerstradinghalthtm.html
[4] See ibid. 3

2. Unlike regulatory trading halts, trading halts in response to a technological malfunction restrict trading on the exchange where the malfunction occurs, but don't preclude other exchanges from trading securities affected by the malfunctioning exchange.

Previous trading halt literature focuses primarily on the costs and benefits of circuit breakers and their efficacy. Early research looks at price movement around trading halts (pre and post) (Hopewell and Schwartz, 1978; Kryzanowski, 1979; King, Pownall, and Waymire, 1992; and Wu, 1998). Schwartz (1982) is one of the first to examine intra-suspension price formation by focusing on indications that estimate post-suspension reopening prices by specialists. Schwartz finds that large permanent stock price movements occur during the suspension period for both news and order imbalance suspensions, suggesting that both types of suspensions are related to the release of new information. He also alludes to the effect trading suspensions may have on market quality. The results of Schwartz establish the importance of a holistic analysis including not only trading around halts but also intra-suspension trading.

Ferris, Kumar, and Wolfe (1992) evaluate the pattern of trading activity in the period surrounding a trading suspension. The authors assess the effectiveness of a trading suspension as a tool of regulatory policy intended to control unusual trading activity, defined by the SEC (at that time) as higher than normal return variance or trading activity.[5] Ferris et al. find that both the variance of returns and trading volume are substantially higher than normal in the pre-suspension period. Likewise, Edelen and Gervais (2003) confirm that that both volume and volatility are

[5]The SEC's Market Surveillance and Evaluation Division maintains a continuous market surveillance program that is designed to closely review the markets in securities in which unusual price and volume changes occur or where there is a large unexplained influx of buy or sell orders. See https://www.sec.gov/rules/sro/nyse/2015/34-75809-ex5.pdf

usually high around trading halts. Edelen and Gervais find about 50% more trades per trading interval, 50% more trade imbalances, and about twice the share volume around trading halts.

In order to provide more insight into pre-halt trading activity, Corwin and Lipson (2000) examine the relation between the limit order book and the post-halt reopening price and between liquidity and post-halt volatility around NYSE trading halts. They find depth near the quotes is unusually low around trading halts. Also, they find a dramatic increase in spreads and a surge in both market order submissions and cancellations just before order imbalance halts. Despite these findings, their results suggest that the change in market conditions is abrupt and there is little time for the NYSE specialist to draw liquidity to the market. The trading halts we study are the result of technical malfunctions and not the result of informational driven events. Because there is no informational event nor published order imbalance to observe prior to these malfunctions, there should be no difference in activity on days prior to the halt relative to all other non-halt days. This conjecture relies heavily on Corwin and Lipson's view that trading halts are enacted after abrupt changes in trading conditions and the fact that technological errors are both sudden and unpredictable expect the following to hold:

> Hypothesis 1: Volume, volatility, depth near-the-quote, and quoted spreads prior to a trading halt will be similar to when there is no trading halt.

Clark-Joseph, Ye, and Zi's (2017) study of two technology-induced trading suspensions provide indications of what to expect from our sample of eight technological malfunctions. Clark-Joseph et al. identify the causal impact of DMM participation on liquidity by examining two trading halts, one at the NYSE and the other at EDGX in 2015. They find that the NYSE shutdown led to a large increase in treatment stocks' spreads, but not in quoted depth (neither volume nor dollar). The authors find that there is no difference in treatment stocks from the

EDGX shutdown compared to control stocks in spreads and depths. Given Clark-Joseph et al.'s results, we expect exchange characteristics to be highly correlated with the halt's effect on trading activity. More specifically, we expect the following:

Hypothesis 2: Overall market liquidity will be more sensitive to technology-driven trading halts that occur on exchanges with a higher presence of market makers, both endogenous liquidity providers and DMMs.

Research shows that the effect of a trading halt depends on the type of activity taking place during the halt. Lee, Ready, and Seguin (1994) investigate the efficacy of trading halts by examining the effect of firm-specific New York Stock Exchange (NYSE) trading halts on volume and price volatility. They find that during a trading halt, trading volume (i.e., the trading volume of the resuming batch trade) is not significantly different from the cumulative volume during price-matched pseudohalts. Lee et al. also go on to find that the absence of recent transaction prices may potentially influence traders to be less willing or able to reveal their demands. This reluctance is especially true during large price adjustments, and thus this reluctance has the potential to lead to noisier reopening prices, accompanied by higher volume and volatility in the post-halt period.

Corwin and Lipson (2000) examine the relation between the limit order book and the reopening price and between liquidity and post-halt volatility. They find market and limit order submissions and cancellations increase significantly during trading halts and that a large portion of the limit book at the reopen is composed of orders submitted during the halt. These findings suggest that traders take advantage of the pause in trading to reposition their trading interests. Christie, Corwin, and Harris (2002) confirm the findings in Corwin and Lipson and Schwartz (1982), as well as provide evidence consistent with Greenwald and Stein (1991) that halt mechanisms allow for an increase in information dissemination during halts and appear to reduce uncertainty. Jiang, McInish, and Upson (2009) investigate the impact of trading halts of NYSE-listed stocks on informationally related securities that continue to trade during the period of the halt. They find both relative and absolute spreads appear to increase, while offer and bid depths also increase. The increase in quoted depth is asymmetrical with offer depth increasing by 27.8% and bid depth increasing by 13.7%. Jiang et al. also report that during a trading halt, spread-to-total depth increases significantly and further report a decline in overall quote-based liquidity during the halt, despite a significant increase in volume.

Additionally, investors may be unable to differentiate trading halts influenced by technological error with non-regulatory trading halts influenced by informational events or order imbalances. Rashes (2001) finds that genuine shifts in sentiment due to misunderstanding of information about a single stock can lead to a deviation of the security's price from its fundamental value. Likewise, Cukierman, Lustenberger, and Meltzer (2018) establish the Permanent-Transitory Confusion (PTC) hypothesis. PTC refers to knowledge of current and past changes in a stochastic variable that leave a margin of uncertainty about how much of those changes will persist into the future versus those that are temporary and will eventually be corrected.

These studies suggest that noisy investors may misinterpret the information surrounding exchange technical errors that result in a trade suspension with non-regulatory trading suspensions stemming from fundamental information changes and order imbalances. Some uncertainty surrounding trading could be reduced if halt mechanisms are in place that allow for increased information dissemination during a technological malfunction. If this information includes why the trading halt is enacted, then traders may choose to reposition their orders; in which case, investor confusion may be lessened and trading during the suspension will be unimpeded. However, if investors are misinterpreting trade suspensions due to a technological error with traditional, non-regulatory trading suspensions, then we expect the following to hold.

Hypothesis 3: Spreads and depth will increase during a trading suspension. Market and limit order submissions and cancellations will increase during a trading halt.

Investor confusion is a salient factor that influences trading activity post-halt and portrays investors as unable to differentiate between permanent and transitory effects resulting from a trading suspension. Cukierman et al. (2018) refer to this dilemma as variable transitory confusion, when investors are aware of current and past changes in a stochastic variable that leaves a margin of uncertainty about how much of the change will persist into the future and how much is temporary. Market glitches appear to be obscure random events with little to no transmission of information regarding one specific stock or industry but simply due to technical issues at the trading exchange. Rashes' (2001) study identifying investor misunderstanding and confusion surrounding an event, is applicable to trading immediately after technological malfunction trading halts. In the period following a trading halt from a technological glitch, we may see abnormal trading activity, resulting from investors misinterpreting the reason for the trading halt when there is little to no information provided. Investors may initially interpret the effects of these events to be permanent and thus post-halt drift effects may result.

Ferris et al. (1992) find that variance of returns and volume revert to normal at a much later date (60 days after a trading halt) and conclude that there is not an immediate elimination of unusual market activity. Lee, Ready, and Seguin (1994) also find that price volatility effects remain for one full trading day after a suspension and higher trading volume is observed for at least three full trading days. Lee et al. interpret investors' reluctance to reveal their trading interests leads to noisier reopening prices and an increase in volatility and volume. Lee et al. also find elevated levels of volume and volatility after halts when matched with pseudohalts. In the first full trading day after a halt, Lee et al. find that volume is 230% greater, while volatility is 50% to 115% larger than price-matched pseudohalts. Kryzanowski and Nemiroff (1998) investigate the price discovery process around exchange-initiated trading halts using stocks on the Montreal Exchange. They find that volatility and measures of trade activity increase significantly around trading halts and return to lower levels in less than two days after the resumption of trading.[6]

Corwin and Lipson (2000) find that both volume and volatility increase significantly after NYSE trading halts and can be explained, in part, by changes in liquidity. The specialist's quoted spread, limit order book spread, 5,000-share spread, and 10,000-share spread increase after a trading suspension, but generally dissipate within two minutes during order imbalance halts and within 30 minutes for news halts. Christie, Corwin, and Harris (2002) find that the uncertainty associated with trading halts on Nasdaq are not resolved by the time the halt is lifted. They find that the median inside spread more than doubles when trading resumes and the post-halt trading period is characterized by unusually high volatility. In addition to these findings, Christie et al. also find that an unusually large number of small trades take place in the period following a suspension. Therefore, we expect:

Hypothesis 4: The post-halt period will be characterized by higher volatility, higher volume, and an increase in the number of small trades relative to the trading period before a technological glitch.

[6] Kryzanowski and Nemiroff (1998) identify "exchange-initiated" trading halts, which are trading halts that are a regulatory response to pre-halt information asymmetry, volatility and/or trade activity on the Montreal Exchange. These "exchange-initiated" trading halts are traditional trading halts and are not in response to technological malfunction.

Hypothesis 5: The post-halt period will be characterized by a drift in abnormal trading activity for an extended period of time, the length of the post-halt drift is unknown.

FRAGMENTATION

Fragmentation continues to be a relevant topic of discussion with three new U.S. stock exchanges having launched operations in later 2020.[7] The Clark-Joseph et al. (2017) study provides the first indication of how the market accommodates trading suspensions in response to a technological malfunction by establishing a link between intra- and post-halt activity with market fragmentation, which is found in their analysis of an EDGX trading malfunction. Several empirical studies document the importance of market fragmentation on market quality. O'Hara and Ye (2011) examine a three-month period in 2008 and proxy for fragmentation via the number of trade reporting facilities (TRFs) reporting trading activity on an asset, finding that fragmentation is associated with lower effective spreads. They conclude that fragmentation appears to provide some benefit to markets.

As of March 2021, trading currently takes place on 16 exchanges, otherwise known as lit venues, and 61 active alternative trading systems (ATS), also referred to as dark venues or dark pools.[8] Gresse (2017) provides empirical evidence that lit fragmentation provides some benefits for large stocks via increased depth but depletes depth for small stocks. Fragmentation on dark venues also appears to deteriorate trading depth and capital formation of small cap stocks (Degryse, De Jong, and Van Kervel, 2015 and Gresse, 2017). The role of fragmentation is instrumental in this study, as it illustrates whether fragmentation is beneficial in the sense that

[7] McCrank, J. (2020, August 21). Competition to heat up among U.S. stock exchanges with new entrants. Reuters. Available at https://www.reuters.com/article/us-usa-exchanges/competition-to-heat-up-among-u-s-stock-exchanges-with-new-entrants-idUSKBN25H23K

[8] See https://www.sec.gov/foia/docs/atslist.htm and https://www.finra.org/filing-reporting/otc-transparency/ats-equity-firms

other markets absorb abnormal trading activity during a technological malfunction that forces a suspension in trading at one of the 16 venues.

Previous studies on trading halts allude to the effect of fragmentation on trading activity on a particular exchange (Lee, Ready, and Seguin, 1994; Christie, Corwin, and Harris, 2002; Jiang, McInish, and Upson, 2009; Clark- Joseph et al. 2017). Clark-Joseph et al. demonstrate effects fragmentation has on particular exchanges by finding that their results are unique to the shutdown of the NYSE. Lee, Ready, and Seguin theorize that one explanation for their results is that price discovery and the batch reopening mechanism employed by the NYSE after a halt is inefficient when compared to a continuous trading process. Corwin and Lipson (2000) find that the proportion of depth contributed by floor participants increases immediately after halts, suggesting that floor participants, including the specialist, step in to provide liquidity after NYSE trading halts. Christie, et al. find that the NASDAQ reopening process is initiated after enough time elapses for information to be widely disseminated and broadcast to investors through various news sources. Jiang, et al. record differences between the effects on the NYSE and Nasdaq to find that both exchanges show significant increases in the number of trades and trade volume, but Nasdaq's trade-based measures of liquidity increase more than those of the NYSE. Additionally, they find that Nasdaq's immediate reaction to the information content of a trading halt is significantly lower, but that information is compounded into the stock price by the close of trading.

The previously mentioned studies present similar findings that imply the location of the technical error is a crucial factor to consider when examining the effects of trading suspensions on intra- and post-halt activity. Unique characteristics of each exchange or exchange group may play a role in the intra- and post-suspension period, as well as in the ability of other markets to supplement trading services of the exchange that is temporarily closed. We then expect:

> Hypothesis 6: The location of the trading halt will have a significant impact on trading activity during and after the halt. Venues with large market shares and/or DMMs presence will have sharper decreases in liquidity following a trading halt.

ALGORITHMIC TRADING

Algorithmic trading (AT), or the use of computer algorithms to automatically execute trading strategies, is another example of technological change over the past few decades and dominates stock markets in the U.S. and globally. The increase in automation greatly reduces the role that human market makers play and as a result AT creates new electronic, endogenous intermediaries also known as high frequency traders (HFTs). HFTs, being a subset of AT, act similar to the traditional human intermediaries they supplanted where HFTs have short holding periods, don't hold large positions, and trade frequently but have substantially smaller latencies than their human counterparts. The benefits that AT and HFTs provide to the market include (1) public information is rapidly incorporated into prices by ATs (Zhang, 2012; Chakrabarty, Moulton and Wang, 2019), and (2) AT orders are more likely to disseminate new information into prices through heightened quoting efficiency (Hendershott, Jones and Menkveld, 2011) and through large permanent price impacts (Brogaard, Hendershott and Riordan, 2014). Kyle (1985) also finds that HFTs also play a beneficial role in price discovery, as these types of traders are informed investors by trading in the direction of permanent price changes and against transitory price movements. Brogaard et al. analyze a sample of 26 HFT firms from the Nasdaq HFT data set and find that not only do HFTs trade in the direction of a permanent price shift, but they also moderate and limit the effect by trading against transitory pricing errors.

In this section we look to see (1) what impact technological malfunction trading halts have on algorithmic trading activity during these trading halts and (2) if algorithmic trading increases following a trading halt that is the result of technological malfunction. Recent literature has shifted its attention to the reliability of endogenous liquidity providers (ELPs), who play an important role in providing liquidity (Hasbrouck and Saar, 2013; Menkveld, 2013; Brogaard et al., 2014; Conrad, Wahal, and Xiang, 2015). During periods of market stress, endogenous liquidity providers often withdraw from the market resulting in liquidity disappearing and transactions costs increasing (Bongaerts and Van Achter, 2015; Korajczyk and Murphy, 2019; Kirilenko, Kyle, Samadi, and Tuzun, 2017). For instance, Kirilenko et al. examine intraday intermediation in the E-mini S&P 500 futures market around the Flash Crash of 2010 and find evidence consistent with the limited risk-bearing capacity hypothesis of intraday intermediaries. This hypothesis contends that these intermediaries do not take on large risky inventories, thereby contributing to large temporary selling pressures when absent from the market.

Brogaard, Carrion, Moyaert, Riordan, Shkilko, and Solokov (2018) examine the activity of common endogenous liquidity providers (ELPs) around extreme price movements (EPMs) and find results that differ from Kyle et al. (2017). EPMs in Brogaard et al. are characterized as stressful periods with large, rapid price fluctuations in small intervals. Brogaard et al. find that HFTs act as endogenous liquidity providers and supply liquidity in the opposite direction during these stressful periods to absorb trade imbalances and correct transitory price movements. The authors conclude that HFTs act to stabilize market during periods of stress and provide net positive effects on liquidity provision. Additionally, Anand and Venkataraman (2016) provide an in-depth analysis of endogenous liquidity providers (ELPs) and find that aggregate ELP participation is sensitive to market conditions, with higher ELP participation associated with an increase in stock volatility.

Hypothesis 3 outlines a state of the market that can also be identified as a period of transitory price movements. This intra-suspension period is characterized by a large increase in order submissions and cancellations that is potentially the result of investors' inability to accurately differentiate trading halts influenced by a technological malfunction from trading halts that are the result of informational events or order imbalances. Potentially, this confusion may not be resolved in the immediate period following the trading halt and investor confusion may remain a salient factor for some time. Hypothesis 4 and 5 characterize post-halt trading activity experiencing higher volatility, higher volume, and a larger amount of small trades. Zhang (2010) examines the implication that HFTs have on stock price volatility and price discovery and characterizes the post-halt environment to be one with increased volatility and uncertainty. Zhang provides additional evidence regarding the relation between HFT activity and periods of high uncertainty by finding that the positive correlation between HFT activity and volatility is stronger in a market with higher uncertainty. Brogaard et al. (2014) have similar findings to Zhang, in that on high volatility days HFTs do not change any of their behaviors, but they also exhibit high volatility themselves.

Weller (2018) studies if algorithmic traders (ATs) impede the acquisition process of price discovery through a newly developed measure termed the "price jump ratio." He examines the amount of information incorporated into a price relative to the overall amount of variance. In Weller, four proxies are related to AT activity: odd lots (positive relation), cancel-to-trade (positive relation), trade-to-order (negative relation), and avg. trade size (negative relation). The relation between AT activity and the proxies used in Weller coincides with the characteristics of post halt trading activity. Given the stressful conditions characterized by higher volume and higher volatility after a trading halt resulting from a technological malfunction, we expect the following:

> Hypothesis 7: In the period during and following a technological malfunction, AT activity will be higher than before the trading halt, as algorithmic traders act as liquidity providers and correct transitory price movements resulting from investor confusion**.**

III. DATA AND METHODS

INSTITUTIONAL DETAILS

Technological malfunctions are events that are not driven by an informational event nor an imbalance in orders and appear to be random and unpredictable. We use a sample of eight technological malfunctions on various equity exchanges that occurred due to a technical issue and either resulted in suspended trading or an attempt to fix the issue while trading continued. Most of the technological malfunctions in our sample resulted in a non-regulatory trading halt but a small proportion of events allowed trading to continue during the glitch. In order to collect the specifics of each of these occurrences, we relied on news clippings and announcements made by the exchange experiencing the malfunction. The following table provides specifics of all technological malfunctions used in our analysis:

DATA AND SAMPLE

Our samples are drawn from TAQ, CRSP, SEC Market Information Data Analytics System (MIDAS), and EODDATA. Given that we analyze multiple exchanges, our sample for each trading is contingent on where the technological malfunction took place and the month in which that exchange experienced the malfunction. Our preliminary samples include NYSE-listed, Nasdaq-listed, and AMEX-listed common stocks and exchange traded funds (ETFs) that are present in the Daily and Monthly TAQ files for their corresponding dates. We use CRSP to identify common stocks (share code 10 and 11) that trade at or above a price of $5.00 dollars every day during the period of analysis for each event.

Once we complete this preliminary procedure, we then eliminate all stocks that are not listed on the exchange where the malfunction occurred. For instance, the sample for the technical malfunction taking place on the Nasdaq exchange in January of 2013 would only consider Nasdaq-listed stocks. All other stocks would be disregarded from the analysis for this event. The exception to this cleaning procedure entails those malfunctions that occurred on the BATS, EDGE-X, Chicago Stock Exchange. And Consolidated Tape Association.[9] For these events, we follow the method used in Clark-Joseph, Ye, and Zi (2017), whereby, if the event took place in the same month that another exchange experienced a malfunction, the sample for both events would then be comprised of the exchange with listed securities. For example, the EDGE-X malfunction took place in the same month that the NYSE experienced a malfunction. Therefore, we use NYSE-listed stocks for both the EDGE-X and NYSE events. We divide this sample of exchange-listed stocks into four quartiles based on market capitalization obtained from CRSP. Within each quartile we include the largest 25 securities based on market capitalization, which produces a final sample size of 100 securities for each event in our sample.

VARIABLES

We use the TAQ dataset to construct the national best bid and offer (NBBO) prices and liquidity measures following the methods prescribed in Holden and Jacobsen (2014) and used in Clark-Joseph, Ye, and Zi (2017). The *quoted spread* is the difference between the best bid and

[9] Since the glitch at the Consolidate Tape Association (CTA) involved errors reporting data for the S$P 500 and Nasdaq, the sample used for this event initially includes all S&P 500 stocks, before the sampling procedure is detailed in section 3.2 is applied.

best ask prices and is weighted by the time. *Effective spread* is defined for a buy as twice the difference between the trade price and the midpoints of the NBBO price. For a sell, effective spread is twice the difference between the midpoints of the NBBO and the trade price. Effective spread in our analysis is weighted by trade size. *Depth* is the time-weighted average of displayed depth at the NBBO. *Volume* is measured daily and within each time interval (described in further detail in the methods sections) and is based on the consolidated volume in all U.S. stock exchanges and off-exchange trading venues.

We follow Lee, Ready, and Seguin (1994) to compute three volatility measures: absolute return (dollar and percentage) and hi – low.[10] *Absolute return* (dollar and percentage) is computed as the difference between the last trade price before the 10-minute interval and the last trade price in a 10-minute interval. *Hi-low* is calculated as the absolute difference between the highest and lowest trade price in a 10-minute interval. In addition to Lee et al., we also provide a price efficiency measure following O'Hara and Ye (2011), which is *short-term volatility*. This variable is computed as the daily standard deviation of 10-minute absolute returns and is a crude measure of the trading irregularities which interprets a lower short-term volatility as being more efficient.

To construct our algorithmic trading (AT) activity measure, we follow the methods of Weller (2018) to compute four measures of AT activity: *odd lot-to-volume, trade-to-order volume*, *cancel-to-trade ratio*, and *average trade size*. Odd lot-to-volume ratio is the total volume executed in quantities smaller than 100 shares divided by the total volume traded. Trade-to-order volume ratio is the total volume traded divided by the total volume from all orders placed. Cancel-to-trade ratio is the number of full or partial order cancellations divided by the total

[10] We forgo the final volatility measure in Lee et al. (1994), that being the spread reversal. Spread reversal is the defined as the number of revisions to the midspread of the NYSE specialist's quote.

number of trades. Average trade size is simply the trade volume in shares divided by the number of trades. Weller found that odd lot-to-volume and cancel to trade ratios are positively related to algorithmic trading activity, while a higher trade-to-order ratio and average trade size are negatively related to algorithmic trading. As per the methods used by Weller and prescribed by SEC Market Information Data Analytics System (MIDAS), odd lot-to-volume, trade-to-order, and cancel-to-trade ratios are adjusted to exclude those orders reported by the NYSE and NYSE MKT.[11] Following Weller, for each stock we compute a rolling average of each algorithmic trading variable across the dates [T – 21, T – 1] and take the log of each measure.

For brevity, the summary statistics are recorded in the appendix for each event in our sample. Panel A of these tables displays summary statistics on the day of a technological malfunction, while Panel B shows statistics for all other days for corresponding event sample. It is important to note the similarity of our summary statistics to that of Clark-Joseph et al. (2017) who analyze two of the events that are included in our study. For instance, in table 2 below, we report the summary statistics for the malfunction that occurred at the NYSE in July of 2015. Our reported mean quoted and effective spreads during the NYSE halt (7.44 cents and 4.11, respectively) closely match that of Clark-Joseph et al. (5.24 cents and 3.62 cents) However, given our sampling technique, by reducing the number of firms in our sample to 100 we report a price and market capitalization that is much larger.[12]

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[11] MIDAS collects around a billion feeds from the proprietary feeds of each of the 13 national equity exchanges and of the 13 exchange feeds the NYSE and NYSE MKT report trade size of the initiating order. The other 11 exchanges, however, separate trades by initiating and contra orders. This results in the NYSE number of trades and trade size to not be comparable with other exchanges. Additional MIDAS details and discussion of exchange exclusions are provided on the MIDAS website at http://www.sec.gov/marketstructure/mar_methodology.html

[12] Clark-Joseph et al. (2017) report an average price of $32.51 and an average market cap of $2.77 billion.

METHODS

Both Clark-Joseph, Ye, and Zi (2017) and Lee, Ready, and Seguin (1994) adjust each interval measure of interest for seasonality. The rationale to do so is drawn from McInish and Wood (1992) who find that liquidity has a reverse J-shaped intraday pattern, whereby spreads at the beginning and end of the day are much higher relative to spreads during the middle of the day. The trading malfunctions recorded in this paper happen at various times throughout the day which makes comparisons of liquidity during the malfunction to all other trading times confounded by the time-of-day effects reported by McInish and Wood. Therefore, we follow the approach of Clark-Joseph et al. and Harvey (1993) to adjust for intraday seasonality using a multiplicative seasonal adjustment.[13] In our sample of events, we divide the trading day (9:30 am – 4:00 pm est.) into 10-minute intervals (39 daily intervals per day) and compute the liquidity measures during each interval.

To assign the correct 10-minute intervals to the period of the day experiencing a halt or malfunction, we use the following as an example of our procedure. The Nasdaq technological malfunction that halted trading on January 3, 2013 started at 1:36 pm and ended at 1:51 pm. If a halt occurs in the middle or ends at the start of a 10-minute interval, the entire interval will be identified as during the trading halt. This method applied to the Nasdaq malfunction in 2013 would identify all 10-minute intervals starting at 1:30 pm and ending at 2:00 pm to be identified as a trading halt currently in place. If no such halt occurred and there was only a technological malfunction, the same method of identifying the affected 10-minute intervals would still apply to this situation We then compute the monthly average of all variables for each stock during each 10-minute interval and further divide the observed values on the treatment days by the

[13] Harvey (1993) method divides each value of the time series by a seasonal index that represents the long-run average value typically observed in each season.

corresponding interval-stock monthly average, which is then referred to as the "normalized measures."[14] The monthly interval averages are taken over all trading days in the month of the halt or malfunction, except for the event dates.[15]

[14] We exclude measures that are measured daily from this procedure, which includes Short-term volatility and the algorithmic trading measures of Weller (2018)

[15] An exemption from this procedure, is in the measurement of any daily measures such as the Weller (2018) AT proxies and the O'Hara and Ye (2011) price efficiency measure. These measures are not calculated during the 10-minute intervals and thus are less susceptible to an intraday seasonality affects.

IV. EMPIRICAL ANALYSIS

As an initial analysis preceding the hypothesis regressions, we perform univariate tests of significance on the desired variables outlined in the previous section. We compute the normalized liquidity, volatility, price efficiency, and algorithmic trading measures for each stock on the date of the technological malfunction and across all other days within the month of the malfunction. We define *During* equal to 1 if the measure is observed on the date of the malfunction, during the time of the malfunction specified in Table 1 and 0 for all other time intervals on that date of the malfunction. For both intra-day periods we compute the average for each measure and analyze the difference between the two. Given that the price efficiency and algorithmic trading variables are computed daily, we exempt these measures from the intraday univariate analysis and construct another mean difference test that compares the average of these variables on the day of the event to all other days in that month. We define *Event* equal to 1 if the date is equal to the date specified in Table 1 and 0 otherwise. For each event, we compare the average of each measure on the day of the malfunction to the average across all other days.[16]

UNIVARIATE

For each stock, we calculate the average normalized measure during the intervals specified above and present the averages across all technological malfunction summarized in

[16] If there are multiple malfunction that occur in a single month, we ignore the observation of other malfunctions when computing the average measure for the entire month. This occurs twice in our datasets – August 2013 and July 2015.

Table 3 and Table 4. *During*, in Table 3, is defined as 1 if there is an ongoing trading halt during a 10-minute interval and 0 otherwise. *Event*, in Table 4, is defined as 1 if the date corresponds to date of the malfunction specified in table 1 and 0 otherwise. To assess statistical significance, we us Welch's t-test of unequal variance due to the difference in sample sizes observed for each event. This is more appropriate than student's t-test as some of the events occur over a short period of time and have substantially fewer observations than across the entire day or entire monthly sample, therefore, we may have starkly different variances.[17]

To assess the effect of a technological glitch on liquidity, we analyze quoted and effective spreads, quoted depth, and interval volume (Table 3, panels A – H). *Interval Volume* is measured following Holden and Jacobsen (2014) as the total volume (share and dollar) that was traded during each 10-minute interval. The univariate results reveal that these technical glitches appear to have a significant impact on spreads, increasing quoted and effective spreads for 5 of the 8 events. Clark-Joseph et al. (2017) find that the NYSE shutdown led to a large and significant increase in stocks' spreads relative to the controls but find no evidence of this effect during the EDGX shutdown. Our results reported in Table 3 support the findings in Clark-Joseph et al., that the NYSE glitch in July of 2015 led to an increase of 19.34% which is on par with the increase that Clark-Joseph et al. find of roughly 17%.

Additionally, Table 3 extends their analysis to observe glitches at other exchanges. In panel E analyzing effective spreads, the largest increase in spreads is observed during the technologic malfunction at Nasdaq in August of 2013 and at the Consolidated Tape Association (CTA) in August of 2019. Effective spreads during the Nasdaq and CTA glitch increased over 250% and 105% relative to their baseline, respectively. While the Nasdaq event led to a

[17] The shortest malfunction takes place at the Nasdaq exchange on January 3, from 1:36 pm – 1:51 pm.

subsequent trading halt, the malfunction at the CTA didn't result in a trading halt. This is notable as the results provide evidence that the effects of the NYSE shutdown in 2015 and reported by Clark-Joseph et al. may not be entirely explained by the presence of Designated Market Makers (DMMs), and warrant further analysis to determine what other contributing factors exacerbate the effects of these events.

While the evidence regarding spreads appears significant and ubiquitous, the results for quoted depth are less clear. Of the eight events, three show a significant decrease interval volume, two show a significant decrease in interval depth, and one reports an increase in depth during the malfunction. This finding also concurs with Clark-Joseph et al. (2017) that depth doesn't appear to change in any discernible way for stocks during these technological malfunctions. The authors attribute this to the widening of quoted spreads during the NYSE shutdown and the increase in spreads implies that depth at the top of the book is for inferior prices compared to when quoted spreads are tighter.

The volatility measures used in our analysis are constructed following Lee et al. (1994): absolute return (dollar and percentage) and Hi-low. Volatility measures are recorded in panels I – K in Table 3 and are used to determine if trading halts resulting from a technological malfunction impact intraday trading similar to trading halts resulting from informational events and order imbalances. Lee et al. and Corwin and Lipson (2000) find that informational trading halts produce an increase in both volume and volatility. However, the results in Table 3 do not show evidence that there is an increase in volatility during these technological malfunctions. We expect an increase in the volatility measures to support both Lee et al. and Corwin and Lipson, but of the eight tech malfunctions, one shows a significant increase in the volatility measure Hi-low. We do observe two of the eight events leading to a decrease in volatility. However, we remain cautious to make a substantive claim until after the regression analysis with other control factors.

Table 4 presents the univariate results using Welch's t-test for the daily variables. These include the algorithmic trading measures of Weller (2018) and the price efficiency measure of O'Hara and Ye (2011). Our expectation, stated in section 2, is that algorithmic trading activity will increase following a trading halt to provide liquidity during a stressful period, such as technological malfunction. Therefore, we should see an increase in odd lot-to-volume and cancel-to-trade ratio, but a decrease in trade-to-order volume and average trade size. First, it appears that there may be evidence that cancel-to-trade increases as two of the eight events show a significant increase. However, the odd lot-to-volume is less clear. Four of the eight events show a significant decrease in this measure, while 3 show a significant increase. The malfunction in August 2013 at the BATS exchange resulted in the largest change of odd lot-to-volume with an increase of eight basis points on the day of the event compared to all other days in the sample.

Trade-to-order volume and average trade size provide an unclear determination in algorithmic trading activity, with both measures showing movement in either direction. Again, we are cautioned to make a strong claim pertaining to hypothesis 8 that there is an increase in algorithmic trading during and after these technical glitches, as this univariate table compares the event day to all other days. For nearly all events, there is a period of the day when trading resumed after a technological glitch and this is the period we are most interested in. However, seeing as though algorithmic trading measures are reported by MIDAS daily, we are unable to observe the intraday movement of these variables. Thus, in the regression analysis we assess the significance of these measures for the day following all technological malfunctions.

REGRESSION ANALYSIS – PRE-MALFUNCTION

Our main empirical analysis is to determine how technological malfunctions that result in trading halts impact intraday trading activity and more specifically how trading activity is impacted before, during, and after the malfunction. Our fist analysis begins with examining activity leading up to the trading halt and formally testing hypothesis 1. We start with a panel regression relating the day before the tech glitch to the normalized measures:

$$Normalized\ Measure_{i,z,t} = \alpha + \beta Event_{t-1} + \delta * controls_{i,z,t} + \epsilon_{i,z,t} \quad (1)$$

where the normalized measure is the average normalized measures of liquidity and volatility for stock *i*, during interval *z*, on day *t*. These normalized measures include quoted spread, effective spread, quoted depth, interval volume, and the Lee et al. (1994) volatility measures. Our indicator variable of interest is $Event_{t-1}$, which we define as equal to 1 if the measure is observed one day before the technological malfunction and 0 for all other times. The control variables in our regression include the log of normalized interval volume, log market capitalization, log daily price, and the interval quoted spread ($). The log volume is included in all regressions, except for normalized volume, as McInish and Wood (1992) show that volume is negatively related to intraday spreads and thus should be included when assessing our interval spread measures. The log of market capitalization and price are obtained from CRSP and are used in all specifications for the remainder of the paper.

An additional set of control variables is used when our dependent variable is volume (dollar and shares). These additional control variables are (1) the market share of the exchange experiencing the malfunction and (2) the market share of the group for which that exchange belongs to. We identify 4 exchange groups through our sample periods. These include the NYSE group, Nasdaq group, CBOE, and IEX group (Spatt 2020). The NYSE group includes the NYSE,

NYSE ARCA, AMEX, NSX, and the Chicago Stock Exchange (now NYSE Chicago). The Nasdaq group includes the Nasdaq, Boston Stock Exchange, and Philadelphia stock exchange (PHLX). The CBOE group includes the Bats-Y, Bats-Z, Edge-X, and Edge-A. The final group, IEX, includes no other member exchanges except for itself. Clark-Joseph et al. identify that listing decisions are not random, and thus may influence the amount of volume that generally takes place on an average day of trading and therefore is used as another control to account for seasonality. To compute the exchange market share and group market shares, we use MIDAS to observe the number of shares traded on each exchange for each stock and divide by the total number of shares traded on all exchanges for each stock. All regressions include day and stock fixed effects and standard errors are clustered by stock to account for correlated errors due to firm characteristics.

Table 5 presents our empirical results from our estimation of equation 1 examining the day preceding a stock glitch. In respect to our liquidity measurements, it's clear that the day before the malfunction is significantly different than the rest of the trading period and rejects our prediction in hypothesis 1. For instance, both quoted and effective spreads are significantly different, but it is unclear whether the day before a malfunction is positively or negatively correlated with spreads. Of the eight events, six events have quoted spreads significantly different than zero and five have effective spreads significantly different than all other days during the sample period. The events occurring on the Nasdaq in August of 2013 and the NYSE in July of 2015, report the largest magnitude for both quoted and effective spreads. The day preceding the Nasdaq halt, stocks experienced a 13.6% and 11.1% increase in quoted and effective spread above the monthly seasonal average, respectively. Likewise, quoted depth at the

NBBO and interval volume are significantly different than zero for four of the eight events in our sample.

As we turn our attention to the Lee et al. (1994) measures in panel D, we see the same evidence from the liquidity measures. Most of the events in our sample report significantly different volatility the day prior to the malfunction than the rest of the month. However, like the liquidity results, the direction of which cannot be determined, as Table 5 shows that we observe both positive and negative differences. Given the significance of our results regarding the liquidity and volatility the day prior to a technological glitch, we cannot confirm hypothesis one to be true that the period before a tech glitch is not significantly different than all other trading days.

REGRESSION ANALYSIS – INTRA- & POST-MALFUNCTION

In this next section, we look to empirically test hypotheses 3, 4, and 5 to determine what effect trading halts that are the result of a technological malfunction have on trading activity during the halt and the period after the halt. To do this we will use the following panel regression:

$$Normalized\ Measure_{i,z,t} = \alpha + \beta_1 During_{I,t} + \beta_2 After_{I,t} + \delta * controls_{i,z,t} + \epsilon_{i,z,t} \quad (2)$$

where the normalized measure is the average normalized measures of liquidity and volatility in stock *i*, during interval *z*, on day *t*, and include quoted spread, effective spread, quoted depth, interval volume, the Lee et al. (1994) volatility measures, and small trades using TAQ trades for each interval. To assess the amount of small trades after a trading halt, we use three proxies. The first two proxies identify the number of odd lot trades and volume in a 10-minute interval. Odd lots are identified as those trades that execute in quantities smaller than 100 shares. Our third measure is an average trade size during the interval and is computed by taking the trade volume during each interval divided by the total number of trades during that interval. We aggregate all odd lot trades and volume in shares with in each 10-minute interval and apply the same intraday seasonality adjustment detailed in section three to normalize all three variables.

In our analysis we follow Clark-Joseph et al. (2017) and segment the trading day into 39 10-minute intervals. Our key variables of interest in equation 2 are *During* and *After*. *During* is defined as 1 if there is an ongoing trading halt during a 10-minute interval and 0 otherwise. *After* is defined as 1 if the 10-minute interval has no trading halt nor malfunction in place and that interval takes place after the trading halt has ended. Our base group of comparison in equation 2 is all trading occurring in 10-minute intervals that take place before the trading halt. We follow the same method outlined in our methods section for identifying the 10-minute intervals to be analyzed as during the trading halt. Our control variables include those applied in equation 1 and all regression standard errors are clustered by stock. For technological malfunctions taking place at the NYSE in May 2016, CHX in November 2018, and at the Consolidated Tap Association (CTA) in August of 2013, we omit the variable *After* from the regression because these malfunctions took place at the open or at the close and thus only have two periods of analysis: during the malfunction and not during.

Table 6 records our empirical test for equation 2 and for brevity we aggregate the coefficients for During and After across all regressions onto a single table and exclude the coefficients for our controls, which can be found in the appendix. First, analyzing all coefficients for the variable During, our results in the regression analysis reflect our initial findings in the univariate analysis and those in Clark-Joseph et al. (2017). In terms of quoted spreads, we can see that four of the eight events show a significant increase and three show a significant decrease. Seeing as these measures are normalized for seasonality, the magnitudes for those events with a positive coefficient are on average larger than those with a negative coefficient. For instance, the malfunction at Nasdaq in August of 2013 increased spreads 146% relative to its baseline during the malfunction compared to the trading period before. Likewise, the results reported for effective spreads are more pronounced as five of the eight events report a significant positive increase during the technological malfunction.

However, quoted depth doesn't appear to change in any discernible way. Panel C in Table 6 shows both significant positive and negative coefficients during the trading halt for two of the eight events. Panel D in Table 6 records the effect of malfunctions on interval volume and as the coefficients during the halt shows for four of the eight events, there is a large reduction in volume relative to its baseline. The reduction in volume during the time a trading halt is in place ranges from 25.9% to 41.4% relative to the baseline. Although there is a substantial reduction in liquidity measured by spreads, we see only a slight deterioration in depth and volume during these events. This finding provides evidence that partially supports hypothesis 3 and the findings in previous trading halt studies such as Jiang et al. (2009) that find a deterioration in quote-based liquidity.

Our next question is to determine the state of trading following a technological malfunction in the short-term and look to see what effects remain persistent in the long-term after a malfunction. First, in table 6, we can interpret the coefficients for the variable *After* as the short-term effect immediately after a malfunction has been corrected. In columns 3 and 4 of table 6, both quoted and effective spreads have positive and significant coefficients. For instance, the after period for the technological malfunction at Nasdaq in August of 2013 shows that quoted and effective spreads remained at 35.9% and 21.1% above their baseline, respectively. Panels C and D in Table 6 provide some evidence that the period following a trading halt has significantly less depth and volume than before an exchange was impacted by a technological malfunction. The deterioration in liquidity in the post-halt period is significant and in part shows evidence that these malfunctions are correlated with irregular trading activity during and after.

Panels F and G of table 6 determine the impact of technological malfunctions on volatility and trade size in the after period. Using the Lee et al. (1994) volatility measures, we can see that the coefficient for *After* is positive and consistently significant across all three measures for two of the events and significant for the variable Hi-Low during the NYSE glitch in July of 2015. However, panel G presents mixed results regarding the number of small trades during this post-halt period. For two of the events there is a significant increase in the number of small trades, while for two halts there is a significant decrease in the number of small trades.

Considering the reduction in liquidity and increase in volatility during the post-halt period, the evidence thus far supports hypothesis 4. During the post-halt period, investor confusion, such as that described in Cukierman et al. (2018) and Rashes' (2001), whereby investors' misunderstanding and confusion surrounding an event produce abnormal trading activity. Cukierman et al. also identify this confusion among investors as their inability to interpret these events as either permanent or transitory. To examine this investor confusion further, we next test the following panel regression:

$$Normalized\ Measure_{i,z,t} = \alpha + \beta_1 Event_{I,t} + \beta_2 Event_{I,t+1} + \beta_3 Event_{I,t+2} + \beta_4 Event_{I,t+3} + \beta_5 Event_{I,t+4} + \beta_6 Event_{I,t+5} + \beta_7 Event_{I,t+6-10} + \delta * controls_{i,z,t} + \epsilon_{i,z,t} \quad (3)$$

Where the same normalized liquidity and volatility measures are used again as dependent variables. However, in equation 3, *Event* is defined equal to 1 if the date corresponds to date of the malfunction specified in table 1 and 0 otherwise. The other variables of interest to determine the long-term effects of technological malfunctions are $\beta_2 - \beta_7$. Each of these is a dummy variable equal to 1 if the date is $t + n$ days after the trading halt and 0 otherwise. The last coefficient, β_7 includes days $[t + 6, t + 10]$ after a technological malfunction.[18] The same controls used in equation 2 are applied here and all regression include stock and day fixed effects and standard errors are clustered by firm.

Table 7 summarizes our empirical analysis of equation 3 for all events in our sample. The liquidity measures summarized in table 7 support our finding in table 6 on the day of the malfunction. The coefficient for *Event* in both quoted and effective spreads are largely positive and significant. The regression analysis for effective spread shows that six of the eight malfunctions are associated with a positive and significant increase above the baseline. However, as we look further past the event date, there are significant sign changes for many of the events as far out as ten days. This reversal in effect is also prominent in the quoted depth coefficients. Initially on the day of the event, the coefficient for Event, matches that of the results in table 6 and table 3. That being, four of the eight events report a significant decrease in depth but two report a significant increase in depth. As we move further from the event, we see negative and significant coefficients as far out as ten days.

The Lee et al. (1994) volatility measures are consistent with prior results in table 6, that on the day of the malfunction many of the events are correlated with a significant increase in volatility across all three measures. However, like the liquidity measures, there is a relatively quick reversal in volatility as early as one day after for a handful of the events. The evidence in this section supports hypothesis 4 and 5, that stocks experience increased volatility and reduction

[18] In two of the event regressions, Event 4 and Event 5, certain days are omitted if another malfunction occurred on that day or if period extends outside the month in question.

in liquidity, the persistence of these effects may be as quick as a day or as long as ten days. We initially see a deterioration in liquidity for both spread and depth on the day of the event. Once we extend this period of analysis out to ten days post-halt, there is a reversal in the liquidity measures, but significance remains. Our analysis of volatility also reflects the same interpretation as our liquidity analysis. On the day that an exchange experiences a technological malfunction, there is a deterioration in liquidity and the market becomes more volatile for those stocks affected by the technological malfunction. The post-malfunction period is characterized by a abnormal trading activity, but these affects appear to be short-term and there is a subsequent reversal in the normalized measures as early as day after the malfunction. The permanent-transitory confusion (PTC) hypothesis of Cukierman et al. (2018) is applicable to the period during and after a technological malfunction. In that, there is a margin of uncertainty about the persistence of these effects, but this confusion does appear to be resolved relatively quickly.

ALGORITHMIC TRADING

Our final analysis we look to determine the impact that technological malfunctions have on algorithmic trading activity. Our main empirical analysis in this section investigates the four algorithmic trading (AT) proxies of Weller (2018) defined in section 3, as the dependent variables in the following panel regression:

$$Weller\ (2018)\ AT\ proxy_{i,t} = \alpha + \beta_1 Event_{I,t} + \beta_2 Event_{I,t+1} + \delta * controls_{i,t} + \epsilon_{i,z,t} \quad (4)$$

Event is defined equal 1 if the date corresponds to the date of the malfunction specified in table 1 and 0 otherwise. $Event_{I,t+1}$ is equal 1 if the date corresponds to one day after a technological malfunction and 0 otherwise. Following the methods of Weller, our controls include the short-term return volatility, quoted spread, log market cap, and log price.

Table 8 summarizes each event regression coefficient for the variables of interest, *Event* and $Event_{I,t+1}$. Our prediction in hypothesis 7 expresses that algorithmic trading activity will be higher during and after a trading halt, as algorithmic traders will act as liquidity providers and correct transitory price movements resulting from investor confusion. Table 6 and 7 indicate that investor confusion is present in the short-term immediately after a technological malfunction. The AT proxies are measured daily rather than intraday and we are unable to exam the immediate impact on AT activity during and after these malfunctions. Regardless, the abnormal trading activity in the form of deteriorated liquidity and higher volatility persists to ten days after for a handful of the events.

The results in Table 8 show evidence consistent with our prediction that the day of (*Event*) and the day following ($Event_{I,t+1}$) a technological malfunction, algorithmic trading activity increases. Consistent with Weller's (2018) stated relation between each proxy and algorithmic trading, all panels in Table 8 show the anticipated direction. Panel A and D of Table 8, report the coefficients for trade-to-order and average trade size and we see that the day of the malfunction, three events report a significant drop in trade-to-order, while four events report a significant drop in average trade size. The effects on algorithmic trading are even stronger the day following the malfunction where five events report both a significant drop in trade-to-order and average trade size. Panel B and C of Table 8 report the coefficients for odd lot-to-volume and cancels-to-trades. We for the coefficients the day of the malfunction, there is a significant increase in four of the events for odd lot-to-volume and three events with a significant increase in cancels-to-trades. Again, we see that the day following reports stronger results consistent with our prediction that algorithmic trading increases, as there are now five events reporting a significant increase in odd lot-to volume and 3 events still reporting a significant increase in cancels-to-trades.

Given the results in table 8, the evidence is consistent with our prediction that algorithmic trading activity increases during and immediately following a technological malfunction. The results show this to be especially true the day after a malfunction occurs as the number of events reporting a significant change in algorithmic trading activity and the magnitude of this change both increase from the day of the malfunction. The strongest evidence in support of hypothesis 7 can be seen during the BATS technological malfunction in August of 2013. The day after the Nasdaq malfunction in August of 2013 we see a significant increase in both the odd lot-to-volume ratio and cancels-to-trade ratio of 8.24 and 15.9 basis points, respectively. We also report significant decrease in trade-to-order and average trade size the day after the Nasdaq malfunction of 16.3 and 3.39 basis points. Given the stressful conditions characterized by higher volume and higher volatility after a trading halt resulting from a technological malfunction, our results are consistent Brogaard et al, (2018) and Anand and Venkataraman (2016) that algorithmic trader become more active during these stressful periods to absorb trade imbalances and correct transitory price movements.

DISCUSSION

The results reported provide insightful analysis regarding the impact that technological malfunctions have on daily trading. The predictions made in our hypothesis are largely upheld, but for hypothesis 2 and 6 we are unable to directly examine these hypotheses. As an observation considering all evidence we report thus far, the results appear to support both hypotheses. The magnitude and effect of the technological glitch at the NYSE in July of 2015 is consistent with the findings of Clark-Joseph et al. (2017). During the NYSE malfunction there is a large deterioration in liquidity compared to other events such as the BATS and EDGE-X malfunctions. Similarly, the deterioration in liquidity is consistent for another malfunction that occurred at the NYSE in May of 2016. Although the scale of this malfunction was smaller both in time length and number of stocks affected, the results still held that NYSE listed stocks during this malfunction were impacted.

Table 6 and 7 show evidence supporting hypothesis 6, in that exchanges with a large market share and presence of DMMs have sharper decreases during and after the trading halt. Of the exchanges with the largest significant magnitude, the Nasdaq and NYSE, whose market shares were the 2 largest within sample (44.7% and 11.10%, respectively), were severely impacted by a technological malfunction. This severity is reflected consistently across all halts that took place at Nasdaq and NYSE and is illustrated by Table 7. During the other shorter event that occurred at Nasdaq in January of 2013, quoted spreads increased by 5.54%, effective spreads increased by 16.1% over their baseline, depth was depleted by 8.8%, and the volatility measures show an increase between 18% and 21%. Likewise, the smaller scale malfunction that took place at the NYSE in May of 2016 shows an increase in quoted spreads of 7.97%, an increase in effective spreads of 91.4%, a depletion in depth of 9.75%, and an increase in volatility on average between 30% and 73%. The smaller scale of these two events, both in terms of length of the halt and the number of securities affected, demonstrate the impact technological malfunctions have on daily trading in the short term but as we extend the analysis further out, we see these effects all but disappear.

V. CONCLUSION

The primary purpose of this study is to examine trading around technological malfunctions, which are events that are neither driven by an informational event nor an order imbalance. These events appear to be random and unpredictable. In this study we present analysis of intraday trading activity before, during, and after a technological malfunction. We find that the day leading up to a technological malfunction experiences abnormal trading, but we are unable to determine if this plays a role in causing the technical glitch. Of the events (8 technological malfunctions) we record in this study, a majority document a reduction in liquidity and an increase in short term volatility during and immediately after a technological malfunction. While nearly all the events we observe result in a trading halt, the malfunction at CTA is interesting, in that trading continued despite a malfunction occurring and yet we still see a partial deterioration in liquidity measured by effective spreads. Additionally, we investigate what impact these events have on algorithmic trading activity and document evidence that shows an increase in algorithmic trading activity during most of the events in our sample, whereby algorithmic traders may be acting as liquidity providers and correcting transitory price movements resulting from investor confusion.

As technological malfunctions have not garnered much academic attention, we contribute to the literature by providing an analysis of various occurrences over a ten-year period to show that these events are not trivial and have a substantial impact on liquidity and volatility. This impact can partially be explained by the permanent-transitory confusion hypothesis, whereby there is a period of uncertainty as to how long these effects will persist. In the short-term it does not appear that there is a proper amount of information dissemination relayed to investors as to the cause of the malfunction, which we see lead to abnormal trading activity immediately after. In comparison to other informationally driven regulatory and nonregulatory trading halts, technological malfunctions that lead to a trading halt do share some characteristics, but largely remain different in terms of the amount of trading activity taking place both during and after a malfunction.

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APPENDIX

APPENDIX 1: EVENT DESCRIPTIONS

Table 1 – Event descriptions: This table provides description such as the place, date, time, and whether there a trading halt for each of events identified in this study.

Exchange	Date	Time	Description	Trading Halt
Nasdaq	January 3, 2013	1:36 pm – 1:51 pm	Nasdaq suffered a technical glitch that caused an outage on real-time prices for all Nasdaq listed securities. Issue involved the Universal Trading Platform's (UTP) centralized securities information processor (SIP) data feeds, which consolidate and display market data for Nasdaq-listed securities from all trading venues. Directed Edge, responded by halting trading in all Nasdaq order books.[19]	No
BATS	August 6, 2013	1:10 pm – 2:00 pm	BATS Global Markets reported an outage due to an internal network problem that did not allow the exchange to accept orders to the BZX exchange. [20]	Yes
Nasdaq	August 22, 2013	12:14 pm – 3:30 pm	Nasdaq released statement regarding "momentary interruption" in its ability to quote prices and attributed the issue to software that manages price quotes for each stock in real time. Technical issues were resolved within 30 minutes, but remaining time was spent coordinating with other exchanges and regulators. [21]	Yes
Edge X	July 6, 2015	9:41 am – 10:20 am	Direct Edge X platform was forced to halt trading due to an unrelated technological issue. [22]	Yes
NYSE	July 8, 2015	11:32 am – 3:10 pm	NYSE suffered an outage due to an internal technical issue and was not due to a cyber breach. Stocks	Yes

[19] See McCrank, John. "Data-Feed Glitch Leaves Some Traders in Dark on Nasdaq Prices." Chicagotribune.com, 7 Sept. 2018, www.chicagotribune.com/news/ct-xpm-2013-01-03-sns-rt-us-exchanges-data-outagebre9020qz-20130103-story.html.

[20] See McCrank, John. "BATS' Largest U.S. Exchange Hit by Technical Outage." Reuters, Thomson Reuters, 6 Aug. 2013, www.reuters.com/article/us-bats-outage/bats-largest-u-s-exchange-hit-by-technical-outage-idUSBRE9750X920130806.

[21] See Yang, Jia Lynn, and Danielle Douglas. "Nasdaq Resumes Trading after Technical Glitch." The Washington Post, WP Company, 22 Aug. 2013, www.washingtonpost.com/business/economy/nasdaq-halts-trading-over-technical-glitch/2013/08/22/7c886348-0b4b-11e3-b87c-476db8ac34cd_story.html; and Mikolajczak, Chuck. "Nasdaq Market Paralyzed by Three Hour Shutdown." Reuters, Thomson Reuters, 22 Aug. 2013, www.reuters.com/article/us-nasdaq-halt-tapec/nasdaq-market-paralyzed-by-three-hour-shutdown-idUSBRE97L0V420130822.

[22] See Mamudi, Sam. "Bats Suffers a 30-Minute Trading Outage on Its EDGX Stock Market." Bloomberg.com, Bloomberg, 6 July 2015, www.bloomberg.com/news/articles/2015-07-06/bats-suffers-a-30-minute-trading-outage-on-its-edgx-stock-market.

			continued to trade elsewhere. No further detail was provided by the NYSE.[23]	
NYSE	May 18, 2016	9:30 am – 1:17 pm	NYSE experienced a technical issue in one trading unit, which affected a subset of symbols. All open orders, including MOC and LOC orders were cancelled. During this halt, the Nasdaq and BATS exchanges declared self-help against the NYSE, which allows trading centers to bypass an automated trading center that is experiencing system problems.[24]	Yes
Chicago Stock Exchange	November 8, 2018	9:20 am – 10:35 am	Chicago Stock Exchange halted trading due to an unspecified "technical issue" [25]	Yes
Consolidated Tape Association	August 12, 2019	3:15 pm – 4:30 pm	The Consolidated Tape Association reported a glitch that was due to a "network component failure" at the exchange's data center.[26] In the Securities Information Processor did not report trades that had already been made and affected indexes such as the S&P 500 and Dow Jones Industrial Average that are calculated off the tape. [27]	No

[23] See Bernard Condon, Associated Press. "How Stocks Kept Trading during NYSE Outage." Https://Www.inquirer.com, The Philadelphia Inquirer, 8 July 2015, 11:58 am, www.inquirer.com/philly/business/20150709_New_York_Stock_Exchange_Suspends_Trading_in_All_Securities.html?outputType=amp.

[24] See https://www.nyse.com/market-status/history#110000023707

[25] See Egan, Matt. "Chicago Stock Exchange Briefly Halts Trade after Glitch." CNN, Cable News Network, 8 Nov. 2018, www.cnn.com/2018/11/08/investing/chicago-stock-exchange-halts-trading/index.html.; and https://www.nyse.com/markets/nyse-chicago/market-status

[26] See Osipovich, Alexander. "Some Stock Prices Delayed as Glitch Hits NYSE-Run Data Feed." The Wall Street Journal, Dow Jones S&P; Company, 13 Aug. 2019, www.wsj.com/articles/some-stock-prices-delayed-as-glitch-hits-nyse-run-data-feed-11565654989.

[27] See Pisani, Bob. "Stock 'Tape' Glitch Means It's Still Not Exactly Clear Where the Dow, S&P 500 Closed on Monday." CNBC, CNBC, 13 Aug. 2019, www.cnbc.com/2019/08/13/stock-tape-glitch-means-its-still-not-exactly-clear-where-the-dow-sp-500-closed-on-monday.html.

APPENDIX 2: SUMMARY STATISTICS

Table 2 Summary Statistics during NYSE July 2015 malfunction (non-normalized measures) - Table reports summary statistics for the sample of data that includes only stocks that with stocks with a price $5.00 or more every day in the sample and are listed on the exchange where the technological malfunction took place. The sampling procedure listed in section 3.2 is used to create a sample size of 100 stocks for each event.

Panel A: NYSE Event day – July 8, 2015					
Variable	Mean	Median	Std Dev	Min	Max
Quoted Spread, $	0.0744	0.0343	0.1248	0.0100	2.5816
Quoted Spread, %	0.0013	0.0008	0.0020	0.0001	0.0724
Effective spread (SW), $	0.0411	0.0204	0.0447	0.0082	0.1640
Effective spread (SW), %	0.0008	0.0005	0.0007	0.0001	0.0026
Depth, share	9,049.95	974.13	41,143.60	237.29	794,300.00
Depth, $	222,606.00	59,477.95	695,557.20	6,015.65	12,900,000.00
Volatility, Abs. return $	0.0980	0.0650	0.0951	0.0003	0.3400
Volatility, Abs. return %	0.1828	0.1359	0.1577	0.0009	0.5607
Volatility, Hi & Low $	1.1743	1.1418	0.5244	0.1742	2.0679
Daily Short-Term Volatility $	1.3066	0.0825	4.7906	0.0225	20.7944
Trade/Order (Midas)	0.0350	342782.0000	0.0103	0.0132	0.0688
Odd lot/Order (Midas)	0.1463	0.1463	0.0737	0.0077	0.2870
Avg. Trade size (Midas)	106.00	90.05	56.60	64.10	486.00
Cancel/Order (Midas)	23.39	21.94	9.42	9.46	69.30
Daily Volume, $ (million)	1,344.91	38.07	11,577.51	2.22	116,626.91
Daily Volume, shares	4,105,807.00	643,342.00	8,787,298.00	44,763.00	69,600,000.00
Market Cap., $ (billion)	51.08	8.84	85.93	0.956	342.83
Price, $	60.35	55.25	42.98	6.10	329.01
NYSE market share	11.11	10.70	3.30	4.19	21.75
NYSE group mrk. share	27.12	26.50	5.41	16.51	45.06
# of firms	100				
Panel B: All other trading days					
Variable	Mean	Median	Std Dev	Min	Max
Quoted Spread, $	0.0618	0.0248	0.1214	0.0100	3.8315
Quoted Spread, %	0.0011	0.0006	0.0020	0.0001	0.0880
Effective spread (SW), $	0.0346	0.0161	0.0407	0.0082	0.1640
Effective spread (SW), %	0.0006	0.0004	0.0006	0.0001	0.0026
Depth, share	14,669.01	1,051.42	66,663.00	200.00	1,341,679.00
Depth, $	366,161.80	66,804.16	1,199,157.00	3,992.00	24,300,000.00
Volatility, Abs. return $	0.0816	0.0500	0.0889	0.0003	0.3400
Volatility, Abs. return %	0.1491	0.1026	0.1468	0.0009	0.5607
Volatility, Hi & Low $	0.9399	0.8696	0.4781	0.1742	2.0679
Daily Short-Term Volatility $	1.3168	0.0686	4.7735	0.0225	20.7944
Trade/Order (Midas)	0.0356	0.0346	0.0104	0.0129	0.1174
Odd lot/Order (Midas)	0.1480	0.1470	0.0750	0.0071	0.3090
Avg. Trade size (Midas)	105.00	90.18	56.10	63.22	493.00
Cancel/Order (Midas)	22.64	20.85	8.53	8.00	69.70
Daily Volume, $ (million)	2,010.32	35.94	23,631.28	675.99	693,326.81
Daily Volume, shares	3,843,789.00	629,477.00	9,251,627.00	33.00	135,000,000.00
Market Cap., $ Billion	51.12	8.83	85.93	0.956	343.83
Price, $	61.21	56.09	44.13	5.75	346.92
NYSE market share	8.53	7.66	4.50	1.26	55.29
NYSE group mrk. share	28.30	27.74	6.60	9.10	64.35
# of firms	100				

APPENDIX 3: UNIVARIATE (INTRADAY METRICS)

Table 3: Univariate statistics – Summarizes the univariate results for each of the normalized measures during the period of the day experiencing the malfunction. These variables include quoted and effective spreads, quoted depth, interval volume, and the Lee et al. (1994) liquidity measures. *During*, in Table 3, is defined as 1 if there is an ongoing trading halt during a 10-minute interval and 0 otherwise. To assess statistical significance, we us Welch's t-test of unequal variance due to the difference in sample sizes observed for each event. Both t-stats and p-values are reported in the column 4 and 5 of each panel. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Interval volume, Shares

Event	(1) During	(2) Not During	(3) Diff. (During – Not)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	.9486156	1.079392	-.1307765	-1.0681	0.2864
Event 2 – August 2013 BATS	1.155602	1.294431	-.1388291	-1.4756	0.1403
Event 3 – August 2013 Nasdaq	.6593503	.9407383	-.281388	-1.5477	0.1229
Event 4 – July 2015 Edge-X	1.030438	1.044293	-.0138546	-0.1979	0.8433
Event 5 – July 2015 NYSE	1.106136	1.190671	-.0845351	-1.7476*	0.0806
Event 6 – May 2016 NYSE	.9127532	1.420158	-.5074044	-10.1550***	0.0000
Event 7 – November 2018 CHX	.9220759	.9510796	-.0290038	-0.4452	0.6566
Event 8 – August 2019 CTA	.5779201	.9293173	-.3513972	-11.0460***	0.0000

Panel B: Interval volume, Dollar

Event	(1) During	(2) Not During	(3) Diff. (During – Not)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	.9517144	1.077633	-.1259189	-1.0257	0.3059
Event 2 – August 2013 BATS	1.169153	1.307159	-.138006	-1.4523	0.1467
Event 3 – August 2013 Nasdaq	.6559692	.9431751	-.287206	1.6151	0.1075
Event 4 – July 2015 Edge-X	1.024464	1.040074	-.0156096	-0.2248	0.8224
Event 5 – July 2015 NYSE	1.089745	1.175631	- .0858867	-1.7874*	0.0740
Event 6 – May 2016 NYSE	.9113362	1.410104	-.4987675	-10.0236***	0.0000
Event 7 – November 2018 CHX	.9547148	.9849401	- .0302252	-0.4492	0.6537
Event 8 – August 2019 CTA	.576697	.929033	-.352336	-11.0301***	0.0000

Panel C: Quoted Spread, $

Event	(1) During	(2) Not During	(3) Diff. (During – Not)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	1.038069	1.099878	.0618097	1.8360*	0.0673
Event 2 – August 2013 BATS	.9033467	.9339111	-.0305645	-1.7323*	0.0838
Event 3 – August 2013 Nasdaq	2.572382	1.102547	1.469834	7.9008***	0.0000
Event 4 – July 2015 Edge-X	1.004215	1.007493	-.0032783	-0.2176	0.8279
Event 5 – July 2015 NYSE	1.401373	1.169884	.2314895	15.9818***	0.0000
Event 6 – May 2016 NYSE	1.037445	1.174712	-.1372664	-7.1394***	0.0000
Event 7 – November 2018 CHX	1.082848	.9739539	.1088944	2.4511**	0.0151
Event 8 – August 2019 CTA	1.035146	.9824658	.0526798	3.4250***	0.0007

Panel D: Quoted Spread, %

Event	(1) During	(2) Not During	(3) Diff. (During – Not)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	1.097935	1.038871	.0590639	1.7274*	0.0851
Event 2 – August 2013 BATS	.8906463	.9217334	-.0310871	1.8065*	0.0714
Event 3 – August 2013 Nasdaq	2.570928	1.106224	1.464704	7.8974***	0.0000
Event 4 – July 2015 Edge-X	1.01096	1.012499	-.0015387	-0.1009	0.9197
Event 5 – July 2015 NYSE	1.421001	1.185741	.2352592	16.0996***	0.0000
Event 6 – May 2016 NYSE	1.040012	1.184066	-.1440546	-7.4514***	0.0000
Event 7 – November 2018 CHX	1.046957	.9360699	.1108872	2.4986**	0.0132
Event 8 – August 2019 CTA	1.039439	.9839907	.0554483	3.6063***	0.0003

Panel E: Effective Spread, $

Event	(1) During	(2) Not During	(3) Diff. (During – Not)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	1.595966	1.151206	.4447594	3.8607***	0.0002
Event 2 – August 2013 BATS	.8882848	.9668494	-.0785646	-3.1244***	0.0019
Event 3 – August 2013 Nasdaq	3.756638	1.178219	2.578418	5.5612***	0.0000
Event 4 – July 2015 Edge-X	1.055252	1.015485	.0397663	1.0759	0.2831
Event 5 – July 2015 NYSE	1.356644	1.163285	.193359	4.6730***	0.0000
Event 6 – May 2016 NYSE	2.222511	1.520691	.70182	4.7745***	0.0000
Event 7 – November 2018 CHX	1.061266	1.12317	-.0619048	-0.6298	0.5291
Event 8 – August 2019 CTA	2.28539	1.206194	1.079196	5.4807***	0.0000

Panel F: Effective Spread, %

Event	(1) During	(2) Not During	(3) Diff. (During – Not)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	1.594451	1.154279	.4401718	3.8410***	0.0002
Event 2 – August 2013 BATS	.8778983	.9553225	-.0774241	-3.1501***	0.0017
Event 3 – August 2013 Nasdaq	3.753829	1.179483	2.574345	5.5764***	0.0000
Event 4 – July 2015 Edge-X	1.063054	1.021395	.0416587	1.1189	0.2644
Event 5 – July 2015 NYSE	1.375935	1.178155	.1977794	4.6948***	0.0000
Event 6 – May 2016 NYSE	2.229231	1.526116	.7031149	4.6903***	0.0000
Event 7 – November 2018 CHX	1.032394	1.087691	-.0552968	-0.5729	0.5670
Event 8 – August 2019 CTA	2.286485	1.213894	1.072591	5.5250***	0.0000

Panel G: Interval Depth, $

Event	(1) During	(2) Not During	(3) Diff. (During – Not)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	.857274	.9609989	-.1037249	-2.1643**	0.0312
Event 2 – August 2013 BATS	1.098652	1.103196	.0045447	0.1175	0.9065
Event 3 – August 2013 Nasdaq	.83297	.9736211	-.1406511	-1.1082	0.2688
Event 4 – July 2015 Edge-X	1.025896	.9936703	.0322258	1.4655	0.1440
Event 5 – July 2015 NYSE	.8006465	.8846718	-.0840253	-7.1842***	0.0000
Event 6 – May 2016 NYSE	.9980131	.7801062	.2179068	7.5307***	0.0000
Event 7 – November 2018 CHX	1.10988	1.015705	.094175	1.1859	0.2370
Event 8 – August 2019 CTA	1.018739	1.015557	.0031819	0.1809	0.8565

Panel H: Interval Depth, Shares

Event	(1) During	(2) Not During	(3) Diff. (During – Not)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	.8587272	.9567598	-.0980325	-2.0285**	0.0433
Event 2 – August 2013 BATS	1.113789	1.122903	-.0091143	-0.2308	0.8176
Event 3 – August 2013 Nasdaq	.8232879	.9713133	-.1480254	-1.2024	0.2303
Event 4 – July 2015 Edge-X	1.020238	.9873539	.0328837	1.4960	0.1359
Event 5 – July 2015 NYSE	.789977	.8731789	-.0832019	-7.1715***	0.0000
Event 6 – May 2016 NYSE	.9964193	.7746296	.2217897	7.6096***	0.0000
Event 7 – November 2018 CHX	1.150149	1.055227	.0949216	1.1714	0.2428
Event 8 – August 2019 CTA	1.014135	1.013696	.0004385	0.0251	0.9800

Panel I: Volatility – Absolute Return, $

Event	(1) During	(2) Not During	(3) Diff. (During – Not)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	1.06001	1.191172	-.1311623	-1.6407	0.1024
Event 2 – August 2013 BATS	1.01494	1.060103	-.0451628	-0.7543	0.4510
Event 3 – August 2013 Nasdaq	1.164998	1.08134	.0836574	0.8560	0.3926
Event 4 – July 2015 Edge-X	.8392938	.9606896	-.1213958	-1.6408	0.1038
Event 5 – July 2015 NYSE	1.173372	1.13959	.0337827	1.1858	0.2358
Event 6 – May 2016 NYSE	1.078396	1.707101	-.6287048	-10.4198***	0.0000
Event 7 – November 2018 CHX	1.046267	.9770841	.0691829	0.9938	0.3214
Event 8 – August 2019 CTA	.6586347	.954164	-.2955293	-7.5217***	0.0000

Panel J: Volatility – Absolute Return, %

Event	(1) During	(2) Not During	(3) Diff. (During – Not)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	1.05759	1.191355	-.1337649	-1.6641*	0.0976
Event 2 – August 2013 BATS	1.007194	1.050025	-.0428312	-0.7135	0.4758
Event 3 – August 2013 Nasdaq	1.170761	1.082246	.0885149	0.9005	0.3685
Event 4 – July 2015 Edge-X	.8471777	.9594002	-.1122225	-1.4881	0.1397
Event 5 – July 2015 NYSE	1.184012	1.147535	.0364773	1.2634	0.2065
Event 6 – May 2016 NYSE	1.082685	1.719294	-.6366091	-10.4992***	0.0000
Event 7 – November 2018 CHX	1.013202	.9414073	.0717948	1.0638	0.2886
Event 8 – August 2019 CTA	.6609824	.9537892	-.2928068	-7.4512***	0.0000

Panel K: Volatility – Hi minus Low, $

Event	(1) During	(2) Not During	(3) Diff. (During – Not)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	1.032373	1.199419	-.1670462	-1.9293*	0.0550
Event 2 – August 2013 BATS	.9410774	1.056391	-.1153138	-1.9963**	0.0464
Event 3 – August 2013 Nasdaq	1.745527	1.124741	.6207858	2.8581***	0.0045
Event 4 – July 2015 Edge-X	.8050027	.9431042	-.1381015	-3.9104***	0.0002
Event 5 – July 2015 NYSE	1.188149	1.158165	.0299839	1.7908*	0.0734
Event 6 – May 2016 NYSE	1.817943	1.709225	.108718	0.2049	0.8377
Event 7 – November 2018 CHX	.9475827	.98325	-.0356673	- 0.8258	0.4097
Event 8 – August 2019 CTA	.8353984	.9499301	-.1145317	- 2.5283**	0.0118

Panel L: Small trades – Interval Odd lots, TAQ trades

Event	(1) During	(2) Not During	(3) Diff. (During – Not)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	1.060066	1.011748	.0483186	0.4940	0.6219
Event 2 – August 2013 BATS	1.082264	1.176323	.094059	1.3854	0.1665
Event 3 – August 2013 Nasdaq	.5153379	.9309763	-.4156385	-4.3280***	0.0000
Event 4 – July 2015 Edge-X	.9560418	.9798279	-.0237861	- 0.5742	0.5664
Event 5 – July 2015 NYSE	1.159518	1.232009	-.0724912	3.4476***	0.0006
Event 6 – May 2016 NYSE	.9826965	1.422118	-.4394214	-10.3357***	0.0000
Event 7 – November 2018 CHX	.9207026	.9538589	-.0331563	- 0.6539	0.5139
Event 8 – August 2019 CTA	.5668289	.8954407	-.3286118	-16.5533***	0.0000

Panel M: Small trades – Interval Odd lots, TAQ shares

Event	(1) During	(2) Not During	(3) Diff. (During – Not)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	1.060066	1.012131	.0479359	0.4901	0.6247
Event 2 – August 2013 BATS	1.082264	1.176289	.0940254	1.3848	0.1667
Event 3 – August 2013 Nasdaq	.5153379	.9307323	-.4153944	-4.3255***	0.0000
Event 4 – July 2015 Edge-X	.9679436	.9717925	-.0038489	-0.0775	0.9383
Event 5 – July 2015 NYSE	1.135379	1.212247	-.0768683	-3.3842***	0.0007
Event 6 – May 2016 NYSE	.980475	1.438644	-.4581691	-9.8471***	0.0000
Event 7 – November 2018 CHX	.9194989	.9537783	- .0342794	- 0.6120	0.5412
Event 8 – August 2019 CTA	.5632303	.9074394	-.3442091	- 16.4158***	0.0000

Panel N: Small trades – Avg. interval trade size, TAQ shares

Event	(1) During	(2) Not During	(3) Diff. (During – Not)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	.9107784	1.001858	-.0910796	-2.0926**	0.0371
Event 2 – August 2013 BATS	1.081843	1.069099	.0127445	0.1748	0.8613
Event 3 – August 2013 Nasdaq	1.09131	1.029481	.0618293	1.1709	0.2419
Event 4 – July 2015 Edge-X	.9675846	1.009593	- .042008	- 1.5578	0.1202
Event 5 – July 2015 NYSE	.9843576	.956839	.0275186	0.4350	0.6636
Event 6 – May 2016 NYSE	.9889856	1.011815	-.0228292	-0.7139	0.4753
Event 7 – November 2018 CHX	.9638674	.9548588	.0090086	0.2809	0.7790
Event 8 – August 2019 CTA	1.020337	1.032547	- .0122104	-0.3892	0.6973

APPENDIX 4: UNIVARIATE (DAILY METRICS)

Table 4: Univariate Statistics – Summarizes the univariate results for each of the normalized measures for the day experiencing the malfunction to all other days in the monthly sample. These variables include the Weller (2018) algorithmic trading proxies and the trade size variable created from the TAQ 10-minute intervals. *Event*, in, is defined as 1 if the date corresponds to date of the malfunction specified in table 1 and 0 otherwise. Non-Event encompasses all other days in the sample with no other technological malfunction. To assess statistical significance, we us Welch's t-test of unequal variance due to the difference in sample sizes observed for each event. Both t-stats and p-values are reported in the column 4 and 5 of each panel. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Trade/Order

Event	(1) Event	(2) Non-Event	(3) Diff. (1-2)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	-2.142295	-2.14358	.0012843	0.1056	0.9159
Event 2 – August 2013 BATS	-2.250477	-2.196908	-.0535689	-4.9687***	0.0000
Event 3 – August 2013 Nasdaq	-2.189635	-2.182772	-.0068633	- 0.4699	0.6385
Event 4 – July 2015 Edge-X	-2.128387	-2.097662	-.0307245	-2.6601***	0.0078
Event 5 – July 2015 NYSE	-2.110503	-2.097662	-.0128411	-1.0930	0.2745
Event 6 – May 2016 NYSE	-1.80542	-1.786434	-.0189865	-1.1797	0.2384
Event 7 – November 2018 CHX	-1.842395	-1.849003	.0066088	0.7100	0.4778
Event 8 – August 2019 CTA	-1.495919	-1.487255	-.0086649	- 0.9452	0.3446

Panel B: Odd lot/Volume

Event	(1) Event	(2) Non-Event	(3) Diff. (1-2)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	-3.230868	-3.247448	.0165807	2.3729**	0.0177
Event 2 – August 2013 BATS	-3.139525	-3.22822	.0886951	11.7045***	0.0000
Event 3 – August 2013 Nasdaq	-3.292457	-3.252699	-.0397584	- 3.6892***	0.0002
Event 4 – July 2015 Edge-X	-3.392172	-3.374877	-.0172951	-3.5381***	0.0004
Event 5 – July 2015 NYSE	-3.396605	-3.374877	-.0217275	-4.4193***	0.0000
Event 6 – May 2016 NYSE	-3.485725	-3.493957	.0082324	0.7940	0.4274
Event 7 – November 2018 CHX	-3.321318	-3.302663	-.0186551	-3.7005***	0.0002
Event 8 – August 2019 CTA	-3.0345	-3.04279	.0082905	2.2200**	0.0265

Panel C: Avg. Trade size

Event	(1) Event	(2) Non-Event	(3) Diff. (1-2)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	3.06878	3.087019	.-0182386	-2.0111**	0.0444
Event 2 – August 2013 BATS	2.996686	3.078343	-.0816562	-10.1763***	0.0000
Event 3 – August 2013 Nasdaq	3.127922	3.099461	.0284616	2.5670**	0.0103
Event 4 – July 2015 Edge-X	3.085575	3.054035	.03154	5.2948***	0.0000
Event 5 – July 2015 NYSE	3.084255	3.054035	.0302195	5.0566***	0.0000
Event 6 – May 2016 NYSE	3.237814	3.244228	-.0064139	- 0.7388	0.4602
Event 7 – November 2018 CHX	3.066263	3.050238	.0160254	3.0201***	0.0025
Event 8 – August 2019 CTA	2.624811	2.629998	-.0051871	-1.2993	0.1939

Panel D: Cancel/Order

Event	(1) Event	(2) Non-Event	(3) Diff. (1-2)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	4.629133	4.629418	-.0002849	-0.0518	0.9587
Event 2 – August 2013 BATS	4.665385	4.643225	.0221594	4.7193***	0.0000
Event 3 – August 2013 Nasdaq	4.637929	4.637039	.0008905	0.1405	0.8883
Event 4 – July 2015 Edge-X	4.59032	4.578651	.0116691	2.0699**	0.0385
Event 5 – July 2015 NYSE	4.585056	4.578651	.0064058	1.1260	0.2602
Event 6 – May 2016 NYSE	4.402441	4.395898	.0065429	1.0145	0.3106
Event 7 – November 2018 CHX	4.411002	4.413037	-.0020351	-0.4712	0.6376
Event 8 – August 2019 CTA	4.264834	4.261975	.0028585	0.4873	0.6261

Panel E: Daily Short-Term Volatility $

Event	(1) Event	(2) Non-Event	(3) Diff. (1-2)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	.105359	.0926215	-.0127375	4.3829***	0.0000
Event 2 – August 2013 BATS	.0930827	.0996458	-.0065632	-2.5659**	0.0103
Event 3 – August 2013 Nasdaq	.1095572	.1006531	.008904	2.6148***	0.0090
Event 4 – July 2015 Edge-X	1.505364	1.307387	.1979768	2.3440**	0.0191
Event 5 – July 2015 NYSE*	1.306554	1.307387	-.0008326	-0.0106	0.9915
Event 6 – May 2016 NYSE*	.0553176	.049312	.0060056	4.9963***	0.0000
Event 7 – November 2018 CHX	.119816	.1273428	-.0075268	- 4.5207***	0.0000
Event 8 – August 2019 CTA	.1681411	.1892346	-.0210935	-4.1114***	0.0000

Panel F: Daily Short-Term Volatility %

Event	(1) Event	(2) Non-Event	(3) Diff. (1-2)	(4) t-stat	(5) p-value
Event 1 – January 2013, Nasdaq	.3043724	.2474457	.0569268	7.6197***	0.0000
Event 2 – August 2013 BATS	.2423571	.2871181	-.044761	-15.0599***	0.0000
Event 3 – August 2013 Nasdaq	.2943817	.2936761	.0007056	0.1355	0.8922
Event 4 – July 2015 Edge-X	3.672129	3.784874	-.1127453	-0.5013	0.6162
Event 5 – July 2015 NYSE	4.040665	3.784874	.2557913	1.0225	0.3066
Event 6 – May 2016 NYSE	.1967735	.1685354	.0282381	19.3239***	0.0000
Event 7 – November 2018 CHX	.2146172	.2280429	-.0134257	-6.4336***	0.0000
Event 8 – August 2019 CTA	.1398514	.1604288	-.0205774	-14.0606***	0.0000

APPENDIX 5: REGRESSION ANALYSIS PRE-MALFUNCTION

Table 5 – Regression Analysis the day before a trading malfunction: Reports the regression analysis for each event testing the normalized measures a day before the technological malfunction. Each dependent variable is adjusted for intraday seasonality, as described in section 3.4 of the study.

$$Normalized\ Measure_{i,z,t} = \alpha + \beta Event_{t-1} + \delta * controls_{i,z,t} + \epsilon_{i,z,t}$$

This table summarizes the coefficients for the variable $Event_{t-1}$, equal to one if the measure is observed one day before the technological malfunction and 0 for all other times. The control variables used in our regression include the log of normalized interval volume, log market capitalization, log daily price, the interval quoted spread ($), exchange and group market share on the day of the event. All regression include stock and day fixed effects and standard errors are clustered by stock. T-stats are reported in parenthesis in column 2 and 4 of each panel. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Quoted Spread

	Quoted Spread - Cents		Quoted Spread – Percentage	
Event	(1) Coefficient $Event_{T-1}$	(2) t-stat	(3) Coefficient $Event_{T-1}$	(4) t-stat
Event 1 – January 2013, Nasdaq	0.0199	(0.707)	0.0258	(0.895)
Event 2 – August 2013 BATS	-0.104***	(-6.481)	-0.121***	(-8.011)
Event 3 – August 2013 Nasdaq	0.136***	(7.411)	0.149***	(8.058)
Event 4 – July 2015 Edge-X	-0.0179	(-1.284)	-0.0194	(-1.355)
Event 5 – July 2015 NYSE	0.0978***	(6.345)	0.106***	(6.900)
Event 6 – May 2016 NYSE	0.0297*	(1.670)	0.0290	(1.639)
Event 7 – November 2018 CHX	-0.00834	(-0.447)	-0.0381**	(-1.996)
Event 8 – August 2019 CTA	0.0676***	(6.457)	0.0565***	(5.308)

Panel B: Effective Spread

	Effective Spread - Cents		Effective Spread – Percentage	
Event	(1) Coefficient $Event_{T-1}$	(2) t-stat	(3) Coefficient $Event_{T-1}$	(4) t-stat
Event 1 – January 2013, Nasdaq	0.0427	(1.166)	0.0469	(1.250)
Event 2 – August 2013 BATS	-0.0770***	(-4.202)	-0.0953***	(-5.470)
Event 3 – August 2013 Nasdaq	0.111***	(5.593)	0.124***	(6.162)
Event 4 – July 2015 Edge-X	0.00480	(0.285)	0.000879	(0.0526)
Event 5 – July 2015 NYSE	0.0928***	(6.121)	0.100***	(6.418)
Event 6 – May 2016 NYSE	-0.0791**	(-2.007)	-0.0813**	(-2.065)
Event 7 – November 2018 CHX	-0.000559	(-0.0118)	-0.0282	(-0.597)
Event 8 – August 2019 CTA	-0.0119	(-0.281)	-0.0206	(-0.482)

Panel C: Quoted Depth

	Quoted Depth - Shares		Quoted Depth - Dollar	
Event	(1) Coefficient $Event_{T-1}$	(2) t-stat	(3) Coefficient $Event_{T-1}$	(4) t-stat
Event 1 – January 2013, Nasdaq	0.00450	(0.0553)	0.00415	(0.0529)
Event 2 – August 2013 BATS	0.0244	(1.184)	0.0439**	(2.067)
Event 3 – August 2013 Nasdaq	-0.0955***	(-5.154)	-0.106***	(-5.614)
Event 4 – July 2015 Edge-X	0.0264	(1.403)	0.0283	(1.442)
Event 5 – July 2015 NYSE	-0.107***	(-6.893)	-0.114***	(-7.330)
Event 6 – May 2016 NYSE	-0.0742***	(-2.881)	-0.0734***	(-2.826)
Event 7 – November 2018 CHX	0.0375**	(2.039)	0.0718***	(3.771)
Event 8 – August 2019 CTA	-0.0210	(-1.200)	-0.0109	(-0.602)

Panel D: Interval Volume

	Interval volume, Shares		Interval volume, Dollar	
	(1)	(2)	(3)	(4)
Event	Coefficient $Event_{T-1}$	t-stat	Coefficient $Event_{T-1}$	t-stat
Event 1 – January 2013, Nasdaq	0.265***	(6.179)	0.257***	(5.984)
Event 2 – August 2013 BATS	0.0596	(1.129)	0.0804	(1.486)
Event 3 – August 2013 Nasdaq	-0.0905**	(-2.580)	-0.101***	(-2.863)
Event 4 – July 2015 Edge-X	-0.0965**	(-2.080)	-0.0951**	(-2.080)
Event 5 – July 2015 NYSE	0.283***	(7.561)	0.275***	(7.360)
Event 6 – May 2016 NYSE	0.0819*	(1.780)	0.0832*	(1.800)
Event 7 – November 2018 CHX	0.0702*	(1.695)	0.102**	(2.445)
Event 8 – August 2019 CTA	-0.166***	(-6.576)	-0.156***	(-5.989)

Panel E: Volatility

	Volatility - Absolute Return Dollar		Volatility - Absolute Return Percentage		Volatility - Absolute Hi-Low Diff Dollar	
	(1)	(2)	(3)	(4)	(5)	(6)
Event	Coefficient $Event_{T-1}$	t-stat	Coefficient $Event_{T-1}$	t-stat	Coefficient $Event_{T-1}$	t-stat
Event 1 – January 2013, Nasdaq	0.196***	(3.744)	0.199***	(3.744)	0.256***	(4.407)
Event 2 – August 2013 BATS	-0.106***	(-3.957)	-0.123***	(-4.699)	-0.114***	(-4.268)
Event 3 – August 2013 Nasdaq	0.192***	(7.130)	0.205***	(7.524)	0.139***	(5.504)
Event 4 – July 2015 Edge-X	-1.158	(-1.558)	-0.558	(-1.512)	-0.0576**	(-2.010)
Event 5 – July 2015 NYSE	-0.917	(-1.109)	-0.214	(-0.641)	0.199***	(6.999)
Event 6 – May 2016 NYSE	0.0877***	(2.881)	0.0866***	(2.849)	0.0356	(1.181)
Event 7 – November 2018 CHX	-0.0101	(-0.502)	-0.0396*	(-1.951)	-0.0148	(-0.810)
Event 8 – August 2019 CTA	0.112***	(7.523)	0.103***	(6.662)	0.0417***	(3.450)

APPENDIX 6: REGRESSION ANALYSIS – INTRA- AND POST-MALFUCNTION

Table 6 – Regression analysis During and After a trading malfunction: Reports the regression analysis for each event Comparing the normalized measures at the time of the malfunction with the all other times during that same trading day. Each dependent variable is adjusted for intraday seasonality, as described in section 3.4 of the study.

$$Normalized\ Measure_{i,z,t} = \alpha + \beta_1 During_{I,t} + \beta_2 After_{I,t} + \delta * controls_{i,z,t} + \epsilon_{i,z,t}$$

This table summarizes the coefficients for the variables *During* and *After*. *During* is defined as 1 if there is an ongoing trading halt during a 10-minute interval and 0 otherwise. *After* is defined as 1 if the 10-minute trading halt has no trading halt during that interval and that interval fall after the trading halt has ended. Our base group of comparison in equation 2 is all trading occurring in 10-minute intervals that take place before the trading halt. The control variables used in our regression include the log of normalized interval volume, log market capitalization, log daily price, the interval quoted spread ($), exchange and group market share on the day of the event. For technological malfunctions taking place at the NYSE in May 2016, CHX in November 2018, and at the Consolidated Tap Association (CTA) in August of 2013, we omit the variable *After* from the regression because these malfunctions took place at the open or at the close and thus only have two periods of analysis: during the malfunction and not during All regression include stock fixed effects and standard errors are clustered by stock. T-stats are reported in parenthesis in column 2 and 4 of each panel. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Quoted Spread

	Quoted Spread - Cents			
Event	(1) Coefficient *During*	(2) t-stat	(3) Coefficient *After*	(4) t-stat
Event 1 – January 2013, Nasdaq	0.0864**	(2.374)	0.145***	(6.757)
Event 2 – August 2013 BATS	-0.0500**	(-2.269)	0.00189	(0.0829)
Event 3 – August 2013 Nasdaq	1.461***	(4.332)	0.359***	(6.172)
Event 4 – July 2015 Edge-X	-0.0929**	(-2.321)	-0.0751**	(-2.126)
Event 5 – July 2015 NYSE	0.250***	(9.022)	0.106***	(3.304)
Event 6 – May 2016 NYSE*	-0.179***	(-4.751)		
Event 7 – November 2018 CHX*	0.106**	(2.166)		
Event 8 – August 2019 CTA*	0.0134	(0.586)		

Panel B: Effective Spread

	Effective Spread - Cents			
Event	(1) Coefficient *During*	(2) t-stat	(3) Coefficient *After*	(4) t-stat
Event 1 – January 2013, Nasdaq	0.589***	(4.656)	0.377***	(5.686)
Event 2 – August 2013 BATS	-0.0827***	(-2.907)	-0.0202	(-0.662)
Event 3 – August 2013 Nasdaq	2.782***	(3.993)	0.211***	(2.854)
Event 4 – July 2015 Edge-X	-0.146*	(-1.905)	-0.0832	(-1.218)
Event 5 – July 2015 NYSE	0.226***	(5.084)	0.106	(1.497)
Event 6 – May 2016 NYSE*	0.708***	(3.324)		
Event 7 – November 2018 CHX*	-0.0440	(-0.414)		
Event 8 – August 2019 CTA*	1.124***	(3.776)		

Panel C: Quoted Depth

	Quoted Depth - Shares			
Event	(1) Coefficient *During*	(2) t-stat	(3) Coefficient *After*	(4) t-stat
Event 1 – January 2013, Nasdaq	-0.104***	(-6.481)	-0.0387***	(-10.49)
Event 2 – August 2013 BATS	0.0345	(0.791)	0.102**	(2.222)
Event 3 – August 2013 Nasdaq	-0.0660	(-0.322)	-0.0350	(-0.784)
Event 4 – July 2015 Edge-X	0.00774	(0.248)	0.0286	(0.947)
Event 5 – July 2015 NYSE	-0.123***	(-5.566)	-0.147***	(-6.094)
Event 6 – May 2016 NYSE*	0.219***	(3.619)		
Event 7 – November 2018 CHX*	0.101	(1.260)		
Event 8 – August 2019 CTA*	0.0736***	(2.826)		

Panel D: Interval Volume

	Interval volume, Shares			
Event	(1) Coefficient *During*	(2) t-stat	(3) Coefficient *After*	(4) t-stat
Event 1 – January 2013, Nasdaq	-0.162	(-1.303)	-0.157	(-1.258)
Event 2 – August 2013 BATS	-0.201	(-1.553)	-0.154	(-1.267)
Event 3 – August 2013 Nasdaq	-0.259**	(-2.419)	0.0514	(0.519)
Event 4 – July 2015 Edge-X	-0.355***	(-2.742)	-0.221**	(-2.071)
Event 5 – July 2015 NYSE	-0.0423	(-0.723)	0.156**	(2.126)
Event 6 – May 2016 NYSE*	-0.414***	(-4.285)		
Event 7 – November 2018 CHX*	0.0487	(0.672)		
Event 8 – August 2019 CTA*	-0.370***	(-7.672)		

Panel E: Volatility – During

	Volatility - Absolute Return Dollar		Volatility - Absolute Return Percentage		Volatility - Absolute Hi-Low Diff Dollar	
Event	(1) Coefficient *During*	(2) t-stat	(3) Coefficient *During*	(4) t-stat	(5) Coefficient *During*	(6) t-stat
Event 1 – January 2013, Nasdaq	0.000905	(0.0114)	-0.00121	(-0.0153)	-0.00112	(-0.0135)
Event 2 – August 2013 BATS	-0.0121	(-0.194)	-0.0189	(-0.307)	-0.0534	(-0.912)
Event 3 – August 2013 Nasdaq	0.371***	(3.003)	0.368***	(2.997)	1.407***	(4.738)
Event 4 – July 2015 Edge-X	-0.164	(-1.508)	-0.165	(-1.505)	-0.252***	(-5.349)
Event 5 – July 2015 NYSE	0.109***	(3.853)	0.116***	(4.107)	0.108***	(5.997)
Event 6 – May 2016 NYSE*	-0.460***	(-6.444)	-0.467***	(-6.488)	0.494	(0.852)
Event 7 – November 2018 CHX*	0.104	(1.604)	0.107*	(1.699)	0.00617	(0.163)
Event 8 – August 2019 CTA*	-0.114	(-1.561)	-0.114	(-1.568)	0.0626	(1.007)

Panel F: Volatility – After

	Volatility - Absolute Return Dollar		Volatility - Absolute Return Percentage		Volatility - Absolute Hi-Low Diff Dollar	
Event	(1) Coefficient $After$	(2) t-stat	(3) Coefficient $After$	(4) t-stat	(5) Coefficient $After$	(6) t-stat
Event 1 – January 2013, Nasdaq	0.268***	(4.323)	0.268***	(4.307)	0.185**	(2.526)
Event 2 – August 2013 BATS	-0.0495	(-1.091)	-0.0522	(-1.164)	-0.0538	(-1.208)
Event 3 – August 2013 Nasdaq	0.517***	(4.723)	0.519***	(4.735)	0.467***	(3.450)
Event 4 – July 2015 Edge-X	-0.0902	(-0.979)	-0.0998	(-1.070)	-0.168***	(-3.736)
Event 5 – July 2015 NYSE	0.193***	(3.997)	0.207***	(4.278)	0.168***	(5.964)

Panel G: Small Orders – After

	Interval Odd lots - trades		Interval Odd lots - Shares		Avg. Interval Trade Size-TAQ	
Event	(1) Coefficient $After$	(2) t-stat	(3) Coefficient $After$	(4) t-stat	(5) Coefficient $After$	(6) t-stat
Event 1 – January 2013, Nasdaq	0.0334	(0.364)	0.0338	(0.369)	-0.118**	(-2.226)
Event 2 – August 2013 BATS	-0.109*	(-1.811)	-0.107*	(-1.766)	-0.207	(-1.308)
Event 3 – August 2013 Nasdaq	0.222***	(3.820)	0.222***	(3.828)	0.0715	(1.395)
Event 4 – July 2015 Edge-X	-0.466***	(-3.336)	-0.392***	(-3.752)	0.197***	(3.688)
Event 5 – July 2015 NYSE	0.260***	(4.035)	0.253***	(4.788)	-0.0440	(-1.337)

APPENDIX 7: POST-MALFUNCTION PERSISTENCE

Table 7 – Regression analysis During and After a trading malfunction: Reports the regression analysis for each event Comparing the normalized measures the day of the malfunction with all other dates in the corresponding monthly sample.

$$Normalized\ Measure_{i,z,t} = \alpha + \beta_1 Event_{I,t} + \beta_2 Event_{I,t+1} + \beta_3 Event_{I,t+2} + \beta_4 Event_{I,t+3} + \beta_5 Event_{I,t+4} + \beta_6 Event_{I,t+5} + \beta_7 Event_{I,t+6-10} + \delta * controls_{i,z,t} + \epsilon_{i,z,t}$$

This table summarizes the coefficients for the variable, *Event* is defined equal 1 if the date corresponds to date of the malfunction specified in table 1 and 0 otherwise. The other variables of interest to determine the long-term effects of technological malfunctions are $\beta_2 - \beta_7$, Each of these is a dummy variable equal to 1 if the date is $t + n$ days after the trading halt. The last coefficient, β_7 includes days $[t + 6, t + 10]$ after a technological malfunction. The control variables used in our regression include the log of normalized interval volume, log market capitalization, log daily price, the interval quoted spread ($), exchange and group market share on the day of the event. For technological malfunctions taking place at the NYSE in May 2016, CHX in November 2018, and at the Consolidated Tap Association (CTA) in August of 2013, we omit the variable *After* from the regression because these events only have two periods of analysis: during the malfunction and not during. All regression include stock and day fixed effects and standard errors are clustered by stock. T-stats are reported in column 2 and 4. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A – Quoted Interval Spreads - Cents

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Event	-0.00213	-0.0797***	0.188***	-0.0106	0.288***	0.0797**	0.0399	-0.00531
	(-0.114)	(-4.001)	(6.476)	(-0.792)	(10.22)	(2.525)	(1.400)	(-0.518)
$Event_{t+1}$	-0.0402*	-0.102***	0.00275	0.0925***	0.00629	0.0531*	0.0222	0.0172*
	(-1.802)	(-5.325)	(0.145)	(5.158)	(0.404)	(1.980)	(1.520)	(1.902)
$Event_{t+2}$	-0.0324	-0.0693***	0.0361**	Omitted	-0.00943	-0.0447**	0.0272**	0.0577***
	(-1.236)	(-3.488)	(2.164)		(-0.777)	(-2.084)	(2.246)	(5.404)
$Event_{t+3}$	-0.0325	-0.0290*	0.0555***	-0.000634	-0.0737***	-0.0389	0.115***	0.0808***
	(-1.380)	(-1.723)	(3.006)	(-0.0369)	(-7.028)	(-1.631)	(5.863)	(7.070)
$Event_{t+4}$	-0.0597**	-0.0654***	0.0352*	-0.0167	-0.0765***	-0.0724***	0.131***	-0.0942***
	(-2.431)	(-4.207)	(1.944)	(-1.222)	(-6.587)	(-3.274)	(6.848)	(-7.599)
$Event_{t+5}$	-0.0199	-0.0570***	0.0455***	-0.0810***	-0.0306**	-0.146***	0.215***	-0.142***
	(-0.871)	(-3.450)	(2.749)	(-6.616)	(-2.544)	(-7.197)	(7.178)	(-11.86)
$Event_{t+6-t+10}$	-0.0169	-0.0165	0.0341***	-0.0569***	Omitted	-0.0857***	0.156***	-0.0119*
	(-0.720)	(-1.348)	(3.695)	(-4.975)		(-5.343)	(8.442)	(-1.844)

Panel B – Interval Effective Spread - Cents								
VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Event	0.109***	-0.0647**	0.508***	-0.0102	0.252***	0.914***	0.179*	0.373***
	(3.477)	(-2.609)	(5.951)	(-0.659)	(7.927)	(5.603)	(1.896)	(4.093)
$Event_{t+1}$	-0.0619**	-0.0881***	0.0135	0.0791***	0.00220	0.0876*	0.0521	0.0733
	(-2.105)	(-4.488)	(0.601)	(4.785)	(0.126)	(1.720)	(1.268)	(1.405)
$Event_{t+2}$	-0.0467	-0.0755***	0.0131	Omitted	-0.0210	-0.0131	0.0346	-0.0709**
	(-1.592)	(-3.356)	(0.702)		(-1.581)	(-0.286)	(0.875)	(-2.247)
$Event_{t+3}$	-0.0390	-0.0336*	0.0509**	-0.0103	-0.0722***	-0.109**	0.150***	0.0142
	(-1.277)	(-1.748)	(2.437)	(-0.546)	(-5.325)	(-2.361)	(3.295)	(0.329)
$Event_{t+4}$	-0.0455	-0.0731***	0.0331	-0.0342**	-0.0922***	-0.0449	0.182***	-0.0582
	(-1.355)	(-4.301)	(1.618)	(-2.408)	(-7.709)	(-0.894)	(3.390)	(-1.313)
$Event_{t+5}$	-0.0299	-0.0780***	0.0439**	-0.0857***	-0.0387**	0.0365	0.292***	-0.141***
	(-0.940)	(-3.917)	(2.374)	(-5.565)	(-2.543)	(0.571)	(5.041)	(-2.850)
$Event_{t+6-t+10}$	-0.0384	-0.0254*	0.0351***	-0.0730***	Omitted	-0.0606*	0.150***	0.0189
	(-1.384)	(-1.848)	(3.710)	(-6.272)		(-1.716)	(4.835)	(0.609)

Panel C - Interval Depth - Shares								
VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Event	-0.0532	0.0866***	-0.0266	-0.0101	-0.172***	-0.0975**	0.00205	0.0308**
	(-0.766)	(2.783)	(-1.075)	(-0.468)	(-7.417)	(-2.258)	(0.146)	(2.502)
$Event_{t+1}$	-0.0383	0.0473**	0.0788**	-0.116***	-0.0438**	-0.0645**	-0.0249	-0.0343***
	(-0.552)	(2.241)	(2.174)	(-7.053)	(-2.622)	(-2.024)	(-1.084)	(-2.781)
$Event_{t+2}$	-0.0837	0.00294	0.0361	Omitted	-0.0334**	0.00464	-0.0461**	-0.0683***
	(-1.233)	(0.124)	(1.217)		(-2.034)	(0.124)	(-2.465)	(-3.774)
$Event_{t+3}$	-0.106	-0.0499**	-0.0200	-0.0431**	0.0231	0.0122	-0.0752***	-0.0748***
	(-1.616)	(-1.994)	(-1.142)	(-2.521)	(1.361)	(0.294)	(-3.063)	(-5.526)
$Event_{t+4}$	-0.0750	0.0248	0.0169	-0.0316*	0.0218	0.0138	-0.0938***	0.0638***
	(-1.101)	(0.687)	(0.699)	(-1.841)	(1.395)	(0.360)	(-7.911)	(4.128)
$Event_{t+5}$	-0.0851	0.00261	0.0281	0.0253	-0.0330**	0.0610	-0.123***	0.102***
	(-1.293)	(0.0879)	(1.034)	(1.426)	(-2.114)	(1.363)	(-10.76)	(5.619)
$Event_{t+6-t+10}$	0.00343	-0.0254	-0.00437	0.0245*	Omitted	0.0702*	-0.0229**	0.00474
	(0.0498)	(-1.512)	(-0.434)	(1.912)		(1.720)	(-2.152)	(0.449)

Panel D – Volatility - Absolute Return Dollar								
VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Event	0.188***	-0.00229	0.165***	0.0129	0.216***	0.301***	0.0464*	-0.0600**
	(4.766)	(-0.0574)	(4.351)	(0.779)	(11.82)	(6.990)	(1.761)	(-2.418)
$Event_{t+1}$	-0.0424	-0.0692**	-0.0606**	0.195***	0.0458**	0.0975***	0.0592***	-0.0307*
	(-1.525)	(-2.097)	(-2.151)	(9.975)	(2.433)	(2.703)	(2.791)	(-1.805)
$Event_{t+2}$	-0.0326	-0.00139	0.00243	Omitted	-0.0596***	-0.00613	0.0689***	0.0211
	(-1.107)	(-0.0344)	(0.0948)		(-3.817)	(-0.207)	(3.971)	(1.274)
$Event_{t+3}$	0.0186	-0.00491	0.0508*	0.0361*	-0.109***	-0.0799***	0.130***	0.0126
	(0.657)	(-0.159)	(1.882)	(1.789)	(-6.361)	(-2.652)	(5.758)	(0.683)
$Event_{t+4}$	-0.0958***	-0.145***	0.0151	-0.0705***	-0.145***	-0.137***	0.209***	-0.261***
	(-3.587)	(-6.059)	(0.626)	(-4.334)	(-9.379)	(-4.923)	(9.256)	(-14.92)
$Event_{t+5}$	0.0104	-0.0716**	0.0109	-0.120***	-0.0861***	-0.139***	0.280***	-0.254***
	(0.386)	(-2.147)	(0.440)	(-6.647)	(-5.283)	(-5.410)	(9.775)	(-15.07)
$Event_{t+6-t+10}$	-0.0415**	-0.0625***	0.127***	-0.110***	Omitted	-0.126***	0.0885***	-0.0861***
	(-2.291)	(-3.694)	(7.272)	(-8.081)		(-6.872)	(6.774)	(-7.813)

Panel E – Volatility - Absolute Return Percentage								
VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Event	0.192***	-0.0155	0.169***	0.0158	0.229***	0.310***	0.0170	-0.0567**
	(5.048)	(-0.395)	(4.435)	(0.962)	(12.33)	(7.164)	(0.663)	(-2.302)
$Event_{t+1}$	-0.0378	-0.0777**	-0.0608**	0.202***	0.0528***	0.120***	0.0437**	-0.0354**
	(-1.399)	(-2.397)	(-2.247)	(10.44)	(2.818)	(3.307)	(2.140)	(-2.034)
$Event_{t+2}$	-0.0240	-0.0166	-0.00290	Omitted	-0.0571***	0.00508	0.0627***	0.0442**
	(-0.847)	(-0.421)	(-0.117)		(-3.615)	(0.172)	(3.668)	(2.604)
$Event_{t+3}$	0.0316	-0.0174	0.0700**	0.0425**	-0.113***	-0.0700**	0.130***	0.0381**
	(1.164)	(-0.570)	(2.598)	(2.120)	(-6.657)	(-2.323)	(5.950)	(2.165)
$Event_{t+4}$	-0.0854***	-0.153***	0.0354	-0.0687***	-0.153***	-0.135***	0.218***	-0.252***
	(-3.253)	(-6.270)	(1.466)	(-4.155)	(-9.921)	(-4.908)	(9.794)	(-14.44)
$Event_{t+5}$	0.0180	-0.0828**	0.0219	-0.124***	-0.0927***	-0.141***	0.292***	-0.255***
	(0.688)	(-2.520)	(0.913)	(-6.941)	(-5.760)	(-5.508)	(10.65)	(-15.56)
$Event_{t+6-t+10}$	-0.0348**	-0.0584***	0.137***	-0.116***	Omitted	-0.132***	0.113***	-0.0820***
	(-1.991)	(-3.441)	(7.945)	(-8.341)		(-7.516)	(8.315)	(-7.504)

Panel F – Volatility Absolute Hi-Low Diff Dollar								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
VARIABLES								
Event	0.208***	-0.0358	0.357***	-0.00292	0.225***	0.729**	0.0511*	-0.0303
	(5.201)	(-0.873)	(6.287)	(-0.232)	(12.62)	(2.356)	(1.870)	(-1.277)
$Event_{t+1}$	-0.0554**	-0.0832**	-0.0269	0.197***	0.0535***	0.110***	0.0540***	0.0369**
	(-2.502)	(-2.493)	(-1.111)	(11.53)	(3.047)	(3.080)	(2.992)	(2.174)
$Event_{t+2}$	-0.00851	-0.0156	-0.0206	Omitted	-0.00784	-0.0282	0.0729***	0.0899***
	(-0.300)	(-0.404)	(-0.951)		(-0.565)	(-1.166)	(4.993)	(6.739)
$Event_{t+3}$	0.00175	-0.0147	0.0748***	0.0447**	-0.0832***	-0.0618**	0.134***	0.0982***
	(0.0599)	(-0.476)	(2.807)	(2.396)	(-6.714)	(-2.606)	(7.519)	(4.793)
$Event_{t+4}$	-0.0847***	-0.0956***	0.00568	-0.0179	-0.129***	-0.108***	0.203***	-0.162***
	(-3.746)	(-3.840)	(0.199)	(-1.195)	(-9.191)	(-3.934)	(7.651)	(-8.937)
$Event_{t+5}$	-0.01000	-0.106***	-0.00778	-0.0935***	-0.0510***	-0.140***	0.298***	-0.197***
	(-0.424)	(-3.984)	(-0.282)	(-6.668)	(-3.874)	(-5.913)	(9.351)	(-12.72)
$Event_{t+6-t+10}$	-0.0585***	-0.0515***	0.114***	-0.0983***	Omitted	-0.0929***	0.0873***	-0.0954***
	(-3.953)	(-2.937)	(3.676)	(-7.666)		(-5.390)	(7.647)	(-10.29)

APPENDIX 8: ALGORITHMIC TRADING ACTIVIY

Table 8 – Regression analysis During and After a trading malfunction: Reports the regression analysis for each event investigates the four algorithmic trading (AT) proxies of Weller (2018) defined in section 3. The Weller measures of AT activity include *odd lot-to-volume*, *trade-to-order volume*, *cancel-to-trade ratio*, and *average trade size*. For each stock, we compute a rolling average of each algorithmic trading variable across the dates [T – 21, T – 1] and take the log of each measure.

$$Weller\ (2018)\ AT\ Proxy_{i,t} = \alpha + \beta_1 During_{I,t} + \beta_2 After_{I,t} + \delta * controls_{i,z,t} + \epsilon_{i,z,t}$$

This table summarizes the coefficients for the variables *During* and *After*. *Event* is defined equal 1 if the date corresponds to date of the malfunction specified in table 1 and 0 otherwise. $Event_{I,t+1}$ is equal 1 if the date corresponds to one day after a technological malfunction and 0 otherwise. Following the methods of Weller, our controls include the short-term return volatility, quoted spread, log market cap, and log price. All regression include day and stock fixed effects and standard errors are clustered by stock. T-stats are reported in parenthesis in column 2 and 4 of each panel. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Trades to Orders

	Trades to Orders			
Event	(1) Coefficient $Event$	(2) t-stat	(3) Coefficient $Event_{t+1}$	(4) t-stat
Event 1 – January 2013, Nasdaq	-0.0220***	(-2.657)	-0.0152*	(-1.816)
Event 2 – August 2013 BATS	-0.0135	(-1.059)	-0.00956	(-0.681)
Event 3 – August 2013 Nasdaq	-0.145***	(-4.815)	-0.163***	(-5.366)
Event 4 – July 2015 Edge-X	-0.00574	(-1.585)	-0.0139***	(-2.988)
Event 5 – July 2015 NYSE	-0.0144**	(-2.031)	-0.0252***	(-3.258)
Event 6 – May 2016 NYSE	-0.0129	(-0.400)	-0.0373	(-1.104)
Event 7 – November 2018 CHX	0.0104	(1.229)	0.0106	(1.096)
Event 8 – August 2019 CTA	0.00987	(1.152)	-0.0164**	(-2.156)

Panel B: Odd lots to Volume

	Odd lots to Volume			
Event	(1) Coefficient $Event$	(2) t-stat	(3) Coefficient $Event_{t+1}$	(4) t-stat
Event 1 – January 2013, Nasdaq	0.0157***	(2.955)	0.0161***	(3.036)
Event 2 – August 2013 BATS	-0.00877	(-1.055)	-0.00757	(-0.834)
Event 3 – August 2013 Nasdaq	0.0756***	(3.646)	0.0824***	(3.976)
Event 4 – July 2015 Edge-X	0.0141***	(5.154)	0.0234***	(6.049)
Event 5 – July 2015 NYSE	0.0342***	(6.174)	0.0383***	(6.246)
Event 6 – May 2016 NYSE	-0.0524*	(-2.078)	-0.0392	(-1.427)
Event 7 – November 2018 CHX	-0.00711**	(-2.615)	-0.00708**	(-2.165)
Event 8 – August 2019 CTA	0.0010	(0.320)	0.0279***	(11.56)

Panel C: Cancels to Trades

	Cancels to Trades			
Event	(1) Coefficient $Event$	(2) t-stat	(3) Coefficient $Event_{t+1}$	(4) t-stat
Event 1 – January 2013, Nasdaq	0.0293***	(3.213)	0.0198**	(2.158)
Event 2 – August 2013 BATS	0.0241**	(2.183)	0.0266**	(2.034)
Event 3 – August 2013 Nasdaq	0.146***	(5.403)	0.159***	(5.602)
Event 4 – July 2015 Edge-X	-0.000539	(-0.136)	0.00561	(1.054)
Event 5 – July 2015 NYSE	0.00312	(0.452)	0.0116	(1.543)
Event 6 – May 2016 NYSE	0.0357	(1.126)	0.0577	(1.639)
Event 7 – November 2018 CHX	0.00289	(0.549)	0.000510	(0.0805)
Event 8 – August 2019 CTA	-0.00545	(-0.840)	-0.00795	(-1.511)

Panel D: Avg. Trade Size

	Avg. Trade Size			
Event	(1) Coefficient $Event$	(2) t-stat	(3) Coefficient $Event_{t+1}$	(4) t-stat
Event 1 – January 2013, Nasdaq	-0.0102***	(-2.858)	-0.00953***	(-2.866)
Event 2 – August 2013 BATS	0.00125	(0.319)	0.00130	(0.284)
Event 3 – August 2013 Nasdaq	-0.0311***	(-3.273)	-0.0339***	(-3.864)
Event 4 – July 2015 Edge-X	-0.00334***	(-2.914)	-0.00583***	(-4.049)
Event 5 – July 2015 NYSE	-0.0100***	(-4.908)	-0.0122***	(-5.220)
Event 6 – May 2016 NYSE	0.0181*	(1.763)	0.0126	(1.017)
Event 7 – November 2018 CHX	0.00325**	(2.106)	0.00325*	(1.826)
Event 8 – August 2019 CTA	-3.54e-05	(-0.0152)	-0.0239***	(-12.42)

PART 2: ALGORITHMIC TRADING IN HIGHLY FRAGMENTED MARKETS

I. INTRODUCTION

The SEC doesn't associate the term fragmentation with negative connotations but simply that it is a dispersal of volume amongst many different venues.[28] A defining feature of modern markets is the growth in fragmentation that forces venues to compete for order flow in more than one dimension whether it be trading fees, pre-trade transparency, single synthesized exchanges, payment for order, and other options.[29] Market fragmentation, especially among lit venues, continues to be the target of discussion as the proliferation of markets can result in increased market complexity (Harris, 1993). These concerns rise as three new U.S. stock exchanges have launched operations in late 2020. These new additions to the stock exchange market include the Members Exchange (MEMX), which began its phased launch on September 4, 2020, and is backed by firms such as Virtu Financial, Morgan Stanley, and TD Ameritrade; the MIAX Pearl Equities run by options exchange operator Miami International Holdings, which debuted on September 25, 2020; and the third being the Long Term Stock Exchange (LTSE), which began trading on August 28, 2020.[30] "I could see us getting to 20-plus," Bryan Harkins, co-head of

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[28] https://www.sec.gov/marketstructure/research/fragmentation-lit-review-100713.pdf
[29] See Budish, E., Lee, R. and Shim, J., 2019. A Theory of Stock Exchange Competition and Innovation: Will the Market Fix the Market?. NBER Working Paper, 25855.
[30] See McCrank, J. (2020, August 21). Competition to heat up among U.S. stock exchanges with new entrants. Reuters. Available at https://www.reuters.com/article/us-usa-exchanges/competition-to-heat-up-among-u-s-stock-exchanges-with-new-entrants-idUSKBN25H23K and https://www.cnbc.com/2020/09/09/a-spate-of-new-stock-exchanges-are-ready-to-launch-that-want-to-compete-for-your-trading-dollar.html

markets at CBOE, stated regarding the U.S. eventually seeing more lit venues in the near future.[31]

The addition of new venues will inevitably lead to more competition amongst venues, which generally forces exchanges to lower trading costs and other fees or offer other benefits. The main objective of this study is to examine to what extent algorithmic trading and highly fragmented markets are related given the growth that has taken place and the upcoming additions set to occur in U.S. markets. As markets are increasingly fragmented, do we see increases/decreases in algorithmic trading for more/less fragmented stocks? We dissect this objective further to determine (1) the relation between lit vs. dark fragmentation and algorithmic trading, and (2) the influence that trading fees have on algorithmic trading and the appeal that different fee venues provide algorithmic traders.

Over the years, financial markets have seen a fluctuating environment between fragmentation and consolidation. Initially, markets were consolidated with two markets being formed in the late 1700s, the Philadelphia Stock Exchange and New York Stock Exchange (NYSE). The NYSE would move on to dominate competition from other exchanges, but leading into the early 1900s there was a substantial amount of fragmentation as regional exchanges were established in many major US cities for trading local securities, evident by the fact that there were over 100 regional exchanges.[32] The initial role of regional exchanges was for the purpose of initial public offerings of growing industries in specific regions of the US. The shift towards a consolidated market began in 1929, after which regional stock exchanges prerogatives changed significantly. Regional exchanges shifted from being a designated market for local securities and

[31] See McCrank, J. (2019, September 19). U.S. stock exchange competition to heat up in 2020 with new entrants. Reuters. Available at https://www.reuters.com/article/us-usa-exchanges/u-s-stock-exchange-competition-to-heat-up-in-2020-with-new-entrants-idUSKBN1W42Y6
[32] See SEC (1963), p. 928

“began to serve as auxiliary markets for New York” (Business Week, 1936). One reason for the change is suggested by Cole (1944) and Doede (1967), whereby regional exchanges sought to reduce communication costs during the 1920s and 1930s in the wake of the Great Crash. New regulations imposed by states in response to the burst of the speculative bubble at the end of the 1920s also prevented regional exchanges from distributing new issues and forced many smaller firms to list and trade on the unregulated over-the-counter market (SEC (1963), p. 916).[33] At the request of the SEC, in 1936 Congress adjusted the Securities Exchange Act to allow Unlisted Trading Privileges that would allow exchanges to trade any security that was already approved for listing on another exchange.

The market was thus becoming a consolidated system and the implications of this consolidation are shown in Arnold, Hersch, Mulherin, and Netter (1999). Arnold et al. examine how regulation and technological changes altered the role of regional exchanges and assess the movement from a fragmented market to one that is much more consolidated. Arnold et al. examine three mergers that took place amongst some of the major regional exchanges: Philadelphia-Baltimore, Midwest mergers (merger of the St. Louis, Cleveland, and Minneapolis exchanges) and Pacific merger (merger of the San Francisco and Los Angeles exchanges). The authors find that the merging exchanges experience a reduction in bid-ask spreads and attract market share away from other regional exchanges rather than from the NYSE.

Prompted by regulation and technological changes throughout the 1900s, U.S. stock markets would continue to consolidate which allowed exchanges to be more effective competitors to the dominant NYSE. Even as recently as the 1990s, markets would remain fairly consolidated. Blume and Goldstein (1997) examine the impacts of the 1975 congressional

[33] See SEC (1963), p. 916

amendment to the Securities exchange Act of 1934, whereby the intention was to create a "strong central market system for securities of national importance, in which all buying and selling in these securities could participate and be represented under a competitive regime."[34] The 1975 amendments included: (1) Consolidated Tape Association (CTA) to oversee dissemination of quotes and trades, (2) Consolidated Quotation System (CQS) to provide quotation information, and (3) Intermarket Trading System (ITS) to allow dealers on NASDAQ to route orders to other markets. At the beginning of the twentieth century there were over 100 regional exchanges, by 1935 there were 35, 15 by 1965, and at the time Blume and Goldstein conducted their analysis in 1995 there were five regional stock exchanges (Boston, Chicago, Cincinnati, Pacific, and Philadelphia). Blume and Goldstein find in this consolidated market that the NYSE is equal to the best displayed price much more frequently than those other markets but regional exchanges contend for order flow by using payment-for-order flow whereby non-NYSE market makers compete with NYSE order flow by paying brokers to send them the order flow of small retail customers.

By the start of the 21st century, we see this paradigm shift from a consolidated market back to a much more fragmentated market and has progressed into the current state the market finds itself in today. Fragmentation and competition increased across venues that are transparent, but fragmentation also exists between transparent and hidden venues. Intermarket competition across venues that are transparent is referred to as "lit" fragmentation and fragmentation that is the result of either trades taking place in alternative trading systems (ATS) or from internalization of orders by dealers is referred to as "dark" fragmentation. As of March 2021,

[34] See The Institutional Investor Study Report of the Securities and Exchange Commission March 10, 1971, Volume 1, Page xxiv

there are 16 lit exchanges and 61 active ATS or dark venues.[35] The newest lit exchanges being the Members Exchange (MEMX), MIAX Pearl Equities, and the Long-Term Stock Exchange (LTSE) which is a Silicon Valley-based startup whose aim is to promote long-term growth rather than short-term profits for the listed companies.[36]

The contribution we make to both the market fragmentation literature and the algorithmic trading literature is to examine the behaviors of algorithmic traders in highly fragmented markets and what relation different algorithmic trading measures have with more/less lit and dark fragmentation. We seek to analyze competition among venues in a different light using the current US market structure that has increasingly grown over the past decade and continues to expand trading with the addition of more venues. Additionally, we examine determinants of daily fragmentation and what factors drive stocks to change the number of daily trading venues over a period.

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[35] See https://www.sec.gov/foia/docs/atslist.htm and https://www.finra.org/filing-reporting/otc-transparency/ats-equity-firms

[36] See Securities and Exchange Act Release No. 34-85828 (May 10, 2019), File No. 10-234 (available at https://www.sec.gov/rules/other/2019/34-85828.pdf) and Posner, Cydney, and Cooley LLP. "Will the Long-Term Stock Exchange Make a Difference?" Harvard Law School Forum on Corporate Governance, June 8, 2019. https://corpgov.law.harvard.edu/2019/06/08/will-the-long-term-stock-exchange-make-a-difference.

II. HYPOTHESIS DEVELOPMENT

Market Competition through consolidation or fragmentation is examined in both theoretical and empirical studies. One of the early models of competition is by Glosten (1994) who demonstrates that in an idealized setting with no friction, market liquidity is invariant to the amount of fragmentation present, suggesting that fragmentation is a trivial factor. Empirically, competition and consolidation are shown in the findings of Arnold et al. (1999) who demonstrate that merging exchanges are effective competitors to the NYSE but recognize that there is increased competition that can arise through fragmentation. Other studies find that markets are deeper and more liquid when consolidated, but a consolidated market might have higher trading costs and other externalities. Christie and Huang (1994) examine stocks which moved from the Nasdaq (a fragmented market) to the NYSE (a consolidated market) and find a reduction in trading costs. Bennet and Wei (2006) also analyze firms that switch from Nasdaq to the NYSE and find they experience improved price efficiency especially for illiquid stocks and suggests that order flow consolidation is valuable for these less liquid stocks.

However, other studies assessing the impact of fragmentation on liquidity, contend that consolidated markets have increased trading costs and other externalities by finding a positive association between fragmentation and liquidity (Demsetz, 1968; Cohen and Conroy, 1990; Battalio, 1997; Mayhew, 2002; Weston, 2000; Boehmer and Boehmer, 2003; De Fontnouvelle, Fishe and Harris, 2003; Battalio, Hatch, and Jennings, 2004; Nguyen, Van Ness and Van Ness, 2007; Foucault and Menkveld, 2008; Chlistalla and Lutat, 2011; O’Hara and Ye, 2011;

Menkveld, 2013; Gresse, 2017). In addition, O’Hara and Ye examine a three-month period in 2008 and using the number of trade reporting facilities (TRFs) reporting trading activity on an asset as a proxy for fragmentation. O’Hara and Ye find that fragmented stocks have lower transaction costs, faster execution speed, and higher short-term volatility but prices appeared to be more efficient by becoming closer to a random walk.

There are also studies which find negative consequences associated with fragmentation, these studies find that increasingly fragmented markets lead to a deterioration in liquidity (Biais, 1993; Madhavan, 1995; Bessembinder and Kaufman, 1997; Arnold, Hersch, Mulherin and Netter, 1999; Amihud, Lauterbach and Mendelson, 2003; Hendershott and Jones, 2005; Bennett and Wei, 2006; Gajewski and Gresse, 2007; Nielsson, 2009). As markets continue to fragment among traditional exchanges (lit venues), competition increases from alternative trading venues and dark pools, otherwise known as dark venue fragmentation, and further illustrates the negative effects of fragmentation. Degryse, Dejong, and Van Kerval (2015); Comerton-Forde and Putnins (2015); and Gresse (2017) portray the negative correlation between fragmentation and liquidity by examining competition from dark venues to find that while empirical evidence suggests that lit fragmentation is beneficial, dark venue fragmentation is harmful to liquidity (diminished price discovery, wider spreads, and higher adverse selection in the lit market), especially when dark volume rises past 10% of all trading volume. Gresse finds that when both algorithmic traders (AT) and lit fragmentation contribute to improving spreads, the economic effect of fragmentation is greater.

The debate surrounding competition and fragmentation is mixed. Recently Baldauf and Mollner (2019) provide a theoretical and empirical analysis to show how the discussion of fragmentation is driven by context. The authors construct a model of imperfect competition among exchanges that demonstrates as competition increases there is a reduction in trading fees but as markets fragment, additional arbitrage opportunities arise for algorithmic trading that intensifies adverse selection. Baldauf and Mollner argue that fragmentation contains two prevailing channels: (1) the competition channel, in which introducing more exchanges can reduce fees and lower spreads and (2) the exposure channel, whereby increasing more exchanges increases both costs and spreads to liquidity providers. Employing this model with order-level data for an Australian security, Baldauf and Mollner find that the benefits of increased competition are overcome by the costs of increased arbitrage. Comparing the spread in prevailing duopoly of Australian markets, the model predicts the counterfactual model spread by 23%.

ALGORITHMIC TRADING

Baldauf and Mollner (2019) provide a theoretical and empirical scenario where imperfect competition among exchanges and an increase in fragmentation produces additional arbitrage opportunities for algorithmic trading, thereby intensifying adverse selection. In this section of the study, we look to determine the relation between fragmentation and algorithmic trading. Do we see more fragmented stocks containing increased levels of algorithmic activity and if so, is this activity evenly dispersed among all available exchanges or do these algorithmic trades concentrate on a few exchanges that drive the increase in overall activity? The Baldauf and Mollner study suggests that a link exists between an increase in algorithmic activity and stocks that experience more fragmentation. Studies such as Menkveld (2013) speak to this link by identifying algorithmic traders like high frequency traders as crucial market makers for new market venues. Menkveld provides an analysis of high frequency trading and its role in increasingly fragmented markets by examining the cross-market making activities in Euronext (the incumbent exchange) and Chi-X (Dutch new entrant). Menkveld determines that high-frequency traders act as modern multi-venue market makers and closely links algorithmic activity to rapidly evolving market structure characterized by the entry of many new and successful trading venues. Menkveld contends that the increased ability of venues to compete, aided by the presence of algorithmic traders, could explain the proliferation of venues in the US and Europe.

Prior to Menkveld (2013), Jovanovic and Menkveld (2010) model HFTs as middlemen in limit order markets with the purpose of cross-market activity and market development. Brogaard, Hendershott, and Riordan (2014b) examine the role of HFT firms in integrating fragmented markets and their role in liquidity provision, managing inventory across multiple trading venues, and transmission of information across exchanges. Boehmer, Li, and Saar (2018) study competition among HFTs and the effects it has on the market environment, as well as the relation between HFT competition and market concentration. Using Order-level data from IIROC, Boehmer et al. identify at least three underlying common strategies: cross-venue arbitrage, market making, and short-horizon directional speculation. The authors determine that HFT competition in each product category, lowers the short-horizon volatility of stocks and enhances the viability of smaller trading venues. The Brogaard et al. and Boehmer et al. both find that HFT play an integral role in tying markets together and lends to the contention that as markets fragment, a need a for synergy among venues may lead to an increase in algorithmic activity.

Latencies, the lag between when an order is sent to an exchange and when the order is processed, ensures traders orders are not always simultaneously processed when sent to several exchanges. Baldauf and Mollner (2020) identify that in a highly fragmented market, faster speeds increase the effectiveness of HFT strategies and allow HFTs to anticipate orders quicker.

Lower latency is a possible mechanism through which, we may see fragmentation lead to more algorithmic activity. Gresse (2017) provides insight into the effects of algorithmic trading and fragmentation by assessing the impact on liquidity. Gresse finds that spreads and depth improve with (or at worst are not affected by) multiple-trading-platforms competition after controlling for endogeneity and AT. More importantly, Gresse suggests that algorithmic traders are the most eager for the low latency offered by alternative trading systems and the most likely to slice and dice orders across markets or to supply liquidity at several venues.[37]

O'Hara, Yao, and Ye (2014) examine odd-lot trading and the effect odd-lot trading has on price discovery and finds that odd-lots, trades that occur with a volume of less than 100 shares, are frequently used by HFTs. Weller (2018) uses the number of odd lots to proxy for the activity of algorithmic trading and finds a positive relation. Brogaard, Carrion, Moyaert, Riordan, Shkilko, and Solokov (2018) examine the activity of common endogenous liquidity providers (ELPs) around extreme price movements (EPM) and find that HFTs acting as ELPs supply liquidity in the opposite direction during these stressful periods, absorb trade imbalances, and correct transitory price movements. The authors conclude that these HFTs act to stabilize markets during periods of stress and provide net positive effects on liquidity provision.

Hypothesis 1: Stocks that are more fragmented will have more algorithmic trading activity than stocks that are less fragmented.

The growth in off-exchange or "dark" trading coincides with the proliferation of "lit" venues and adds complexity to the market that influences participant order preferencing. This raises the question of the relation between the amount of lit-to-dark trading and the amount of

[37] O'Hara, Yao, and Ye (2014); Weller (2018); Brogaard, Carrion, Moyaert, Riordan, Shkilko, and Solokov (2018); Boehmer, Li, and Saar (2018); and Hasbrouck (2018) support this claim.

algorithmic trading activity. Aitken, Harris, and Harris (2015) examine the impact that algorithmic trading and dark venue separately can have on market quality. Aiken et al. find that dark trading increases the incidence of successful manipulation by taking liquidity from the lit market, while profiting by closing positions at lower prices, similar to the model by Klock, Schied, and Sun (2011) where dark trading in cross-venues create the opportunity for trade-based manipulation. Aitken et al. find that dark trading makes quote competition less transparent thereby reducing front running but also reducing the strategies of algorithmic trading that Brogaard et al. (2014a) document (contrarian and arbitrage directional strategies such as short-horizon speculation, market making, and cross-venue arbitrage).[38] Additionally, Aitken et al. find that off-exchange trading reduces the level of algorithmic trading, indirectly widening effective spreads by increasing the incidence of end-of-day dislocations. Buti, Consonni, Rindi, Wen, and Werner (2015) examine dark trading and more specifically sub-penny trading that allows traders to undercut displayed liquidity. Buti et al. argue that as queue-jumping activity increases, high-frequency trading activity will decrease. In congruence, with hypothesis 1, we should see that not only does algorithmic trading increase with lit fragmentation but that the amount of dark fragmentation will likely decrease the amount of algorithmic trading due to the reduction in front running and other strategies used by algorithmic traders. Therefore, we may expect the following hypothesis to hold:

> Hypothesis 2: Stocks that have more lit-to-dark fragmentation will have more algorithmic trading activity than stocks with less lit-to-dark fragmentation.

[38] See Securities and Exchange Commission (2010). Concept Release on Equity Market Structure, Release No. 34-61358; File No. S7-02-10 for further description of HFT arbitrage strategies.

DARK TRADING

Generally, research focuses on the role that dark trading plays in price discovery and the overall impact on market quality. Buti, Rindi, and Werner (2017) build a theoretical model that predicts the effects of introducing a continuous dark pool on a lit market that operates as a limit order book where traders have the option to submit orders to the fully transparent limit order book or to a dark pool. The model by Buti et al. predicts higher consolidated fill rates and higher limit order book fill rates. The authors also find that dark pool activity is less when prices are higher and relative tick size is smaller. In this section we seek to answer the question of what type of relation exists between lit fragmentation and dark fragmentation, and whether prior findings regarding the relation between market quality and dark trading hold in U.S. markets which are considerably more fragmentated than their counterparts in Europe and Asia. The main objective is to determine how the degree of lit fragmentation relates to the amount of dark trading or fragmentation. Is there more/less dark trading for stocks with more/less fragmentation? Our contribution is to establish what this relation between lit fragmentation and dark trading is and differs from other works (DeGryse et al., 2015; Gresse, 2017; Hatheway et al., 2017) whose concentration is fixated on the effects that dark trading has on market quality.

Gresse (2017) provides empirical evidence that (1) spreads and depth improve with or at worst are not affect by lit fragmentation, while (2) dark trading via over the counter (OTC) and internalized trading do not harm liquidity on lit exchanges but may actually improve some of its dimensions. Gresse's main objective is to investigate the link between lit/dark fragmentation and liquidity by using a large sample of stocks that trade on eight European exchanges (Euronext, the LSE, Deusche Boërse, Chi-X, BATS Europe, Turquoise, Nasdaq OMX Europe, and PLUS) and by using MiFID as a natural experiment for the dramatic rise in competition between marketplaces. The U.S. market, compared the European market analyzed in Gresse, is considerably more fragmented with 16 lit exchanges and 61 active ATS or dark venues, and begs the question of whether the effects presented by Gresse hold in a much more fragmented market and speaks to what the relation may be between the amount of lit fragmentation observed and the amount of dark trading taking place.

Kwan, Masulis, and McInish (2015), Hatheway, Kwan, and Zheng (2017), and Menkveld, Yushen, and Zhu (2017) provide evidence of the nature of dark trading on U.S. exchanges. Kwan et al. investigate competition between traditional stock exchanges and dark trading venues for a sample of NYSE- and Nasdaq-listed securities that cross the $1.00 threshold. The objective of Kwan et al. is to show that differential market regulation provides dark trading venues an economic advantage and results in the proliferation of the U.S. market. The authors find that dark pools attract order flow when the NBBO is constrained and the market share of dark pools nearly doubles when the stock crosses the $1.00 threshold. Furthermore, the authors find that price discovery is taking place in off-exchange venues when bid-ask spreads on traditional exchanges are tick constrained by government regulation. Similarly, Hatheway, Kwan, and Zheng examine competition between dark venues and lit exchanges to determine how lit and dark market structures affect an investor's trading choices and the resulting impact on market quality. Hatheway et al. also apply their analysis to U.S. markets using a sample of NYSE- and Nasdaq-listed securities to find that dark venues attract uninformed order flow, leading to higher adverse selection risk on lit markets. Contrary to Kwan et al., Hatheway et al. determine that lit markets contribute more to price discovery as informed traders concentrate on lit markets and dark fragmentation is associated with higher transaction costs and lower price efficiency. Menkveld et al. (2017) examine dynamic fragmentation of U.S. equity markets to test the “pecking order” hypothesis which states that as investors’ trading needs become more urgent, they move from low-cost, low-immediacy venues to high-cost, high-immediacy venues. Using a stratified sample of Nasdaq’s HFT data (120 stocks), the authors confirm the hypothesis.

The previous studies don’t directly address the point of emphasis in this paper, which is to determine the relation and interaction between lit fragmentation and dark fragmentation. More specifically, do stocks that are have more lit fragmentation have higher or lower levels of dark trading? Degryse, DeJong, and Van Kerval (2015) show that investors are more likely to trade in the dark when the visible markets are illiquid (substitution effect), this suggests that more consolidated markets push investors to trade in the dark given that these markets are shown to be less liquid than fragmented markets. Zhu (2014) finds that by disproportionately reducing the number of uninformed trades in the lit market, high levels of dark trading increase adverse selection risk in the lit market and lead to wider bid-ask spreads, another characterization of less fragmented markets.

Comerton-Forde, Gregoire, and Zhong (2019) suggest the benefits to pricing efficiency of trading on inverted venues are likely the result of a substitution effect, where inverted venues replace dark venues as the preferred platform. As markets continue to fracture and give rise to opportunities for traders to route orders through inverted venues when stocks are tick constrained, we may see that higher levels of lit fragmentation result in less dark trading. This could also be implied by the pecking order theory (Menkveld et al., 2017), that as markets fragment and higher-immediacy venues are available to investors, the high-cost component may be avoided with different fee structures. Thus, we contend that the following hypotheses will hold.

Hypothesis 3: Stocks that have more lit fragmentation will have lower levels of dark fragmentation (negative relation).

DETERMINANTS OF LIT FRAGMENTATION

A key concept of fragmentation is that every market reflects different participant needs and wants at various times and is composed of different segments. The addition of new markets is also born out of competition among venues, which promotes venues to adapt to the increased competition in order to be more effective competitors. Specifically, lit fragmentation often arises between trading venues that charge different fees or offer different market access to participants that addresses the needs of diverse categories of investors in a more efficient manner (Harris, 1993; Hendershott and Mendelson, 2000; Gresse, 2006). A question we examine in this section is what determinants influence changes in daily fragmentation for securities. For stocks that trade on a different number of market venues over a given period, what are some driving factors to these changes? If increasing fragmentation provides increasing marginal benefits, such as lowering trading charges and other fees (O'Hara and Ye, 2011; Gresse, 2017), then should all securities trade on all available exchanges to maximize these benefits? These questions create an opportunity to provide a contribution to the fragmentation literature through the examination of determinants that cause stocks to change the number of daily venues that trading takes place on over a given period.

A crucial concept of fragmentation relies upon markets providing participants a host options that satisfy their needs and implies that a potential determinant of fragmentation is a clientele effect. Disseminated further, this clientele effect is influenced by factors such as trading costs (spreads and venue fees), volatility, price, competition, and informational advantages. Various changes in these factors may influence where investors preference their orders and if there aren't enough venues to meet those preferences, investors may seek out other venues to address their needs. Hendershott and Menkveld (2014) study price pressures or deviations from the efficient price due to risk-averse intermediaries supplying liquidity to asynchronously arriving investors. The authors measure transitory effects using NYSE data to find that this price pressure is significantly larger for small-cap stocks compared to large-cap stocks. Hendershott and Menkveld also state that institutional investors often care more about the marginal pressure, price elasticity of liquidity demand, than the spread.

Hendershott and Menkveld (2014) in their study of transitory price pressure allude to a subset of investors having different preferences that influence trading behavior. Similarly, Harris (1993) states that markets fragment in part because traders differ in the types of trading problems that they confront. He determines that traders are heterogenous and demand a variety of market mechanisms to satisfy their needs. Chowdry and Nanda (1991) find that large liquidity traders such as institutions split their trades across markets to minimize costs, typically carried out via intermarket sweep orders. Small liquidity traders, in their model, however, do not split their trades across markets and will choose the market they expect trading costs to be the smallest. Chowdry and Nanda determine that these small liquidity traders will concentrate in the market with the largest number of liquidity traders who are unable to move between markets like institutions and, in turn, will attract more trading by the informed as well as the large liquidity traders. The evidence from the aforementioned studies suggests that trading patterns of investors are sensitive to the relative distribution of traders across available venues who don't have the flexibility that large traders like institutions have.

Hendershott and Mendelson (2000) show that fragmentation can reduce the inventory risk of individual dealers and that institutional investors have a need for alternative markets that provide low-cost execution while sacrificing immediacy and has led to the evolution of crossing networks. O'Hara, Yao, and Ye (2014) provide determinants of what may cause institutional investors to seek other markets and split orders. O'Hara et al. find that higher prices, illiquidity, and lower volatility are associated with increased odd-lot trading and that odd-lot trading is positively correlated with informed trading. Odd-lots are also much more likely to come from algorithmic trading and high frequency traders (O'Hara et al., 2014; Weller, 2018). More recently, Comerton-Forde et al. (2019) describe a substitution effect between inverted lit venues and dark trading venues that results in competition for sub-tick liquidity provision reduces trading costs and improves liquidity. The rise and availability of inverted venues enables participants to adapt to the regulatory change and enhance competition among liquidity providers (Comerton-Forde et al.). Given the impact liquidity, price, volatility, and order imbalances have on investors preferencing orders to certain venues, we would expect these factors to also be determinants of daily fragmentation changes for stocks.

> Hypothesis 4: Stocks that have transitory spikes in trading costs, volatility, and trading imbalances (large volume) will increase the number of daily markets the stock currently trades on.

VENUE FEE STRUCTURE

Trading fees are a mechanism through which venues can compete for order flow. The most common fee model among lit venues is the make-take fee structure, with 8 of the 16 U.S. exchange models using this model. In this structure, liquidity suppliers (makers) receive a rebate, while liquidity demanders (takers) are charged a fee. There are also venues called taker-maker or an inverted venue. In these fee venues, liquidity demanders (takers) receive the rebate while liquidity suppliers (makers) receive a fee. Of the 16 lit venues, five exchanges, BATS-Y, NASDAQ BX, EDGA, MEMX, and NYSE Market, have adopted this fee structure.[39] The remaining lit venues contain 2 exchanges (IEX and NYSE Chicago) that have a fee only model where no rebates are paid. The Long-Term Stock Exchange will operate with no transaction or market data fees.[40]

The objective of this section is to determine what affects differing venue fees have on market fragmentation. We seek to shed light on the relation between venue fees and the relation they have to various levels of fragmentation and algorithmic trading activity. First, do varying levels of inverted trading have a larger impact on market quality for highly fragmented stocks versus stocks that are more consolidated? Second, how does fee structure influence algorithmic trading in highly fragmented markets, given that different fee venues appeal to different clientele in highly fragmented markets. Lastly, what role do inverted venues play in price discovery in the presence of highly fragmented markets and larger algorithmic trading activity?

Fee models add to the complexity to fragmentation, because, despite the differences between maker-take and inverted venues, participant activity level and stock price also factor into the different fees offered by these exchanges. Inverted venues not only change the preferences for making versus taking liquidity but also influence traders' routing preferences and the probability of executing orders (Comerton-Forde, Gregoire, and Zhong, 2019; Battalio, Corwin, and Jennings, 2016; Cox, VanNess, and VanNess, 2017; Foucault, Kadan, and Kandel, 2013; Angel, Harris, and Spatt, 2015; and Harris, 2013). This is in part due the sub-tick pricing

[39] See https://www.businesswire.com/news/home/20180518005767/en/NYSE%E2%80%99s-Newest-Market---NYSE-National-Commence. The Members Exchange plans to provide a standard rebate of $0.0005 per share for orders that remove liquidity and assesses a fee of $0.0019 per share for orders that add liquidity. See Securities and Exchange Act Release No. 34-87960 (January 14, 2020), File No. SR-CboeBYX-2020-001 (available at https://www.sec.gov/rules/sro/cboebyx/2020/34-87960.pdf)
[40]See https://www.sec.gov/rules/sro/ltse/2020/34-88133.pdf

grid offered by inverted markets and the case where nonmarketable limit orders displayed on inverted markers will execute before orders displayed at the same price on tradition make-take venues (Comerton-Forde et al.). Gresse (2017) shows that the markets for large stocks are usually more fragmented than markets for small stocks. Comerton-Forde et al. add to this by finding that stocks with higher volume, better liquidity, tighter spreads, and larger market capitalization have more inverted trading. The empirical findings suggest that a positive relation may exist between highly fragmented stocks and inverted trading and holding hypothesis 3 to be true lends additional evidence to this contention given the substitutional benefits of inverted venues found in Comerton-Forde et al.

Hypothesis 5: Stocks that are highly fragmented will have higher levels of inverted trading than those that are less fragmented (positive relation).

Next, we examine what additional effects higher levels of inverted trading may have on market quality for highly fragmented stocks. Foucault, Kadan, and Kandel (2013) argue that the maker-taker fee model gives market makers incentives to price improve and increase overall liquidity provision. Malinova and Park (2015) analyze the impact of the maker-taker pricing model on market liquidity, trader behavior, and trading volume to find that market quality improves with introduction of make-take fees at the TSX. Other works such as Anand, Hua, and McCormick (2016) and Battalio, Shkilko, and Van Ness (2016) show that maker-taker fee venues are associated with reduced transaction costs for options, depending on the price of option but the reduction of costs is not as great as venues with payment for order flow.

Another differencing factor for inverted venues compared to maker-take venues is that the inverted fee venue can be viewed as an extension of the payment-for-order-flow model in which participants want to transact against lesser informed retail order flow (O’Hara, 2015). Not only do these venues resemble payment-for-order-flow models but the competition between these venues can produce additional benefits to market quality. Lin, Swan, and Harris (2019) find that competition between venues differentiated by maker-taker fee structures can lead to greater liquidity, higher trade volume, more informed trading, and better price discovery in venues with the highest permitted fee rebates. Lin et al. further find that informed traders will allocate trades to adjust their information content upwards to capture the full make rebate in traditional (maker-take) venues, and downward in inverted venues in a process of rent extraction. Given this evidence we expect the following hypothesis to hold:

> Hypothesis 6: More inverted trading will increase market quality (spread, depth, and price discovery) in highly fragmented securities.

We anticipate that in highly fragmented stocks, algorithmic activity will be large but concentrated the highest on exchanges with the lowest fees. Foucault, Kadan, and Kandel (2013) suggest a similar idea to hypothesis 1b, that trading fees on different market fee venues may influence the level of algorithmic activity, indicating a positive relation. Likewise, Brogaard, Hendershott, and Riordan (2014a) find that maker-taker fee venues benefit algorithmic traders acting in a market making capacity. Our objective in this section is to not only determine the relation between venue fees and fragmentation but also establish what effect this may have on algorithmic trading activity as a result. The relation between market venue fees and algorithmic trading can be found in other studies such as Yao and Ye (2014). Yao and Ye provide evidence of the level of high frequency trading across market fee venues to find that HFTs tend to be more active in maker-taker models where they are paid to provide liquidity, whereas non-HFTs will migrate toward taker-maker venues to bypass the tick size constraints and jump ahead of the queue. Buti, Consonni, Rindi, Wen, and Werner (2015) also argue that as this queue-jumping activity increases, high-frequency trading activity will decrease.

Angel, Harris, and Spatt (2015) also provide an incentive for not only non-algorithmic traders but for brokers to route orders to taker-maker venues despite other exchanges offering the same price. The inverted pricing model allows dealers and traders a path to avoid sub-penny pricing restrictions. In doing so, this practice has allowed traders and brokers to violate one of the goals of Reg NMS in reducing "pennying" practices. While this option is available for brokers, Battalio, Corwin, and Jennings (2016) show that brokers fail to take advantage of the sub-penny pricing benefits of inverted venues and rather try to capture the venue with the largest liquidity rebate. Despite Battalio et al., we expect the findings of the previously stated literature analyzing participant activity and venue selection to hold.

Hypothesis 7: In highly fragmented stocks, algorithmic activity will be largely concentrated on the maker-take venues as inverted venue trading increases.

III. DATA AND METHODS

DATA AND SAMPLE

Our sample is drawn from Daily TAQ, CRSP, and SEC Market Information Data Analytics System (MIDAS) for all common stocks in the year 2019. To draw stocks to be included in our sample, we use CRSP to initially identify common stocks (share code 10 and 11) that trade at or above a price of $5.00 dollars every day during December 2018, the month preceding the sample period. Once we complete this preliminary procedure, we then use the SEC's MIDAS dataset to construct the Herfindahl-Hirschman Index (HHI) as a market fragmentation metric for December 2018. Midas allows us to observe daily trading volume by stock exchange for each ticker. To compute the HHI we first compute the HHI for each stock using the following equation:

$$HHI = \sum_{i=1}^{13} (MS_i)^2 \quad (1)$$

where MS_i equals the market share of volume for each of the 13 lit exchanges identified by the Midas data set. Once HHI is calculated, we calculate one minus the usual HHI to allow for an easier interpretation, this being that larger values of 1-HHI now correspond to a greater degree of market fragmentation (from here on, we refer to this measure as simply the HHI). We then calculate the average HHI over the month of December 2018, and segment stocks into quartiles. Quartile 1 contains stocks with the least amount of fragmentation and quartile 4 being stocks with the most fragmentation. We then eliminate stocks in quartiles two and three and all quartile four stocks not in top 100 ranked by HHI. To create a matched sample of highly fragmented and consolidated stocks we follow Shkilko and Sokolov (2020) to compute the matching error between the top 100 HHI stocks in quartile four and all stocks in quartile one:

$$match\ error = \sum_{k=1}^{3} \left(\frac{c_k^i - c_k^j}{c_k^i + c_k^j}\right)^2 \qquad (2)$$

C_k is one of three stock characteristics: price, volume, and market capitalization (Shkilko and Sokolov). We then select pairs with the smallest matching errors (without replacement) and label stocks in quartile four as the fragmented group and those in quartile one as the consolidated group, which produces a final sample size of 200 securities.[41] The objective of classifying stocks into fragmented and consolidated categories is to determine the relation that each has with algorithmic trading activity and what market frictions exist that may be unique to each group. The results of the matching procedure are found in Panel A of Table 2 and appear to be mixed – prices and market capitalization do not differ statistically, while volume is significantly different.

VARIABLES

We use the TAQ dataset to construct the national best bid and offer (NBBO) prices and liquidity measures following the methods prescribed in Holden and Jacobsen (2014). The liquidity measures used in this study follow with the standard measures of market quality. These include the quoted spread and depth, effective spread, realized spread, and price impact. The *quoted spread* is the difference between the best bid and best ask prices and is weighted by the time. *Effective spread* is defined for a buy as twice the difference between the trade price and the midpoints of the NBBO price. For a sell, effective spread is twice the difference between the

[41] Due to stocks delisting and acquisitions over the year 2019, we have a reduction in sample size over our period to 181 securities. 98 securities in the fragmented group and 83 securities in the consolidated group.

midpoints of the NBBO and the trade price. Effective spread in our analysis is weighted by trade size. *Depth* is the time-weighted average of displayed depth at the NBBO. *Volume* is measured daily and is based on the consolidated volume in all U.S. stock exchanges and off-exchange trading venues.

The *realized spread* is constructed to proxy as the temporary component of the dollar effective spread and is defined as twice the difference between the execution price and the midpoint of the spread prevailing five minutes after a trade. The dollar *price impact* is the permanent component of the dollar effective spread and is defined as twice the difference between the midpoint of the spread prevailing five minutes after a trade and the midpoint of the NBBO quotes of the current trade. For both realized spread and price impact we multiple each difference by +1 if the trade is a buy or by -1 if the trade is a sell. In order to identify whether a trade is a liquidity-demander "buy" or a liquidity-demander "sell", we consider the Lee and Ready (1991) version used by Holden and Jacobsen (2014).

To assess volatility and price efficiency, we use several different measures. We follow Lee, Ready, and Seguin (1994) to first compute the absolute return (dollar and percentage) over a 10-minute interval. Absolute return (dollar and percentage) is computed as the difference between the last trade price before the 10-minute interval and the last trade price in a 10-minute interval. We then follow O'Hara and Ye (2011), to compute the short-term volatility of these returns by finding the daily standard deviations of the absolute returns. This variable is a crude measure of the trading irregularities which we interpret to be that lower levels the short-term volatility correspond to a more efficient market. Additionally, we also compute the daily standard deviation of prices as another crude measure of price efficiency and can be interpreted in the same manner as the short-term return volatility. Our third measure of price efficiency is the variance ratio test (Lo and MacKinlay, 1988). The variance ratio is defined as:

$$Variance\ Ratio_{kl} = \left|\frac{\sigma_{kl}^2}{\sigma_l^2} - 1\right| \quad (3)$$

Where σ_l^2 and σ_{kl}^2 are the variances of 30-minute returns divided by 3 times the variance of the 10-minute returns for a given stock-day, respectively. As this number moves closer to zero, we interpret this to be that prices increasingly behave like a random walk and thus correspond to a more efficient market.

To construct our algorithmic trading (AT) activity measure, we follow the method of Weller (2018) to compute four measures of AT activity: *odd lot-to-volume*, *trade-to-order volume*, *cancel-to-trade ratio*, and *average trade size*. Odd lot-to-volume ratio is the total volume executed in quantities smaller than 100 shares divided by the total volume traded. Trade-to-order volume ratio is the total volume traded divided by the total volume from all orders placed. Cancel-to-trade ratio is the number of full or partial order cancellations divided by the total number of trades. Average trade size is simply the trade volume in shares divided by the number of trades. Weller finds that odd lot-to-volume and cancel to trade ratios are positively related to algorithmic trading activity, while a higher trade-to-order ratio and average trade size are negatively related to algorithmic trading. As per the methods used by Weller and prescribed by SEC Market Information Data Analytics System (MIDAS), odd lot-to-volume, trade-to-order, and cancel-to-trade ratios are adjusted to exclude those orders reported by the NYSE and NYSE MKT.[42]

[42] During this sample period, 13 of the 16 exchange venues were active, while the remaining 3 were still in the process of completing SEC approval. MIDAS collects around a billion feeds from the proprietary feeds of each of the 13 national equity exchanges and of the 13 exchange feeds the NYSE and NYSE MKT report trade size of the

Panels A and B of Table 1 report summary statistics for both our fragmented and consolidated sample, respectively. The average market capitalization for our fragmented sample is $10.65 billion while our consolidated sample is 5.00 billion. Despite the differences in size between the samples, both samples consist of large-, mid-, and small-cap stocks, as can be seen by the min and max. As we expect, the HHI index of our fragmented group throughout the sample period is on average larger than our consolidated sample. Initial examination of the AT proxies between the two groups shows mixed evidence relating to the amount of algorithmic trading activity occurring in the highly fragmented sample of stocks. Cancel-to-trade ratio is larger for the fragmented sample while the trade-to-order volume is smaller. This is in line with the relation reported by Weller (2018), however, both odd lot-to-volume and avg. trade size are opposite of expectations. Lastly, market quality for fragmented stocks appears to be better than consolidated stocks, in terms of spreads and depth. The quoted and effective spread for our fragmented sample is $0.754 and $0.0333, respectively, and has an average daily NBBO depth of 2,269 shares. Whereas, the consolidated sample has a mean quoted and effective spread of $0.0994 and $0.0457, respectively, and has an average daily of 1,582 shares.

MARKET FRAGMENTATION

We use multiple methods to determine the level of fragmentation experienced by each stock in the sample. Our first metric is the HHI index, which is computed daily over our sample period using trading volume from the Midas dataset. Despite the fluctuation in the HHI for each stock, the identification established in the pre-sample period as either "fragmented" or

initiating order. The other 11 exchanges, however, separate trades by initiating and contra orders. This results in the NYSE number of trades and trade size to not be comparable with other exchanges. Additional MIDAS details and discussion of exchange exclusions are provided on the MIDAS website at http://www.sec.gov/marketstructure/mar_methodology.html “consolidated” does not change over the sample period. Our second market fragmentation metric is the number of daily Midas venues that a security trades on each day. Midas captures the number of trades and total trade volume for each of the 13 market venues for each stock daily. For a market venue to be included in the daily count of venues, we require that at least one share execute at that exchange. As the number of daily venues that a stock records trading volume at increases, we consider this to be an increase in fragmentation. Likewise, stocks that record trading at a small number of daily venues, we interpret to be an example of a consolidated market.

In figure 1 (a), we record the number of the number of daily venues by the number of daily stock observations.[43] As shown in Figure 1 (a), we see that although all stocks can trade on all thirteen venues every day, the majority of our sample trades on twelve venues each day. However, not every observation congregates on twelve and thirteen venues. As figure 1(a) shows, there is variation in the number of venues a stock trades on each day, albeit the frequency of these smaller number of daily venues is substantially smaller. As we dissect this further and look to see the difference between the fragmented group of stocks, those being stocks with the highest HHI, and the consolidated group, stocks with the lowest HHI, the results in Figure 1 (b) mimic the construction of these groups. Stocks included in the fragmented group trade on no less than 8 daily venues, while stocks in the consolidated group report an observation at each unique number of daily venues. Again, we also see that consistent across both groups, the most common number of daily venues is twelve.

To assess the impact of fragmentation on market liquidity and algorithmic trading, we start by looking at the relation between daily venues reported by Midas and the variables of

[43] For instance, if AAPL were to trade on all thirteen stock venues one day and then trade on twelve stock venues the next day, there would be one observation for twelve venues and one observation for thirteen venues.

interest (spreads, depth, and Weller 2018 AT proxies). Figures 2 (a) through 2 (f) graph the average spread, depth, and AT proxies by each unique number of daily Midas venues for the entire sample (all 200 stocks). First, we see in Figure 2 (a) there is a dynamic shape to increasing fragmentation that characterizes spreads to be deteriorating as the number of daily venues grows from trading at one venue to trading at eight venues. However, after the eighth venue, liquidity measured by both spreads improves before leveling off after the eleventh daily exchange. Although this figure shows a dynamic relation between fragmentation and liquidity, this trend is largely driven by consolidated sample, as this sample, shown in Figure 1(b), reports an observation at each of the unique number of daily venues. To assess the relation between fragmentation and liquidity for the fragmented sample, we would need to only consider spreads from eight daily venues onward. In which case, fragmentation has a beneficial impact on spreads, as we can see spreads decline the more venues a stock executes a trade at.

The tradeoff of consolidation versus fragmentation is an ongoing discussion and the results presented in figure 2 (a) provide a visual of this dynamic. For the consolidated sample of stocks, as the number of daily venues a stock trades on increases from one, liquidity begins to deteriorate until spreads are at their largest when stocks are trading at eight daily venues. This suggest that lower levels of fragmentation may provide evidence consistent with studies such as Arnold et al. (1999), Bennett and Wei (2006), and Baldauf and Mollner (2019), that consolidation yields better liquidity as the benefits of competition are outweighed by the cost of increased arbitrage. However, Figure 2(a) also presents evidence consistent with O'Hara and Ye (2011) and Gresse (2017), that there exists a linear relationship between fragmentation and market liquidity, where stocks experience better liquidity when executing trades at every venue available.

Consistent with our previously stated conjecture that the number of daily venues recording trading is another proxy for fragmentation, Figure 2(b) shows the positive relation between daily venues and HHI. Furthermore, as we graph daily venues by Weller's (2018) AT proxies in Figures 2(c) through 2(f), we gain initial insight into the relation between fragmentation and algorithmic trading. Our expectation that algorithmic trading increases with fragmentation is consistent with these figures. We should see a negative linear relation between trade-to-order volume (figure 2c), average trade size (figure 2e), and daily venues. However, we should see a positive linear relation between odd lot-to-volume (figure 2d), cancel-to-trade ratio (figure 2f), and daily venues. These expectations appear to hold for odd lot-to-volume, trade-to-order volume, and average trade size; but cancel-to-trade ratio is not consistent. Despite cancel-to-trade ratio displaying the opposite trend, we find supporting evidence of hypothesis 1, that stocks with higher levels of fragmentation will have more algorithmic trading activity.

IV. EMPIRICAL ANALYSIS

As an initial analysis preceding the hypothesis regressions, we perform a univariate test of differences on the desired variables outlined in the previous sections. We are looking to determine the differences between our fragmented and consolidated samples to answer the main question of this study, whether algorithmic trading activity is more active in highly fragmented stocks. Additionally, we assess the impact that fragmentation has on market quality and to determine what venues algorithmic traders concentrate on or if there is a preference of routing orders to certain venues.

UNIVARIATE

In panel A of Table 2, we first report the univariate differences between our matched fragmented and consolidated samples for the matching variables: price, market capitalization, and volume. The pre-sample period matching shows that price is not statistically different, but the fragmented sample is twice the size and has nearly twice the average daily volume (1.631 million shares) as the consolidated sample (.826 million shares), which are both significant.

Panel B of Table 2 reports our results for the differences between the fragmented and consolidates samples for the variables of interest including: the Weller (2018) AT proxies, quoted and effective spreads, quoted depth, volatility measures, and dark venue market share. First, the Weller AT measures show mixed evidence of more AT activity for the fragmented sample, as only two of the four proxies are significant. Trade-to-order is significantly smaller for the fragmented sample compared to the consolidated sample and matches the expected relation to AT activity set forth by Weller. However, odd lot-to-volume means are significantly different, but this difference is contrary to expectations. We should expect the fragmented sample report a larger odd lot-to-volume ratio than the consolidated sample if there is more algorithmic trading activity in highly fragmented stocks, but Table 2 reports the opposite. Seeing as this is merely a univariate analysis with no controls, we will look to further assess these relations in the next section.

Next, we look to compare the market quality differences between our fragmented and consolidated samples. In Figure 2(a) we initially show evidence that supports the literature attributing a positive correlation between fragmentation and market quality (O’Hara and Ye, 2011 and Gresse, 2017). Panel B of Table 2, however does not support our initial findings. Although both quoted and effective spreads for the highly fragmented sample have smaller spreads by $0.028 cents and $0.016 cents, and larger depth by about 713 daily shares, but these differences are not significant. However, market quality, as measured by volatility, does appear to improve with more fragmentation. We see that across two of the three volatility and price efficiency measures, fragmented stocks have significantly lower levels of volatility and better price efficiency. The differences in average variance ratios between the two samples, although not significant, show that the smaller variance ratio of fragmented stocks may imply better price efficiency as prices behave more like a random walk. The results thus far are similar to O’Hara and Ye (2011), who find that fragmented stocks have lower transactions and are more price efficient by becoming closer to a random walk. However, unlike O’Hara and Ye, we show that there is lower short-term volatility, suggesting that overall market quality is improved via fragmentation.

Lastly, we look to determine the amount of dark trading occurring in fragmented stocks and the relation between dark and lit fragmentation. Our two measures of interest are *Dark Market Share* and *Dark/Lit Volume*. To calculate the amount of dark volume and trades we us "D" TAQ to identify all dark trades. Consistent to hypothesis 3 that highly fragmented stocks will have lower levels of dark fragmentation, Panel B of Table 2 shows significantly less dark trading occurring in fragmented stocks compared to the consolidated sample. Dark/Lit Volume is substantially higher in the consolidated sample and is roughly 60% of all lit volume, whereas fragmented stocks dark volume represents just under 50% of all lit volume. In the next section we look to assess this finding using a multivariate analysis that may be able to determine if the higher levels of dark trading result in less dark trading.

ALGORITHMIC TRADING

The objective of this study is to determine to what extent algorithmic trading and highly fragmented markets are related and to dissect this dynamic interaction further in order to see how differing market venue structures influence algorithmic trading routing preferences. Our first analysis begins with establishing a link between fragmentation and algorithmic trading in U.S. markets. We use a Difference-in-Difference panel regression to establish a relation between the amount of fragmentation measured by the HHI and the four algorithmic trading proxies following Weller (2018):

$$Weller\ (2018)\ AT\ proxies = \alpha_0 + \beta_2 HHI_{it} + \beta_2 Fragmented_{it} + \beta_3 Frag * HHI_{it} + \delta_1 Controls_{it} + \varepsilon_{it} \qquad (4)$$

Our dependent variables include log of odd lot-to-volume, trade-to-order volume, cancel-to-trade ratio, and average trade size for stock *i*, on day *t*. Our indicator variables of interest are the (1) HHI; (2) $Fragmented_{it}$, which is constructed the same as in the previous section and is defined as 1 if the security belongs to the fragmented sample, and 0 other wise; and (3) the interaction, $Frag * HHI_{it}$, which gives the difference-in-difference estimate for each variable. The control variables we use follow the method of Weller and include the short-term return volatility, quoted spread ($), log market capitalization, and log price. We also control for day and stock fixed effects and the errors are clustered by stock. Panel A of Table 3 reports the results of equation 4 and shows conflicting evidence with hypothesis 1, that there exists a positive relation between algorithmic trading and fragmentation.

Stocks included in the fragmented sample have significantly more algorithmic trading activity than those in the consolidated sample. The coefficients for trade-to-orders and average trade size report a significant negative relation with the highly fragmented sample of 1.65% and 1.095%, respectively, while the coefficients for odd lot-to-volume and cancels-to-trades report a significant positive relation of 1.23% and 1.37%, respectively. As we would expect, the directions of these relations follow with the findings of Weller (2018) and we can conclude that the fragmented sample has significantly more algorithmic trading activity than the consolidated sample. However, if we next look at the coefficient for HHI, we see that there is no significance across all coefficients. The lack of significance for the HHI coefficients may be the result of including the interaction term, $Frag * HHI_{it}$, which does have significance across all coefficients.

Contradictory to our expectations that fragmentation and algorithmic trading are positively correlated, the interaction term shows evidence of just the opposite. Although there is more algorithmic trading activity in the fragmented sample, the interaction term suggests that high levels of fragmentation are associated with a reduction in algorithmic trading activity. As per our first prediction, we propose that as fragmentation increases so should algorithmic trading activity as a result of highly fragmented markets allowing for faster spreads that increase the effectiveness of HFT strategies (Brogaard et al., 2014; Baldauf and Mollner, 2020).[44] The evidence in Table 3 partially supports this conjecture, but at higher levels of HHI, algorithmic trading activity begins to decrease.

Panel B of Table 3 provides a Difference-in-Difference panel regression to determine what effect fragmentation has as on market quality. The market quality variables used in this analysis include quoted spread, effective spread, quoted depth, interval volume, short-term volatility, price volatility, and variance ratio. To examine the relation between fragmentation and market quality, our indicator variables of interest remain the same fragmentation metrices as in Panel A of Table 3 and include: HHI, $Fragmented_{it}$, and the interaction, $Frag * HHI_{it}$. We use common control metrics that influence daily spreads, which include daily volume, market capitalization, price, and quoted spread (McInish and Wood, 1992; Chung and Zhang, 2014; Gresse, 2017). First, the coefficients for fragmentation show no significance except for the short-term return volatility metric, which attributes the fragmented sample to have less return volatility by 103 basis points. Despite the reduction in volatility, no other market quality measures differ significantly between the two samples. However, as we observe the coefficients for HHI and the interaction term, we see a dynamic relation between spreads and the fragmentation proxies. A

[44] Our results using Daily Venues as the explanatory variable are qualitatively the same as using HHI. We see a positive relation between the number of daily venues, odd lots-to-orders, and cancel-to-trades; but there is a negative relation with trade-to-orders and average trade size. The findings from this regression strengthen the results found in Table 3 and correct the univariate results from Table 2. More Specifically the odd lot-to-order and avg. trade size change from the univariate analysis to the direction that is consistent with the prediction in this study and the findings of Weller (2018).

one standard deviation increase in HHI is associated with a decrease in quoted and effective spreads of $0.0019 and $0.0021 cents.

Despite an improvement in spreads at moderate levels of fragmentation, the interaction term shows the opposite. The positive coefficient for $Frag * HHI_{it}$ suggests that stocks in the fragmented sample have a deterioration in liquidity as they continue to fragment more. Both quoted and effective spread increase by $0.0065 and $ 0.0028 cents with an increase in HHI by one standard deviation. The larger magnitude for the interaction term over the coefficient for HHI may suggest that there is an overall deterioration in liquidity at high levels of fragmentation and support similar findings in Baldauf and Mollner (2019, 2020) appearing in U.S. markets. The positive relation between market fragmentation and algorithmic activity from Panel A of Table 3, whereby algorithmic trading activity substantially increases as the market fragments, may partially explain the deterioration in liquidity at higher levels of fragmentation. Baldauf and Mollner find that increasing fragmentation produces additional arbitrage opportunities, thus causing spreads to widen.

The results in Panel B of Table 3, support this conjecture by Baldauf and Mollner (2019, 2020) that at high levels fragmentation, measured by HHI in the highly fragmented sample, market quality begins to deteriorate. However, at moderate levels of fragmentation, measured by the consolidated sample, fragmentation is beneficial and provides a sub-penny price improvement in spreads. At moderate levels of fragmentation, evidence from Table 3 may show the benefits of algorithmic trading by other studies that link ATs/HFTs as crucial market makers who act as endogenous liquidity providers, compounding information into prices and correcting transitory price movements during periods of stress (Menkveld, 2013; Brogaard et al., 2014b; Boehmer et al., 2018; Brogaard et al., 2018).

Our next question aims to determine the effects that market fragmentation has on algorithmic trading activity and to what relation exists between the amount of off-exchange trading (i.e. dark trading) and amount of algorithmic trading activity taking place on lit venues. To measure the amount of dark trading we look at all off-exchange volume and trades reported under the 'D' exchange in the TAQ trades file. To answer this questions, Table 4 presents the results following a similar model used Weller (2018):

$$Weller\ (2018)\ AT\ proxies = \alpha_0 + \beta_1 \frac{Dark}{Lit} volume_{it} + \delta_1 Controls_{it} + \varepsilon_{it} \quad (5)$$

However, in our model, the variable of interest is the $\frac{Dark}{Lit} volume$ for stock *i*, on day *t*, and the dependent variables include the AT proxies. The coefficients for $\frac{Dark}{Lit} volume_{it}$, in spite of our prediction, are not significant and imply that the amount of dark trading has little effect on the amount of algorithmic trading. Additionally, we exchange $\frac{Dark}{Lit} volume_{it}$ for dark market share and find similar results, reported in the appendix. The coefficients are not significant for $\frac{Dark}{Lit} volume_{it}$ nor dark market share and suggest that there is no significant relation between dark fragmentation and algorithmic trading activity. This finding, or lack of significance, is contrary to the findings in Aitken et al. (2015), where off-exchange trading increases the incidence of successful manipulation and makes quote competition less transparent thereby reducing algorithmic trading strategies.

DARK TRADING & LIT FRAGMENTATION

In this section, we look to answer the question of what type of relation exists between lit fragmentation and dark fragmentation in U.S. markets. To do so we propose the following difference-in-difference panel regression that resembles the first stage regression method of Gresse (2017), whereby our control variables include market capitalization, price, and volume. Additionally, our model includes short-term volatility and quoted spreads ($) in response to findings of Buti et al. (2017) and Menkveld et al. (2017) who show that off-exchange trading fluctuates in response to varying levels of volatility and spreads.

$$Dark\ Trading_{i,t} = \alpha_0 + \beta_2 HHI_{it} + \beta_2 Fragmented_{it} + \beta_3 Frag * HHI_{it} + \delta_1 Controls_{it} + \varepsilon_{it} \quad (6)$$

$Fragmented_{it}$ is constructed the same as in the previous section, which is defined as 1 if the security belongs to the fragmented sample, and 0 other wise. The interaction, $Frag * HHI_{it}$, gives the difference-in-difference estimate for each variable. We include three dependent variables to identify the amount of dark trading taking place: dark volume, number of dark venue trades, and dark venue market share. Each measure is constructed from all off-exchange volume and trades reported under the 'D' exchange in the TAQ trades file. In Panel A of Table 5, for stocks that belong to the fragmented sample, we see a positive correlation with number of dark trades. Stocks that belong to the highly fragmented group execute 5,797 more trades at dark venues than the consolidated group. Although dark venue activity is increasing via trades, neither dark venue market share nor dark volume are significantly different between the two samples. Furthermore, there is a lack of significance for the other two variables of interest. Both HHI and $Frag * HHI_{it}$ report coefficients in both directions but with no significance.

Hypothesis 3 proposes that stocks which have more lit fragmentation (higher HHI), will produce lower levels of dark fragmentation. The findings in Table 5 weakly contradict this hypothesis, as the only coefficient providing evidence against our prediction is the significantly larger number of dark venue trades for the highly fragmented sample compared to the consolidated sample.

To further analyze the correlation between lit fragmentation and dark fragmentation, we make use of our proxy for fragmentation that measures the daily number of market venues a stock executes a trade on. Panel B of the Table 5 reports the cross-sectional coefficients for percentage changes in dark volume measures and a percentage change in the number of market venues a security executes a trade at. Our dependent variables include those used in equation 6 and our independent variable of interest is the percentage change in daily venues reported by Midas. In column 3 of Panel B, we can see that coefficient for daily venues changes is significant and implies that a percentage change increase in daily venues is correlated with an increase in dark venue market share of 3.70%.

The evidence provided in Panel B of Table 5, is consistent with our results in Panel A of Table 5 that lit fragmentation is positively associated with dark fragmentation and provides further evidence against Hypothesis 3 (negative relation). A possible explanation to these findings may be that as highly fragmented stocks add another venue of trading (change in fragmentation) dark market share increases. This may partially be explained not by an increase in dark volume but because of a decrease in the amount of lit volume. In other words, there appears to be less volume which executes (smaller average trade size) on lit exchanges mechanically increasing the dark market share as fragmentation increases. This is also consistent with Table 1 in the Appendix which shows that algorithmic trading increases with fragmentation, and more specifically, as the number of venues increases, odd-lots or orders with less than 100 shares is also increasing.

Furthermore, we also provide evidence regarding the impact that dark fragmentation has on market quality. In Panel C of Table 5, we record the coefficients for our panel regression where the dependent variables are market quality measures, and the independent variables are log of dark volume and dark venue market share. As we can see from the coefficients from both variables of interest there is a distinction between increasing dark volume and increasing dark venue market share. On one hand, as the amount of dark volume increases there is negative correlation with tightening spreads and increasing price efficiency measured by variance ratio. We also record a decrease in NBBO depth and an increase in volatility. On the other hand, we see the opposite occur with an increase in dark venue market share associated with an decrease in quoted spreads, increase in depth, decrease in volatility, and a reduction in price efficiency. These finding suggest that although dark volume is harmful for market quality, as the market share of dark venues grows (i.e. an increase in dark fragmentation) the drawbacks from an increase dark volume are overcome and there is an overall improvement in liquidity.[45]

DETERMINANTS OF LIT FRAGMENATION

Competition across markets for order flow produces differing market structures to offer participants a host of trading products or features such as different liquidity providing/taking fees or different market access. In this section, we look to determine what influences daily fragmentation for securities. For instance, Figure 1 highlights the variation that occurs in our sample of securities for stocks trading on one daily venue to trading on all thirteen venues. The findings in Figure 1 are contrary to O'Hara and Ye (2011) and Gresse (2017), in that, if increasing fragmentation provides increasing marginal benefits, then all securities should trade

[45] The results obtained using the full sample of stocks do not qualitatively differ from using only the treated sample of stocks.

on all available exchanges to maximize benefits. Seeing as there are observations across the spectrum of daily venues, this shows that securities will trade at less than all thirteen venues. However, the largest number of observations occurs at either twelve or thirteen daily venues.

Given that fragmentation relies upon markets providing participants a host of options to satisfy their needs, certain stock characteristics may influence the preferencing of orders to certain venues or to a certain number of venues. Therefore, in this section we look to formally test hypothesis 4 and assess what role trading costs (spreads), volatility, price, competition, and informational advantages play in causing traders to seek out other venues and causing fluctuations in the number of daily venues a stock trades on. To test hypothesis 4, we follow Chordia, Roll, and Subrahmanyam (2001) to determine if changes in trading costs, volatility, and price efficiency are correlated with changes in fragmentation measured by daily venue changes. We us the following panel regression model:

$$\%\Delta Daily\ Venues = \alpha_0 + \beta_1 \%\Delta Trading\ cost_{it} + \beta_2 \%\Delta Volatility\ _{it} + \beta_3 \%\Delta Price\ Efficency_{it} + \delta_1 Controls_{it} + \varepsilon_{it} \tag{7}$$

where our dependent variable is the percentage change in the number of trading venues recording an execution on day *t*, for stock *i*. Our variables of interest include percentage changes in quoted and effective spreads, volatility, price efficiency measures, and return. Our control variables remain constant with previous models, as well as a control for the fragmentation measured by HHI. Figure 2b demonstrates that HHI and Daily venues have a positive association and thus should be included to account for changes in daily venues. All regressions include day and stock fixed effects and standard errors clustered by stock.

In Column 1 of Table 6 we include all variables of interest from equation 8 and we see that by including all variables, changes in quoted spreads, return, and the current level of fragmentation measured by HHI significantly increase the number of daily venues a stock trades on. In other tabulated statistics regarding the variables of interest, the reported standard deviations in %Δreturn, %Δquoted spread, and HHI are 1.45, .41, and .076, respectively. Column 1 of Table 6 shows that a one standard deviation increase in quoted spreads and return is associated with an increase in daily venues of 29.37 and 40.06 basis points, respectively. Similarly, one standard deviation increases in HHI is associated with an increase in daily venues by 1.98% and suggests that stocks which are already highly fragmented are associated with a larger increase in number of daily venues and more likely to continue to fragment.[46] As we can see in table 6, the findings suggest that determinants to an increase in market fragmentation measured by daily venues include trading costs, return, and current level of fragmentation. All of which, are consistent with the prediction made in hypothesis detailing possible determinants of market fragmentation. These findings can be partially explained by the splitting of larger trades across markets to minimize costs (Chowdry and Nanda, 1991), and the influence that deviations from efficient prices have risk-averse intermediaries supplying liquidity to asynchronously arriving investors (Hendershott and Menkveld, 2014).

VENUE FEE STRUCTURES

Differing fee structures add to the complexity of market fragmentation by introducing another mechanism through which venues compete for order flow. In this section we look to shed light on the relation between venue fees and algorithmic trading activity. The main questions in this section are how does fee structure influence algorithmic trading activity in

[46] Appendix table A(2) reports the Tobit analysis and likelihood estimates regarding possible determinants of fragmentation and the number of daily venues.

highly fragmented markets and do varying levels of inverted trading have a larger impact on market quality for highly fragmented stocks versus stocks that are more consolidated. The two main current fee structures in U.S. markets are make-take and inverted fee models. Inverted models, in addition to make-take fees, change both investor preferences for making versus taking liquidity and routing preferences (Comerton-Forde et al., 2019; Battalio et al., 2016; Cox et al., 2017; Foucault et al., 2013; Angel et al., 2015; and Harris, 2013). The empirical findings suggest that a positive relation may exist between highly fragmented stocks and inverted trading. To test hypothesis 5, we propose the following difference-in-difference model:

$$Inverted\ Venues_{i,t} = \alpha_0 + \beta_1 HHI_{it} + \beta_2 Fragmented_{it} + \beta_3 Frag * HHI_{it} + \delta_1 Controls_{it} + \varepsilon_{it} \quad (8)$$

where our dependent variables are the amount of volume, number of trades, and market share of inverted venues for stock *i*, on day *t*. Using the SECs Midas data set, we are able to observe the amount of volume and trades at each of the four inverted venues.[47] $Fragmented_{it}$ equals 1 if a stock belongs to the fragmented sample and 0 otherwise. The main variable of interest is the difference-in-difference interaction between $Fragmented_{it}$ and HHI_{it}. In Panel A of Table 7, the coefficients for HHI and $Frag * HHI_{it}$ are all statistically and economically significant. An increase in HHI by one standard deviation is associated with a rise in inverted volume by 25.52 basis points, inverted venue trades by 67.44 trades, and inverted venue market share by 2.79%. The coefficients for $Frag * HHI_{it}$ further show that the most fragmented stocks included in the fragmented sample and measured by their HHI have a positive correlation with inverted volume, trades, and market share. This evidence supports not only hypothesis 5, which

[47] Of the 14 U.S. lit exchanges, there are four venues using an inverted fee schedule. These exchanges include the Bats-Y, Nasdaq Boston, Edge-A, and NYSE National.

states that there exists a positive relation between fragmentation and the amount of inverted trading activity, but also reflects results similar to Comerton-Forde et al. (2018) where larger stocks have more inverted trading.

The next question we look to answer is what impact an increase in inverted trading has on market quality for highly fragmented securities. In Panel B of Table 7, we follow our earlier models relating fragmentation to market liquidity, as well as the method of Gresse (2017), and regress inverted volume and market share on specified market liquidity variables to assess whether more inverted trading is correlated with better market liquidity. The coefficients for both variables of interest, log inverted volume and inverted market share, present conflicting evidence with our expectation that more inverted venue trading is correlated with better market quality. As the amount of inverted venue volume increase there is a positive correlation with smaller spreads. An increase in inverted venue volume is correlated with a $0.0191 decrease in quoted spreads and a $0.009 decrease in effective spreads. However, as inverted venue market share increases, we see a deterioration in spreads and an increase in price volatility. The findings in this section show that inverted volume is beneficial for market quality, but as the inverted venue market share grows the benefits provided by the additional inverted volume are essentially muted. In light of the findings of Lin et al. (2019) where venues differentiated by maker-taker fee structure leads to greater liquidity, our results in Panel B of Table 7 present conflicting evidence of improved market liquidity via transaction action costs and show an increase in price volatility as inverted volume and trading increase. We find this evidence to be inconsistent with hypothesis 6 that more inverted venue trading will improve market quality and conclude that there are trivial effects of increasing/decreasing the amount of inverted venue activity.

In our last section of analysis, we initially anticipate that in highly fragmented stocks, algorithmic activity will be large but concentrated on exchanges with the lowest fees. To further disseminate the relation between venue fee schedules and fragmentation, we next establish to what degree does this relation influence algorithmic trading activity. In hypothesis 7, we propose that highly fragmented stocks will largely concentrate on the maker-taker venues due the liquidity rebate received for acting in a market making capacity. To test this prediction, we propose the following OLS regression model:

$$Weller\ 2018\ AT\ proxy_{it} = \alpha_0 + \beta_1 Make - take\ Activity_{it} + \delta_1 Controls_{it} + \varepsilon_{it} \qquad (9)$$

Table 8 reports the results of the regression for the sample of stocks in our fragmented sample. The variables of interest are the log of maker-taker volume and maker-taker venue market share for stock *i*, on day *t*. In the U.S. market, 8 of the 16 lit exchanges employ a make-take fee structure. Using the SEC's Midas dataset, we record the cumulative volume and trades from all make-take fee venues. The coefficients are then recorded in Table 8, which we can see present conflicting results. Following the findings of Weller (2018), algorithmic trading activity has a positive correlation with odd lot-to-order and cancels-to-trades, but a negative association with trades-to-orders and average trade size. Economically, an increase in maker-taker market share is associated with an increase of 4.94 basis points and 7.78 basis points for odd lots-to-order and cancels-to-trades, respectively. An increase in maker-taker market share is correlated with a decrease in trades-to-orders and avg. trade size of 9.65 and 3.34 basis points. The coefficients collectively suggest that there exists a positive and significant correlation with algorithmic trading activity and the amount of make take activity measured by maker-taker market share for highly fragmented stocks.

Although the aforementioned effects of increasing maker-taker market share support our prediction that algorithmic trading activity is positively associated with maker-taker activity, the coefficients for maker-taker volume suggest otherwise. The signs for each coefficient are flipped and give the interpretation that as maker-taker volume rises we see a decrease in algorithmic trading activity. The dynamic association of make -take venue activity and algorithmic trading activity may imply that the ATs/HFTs are more active in make-take models where they are paid a rebate to provide liquidity and thus act in a market making capacity (Yao and Ye, 2014). On the other hand, the negative association between marker-taker volume and algorithmic trading activity provides evidence consistent with Comerton-Forde et al. (2017) who posit that high frequency traders benefit by sending orders to inverted venues and thus could explain the negative association for maker-taker volume in Table 8.

V. CONCLUSOION

In light of the expected proliferation of U.S. markets in the upcoming years, the initiative of this study is to analyze how differing degrees of market fragmentation influences participant trading behaviors, specifically the activity of algorithmic traders. As markets continue to fragment, do we see increases/decreases in algorithmic trading? Likewise, if we dissect further the competition among market venues to capture order flow, do certain venue structures attract more algorithmic trading activity than others? In this study, we use multiple methods to determine the level of fragmentation experienced by each stock included in the sample. We demonstrate using the number of daily venues recorded by the SEC's Midas dataset that there exists an inverted 'U' shape pattern between the number of daily venues and trading costs. As fragmentation increases from one trading venue to eight, both effective and quoted spreads widen. However, after the eighth venue, trading costs improve before leveling off. This dynamic shape to market quality and fragmentation supports evidence from both sides of the discussion regarding the effects that fragmentation has on market quality. On one hand, fragmentation does negatively impact market quality as the number of venues begins to increase from one which supports studies such as Arnold et al. (1999), Bennett and Wei (2006), and Baldauf and Mollner (2019). On the other hand, fragmentation past the eighth exchange produces better liquidity consistent with O'Hara and Ye (2011) and Gresse (2017).

Additionally, we show evidence consistent with our predictions that more fragmented stocks will have more algorithmic trading activity, and that this activity will be concentrated on make-take venues where ATs are paid a rebate to provide liquidity. We show mixed evidence regarding the correlation between algorithmic trading proxies provided by Weller (2018) and all fragmentation proxies used in this study. The strongest evidence we provide is the relation between the fragmented sample and algorithmic trading activity. Across all four AT proxies developed by Weller, there is a significant positive relation to the highly fragmented sample. Our findings regarding the positive relation between fragmentation and algorithmic trading are consistent with Baldauf and Mollner (2019,2020) where increasing fragmentation may produce additional arbitrage opportunities for algorithmic trading and thus the results would share a positive relation between lit fragmentation and algorithmic trading. Although we provide evidence that supports this conjecture, at higher levels of HHI, algorithmic trading activity begins to decrease. Unlike Baldauf and Mollner, securities in U.S. markets don't experience a deterioration in liquidity but just the opposite. We show that there are improved trading costs as algorithmic trading activity and fragmentation increase, lending evidence to the benefits of algorithmic trading by other studies (Menkveld, 2013; Brogaard et al., 2014b; Boehmer et al., 2018; Brogaard et al., 2018).

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APPENDIX

APPENDIX 1: SUMMARY STATISTICS

Table 1 Summary Statistics – Table reports summary statistics for the sample of data that includes only stocks that with stocks with a price $5.00 or more every day in the sample. We partition into quartiles based on the Hirschman Herfindahl Index in the month preceding the sample period. We draw two samples from Q4 (Highly Fragmented stocks) and Q1 (Highly Consolidated Stocks). We create a matched sample of stocks based on market capitalization, price, and trading volume.

Panel A: Highly Fragmented Stocks					
Variable	Mean	Median	Std Dev	Min	Max
HHI (1-HHI)	0.8211	0.8282	0.0375	0.3524	0.8971
Dark Market share	0.3070	0.2940	0.1075	0.0003	0.9579
Quoted Spread, $	0.0754	0.0275	0.1719	0.0005	3.4978
Effective Spread, $	0.0333	0.0144	0.0733	0.0007	1.4591
NBBO Depth, Shares	2,268.65	542.0311	5,586.56	171.55	217,142.20
NBBO Depth, Dollar	39,223.72	17,993.99	70,196.72	464.10	2,822,569
Realized Spread, $	0.0125	0.0040	0.0493	-0.1588	1.2000
Price Impact, $	0.0207	0.0097	0.0350	-0.3143	0.8245
Price Volatility, TAQ bps	0.1930	0.1146	0.2553	0.0026	8.0586
S-T Return Volatility, (LRS 94) bps	0.2380	0.1878	0.1786	0.0085	3.8500
Return, (Amihud et al., 94) bps	0.0004	0.0010	0.0249	-0.4910	0.3704
Variance Ratio	0.4892	0.5000	0.2702	.0000398	2.1360
Odd lot/Order (Midas)	0.2057	0.1934	0.1217	0.0058	1.00
Trade/Order (Midas)	0.0463	0.0420	0.0243	0.00	0.3443394
Cancel/Order (Midas)	15.65	14.17	8.0035	1.47	283.70
Avg. Trade size (Midas)	88.89	77.05	49.38	4.29	629.83
Daily Volume, $	51,153,219	10,639,133	118,310,120	13,765	5,349,996,342
Daily Volume, shares	1,640,142	366,548.5	3,173,704	1,326	80,149,581
Market Capitalization, $	10,647,771,160	51,153,219	26,395,663,993	11,256,016	168,860,401,764
Price, $	42.11	23.85	48.58	0.34	317.42
# of firms	98				

Panel B: Highly Consolidated Stocks					
Variable	Mean	Median	Std Dev	Min	Max
HHI (1- HHI)	0.7323	0.7435	0.0784	0.00	0.8988
Dark Market share	0.3241	0.3103	0.1207	0.0025	1.00
Quoted Spread, $	0.0994	0.0531	0.1754	0.0082	3.1806
Effective Spread, $	0.0457	0.0264	0.0761	0.0045	1.6388
NBBO Depth, Shares	1,578.192	424.7965	7,282.651	208.8803	229,019.3
NBBO Depth, Dollar	32,343.1	12,661.56	144,353.2	723.2275	10,947,635
Realized Spread, $	0.0160	0.0070	0.1470	-14.9866	4.1002
Price Impact, $	0.0297	0.0182	0.1415	-4.0814	15.0166
Price Volatility, TAQ bps	0.2585	0.1466	0.3662	0.00	8.2112
S-T Return Volatility, (LRS 94) bps	0.3038	0.2238	0.2794	0.00	3.8342
Return, (Amihud et al., 94) bps	0.0003	0.0006	0.0360	-1.9870	0.9952
Variance Ratio	0.4953	0.5107	0.2693	0.00000443	2.3297
Odd lot/Order (Midas)	0.2455	0.2374	0.1090	0.00	1.00
Trade/Order (Midas)	0.0562	0.0498	0.0419	0.00	1.5613
Cancel/Order (Midas)	14.13	11.92	14.04	0.47	600.79
Avg. Trade size (Midas)	83.75	69.35	192.38	7.47	16,884.67
Daily Volume, $	42,996,973	8,573,722	105,548,542	20	2,757,767,612
Daily Volume, shares	847,193.7	342,933	2,493,990	2.00	180,639,993
Market Capitalization, $	5,004,296,995	1,202,255,400	13,615,743,119	10,322,819	123,153,590,820
Price, $	43.12	25.00	50.90	0.77	366.39
# of firms	83				

APPENDIX 2: UNIVARTIATE

Table 2: Univariate statistics –. Both t-stats and p-values are reported in the column 4 and 5 of each panel. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Matching variables, Sample period (2019)

Variables	(1) Fragmented	(2) Consolidated	(3) Diff. (Frag. – Cons.)	(4) t-stat	(5) p-value
Market Capitalization	10,266,584,755	4,857,719,060	5,408,865,695	1.8311*	0.0686
Price	41.1884	42.4040	-1.2156	-0.1740	0.8620
Volume, shares	1,631,558	825,651.5	805,907	3.0483***	0.0027

Panel B:

Variables	(1) Fragmented	(2) Consolidated	(3) Diff. (Frag. – Cons.)	(4) t-stat	(5) p-value
HHI (1-HHI)	.8213438	.7269147	.0944291	12.7469***	0.0000
Trade/Order (Midas)	.047271	.0564758	-.0092048	-3.2753***	0.0013
Odd lot/Volume (Midas)	.2003239	.2384241	-.0381002	-2.8221***	0.0053
Avg. Trade size (Midas)	90.21703	93.83577	-3.618748	-0.2237	0.8233
Cancel/Order (Midas)	16.0572	14.8649	1.192299	1.2256	0.2218
Quoted Spread, $	.0733238	.1017203	-.0283965	-1.3242	0.1870
Effective Spread, $	.0324706	.0483339	-.0158633	-1.5693	0.1182
NBBO Depth, Shares	2,250.295	1,537.546	712.7497	1.3848	0.1678
Daily Price Volatility	.1910331	.2553561	-.0643231	-2.1506**	0.0328
S-T return Volatility, LRS 94 bps	.2382738	.304244	-.0659701	-3.1147***	0.0021
Variance Ratio	.4899404	.4946491	-.0047086	-0.8767	0.3818
Dark Market share	.3070666	.3224713	-.0154047	-2.0254**	0.0443
Dark/Lit volume	.4967294	.5995484	-.102819	-2.1503**	0.0329

APPENDIX 3: MARKET FRAGMENTATION & ALGORITHMIC TRADING

Table 3 – Regression analysis – Market Fragmentation and Algorithmic Trading Activity: Reports the regression analysis investigating the four algorithmic trading (AT) proxies of Weller (2018) defined in section 3 and market fragmentation. The Weller measures of AT activity include *odd lot-to-volume*, *trade-to-order volume*, *cancel-to-trade ratio*, and *average trade size*. For each AT measure we use the log as the dependent variable. Table A reports the coefficients for equation 5 for all securities in the sample. HHI_{it} is Herfindahl-Hirschman Index, calculated as 1-HHI_{it}, and can be interpreted to be that larger values equate to more fragmentation. The Control variables include short-term return volatility, quoted spread, log volume, log market cap, and log price. Panel B the dependent variables measure market quality, volatility, and price efficiency. These include quoted spread, effective spread, quoted depth, daily volume, short-term return volatility, price volatility, and variance ratio. $Fragmented_{it}$ is defined as equal to 1 if the stock belongs to the fragmented sample and 0 otherwise. The interaction, $Frag * HHI_{it}$, gives the difference-in-difference estimate for each variable. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Algorithmic Trading Activity and HHI

VARIABLES	(1) Trades to Orders	(2) Odd lot to Volume	(3) Cancels to Trades	(4) Avg. Trade Size
$Fragmented_{it}$	-1.648*	1.225*	1.372**	-1.095***
	(-1.969)	(1.678)	(2.495)	(-2.706)
HHI_{it}	0.0714	-0.0928	-0.266	-0.150
	(0.281)	(-0.436)	(-1.474)	(-0.843)
$Frag * HHI_{it}$	1.968***	-0.679**	-1.400***	0.809***
	(5.077)	(-2.293)	(-4.815)	(3.589)
Log Market Capitalization	0.182	-0.437**	-0.143	0.314***
	(0.699)	(-2.015)	(-0.847)	(2.675)
Log price	-0.278	0.838***	0.248	-0.598***
	(-1.011)	(3.558)	(1.411)	(-4.617)
Log Volume, shares	-0.0124	0.00331	0.00365	-0.00507
	(-1.644)	(0.436)	(0.571)	(-1.172)
Quoted Spread, $	-0.00619	-0.0367	-0.0269	0.00357
	(-0.141)	(-0.737)	(-0.836)	(0.121)
S-T Return Volatility (LRS, 94)	0.0229	-0.0190	0.0246	0.0204
	(0.937)	(-0.829)	(1.008)	(1.287)
Constant	-6.488	4.864	5.484*	-0.247
	(-1.398)	(1.273)	(1.821)	(-0.120)
Observations	39,007	38,985	39,007	39,007
R-squared	0.434	0.805	0.437	0.752
Day Fixed Effects	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes

Panel B: Market Fragmentation and Market Quality

VARIABLES	(1) Quoted Spread - $	(2) Effective Spread - $	(3) NNBO Depth - Shares	(4) Price Volatility – bps	(5) S-T Return Volatility (LRS, 94)- bps	(6) Variance Ratio
$Fragmented_{it}$	-0.0989	-0.0139	-14,948	-0.148	-1.031**	0.127
	(-1.193)	(-0.321)	(-0.253)	(-0.235)	(-1.998)	(0.667)
HHI_{it}	-0.0247*	-0.0278**	-566.6	0.00118	0.00286	0.0398
	(-1.813)	(-2.508)	(-0.367)	(0.0179)	(0.0651)	(1.403)
$Frag * HHI_{it}$	0.0864*	0.0376**	2,155	0.0373	0.0506	0.0511
	(1.817)	(2.092)	(0.798)	(0.293)	(0.683)	(0.799)
Log Volume, shares	0.000944	0.000637	1,353***	0.0252**	0.00644	-0.0151***
	(0.917)	(1.310)	(4.365)	(2.294)	(0.987)	(-7.099)
Log Market Capitalization	-0.0275	-0.0156	2,900	0.0455	0.250	-0.0160
	(-1.282)	(-1.327)	(0.166)	(0.258)	(1.635)	(-0.306)
Log price	0.0647***	0.0306**	-3,123	-0.0625	-0.236	0.0212
	(2.918)	(2.577)	(-0.176)	(-0.340)	(-1.481)	(0.395)
Quoted Spread, $			-259.2			
			(-0.604)			
Constant	0.497	0.288	-67,009	-0.875	-4.305	0.853
	(1.269)	(1.349)	(-0.216)	(-0.279)	(-1.606)	(0.928)
Observations	39,302	39,302	39,025	39,301	39,285	39,272
R-squared	0.794	0.846	0.339	0.441	0.405	0.026
Day Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes	Yes	Yes

APPENDIX 4: OFF-EXCHANGE TRADING & ALGORITHMIC TRADING ACTIVITY

Table 4 – Off-Exchange Trading and Algorithmic Trading Activity: Reports the regression analysis investigating the four algorithmic trading (AT) proxies of Weller (2018) defined in section 3 and the Off-exchange trading. The Weller measures of AT activity include *odd lot-to-volume*, *trade-to-order volume*, *cancel-to-trade ratio*, and *average trade size*. For each AT measure we use the log as the dependent variable.

$$Weller\ (2018)\ AT\ proxies = \alpha_0 + \beta_1 \frac{Dark}{Lit} volume_{it} + \delta_1 Controls_{it} + \varepsilon_{it}$$

$\frac{Dark}{Lit} volume$ is all off-exchange volume reported by D'TAQ for stock i, on day t, divided by remaining on-exchange volume. The Control variables include short-term return volatility, quoted spread, log volume, log market cap, and log price. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

VARIABLES	(1) Trades to Orders	(2) Odd Lot to Volume	(3) Cancels to Trades	(4) Avg. Trade Size
$\frac{Dark}{Lit} Volume_{it}$	0.00365	0.00544	0.00348	0.00170
	(0.215)	(0.379)	(0.236)	(0.195)
Log Market Capitalization	0.166	-0.428**	-0.131	0.308***
	(0.674)	(-2.007)	(-0.827)	(2.701)
Log price	-0.247	0.821***	0.225	-0.589***
	(-0.945)	(3.542)	(1.351)	(-4.658)
Log Volume, shares	-0.0119	0.00431	0.00431	-0.00512
	(-1.486)	(0.560)	(0.643)	(-1.158)
Quoted Spread, $	-0.383***	0.202***	0.269***	-0.128***
	(-6.701)	(5.307)	(4.408)	(-4.022)
S-T Return Volatility (LRS, 94)	0.0205	-0.0253	0.0219	0.0211
	(0.858)	(-1.110)	(0.944)	(1.374)
Constant	-5.878	4.507	4.853*	-0.209
	(-1.317)	(1.180)	(1.683)	(-0.102)
Observations	39,275	39,253	39,275	39,275
R-squared	0.075	0.183	0.101	0.150
Day Fixed Effects	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes

APPENDIX 5: OFF-EXCHANGE TRADING AND LIT FRAGMENATION

Table 5 – Off-Exchange Trading and Lit Fragmentation: Reports the regression analysis investigating the relation between off-exchange trading (dark venue trading) and both the amount of lit fragmentation and changes in lit fragmentation. In Panel A, Dark Trading is composed of three dependent variables to identify the amount of dark trading taking place: dark volume, number of dark venue treads, and dark venue market share. $Fragmented_{it}$ is constructed the same as in the previous section, which is defined as 1 if the security belongs to the fragmented sample, and 0 other wise. HHI_{it} is Herfindahl-Hirschman Index, calculated as 1-HHI_{it}. The interaction, $Frag * HHI_{it}$, gives the difference-in-difference estimate for each variable. The Control variables include short-term return volatility (constructed from Lee et al. (1994) absolute returns), quoted spread, log volume, log market cap, and log price. In Panel B, our dependent variables are the three measures of dark trading (dark volume, number of dark venue treads, and dark venue market share) but now we compute the percentage change in each for stock *i*, on day *t*, and is denoted by %Δ. The explanatory variables include $Daily\ Venues_{it}$, calculated as the number of daily venues executing a trade for stock *i*, on day *t*. Panel C, reports the coefficients for our panel regression where the dependent variables are market quality measures, and the independent variables are log of dark volume and dark venue market share. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Dark Venue Trading and Lit Fragmentation

VARIABLES	(1) Dark Volume	(2) Dark Trades	(3) Dark Market Share
$Fragmented_{it}$	-690,933	5,797**	23.09
	(-0.294)	(2.584)	(1.133)
HHI_{it}	-67,451	-226.3	-0.208
	(-0.566)	(-0.767)	(-0.128)
$Frag * HHI_{it}$	35,520	293.7	-4.383
	(0.104)	(0.367)	(-0.993)
Log Market Capitalization	534,266	760.4	-5.210
	(0.801)	(1.252)	(-0.883)
Log price	-683,971	-1,034	7.204
	(-1.025)	(-1.599)	(1.167)
Log Volume, shares	482,577***	1,403***	2.294***
	(7.938)	(15.18)	(8.721)
Quoted Spread, $	460,027***	1,198***	0.445
	(3.126)	(4.225)	(0.676)
S-T Return Volatility (LRS, 94)	519,942***	1,304***	7.005***
	(4.963)	(5.327)	(6.090)
Constant	-1.502e+07	-29,651***	83.29
	(-1.276)	(-2.787)	(0.809)
Observations	39,001	39,001	39,276
R-squared	0.432	0.671	0.248
Day Fixed Effects	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes

Panel B: Change in Dark Venue Trading and Change in Lit Fragmentation			
	(1)	(2)	(3)
VARIABLES	%Δ Dark Volume	%Δ Dark Trades	%Δ Dark Market Share
$\%\Delta Daily\ Venues_{it}$	0.0463	0.0160	0.0370*
	(1.021)	(0.415)	(1.708)
Log Market Capitalization	-0.317	-0.224**	-0.0351
	(-1.586)	(-2.303)	(-0.929)
Log price	0.361*	0.249**	0.0465
	(1.798)	(2.536)	(1.210)
Log Volume, shares	0.190***	0.103***	0.0307***
	(10.76)	(11.74)	(7.743)
Quoted spread, $	-4.926***	-8.493***	-2.262***
	(-3.040)	(-6.435)	(-2.664)
S-T Return Volatility (LRS, 94)	0.468***	0.405***	0.0750***
	(10.98)	(13.53)	(4.628)
Constant	3.045	2.597	0.170
	(0.841)	(1.474)	(0.248)
Observations	38,186	38,186	38,186
R-squared	0.077	0.070	0.014
Day Fixed Effects	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes

Panel C: Market Quality and Dark Venue Trading						
VARIABLES	(1) Quoted Spread - $	(2) Effective Spread - $	(3) NNBO Depth - Shares	(4) Price Volatility	(5) S-T Return Volatility (LRS, 94)- bps	(6) Variance Ratio
Log Dark Volume	0.00926**	0.00254	-2,911*	0.0902***	-0.0275	-0.0263**
	(2.540)	(1.185)	(-1.931)	(5.224)	(-0.629)	(-2.467)
Dark Market share	-0.0269**	-0.00933	9,752*	-0.223***	0.311*	0.107***
	(-2.174)	(-1.252)	(1.781)	(-3.663)	(1.870)	(2.995)
Log Volume, shares	-0.00852**	-0.00192	4,307***	-0.0672***	0.0287	0.0110
	(-2.209)	(-0.870)	(2.720)	(-3.763)	(0.660)	(1.000)
Log Market Capitalization	-0.0294	-0.0164	3,781	0.0374	0.255*	-0.00983
	(-1.377)	(-1.395)	(0.224)	(0.214)	(1.656)	(-0.208)
Log price	0.0667***	0.0313***	-4,135	-0.0537	-0.246	0.0138
	(3.017)	(2.642)	(-0.242)	(-0.295)	(-1.537)	(0.285)
Quoted Spread, $			-165.3			
			(-0.413)			
Constant	0.537	0.301	-89,953	-0.536	-4.519*	0.765
	(1.359)	(1.387)	(-0.296)	(-0.169)	(-1.661)	(0.902)
Observations	39,296	39,291	39,021	39,301	39,294	39,282
R-squared	0.213	0.032	0.070	0.018	0.027	0.009
Day Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes	Yes	Yes

APPENDIX 6: DETERMINANTS OF MARKET FRAGMENTATION

Table 6 – Determinants of Market Fragmentation: Reports the regression analysis to determine if changes in trading costs, volatility, and price efficiency are correlated with changes in fragmentation measured by daily venue changes. We us the following panel regression model:

$$\%\Delta Daily\ Venues = \alpha_0 + \beta_1 \%\Delta Trading\ cost_{it} + \beta_2 \%\Delta Volatility_{\ it} + \beta_3 \%\Delta Price\ Efficency_{it} + \delta_1 Controls_{it} + \varepsilon_{it}$$

where our dependent variable is the percentage change in t$Daily\ Venues_{it}$, calculated as the number of daily venues executing a trade for stock i, on day t, denoted by %Δ. Our variables of interest include percentage changes in quoted and effective spreads, volatility, price efficiency measures, and return. The Control variables include the Herfindahl-Hirschman Index (1-HHI_{it}), log volume, log market cap, and log price. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

VARIABLES	(1) $\%\Delta Daily\ Venues$	(2) $\%\Delta Daily\ Venues$	(3) $\%\Delta Daily\ Venues$
%Δ Effective Spread, $	-0.00456	-0.0126	
	(-0.506)	(-1.560)	
%Δ Quoted Spread, $	0.00721***	0.00793***	
	(2.685)	(2.647)	
%Δ S-T Volatility (LRS, 94)	-0.000300		-0.000321
	(-0.253)		(-0.270)
%Δ Return (Amihud et al., 94), bps	0.00277***		0.00277***
	(5.059)		(5.183)
%Δ Variance Ratio	0.000334		0.000310
	(0.791)		(0.725)
%Δ Price Volatility, bps	-6.34e-05		-3.99e-05
	(-0.0664)		(-0.0411)
HHI_{it}	0.261***	0.301***	0.262***
	(3.932)	(3.624)	(3.937)
Log Market Capitalization	-0.00254	-0.00224	-0.00252
	(-0.518)	(-0.388)	(-0.515)
Log price	0.00152	0.000732	0.00152
	(0.299)	(0.119)	(0.299)
Log Volume, shares	0.000114	-0.000192	0.000110
	(0.386)	(-0.548)	(0.372)
Constant	0.141	-0.0585	0.140
	(1.272)	(-0.361)	(1.264)
Observations	37,120	39,103	37,120
R-squared	0.030	0.032	0.029
Day Fixed Effects	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes

APPENDIX 7: VENUE FEE STURUCTURES – INVERTED VENUES

Table 7 – Venue Fee Schedule – Inverted Fees: Reports the regression analysis to what relation exists between Lit fragmentation and the amount of trading on venues that have an inverted fee schedule. We us the following panel regression model:

$$Inverted\ Venues_{i,t} = \alpha_0 + \beta_1 HHI_{it} + \beta_2 Fragmented_{it} + \beta_3 Frag * HHI_{it} + \delta_1 Controls_{it} + \varepsilon_{it}$$

where the dependent variables are the amount of volume, number of trades, and market share of inverted venues for stock *i*, on day *t*. Using the SECs Midas data set, we're able to observe the amount of volume and trades at each of the four inverted venues (These exchanges include the Bats-Y, Nasdaq Boston, Edge-A, and NYSE National). $Fragmented_{it}$ is constructed the same as in the previous section, which is defined as 1 if the security belongs to the fragmented sample, and 0 other wise. HHI_{it} is Herfindahl-Hirschman Index, calculated as 1-HHI_{it}. The interaction, $Frag * HHI_{it}$, gives the difference-in-difference estimate for each variable. The Control variables include, quoted spread, log volume, log market cap, and log price. In Panel B, the dependent variables are the various market quality metrics, and the independent variables include the log of inverted volume and market share of inverted venues. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Inverted Trading and Market Fragmentation

VARIABLES	(1) Log Inverted Volume	(2) Inverted Trades	(3) Inverted Market Share
$Fragmented_{it}$	-1.652	572.6	-29.18***
	(-1.456)	(0.507)	(-3.970)
HHI_{it}	3.369***	890.5***	36.83***
	(9.600)	(4.039)	(9.274)
$Frag * HHI_{it}$	0.908**	3,876***	30.62***
	(2.108)	(3.946)	(5.657)
Log Market Capitalization	1.013***	477.4*	-1.350
	(3.124)	(1.678)	(-0.862)
Log price	-1.195***	-995.3***	0.998
	(-3.300)	(-3.086)	(0.617)
Log Volume, shares	-0.00836	-17.53	-0.0110
	(-0.932)	(-1.048)	(-0.158)
Quoted Spread, $	-0.659***	-78.92	1.674*
	(-4.772)	(-0.878)	(1.754)
Constant	-9.398*	-6,322	14.97
	(-1.665)	(-1.214)	(0.537)
Observations	39,217	39,301	39,301
R-squared	0.910	0.802	0.554
Day Fixed Effects	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes

Panel B: Inverted Trading and Market Quality						
VARIABLES	(1) Quoted Spread - $	(2) Effective Spread - $	(3) NNBO Depth - Shares	(4) Price Volatility	(5) S-T Return Volatility (LRS, 94)- bps	(6) Variance Ratio
Log Inverted volume	-0.0191***	-0.00893***	241.7	0.00367	-3.40e-05	0.00373
	(-4.869)	(-4.468)	(1.420)	(0.586)	(-0.00587)	(1.453)
Inverted Market share	0.00100**	0.000417**	-10.41	0.000869*	2.92e-05	0.000195
	(2.452)	(2.516)	(-0.796)	(1.805)	(0.0643)	(0.706)
Log Volume, shares	0.000764	0.000578	1,355***	0.0253**	0.00638	-0.0151***
	(0.779)	(1.259)	(4.365)	(2.297)	(0.977)	(-7.067)
Log Market Capitalization	-0.00641	-0.00619	2,634	0.0424	0.250	-0.0193
	(-0.329)	(-0.557)	(0.151)	(0.246)	(1.633)	(-0.374)
Log price	0.0398**	0.0194*	-2,824	-0.0589	-0.237	0.0253
	(2.135)	(1.830)	(-0.159)	(-0.328)	(-1.487)	(0.474)
Quoted Spread, $			-70.03			
			(-0.183)			
Constant	0.261	0.202	-78,223	-0.988	-5.284*	1.077
	(0.628)	(0.854)	(-0.212)	(-0.266)	(-1.653)	(0.995)
Observations	39,217	39,217	38,940	39,216	39,200	39,187
R-squared	0.797	0.853	0.339	0.441	0.405	0.026
Day Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Stock Fixed Effects	No	No	No	No	No	No
Stock Clustered Errors	Yes	Yes	Yes	Yes	Yes	Yes

APPENDIX 8: VENUE FEE STURUCTURE & ALGORITHMIC TRADING

Table 8 – Venue Fee Schedule and Algorithmic Trading: Reports the regression analysis investigating the four algorithmic trading (AT) proxies of Weller (2018) defined in section 3 and venues that have a make-take fee schedule. The Weller measures of AT activity include *odd lot-to-volume*, *trade-to-order volume*, *cancel-to-trade ratio*, and *average trade size*. For each measure we use the log as the dependent variable.

$$Weller\ 2018\ AT\ proxy_{it} = \alpha_0 + \beta_1 Log\ Make - Take\ Volume_{it} + \beta_1 Log\ Make - Take\ Mrk.Share_{it} + \delta_1 Controls_{it} + \varepsilon_{it}$$

Panel A reports the results of the regression for the sample of stocks in our fragmented sample. The variable of interest is the log of make-take volume for stock *i*, on day *t*. In the U.S. market, 7 of the 14 lit exchanges employ a make-take fee structure. Using the SEC's Midas dataset, we record the cumulative volume and trades from all make-take fee venues. Panel B reports the same regression but for the full sample of securities in the study. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Algorithmic Trading and Make-Take Trading (Fragmented sample)

VARIABLES	(1) Trades to Orders	(2) Odd Lot to Volume	(3) Cancels to Trades	(4) Avg. Trade Size - Midas
Log Make-Take volume	0.501***	-0.355***	-0.328***	0.259***
	(29.66)	(-15.29)	(-19.02)	(18.61)
Marker-Taker Market Share	-0.0121***	0.00692***	0.00976***	-0.00419***
	(-13.49)	(6.564)	(13.45)	(-5.357)
Log Market Capitalization	-0.421**	-0.0144	0.252*	0.00670
	(-2.469)	(-0.136)	(1.907)	(0.146)
Log price	0.425**	0.347***	-0.213	-0.242***
	(2.082)	(2.918)	(-1.473)	(-4.491)
Log Volume	-0.00763	0.00170	0.00167	-0.00303
	(-0.994)	(0.251)	(0.272)	(-0.798)
Quoted Spread, $	-0.0206	-0.0568*	0.0347	0.0616*
	(-0.374)	(-1.793)	(0.662)	(1.861)
S-T Return Volatility (LRS, 94)	0.0206	-0.0245	0.0229	0.0213*
	(0.837)	(-1.335)	(0.905)	(1.730)
Constant	-0.859	1.121	1.442	2.199***
	(-0.279)	(0.604)	(0.608)	(2.778)
Observations	21,509	21,509	21,509	21,509
Number of Tickers	0.412	0.479	0.360	0.440
Day Fixed Effects	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes

APPENDIX 9: MARKET FRAGMENTATION (DAILY VENUES) & ALGORITHMIC TRADING ACTIVITY

Table A (1) – Regression analysis – Market Fragmentation (Daily Midas Venues) and Algorithmic Trading Activity: Reports the regression analysis investigating the four algorithmic trading (AT) proxies of Weller (2018) and market fragmentation. The Weller measures of AT activity include *odd lot-to-volume*, *trade-to-order volume*, *cancel-to-trade ratio*, and *average trade size*. For each AT measure we use the log as the dependent variable. The variables of interest include $Daily\ Venues_{it}$ and $Daily\ Venues_{it}{}^2$. The Control variables include short-term return volatility, quoted spread, log volume, log market cap, and log price. Panel B the dependent variables measure market quality, volatility, and price efficiency. These include quoted spread, effective spread, quoted depth, daily volume, short-term return volatility, price volatility, and variance ratio. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

VARIABLES	(1) Trades to Orders	(2) Odd Lot to Volume	(3) Cancels to Trades	(4) Avg. Trade Size
$Daily\ Venues_{it}$	0.337***	-0.266***	-0.218***	0.141***
	(6.009)	(-6.157)	(-5.181)	(5.006)
$Daily\ Venues_{it}{}^2$	-0.0120***	0.00932***	0.00781***	-0.00467***
	(-5.114)	(5.200)	(4.375)	(-3.784)
Log Market Capitalization	0.147	-0.408**	-0.121	0.294***
	(0.604)	(-1.995)	(-0.770)	(2.696)
Log price	-0.265	0.827***	0.239	-0.590***
	(-1.033)	(3.752)	(1.472)	(-4.944)
Log Volume, shares	-0.0120	0.00451	0.00505	-0.00510
	(-1.553)	(0.599)	(0.789)	(-1.184)
Quoted Spread, $	0.0735	-0.201	0.253	0.0250
	(0.124)	(-0.388)	(0.385)	(0.0694)
S-T Return Volatility (LRS, 94)	0.0190	-0.0189	0.0198	0.0191
	(0.819)	(-0.914)	(0.883)	(1.335)
Constant	-7.754*	5.934	6.102**	-0.939
	(-1.755)	(1.609)	(2.131)	(-0.476)
Observations	39,007	38,985	39,007	39,007
R-squared	0.098	0.208	0.114	0.168
Day Fixed Effects	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes

APPENDIX 10: OFF-EXCHANGE TRADING & ALGORITHMIC TRADING ACTIVITY

Table A (2) – Off-Exchange Trading and Algorithmic Trading Activity: Reports the regression analysis investigating the four algorithmic trading (AT) proxies of Weller (2018) defined in section 3 and the Off-exchange trading. The Weller measures of AT activity include *odd lot-to-volume*, *trade-to-order volume*, *cancel-to-trade ratio,* and *average trade size*. For each AT measure we use the log as the dependent variable.

$$Weller\ (2018)\ AT\ proxies = \alpha_0 + \beta_1 Dark\ Market\ Share_{it} + \delta_1 Controls_{it} + \varepsilon_{it}$$

Dark Market Share is computed from off-exchange volume to total trade volume from D'TAQ for stock *i*, on day *t*. The Control variables include short-term return volatility, quoted spread, log volume, log market cap, and log price. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

VARIABLES	(1) Trades to Orders	(2) Odd Lot to Volume	(3) Cancels to Trades	(4) Avg. Trade Size
Dark Market Share	0.000220	0.0266	-0.00114	-0.00945
	(0.00311)	(0.455)	(-0.0183)	(-0.257)
Log Market Capitalization	0.614***	-0.802***	-0.385***	0.419***
	(4.899)	(-7.956)	(-3.879)	(5.292)
Log price	-0.841***	1.218***	0.581***	-0.735***
	(-4.556)	(9.440)	(4.992)	(-8.611)
Log Volume, shares	-0.0165	0.00191	0.0106	-0.00446
	(-1.322)	(0.156)	(1.036)	(-0.640)
Quoted Spread, $	-0.259***	0.123***	0.243***	0.00572
	(-3.315)	(2.785)	(2.810)	(0.209)
S-T Return Volatility (LRS, 94)	-0.00131	0.0306	0.0263	-0.00369
	(-0.0375)	(0.972)	(0.787)	(-0.188)
Constant	-13.79***	11.34***	9.319***	-2.185
	(-6.228)	(6.274)	(5.272)	(-1.499)
Observations	21,509	21,509	21,509	21,509
R-squared	0.115	0.270	0.149	0.223
Day Fixed Effects	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes

APPENDIX 11: DETERMINANTS OF DAILY MIDAS VENUES (FRAGMENATION)

Table A (3) – Determinants of Market Fragmentation (Tobit Model – Censoring): Reports the Tobit regression accounting for censored number of trading venues [1,13] to determine if changes in trading costs, volatility, and price efficiency are correlated with changes in fragmentation measured by daily venue changes. Control Variables include the Herfindahl-Hirschman Index (1-HHI), Log Market Capitalization, Log Price, and Log Daily Volume. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

VARIABLES	(1) $Daily\ Venues$	(2) $Daily\ Venues$	(3) $Daily\ Venues$
Effective Spread, $	-1.638***	-1.619***	
	(-4.023)	(-4.007)	
Quoted Spread, $	0.307**	0.304**	
	(2.058)	(2.027)	
S-T Volatility (LRS, 94)	0.118		0.111
	(1.461)		(1.366)
Return, bps	-0.0245		-0.0258
	(-1.309)		(-1.372)
Variance Ratio	-0.175		-0.140
	(-1.377)		(-1.098)
Price Volatility, bps	0.0267		0.0164
	(0.523)		(0.319)
HHI_{it}	2.785***	2.764***	2.760***
	(3.095)	(3.059)	(3.039)
Log Market Capitalization	0.160***	0.159***	0.185***
	(5.426)	(5.368)	(6.110)
Log price	0.176***	0.174***	0.126***
	(3.866)	(3.826)	(2.801)
Log Volume, shares	0.00745	0.00433	0.00920
	(0.599)	(0.339)	(0.725)
Observations	39,271	39,302	39,271
Day Fixed Effects	Yes	Yes	Yes
Stock Fixed Effects	No	No	No
Stock Clustered Errors	Yes	Yes	Yes

APPENDIX 12: DAILY VENUES (FRAGMENTATION) & MARKET QUALITY

Table A(4) – Regressions for Figure 2(a) – Daily venues and Market Quality: This table reports the regression analysis complimentary to Figure 2(a), where the various market quality metrics are treated as the dependent variables and the variables of interest include $Daily\ Venues_{it}$ and $Daily\ Venues_{it}^{2}$. Control Variables include the Herfindahl-Hirschman Index (1-HHI), Log Market Capitalization, Log Price, and Log Daily Volume. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

VARIABLES	(1) Quoted Spread - $	(2) Effective Spread - $	(3) NBBO Depth - Shares	(4) Price Volatility	(5) S-T Return Volatility Lee 94	(6) Variance Ratio
$Daily\ Venues_{it}$	-0.0204**	-0.0174***	242.8	0.00186	0.00324	-0.00206
	(-2.183)	(-3.499)	(0.820)	(0.172)	(0.164)	(-0.274)
$Daily\ Venues_{it}^{2}$	0.000826**	0.000708***	-10.30	-0.000123	-2.68e-05	9.80e-05
	(2.040)	(3.379)	(-0.811)	(-0.259)	(-0.0328)	(0.259)
HHI_{it}	0.0174	0.000886	-402.9	0.00877	0.00390	0.0519*
	(1.275)	(0.158)	(-0.253)	(0.147)	(0.100)	(1.788)
Log Volume	0.000976	0.000658	1,373***	0.0252**	0.00645	-0.0151***
	(0.958)	(1.378)	(4.305)	(2.298)	(0.989)	(-7.106)
Log Market Capitalization	-0.0270	-0.0152	2,827	0.0459	0.248	-0.0162
	(-1.286)	(-1.320)	(0.163)	(0.260)	(1.624)	(-0.311)
Log price	0.0648***	0.0306***	-3,074	-0.0628	-0.235	0.0213
	(2.978)	(2.643)	(-0.174)	(-0.342)	(-1.476)	(0.398)
Quoted Spread, $			16,562			
			(0.623)			
Constant	0.581	0.375*	-67,260	-0.908	-4.386	0.909
	(1.507)	(1.781)	(-0.214)	(-0.283)	(-1.602)	(0.967)
Observations	39,302	39,302	39,025	39,301	39,285	39,272
R-squared	0.214	0.041	0.064	0.015	0.012	0.009
Number of Stocks	181	181	181	181	181	181
Day Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes	Yes	Yes

APPENDIX 13: ALGORITHMIC TRADING ACTIVITY BY VENUE FEE STRUCTURE

Table A (5) – Algorithmic Trading Proxies by Venue Fee Type: Reports the regression analysis to analyze algorithmic trading proxies and venue fee structures in relation the level of fragmentation measured by the Herfindahl-Hirschman Index (1-HHI). The dependent variables are computed by venue type structure, Maker-Taker and Inverted (Taker-Maker) and then the logs of each measure are used. These variables include the Weller (2018) AT proxies: Odd lot volume-to-Trade, Trade-to-Order, Cancels-to-Trades, and Avg. Trade Size. Panel A reports the daily AT measures computed from Maker-Taker venues. Panel B reports the daily AT measures computed by Taker-Maker Venues. Control Variables include Log Market Capitalization, Log Price, and Log Volume. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Maker-Taker Venues				
VARIABLES	(1) Trades to Orders	(2) Odd lots to Volume	(3) Cancels to Trades	(4) Avg. Trade Size
HHI_{it}	-0.716***	0.114	0.380**	-0.328***
	(-4.242)	(0.749)	(2.305)	(-2.772)
Log Market Capitalization	-0.0238	-0.446**	0.0568	0.282**
	(-0.104)	(-2.087)	(0.342)	(2.524)
Log price	-0.0706	0.851***	0.0728	-0.576***
	(-0.288)	(3.596)	(0.405)	(-4.593)
Log Volume	-0.0130*	0.00924	0.00236	-0.00761
	(-1.755)	(1.083)	(0.371)	(-1.552)
Quoted Spread, $	-0.413***	0.196***	0.280***	-0.130***
	(-7.508)	(4.657)	(4.680)	(-3.790)
S-T Return Volatility (LRS, 94)	0.0220	-0.0271	0.0305	0.0226
	(0.899)	(-1.086)	(1.186)	(1.274)
Constant	-1.995	4.693	1.138	0.596
	(-0.483)	(1.227)	(0.380)	(0.298)
Observations	39,285	39,265	39,285	39,285
R-squared	0.075	0.173	0.097	0.150
Number of Stocks	181	181	181	181
Day Fixed Effects	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes

Panel B: Inverted Venues				
VARIABLES	(1) Trades to Orders	(2) Odd lots to Volume	(3) Cancels to Trades	(4) Avg. Trade Size
HHI_{it}	2.292***	-0.548***	-1.719***	0.781***
	(10.69)	(-3.590)	(-7.759)	(6.940)
Log Market Capitalization	-0.0110	-0.235	0.265**	0.247**
	(-0.0756)	(-1.007)	(2.500)	(2.239)
Log price	-0.225	0.575**	-0.138	-0.441***
	(-1.414)	(2.326)	(-1.152)	(-3.695)
Log Volume	-0.00581	0.00191	-0.000202	-0.00480
	(-0.769)	(0.281)	(-0.0268)	(-1.311)
Quoted Spread, $	-0.0316	0.164***	0.0419	-0.0486
	(-0.518)	(4.297)	(0.528)	(-0.769)
S-T Return Volatility (LRS, 94)	-0.0176	-0.00810	0.0334	0.00769
	(-0.779)	(-0.351)	(1.579)	(0.575)
Constant	-4.004	1.713	-0.897	-0.0484
	(-1.514)	(0.407)	(-0.463)	(-0.0244)
Observations	39,199	39,111	39,199	39,200
R-squared	0.149	0.159	0.138	0.137
Number of Stocks	181	181	181	181
Day Fixed Effects	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes

APPENDIX 14: DAILY FRAGMENATION BY STOCK OBSERVATIONS

Figure 1: Daily Midas Venues and Unique Stock Observations

Figures 1(a) and 1(b) provide the relation between the number of unique stock observations the number of daily Midas venues [1, 13]. For instance, if Apple (APPL) were to trade at all 13 lit venues on day t, I report one observation for 13 daily Midas venues and continue this process for all stocks on every day in the sample (2019).

Figure 1a – Daily Venues by Daily Stock Observations

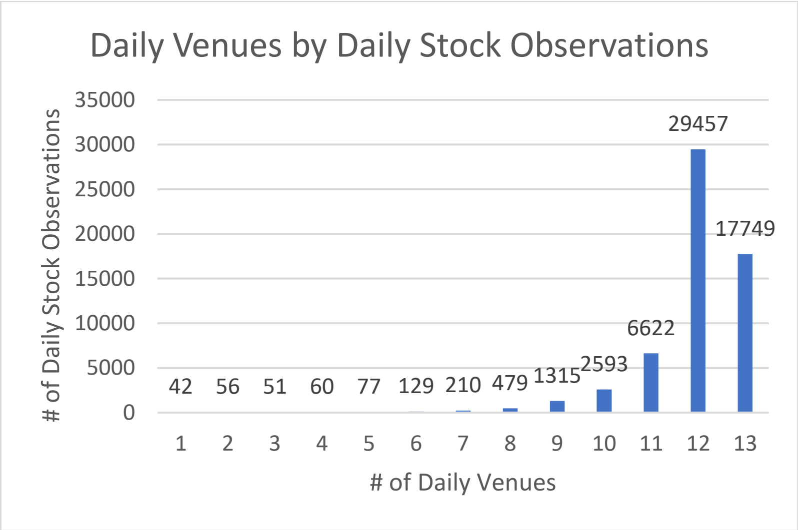
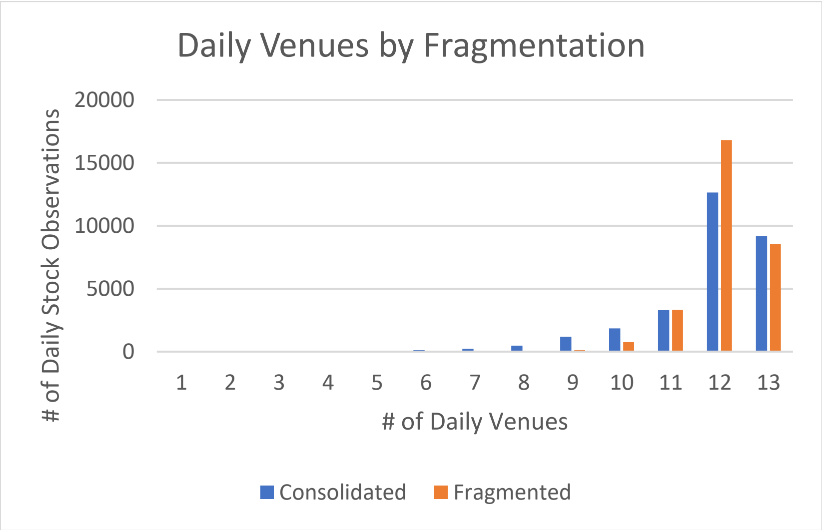


Figure 1b – Daily Venues by fragmentation



APPENDIX 15: DAILY MIDAS VENUES (LIT FRAGMENTATION) BY MARKET QUALITY & ALGORITHMIC TRADING ACTIVITY

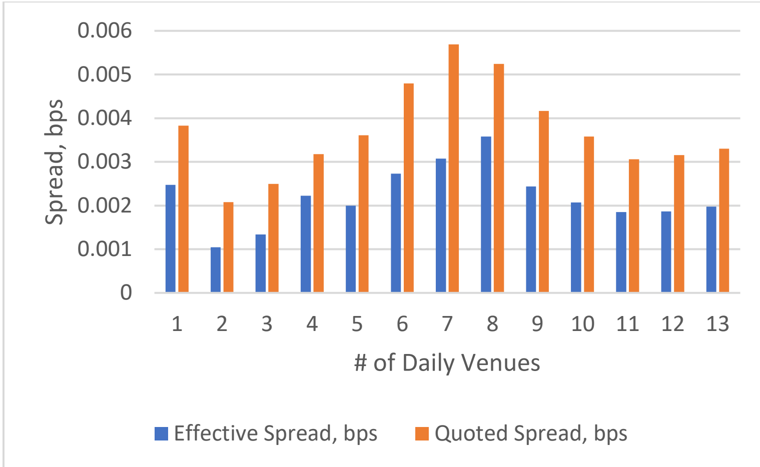


Figure 2a – Market liquidity

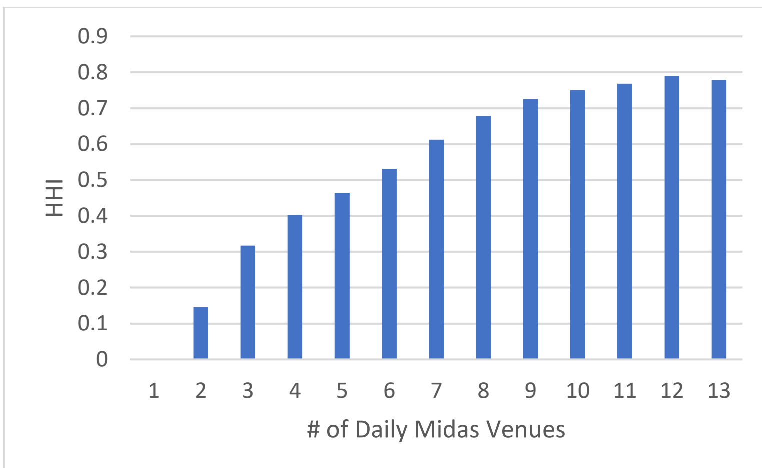


Figure 2b – Herfindahl-Hirschman Index

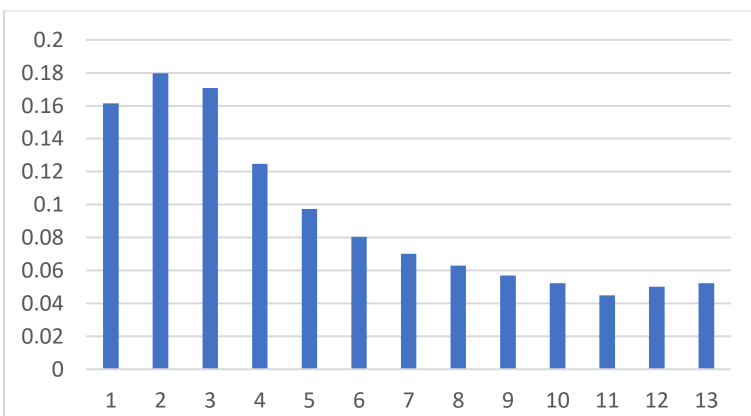


Figure 2c – Trades to Orders

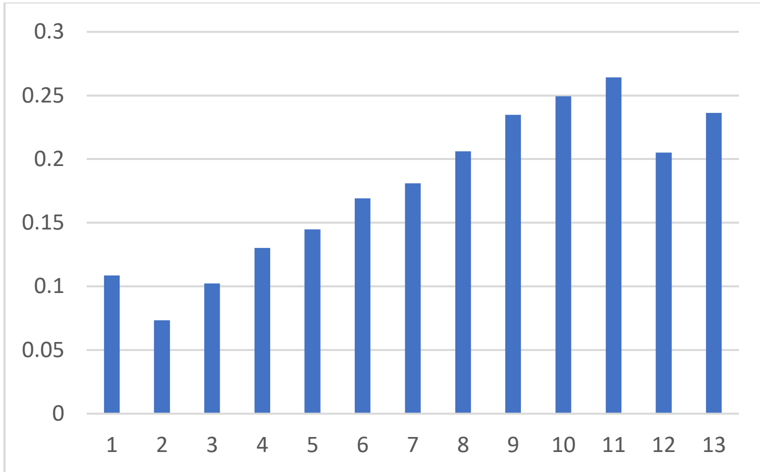


Figure 2d – Odd Lot to Volume

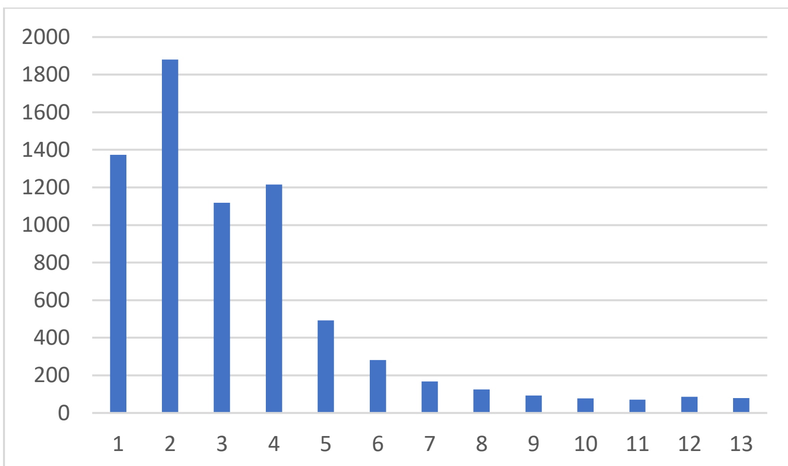


Figure 2e – Average Trade Size

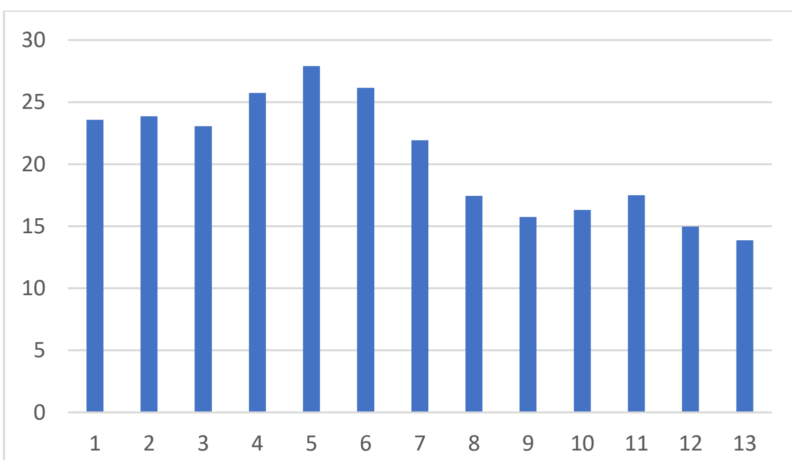


Figure 2f – Cancels to Trades

PART 3: DETERMINING FACTORS IN THE ILLIQUIDITY OF THINLY TRADED SECURITIES

I. INTRODUCTION

Market quality and fragmentation remains at the forefront of discussions as U.S. markets continue to fragment and new concerns are raised to determine if a "one size fits all" approach to securities market structure is optimal for liquidity among various securities. Market structure remains a salient topic among academics but a new effort by the Securities and Exchange Commission (SEC) has refocused the discussion to determine what effects fragmentation may have on different classifications of stocks and what resolutions may be needed to address potential negative consequences on market liquidity and efficiency. The SEC seeks to reevaluate the current market structure for securities that are classified as "thinly traded", securities whose average daily trading volume (ADV) is below 100,000 shares.[48] In the SEC's statement regarding market structure for thinly traded securities on October 17, 2019, the SEC announced that it is seeking proposals to address the impact that lower trading volumes may drive higher transaction costs for investors and ultimately impact investors seeking to unwind meaningful positions.[49] Thinly traded securities often face liquidity problems that include higher transaction costs, fewer market makers, and potential difficulties for investors seeking to unwind their positions or take advantage of new information. Ultimately, these issues have the potential to discourage small- and medium-sized enterprises from accessing the capital markets, a recent problem that the SEC and SEC chairmen, Jay Clayton, have been anxious to address as of late.

[48] See Securities and Exchange Commission Statement on Market Structure Innovation for Thinly Traded Securities, Securities Exchange Act Release No. 87327 (October 17, 2019), available at https://www.sec.gov/rules/policy/2019/34-87327.pdf (the "Commission Statement").
[49] SEC release No. 87327 (October 17, 2019)

"As we have heard from issuers, exchanges, and other market participants, a one-size-fits-all approach to market structure does not work for many of our public issuers, particularly small and medium sized companies. We want to know if more can be done to improve secondary market quality for thinly traded securities, and we look forward to seeing proposals geared to enhance trading and liquidity for this segment of the market while maintaining or improving market integrity."[50]

In light of the SEC's recent focus on addressing liquidity concerns for thinly traded securities, the objective of this paper is to determine what contributing factors predominantly deteriorate liquidity in "thinly traded" stocks and how do these determinants influence daily trading in thinly traded securities that differs from actively traded securities. Preceding the statement on market structure innovation for thinly traded securities by the SEC in October of 2019, the SEC conducted its own analysis of thinly traded securities and hosted a roundtable discussion where attendees were able to comment on the current state of these stocks.[51] Among those participating in the round table, there appeared to be a general consensus among a majority of the participants and those that submitted comments to the SEC, contending that the poor liquidity for thinly traded stocks is largely due to fragmentation. More specifically, spatial fragmentation, or the number of trading venues in U.S. markets is the largest contributing factor to the reduction in liquidity. Daniel Schlaepfer, president of Select Vantage, a proprietary trading

[50] See "SEC Issues Statement on Market Structure Innovation for Thinly Traded Securities." Securities and Exchange Commission, 17 Oct. 2019, https://www.sec.gov/news/press-release/2019-217 . Press release.

[51] See Equity Market Structure Roundtables: Roundtable on Market Structure for Thinly-Traded Securities, April 23, 2018, available at https://www.sec.gov/spotlight/equity-market-structure-roundtables (providing press release, agenda, transcript, comment letters, and other Roundtable materials).

firm, agreed with the sentiment that trading venue fragmentation is a reason why thinly traded stocks have wide spreads and low trading volume.[52]

Fragmentation, however, was not agreed upon by all participants as the main contributor of liquidity issues, as others stated alternative contending explanations to the difference in liquidity between thinly traded and actively traded securities. Don Ross, Chief Executive Office of PDQ Enterprises, an equity trading platform, contends that "temporal fragmentation" rather than spatial fragmentation is the root cause of small capitalization stock illiquidity.[53] Temporal fragmentation being buy and sell orders that don't match, owing to the arrival at different points in time and, thus, impacts investors as they are reluctant to place limit orders that may suffer from information leakage and adverse selection while waiting to execute. Steve Cavoli, a Senior Vice President of Virtu Financial, a large market maker, reiterated this view that the challenges of accessing liquidity were caused more by timing dislocation, where there is a limited number of, or a lack of, diverse holders of the name at any given time, rather than geographic (spatial) fragmentation caused by multiple venues.[54]

Another competing factor determining the illiquidity of thinly traded stocks, is the difficulty of market making in these securities coupled with a clientele effect. Using automated systems for maker making may lend to illiquidity as these endogenous liquidity providers seek more actively traded securities because of lower per-share profitability and higher trading frequency.[55] Dr. Kumar Venkataraman adds to this view point by expressing that there is a lack

[52] See Letter from Daniel Schlaepfer, President, Select Vantage (April 20, 2018), available at https://www.sec.gov/comments/265-31/26531-3489072-162255.pdf ("Select Vantage Letter").
[53] See Letter from Don Ross, Chief Executive Officer, PDQ Enterprises, LLC (May 10, 2018), available at https://www.sec.gov/comments/265-31/26531-3619683-162360.pdf ("PDQ Letter").
[54] See Equity Market Structure Roundtables April 23, 2018, (Mr. Steve Cavoli, Senior Vice President, Global Execution Services, Virtu Financial).
[55] Letter from Daniel Schlaepfer to SEC, April 20, 2018

of affirmative market making obligations, like you would see with human market makers, compelling market makers to be highly correlated with each other rather than the needs of individual securities.[56] The SEC is to seeking proposals aimed at enhancing liquidity in thinly traded securities, however, it remains unclear what the driving determinant of illiquidity is in thinly traded stocks and thus produces an opportunity to identify factors with the largest influence. The goal of this paper is to examine thinly traded securities and determine if the illiquidity of these securities can be mainly attributed to spatial fragmentation or do other factors such as temporal fragmentation, market making, and clientele play a role as well.

Past attention to thinly traded securities has had difficulty overcoming how to properly define "thinly traded." Prior opinions and classifications vary depending on the individual or organization, who may define thinly traded securities as stocks that trade less than 20,000 shares a day, stocks that cannot sell $1 million worth of shares a day, or stocks that have a spread of 50 cents or more.[57] However previously defined, the SEC, given its recent statements, presents a formal definition for these types of stocks and thus allows us in this paper to analyze these stocks based on those parameters set forth by the SEC. In this study we also seek to extrapolate the effect that spatial fragmentation has on thinly traded securities by disseminating fragmentation further into the effect of transparent, "lit" market venues in comparison to fragmentation via "dark", non-transparent trading resulting from alternative trading systems (ATS) and internalization of orders by brokers/dealers.

The recent solicitation of proposals to address issues of liquidity for thinly traded stocks by the SEC conveys the importance of this topic. Answering the call for proposals by the SEC,

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[56] See Equity Market Structure Roundtables April 23, 2018, Dr. Kumar Venkataraman, Professor of Finance, Cox School of Business, Southern Methodist University)

[57] See https://www.marketwatch.com/story/thinly-traded-stocks-proceed-with-caution

Nasdaq on February 5, 2020, filed a proposal to suspend "unlisted trading privileges" (UTP) for Nasdaq stocks that trade below 100,000 shares or less in ADV.[58] UTP allows for shares to trade on any national exchange regardless of which ever exchange they are listed. In roundtable discussions the SEC recognized that trading on multiple exchanges permitted by UTP could adversely affect other proposals to address the market quality concerns regarding thinly traded securities. Nasdaq believes that restricting UTP would incentivize exchanges to implement innovations in structure for thinly traded stocks and that UTP may "exacerbate market fragmentation."[59] However, others believe that the market is capable of manufacturing liquidity on its own without special advantages or protections given to select exchanges.[60] Whether or not this proposal is upheld and approved by the SEC remains to be seen, but regardless, questions have been raised as to whether a one size fits all for securities or if different market structures should be implemented that are tailored to different classification of stocks.

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[58] See https://www.nasdaq.com/docs/2020/02/05/2020%20UTP%20Termination%20Application.pdf

[59] See id. at 4 (Mr. Frank Hatheway, Chief Economist, Nasdaq OMX Group, Inc.). See Application to Permit Issuer Choice to Consolidate Liquidity by Suspending Unlisted Trading Privileges (April 25, 2018), available at https://www.sec.gov/comments/265-31/26531-3515735- 162293.pdf, at 11. The Nasdaq Application requests that the Commission suspend, for a period of up to 12 months, UTP for certain Nasdaq-listed securities. More specifically, Nasdaq requested that the Commission restrict UTP for Nasdaq-listed securities that are: (1) issued by an operating company; (2) have an initial market capitalization of $700 million or less or a continued market capitalization of $2 billion or less; (3) have an initial ADV of 100,000 shares or less; and (4) have a bid price greater than $1.

[60] See Letter from Don Ross, Chief Executive Officer, PDQ Enterprises, LLC (May 10, 2018), available at https://www.sec.gov/comments/265-31/26531-3619683-162360.pdf ("PDQ Letter").

II. HYPOTHESIS DEVELOPMENT

MARKET FRAGMENTATION

Per comments made in the SEC round table and those submitted to the SEC, spatial fragmentation, the number of geographical exchanges, is touted as a contributor to illiquidity in thinly traded stocks. Many participants, including Nasdaq, have proposed to address this issue by suspending UTP for thinly traded stocks. The discussion among theoretical and empirical studies regarding the overall effect of fragmentation, across all stocks, is an ongoing conversation with evidence presented on both sides to determine if competition is better through consolidation or fragmentation. Numerous studies assessing the impact that fragmentation has on liquidity, find a positive association between fragmentation and liquidity and that consolidated markets increase trading costs and other externalities (Demsetz, 1968; Cohen and Conroy, 1990; Battalio, 1997; Mayhew, 2002; Weston, 2000; Boehmer and Boehmer, 2003; De Fontnouvelle, Fishe and Harris, 2003; Battalio, Hatch, and Jennings, 2004; Nguyen, Van Ness and Van Ness, 2007; Foucault and Menkveld, 2008; Chlistalla and Lutat, 2011; O'Hara and Ye, 2011; Menkveld, 2013; Gresse, 2017).

On the other side of the discussion, some studies find negative consequences of fragmentation such as a deterioration in liquidity but deeper and more liquid markets when trading is consolidated (Biais, 1993; Madhavan, 1995; Bessembinder and Kaufman, 1997; Arnold, Hersch, Mulherin and Netter, 1999; Amihud, Lauterbach and Mendelson, 2003; Hendershott and Jones, 2005; Bennett and Wei, 2006; Gajewski and Gresse, 2007; Nielsson, 2009). Christie and Huang (1994) examine stocks that move from the Nasdaq (a fragmented market) to the NYSE (a consolidated market) and find a reduction in trading costs. Bennet and Wei (2006) also analyze firms that switch from Nasdaq to the NYSE and find improved price efficiency especially for illiquid stocks and suggest that order flow consolidation is valuable for these less liquid stocks. Most recently, Baldauf and Mollner (2019) argue that fragmentation contains two prevailing channels: (1) the competition channel, in which introducing more exchanges can reduce fee and lower spreads and (2) the exposure channel, whereby increasing more exchanges increases both costs and spreads to liquidity providers. The Baldauf and Mollner study implies that consolidation yields better liquidity as the benefits of increased competition are overcome by the costs of increased arbitrage.

Views regarding market fragmentation effects on small stocks reflect the same division in opinion, in that some studies find beneficial effects on certain aspects of liquidity (Hatheway, Kwan, and Zheng, 2017) while others find, as a whole, that fragmentation is detrimental to market quality for small stocks (O'Hara and Ye, 2011; DeGryse, DeJong, and Van Kervel, 2015; Gresse, 2017). O'Hara and Ye (2011) use the number of trade reporting facilities (TRFs) recording trading activity on a stock as a proxy for fragmentation. They find that fragmented stocks have lower transaction costs, faster execution speed, and higher short-term volatility but prices appear to be more efficient by becoming closer to a random walk. O'Hara and Ye also find that TRF fragmentation is more important for NASDAQ and it affects small stocks more so than large stocks and plays a smaller role for NYSE stocks. Gresse (2017) provides empirical evidence that lit fragmentation is beneficial for large stocks, as depth appears to improve but for small stocks depth is depleted. Gresse also determines that the increased adverse selection risk inherent in small stocks magnifies the adverse effects of fragmentation.

In congruence with these studies, both the U.S. Department of Treasury and the SEC released separate reports examining the effect that excessive fragmentation can have on liquidity in thinly traded securities. The U.S. Department of the Treasury's Capital Markets Report in October of 2017, identifies the liquidity differences between small- and mid-capitalization stocks with large-capitalization stocks, and notes that "venue fragmentation can be particularly problematic for thinly traded stocks because relatively small volumes of trading are spread out among a number of different venues."[61] Likewise, the SEC's Division of Trading and Markets Data (OAR) study finds that thinly traded securities have, on average, fewer exchanges quoting at the national best bid ("NBB") or national best offer ("NBO") than more actively traded securities.[62] The OAR study using a daily Herfindahl-Hirschman Index (HHI) to measure fragmentation, finds that thinly traded stocks are substantially less fragmentated than actively traded stocks. Given the findings presented in both the Capital Markets Report and the SEC OAR study, as well as the findings presented in the previous identified literature, it stands to reason that thinly traded securities are less likely to fragment because of the benefits provided by consolidated order flow. This implies that the literature supports the comments made by participants crediting the illiquidity of thinly traded stocks to the fragmentation of the market. This paper, therefore, believes the following hypothesis identifies spatial fragmentation as a possible contributor to the illiquidity of thinly traded stocks:

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[61] A Financial System That Creates Economic Opportunities: Capital Markets, October 2017, at 59-60, available at https://www.treasury.gov/press-center/press-releases/Documents/A-Financial-System-Capital-MarketsFINAL-FINAL.pdf ("Capital Markets Report").

[62] See Division of Trading and Markets Data Paper: Empirical Analysis of Liquidity Demographics and Market Quality, April 10, 2018, available at https://www.sec.gov/files/thinly_traded_eqs_data_summary.pdf (summarizing the quoting and trading characteristics of NMS stocks on the lower end of the liquidity spectrum) ("OAR Study"); A Financial System That Creates Economic Opportunities: Capital Markets, October 2017, at 59-60, available at https://www.treasury.gov/press-center/press-releases/Documents/A-Financial-System-Capital-MarketsFINAL-FINAL.pdf ("Capital Markets Report").

Hypothesis 1: Thinly traded stocks that are more consolidated will have better liquidity than thinly traded stocks that are more fragmented.

The SEC, in its "Division of Trading and Markets: Background Paper on the Market Structure for Thinly Traded Securities," ("OAR" study) states that "market capitalization and trading volume are positively related, although not perfectly so."[63] Considering this finding by the SEC that small-cap stocks and thinly traded securities are positively correlated, we pose the question that if consolidation is beneficial to thinly traded securities, what factors drive traders to seek out other venues thereby increasing daily fragmentation? This process is highlighted in the roundtable's discussion by Jason Vedder, Director of Trading and Operations for an institutional investment management firm. Daily trading in thinly traded securities, he notes, becomes a "cat and mouse game," whereby traders are forced to "hunt across venues for limited pockets of liquidity."[64]

Hendershott and Menkveld (2014) provide an empirical illustration of what may be driving this need to seek out other venues for order execution. Hendershott and Menkveld study price pressures and deviations from efficient prices due to risk-averse intermediaries supplying liquidity to asynchronously arriving investors. The authors find that price pressure is larger for small-cap stocks compared to large-cap stocks and that the volatility of these daily transitory price pressures reflects the same. Likewise, Chowdry and Nanda (1991) argue that in the presence of asymmetric information, adverse selection costs increase with the number of markets trading the asset. The authors suggest that large liquidity traders (such as institutions similar to Mr. Vedder) split their trades across markets to minimize costs. Seeing that thinly traded stocks

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[63] See https://www.sec.gov/rules/policy/2019/thinly-traded-securities-tm-background-paper.pdf

[64] See Equity Market Structure Roundtables April 23, 2018, Mr. Jason Vedder, Director of Trading and Operations, GTS Capital Management

suffer from large price pressures outlined in Hendershott and Menkveld, and the increase in fragmentation by participants seeking to minimize costs in Chowdry and Nanda, we would expect the following hypothesis to hold:

> Hypothesis 2: Thinly traded securities have more daily venue changes over a given period than actively traded securities.

The current sentiment described previously regards fragmentation born from geographical location (special fragmentation) as the determining factor of illiquidity in thinly traded stocks. However, a competing proposition presented at the SEC roundtable is that illiquidity is caused more by timing dislocation, where there is a limited number of, or a lack of, diverse holders of the name at any given time, rather than geographic fragmentation caused by multiple venues.[65] Temporal fragmentation, defined by Mr. Ross, being buy and sell orders that don't match, owing to the arrival at different points in time and, thus, impacts investors as they are reluctant to place limit orders that may suffer from information leakage and adverse selection while waiting to execute.[66] A key concept of fragmentation is that every market reflects different participant needs and wants at various times and is composed of different segments. Lit fragmentation often arises between trading venues that charge different fees or offer different market access to participants that addresses the needs of a diverse range of investors in a more efficient manner (Harris, 1993; Hendershott and Mendelson, 2000; Gresse, 2006).

Trading fees are a mechanism through which venues can compete for order flow and add another layer of complexity when determining how fragmentation affects market quality. Venue fee differences are one way for exchanges to address the specific needs of investors such as

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[65] See Equity Market Structure Roundtables April 23, 2018, (Mr. Steve Cavoli, Senior Vice President, Global Execution Services, Virtu Financial).

[66] See Letter from Don Ross, Chief Executive Officer, PDQ Enterprises, LLC (May 10, 2018), available at https://www.sec.gov/comments/265-31/26531-3619683-162360.pdf (“PDQ Letter”).

providing more timely execution and reducing the risk of information leakage and adverse selection. Next, we examine whether inverted venues alleviate some of the issues with temporal fragmentation by allowing investors to place limit orders and reduce the wait time. Additionally, we examine if inverted venues reduce the costs of information leakage and adverse selection that typically outweigh the benefits of displayed patience. Do thinly traded stocks tend to concentrate more on these types of exchanges than actively traded stocks? The contribution of this section is to indirectly test whether temporal fragmentation plays a considerable role, aside from spatial fragmentation, in determining the illiquidity of thinly traded stocks.

The most common fee model among lit venues is the make-take fee structure, with 8 of the 16 U.S. exchange models using this model. In this structure, liquidity suppliers (makers) receive a rebate, while liquidity demanders (takers) are charged a fee. There are also inverted venues called taker-maker or an inverted venue. In these fee venues, liquidity demanders (takers) receive the rebate while liquidity suppliers (makers) receive a fee. Of the 16 lit venues, five exchanges, BATS-Y, NASDAQ BX, EDGA, MEMX, and NYSE Market, have adopted this fee structure.[67] Inverted venues not only change the preferences for making versus taking liquidity but also influence traders' routing preferences and the probability of executing said orders (Comerton-Forde, Gregoire, and Zhong, 2019; Battalio, Corwin, and Jennings, 2016; Cox, VanNess, and VanNess, 2017; Foucault, Kadan, and Kandel, 2013; Angel, Harris, and Spatt, 2015; and Harris, 2013). This change in routing preferences is in part due the sub-tick pricing grid offered by inverted markets and the case where nonmarketable limit orders displayed on inverted markers will execute before orders displayed at the same price on tradition make-take venues (Comerton-

[67] See https://www.businesswire.com/news/home/20180518005767/en/NYSE%E2he%80%99s-Newest-Market---NYSE-National-Commence

Forde et al., 2019). Comerton-Forde et al. find that inverted venues enhance competition among liquidity providers which reduces trading costs and improves liquidity.

Battalio, Corwin, and Jennings (2016) examine the impact of differential fee schedules on broker routing decisions and limit order execution quality, by identifying retail brokers that route orders to maximize order flow payments by selling market orders and sending limit orders to venues paying large liquidity rebates. The authors determine that fee structure has an impact on limit order execution quality measured by the fill rate, time to execution, realized spread, and good fill ratio; and find that inverted venues have a positive relation with limit order execution quality. The evidence provided by Battalio et al. suggests that inverted venues have higher fill rates, faster time to execution, higher average realized spreads, and larger good fill ratios. Inverted venues provide an opportunity to indirectly examine the impact that temporal fragmentation may have on thinly traded stocks. As per the findings in the aforementioned studies, inverted venues are one possible mechanism through which thinly traded securities reduce wait time and offer faster execution by providing liquidity takers a rebate, while liquidity makers incur a small trading fee. In light of the ability of inverted venues to reduce information leakage and adverse selection, we contend the following hypotheses to hold.

Hypothesis 3: Thinly traded securities with higher inverted venue trading to overall trading will have better liquidity (narrower spreads and lager depth) than those thinly traded stocks with less inverted venue trading.

Hypothesis 4: Thinly traded securities have a higher proportion of trades (volume) to overall trades (volume) executed at inverted venues than actively traded securities.

DARK TRADING

Lit fragmentation is generally defined as intermarket competition on venues that display quotes, however, another factor to examine regarding thinly traded stocks is the effect of fragmentation that is the result of internalization of orders by dealers, execution by alternative trading systems (ATS) such as dark pools, or off-exchange transactions that are classified as "other OTC transactions. This type of fragmentation is regarded as "dark" fragmentation, and as of February 2021 there are 61 active ATS or dark venues.[68] The growth in dark trading coincides with the proliferation of lit venues and the literature comparing these two types of fragmentation focuses on the role that dark trading plays in price discovery and the overall impact on market quality. The discussion regarding dark trading and the effect on market quality is extensive and the objective of this section is to determine what effects this may have on thinly traded stocks. We examine the questions of whether fragmentation is detrimental to thinly traded securities and whether it is a contributor to the illiquidity of these types of stocks. By analyzing an alternate type of fragmentation, dark fragmentation, we look to see if the similar conclusions from the previous section holds, that being if thinly traded stocks experience optimal liquidity when there is greater consolidation?

The SEC's OAR report finds that thinly traded securities execute a larger proportion of overall volume on Trade Reporting Facilities (TRFs), or on non-exchange venues. For common stocks with an ADV below 100,000 shares but greater than 50,000 shares, 37% of the share volume executes off-exchange. Whereas stocks with an ADV above 100,000 shares have 34% of their share volume executing off-exchange.[69] Although no indication is given to the significance

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[68] See https://www.sec.gov/foia/docs/atslist.htm and https://www.finra.org/filing-reporting/otc-transparency/ats-equity-firms

[69] See Division of Trading and Markets Data Paper: Empirical Analysis of Liquidity Demographics and Market Quality, April 10, 2018.

of this difference, the OAR study provides initial evidence that stocks with lower ADV exhibit different trading characteristics compared to stocks with higher ADV. This difference is larger when compared to stocks that have an ADV below 50,000 shares, which execute 41% of total volume off-exchange. The SEC's Division of Trading and Markets background paper suggests that this difference in off-exchange trading indicates that "relative to actively traded securities, investors view exchanges as less appealing venues on which to transact.'[70] Comerton-Forde et al. (2019) allude to this in their examination of Tick Size Pilot implemented by FINRA and the SEC in 2016 that essentially eliminated trading in the dark for a subset of stocks. The authors find that the benefits of pricing efficiency of trading on inverted venues is likely the result of a substitution effect where inverted venues replace dark venues as the preferred platform. Comerton-Forde et al. find that the sub-tick trading in both dark and inverted venues suggests that liquidity providers see these two types of venues as substitutional and are willing to offer sub-tick price improvement. The findings in Comerton-Forde et al. coincide with the findings from the SEC OAR report, in that, the competition for sub-tick liquidity provision may offer larger reductions in trading costs and improved liquidity for thinly traded securities than actively traded securities.

Buti, Rindi, and Werner (2017) build a dynamic equilibrium model to study the consequences of introducing a dark pool and the effect it has on a multi-period limit order book (LOB). The model of Buti et al. shows when a dark pool is introduced in an illiquid market, the expected fill rate for limit orders declines, thus prompting limit orders to switch to market orders or migrate to the dark pool where fill rates and volume increase. Similarly, the authors find that

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[70] See SEC's Division of Trading and Markets: Background Paper on the Market Structure for Thinly Traded Securities, October 17, 2019, available at https://www.sec.gov/rules/policy/2019/thinly-traded-securities-tm-background-paper.pdf

as the probability of dark order executions increases, participants with access to the dark pool are better off. Temporal fragmentation and the risk that limit orders of thinly traded stocks may suffer from information leakage and adverse selection in an illiquid market on exchange, may prompt limit orders to migrate to the dark in search of faster execution, larger depth, and finer price increments. Kwan, Masulis, and McInish (2015) examine competition between traditional stock exchanges and dark venues for a sample of small-cap, U.S. stocks that trade just above and below a price of $1.00. The authors find that dark pools provide an important economic advantage which leads to more dark pool trading at the expense of traditional exchanges. Additionally, they find that as order flow is pulled to the dark pools, the probability of execution in dark pools rises and encourages more traders to submit orders off-exchange. Kwan et al. suggests that queue jumping in dark venues discourages from providing liquidity to traditional exchanges thereby widening spreads and decreasing depth but increasing price discovery in off-exchange venues.

An advantage of dark venues is the lack of transparency that allows traders to hide orders since dark venues provide participants limited or no pre-trade transparency and best-priced bids and offers are not required to be included in publicly distributed consolidated quotation data. The ability to hide order intentions through dark venues may provide additional motivation for thinly traded securities to trade off-exchange. Fang and Peress (2009) examine the relation between mass media coverage and stock returns. The authors find that stocks not covered in the media earn significantly higher future returns, which is larger for small stocks, stocks with high individual ownership, and low analyst following. Low analyst following of small stocks may partially explain the risk of information leakage and adverse selection in thinly traded stocks as these informed investors may seek to hide information given the lack of coverage. Boulatov and

George (2013) find that when liquidity is displayed, informed participants are drawn to trade as liquidity demanders than as liquidity providers, which implies that hidden liquidity has a favorable impact on market quality by intensifying competition among the informed. Friederich and Payne (2014) find that anonymity has a large and positive impact on liquidity in terms of spreads, depths, price impact, and dynamic price drift. Friederich and Payne find that small stocks, stocks with naturally shallow order books, and stocks where trading is highly concentrated benefit the most from anonymity. Similarly, Gozluklu (2016) finds that both liquidity and informed traders compete for liquidity provision and make use of undisclosed orders in opaque markets.

The benefits that opaque, non-transparent dark venues offer for price improvement and quicker order executions may explain the SECs OAR findings that thinly traded stocks have more off-exchange trading. The aforementioned articles suggest that expanding dark fragmentation may have positive effects on liquidity for thinly traded securities and that dark venues resolve issues related to temporal fragmentation rather than spatial fragmentation. The increased competition among liquidity providers in opaque markets and the reduced risk of information leakage of limit orders in dark venues lead to the following hypothesis:

> Hypothesis 5: Thinly traded securities with more dark trading will have better liquidity (narrower spreads and larger depth) and higher price discovery than thinly traded securities with low levels of dark trading.

Although dark venues may provide improved liquidity and price discovery for thinly traded securities, high levels of dark trading may be detrimental to market quality. Comerton-Forde and Putnins (2015) examine dark/block trading on price discovery to find that high levels of dark trading increases adverse selection risks in the lit market, but low levels of dark trading are either benign or beneficial to information efficiency. Comerton-Forde and Putnins establish that when dark trading exceeds 10% of total volume the benefits to information efficiency deteriorate and dark trading becomes harmful to market quality, but in their sample of stocks listed on the Australian Securities Exchange (ASX), the typical level of dark trading is below harmful levels. However, the sample used in Comerton-Forde and Putnins is of the 500 largest stocks on the ASX with an average market cap of $2.75 billion. This is substantially higher than the SEC OAR report for thinly traded securities who have an average market cap of $290 million. Given the amount of off-exchange trading reported by the SEC OAR report (37% of overall volume) that breaks the 10% threshold of Comerton-Forde and Putnins, and accounting for the findings in Kwan et al. (2015) whose sample specifically examines small cap stocks to find more price discovery taking place off-exchange; we contend that the "tipping point" for information efficiency in thinly traded securities is higher than that reported in Comerton-Forde and Putnins.

Hypothesis 6: The threshold of dark trading to overall volume where the liquidity and price discovery begins to deteriorate is higher for thinly traded securities than for actively traded securities.

MARKET MAKING

We examine the effect that market making in thinly traded securities can have on the lack of liquidity, as well as what clientele impact exists that could potentially exacerbate these effects. Using automated systems for market making may lend to the illiquidity as these endogenous liquidity providers seek more actively traded securities because of lower per-share profitability and higher trading frequency.[71] Dr. Kumar Venkataraman, a participant of the SEC round table, expressed his view that a lack of affirmative market making obligations, like what you would see from human market makers, results in automated market makers being highly correlated with each other rather than addressing the needs of individual securities.[72] Dr. Venkataraman's comments bring to light the conflict between market maker obligations and institutional trading, which we look to address further and determine whether the lack of institutional trading in thinly traded stocks can, in part, explain the illiquidity these types of securities.

Jason Vedder, Director of Trading and Operations for GTS Capital Management, an institutional investment management firm, and Brian Frambes, Co-Head Global Cash Trading for Fidelity Management & Research Co., expressed the challenges that institutional investors may have in trading thinly traded securities. These factors include small floats, highly convicted owners of those securities (i.e., owners that are inclined to hold), and the lack of index inclusion that may make institutional investors reluctant to invest in thinly traded securities.[73] Likewise, Brian Fagen, Head of Execution Strategy for Equities for Deutsche Bank, noted that a perceived difficulty for institutional investors is attempting to unwind a position taken in thinly traded securities, in that, the "demand to acquire a position is generally more patient than the demand to unwind a position."[74] Frank Hatheway, Chief Economist for Nasdaq OMX Group, reiterated this

[71] Letter from Daniel Schlaepfer to SEC, April 20, 2018

[72] See Equity Market Structure Roundtables April 23, 2018, at 193 (Dr. Kumar Venkataraman, Professor of Finance, Cox School of Business, Southern Methodist University)

[73] See Equity Market Structure Roundtables April 23, 2018, at 34 (Mr. Jason Vedder, Director of Trading and Operations, GTS Capital Management), and at 108 (Mr. Brian Frambes, Co-Head Global Cash Trading, Fidelity Management & Research Co

[74] See Equity Market Structure Roundtables April 23, 2018, at 37 (Mr. Brian Fagen, Head of Execution Strategy for Equities, Deutsche Bank).

point that the difficulty to trade in and out of these securities is an impediment to investing and perpetuates the limitations of marketability in thinly traded securities.[75]

Empirically, the literature examining small stocks agrees with the contention that small-cap, illiquid stocks are unattractive for indexing due to the excessive costs (Keim, 1999). Keim examines a passive mutual fund launched by Dimensional Fund Advisors (DFA), that is designed to capture the returns and risks of small-cap stocks. Keim finds that by excluding illiquid, low-priced stocks and maintaining a hold range of more liquid, larger-cap stocks trading costs are reduced. Kiem notes that the excessive costs of a pure indexing strategy of illiquid small-cap stocks demand immediacy of execution. Kiem provides empirical evidence that matches the sentiment of the roundtable discussants that the difficulties of indexing may dissuade institutional investors from investing in thinly traded securities. O'Hara, Yao, and Ye (2014) provide indirect evidence of the lack of institutional investors trading these types of securities in their study examining odd-lot trading and the effect odd-lot trading has on price discovery and market measures. O'Hara et al. find that odd-lots are trades that occur with a volume of less than 100 shares and are frequently used by high frequency traders (HFTs). The authors also find that informed trading is positively associated with odd-lot trading and that stocks with higher prices, illiquidity, and lower volatility is positively correlated with odd-lot trading.

Weller (2018) also examines the number of odd-lots and uses this as a proxy for the activity of algorithmic trading to find a positive relation between the two. Aitken, Harris, and Harris (2015) examine the impact that algorithmic trading and dark venue separately can have on market quality. Aitken et al. finds that off-exchange trading reduces the level of algorithmic

[75] See Equity Market Structure Roundtables April 23, 2018, at 45 (Mr. Frank Hatheway, Chief Economist, Nasdaq OMX Group, Inc.)

trading, indirectly widening effective spreads by increasing the incidence of end-of-day dislocations. Buti, Consonni, Rindi, Wen, and Werner (2015) also examine dark trading and more specifically sub-penny trading that allows traders to undercut displayed liquidity. Buti et al. argue that as queue-jumping activity increases, high-frequency trading activity decreases.

In light of the excessive costs to indexing thinly traded securities and the negative relation between the amount of AT/HFT activity and stocks that have more off-exchange trading that permits queue-jumping activity, we should see less institutional trading, measured by the amount of algorithmic trading, in thinly traded securities that contributes to the illiquidity of these stocks. Algorithmic and high frequency traders play a crucial role in market making activity. These types of traders tend to improve efficiency of the price discovery process, reduce trading costs, lower short-horizon volatility, absorb trade imbalances, increase depth, correct transitory price movements, and provide net positive effects on liquidity provision (Hendershott, Jones, and Menkveld, 2011; Menkveld, 2013; Brogaard, Hendershott, and Riordan, 2014a and 2014b; Conrad, Wahal, Xiang, 2015; Boehmer, Li, and Saar, 2018; Brogaard, Carrion, Moyaert, Riordan, Shkilko, and Solokov, 2018).

Hypothesis 7: Thinly traded securities will have less algorithmic traders, who act as endogenous liquidity providers or market makers, than actively traded securities.

Although algorithmic traders may take on the role of market making, these traders are not affirmative market makers, which explains the highly correlated activity amongst this group of traders (Benos, Brugler, Hjalmarsson, and Zikes, 2017; Boehmer et al., 2018). The unfavorable trading conditions in thinly traded securities and no affirmative obligations placed on these institutional traders leads to the lack of market makers as stated in hypothesis 7a. To alleviate this problem, one resolution is to create affirmative market makers; however, the sole exchange with these types of market makers is the NYSE, who classifies these specialists as designated market makers (DMMs). DMMs have the responsibility to maintain fair and orderly markets for assigned securities both manually and electronically, and to facilitate price discovery during periods of trading imbalances or instability.[76] DMM presence on only the NYSE and not on the other active exchanges may explain the lack of liquidity in thinly traded securities. This viewpoint is also echoed in the SEC roundtable by Daniel Schlaepfer, president of Select Vantage. He stated that, previously, human market makers were obligated to make markets in a range of stocks, including less liquid stocks, but currently, market liquidity largely is provided by firms operating automated systems with no obligation to support less liquid stocks.[77]

Clark-Joseph, Ye, and Zi (2017) provide empirical evidence on the impact that DMM participation has on market liquidity by examining technological glitches on the NYSE and EDGX that shut down trading at both exchanges for several hours. The authors find that NYSE-listed stocks experience significantly higher quoted and effective spread shocks during the shutdown on the NYSE, but the shutdown of the EDGX experienced no such change in liquidity. Clark-Joseph et al. find that this reduction in liquidity on the NYSE is more sensitive for stocks that have a higher proportion of DMM activity on the NYSE. The evidence in Clark-Joseph et al. suggests that removing DDMs substantially reduces liquidity but removing voluntary liquidity providers has trivial effects on liquidity. DMMs have a formal obligation that yields significantly improved liquidity for securities and as it applies to thinly traded securities, we may see a significant difference in NYSE-listed thinly traded stocks compared to other exchange-listed stocks. Therefore, we propose the following hypothesis:

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[76] See How the NYSE Market Model Works at https://www.nyse.com/market-model

[77] Letter from Daniel Schlaepfer to SEC, April 20, 2018 at 2.

Hypothesis 8: Thinly traded, NYSE-listed stocks will have better liquidity (lower spreads, larger depth, less volatility) than other exchange-listed thinly traded stocks.

III. DATA AND METHODS

DATA AND SAMPLE

Our data comes from Daily TAQ, CRSP, and SEC Market Information Data Analytics System (MIDAS), and EODDATA, for all common stocks in the year 2019. [78] For stocks to be included in our sample, we use CRSP to initially identify common stocks (share code 10 and 11) that trade at or above a price of $5.00 dollars every day during December 2018, the month preceding the sample period. Once we complete this preliminary procedure, we then identify stocks that have an average daily volume (ADV) in December 2018 below 100,000 shares (reported by CRSP) as thinly traded securities, as defined by the SEC. All other securities with an ADV equal to or greater than 100,000 shares as actively traded. We divide both samples of stocks into four quartiles based on market capitalization obtained from CRSP. Within each quartile we include the largest 25 securities based on market capitalization, which produces a final sample size of 200 securities, 100 from both thinly traded and actively traded samples.

VARIABLES

We use the TAQ dataset to construct the national best bid and offer (NBBO) prices and liquidity measures following the methods prescribed in Holden and Jacobsen (2014). The liquidity measures used in this study follow with the standard measures of market quality. These

[78] EODDATA is an end of day stock market data system that is used to obtain the listing of each stocks. We obtain all NYSE listed securities used to test hypothesis 8.

include the quoted spread and depth, effective spread, realized spread, and price impact. The *quoted spread* is the difference between the best bid and best ask prices and is weighted by the time. *Effective spread* is defined for a buy as twice the difference between the trade price and the midpoints of the NBBO price. For a sell, effective spread is twice the difference between the midpoints of the NBBO and the trade price. Effective spread in our analysis is weighted by trade size. *Depth* is the time-weighted average of displayed depth at the NBBO. *Volume* is measured daily and is based on the consolidated volume in all U.S. stock exchanges and off-exchange trading venues.

The *realized spread* is constructed to proxy as the temporary component of the dollar effective spread and is defined as twice the difference between the execution price and the midpoint of the spread prevailing five minutes after a trade. The dollar *price impact* is the permanent component of the dollar effective spread and is defined as twice the difference between the midpoint of the spread prevailing five minutes after a trade and the midpoint of the NBBO quotes of the current trade. For both realized spread and price impact we multiple each difference by +1 if the trade is a buy or by -1 if the trade is a sell. In order to identify whether a trade is a liquidity-demander "buy" or a liquidity-demander "sell", we consider the Lee and Ready (1991) version used by Holden and Jacobsen (2014).

To assess volatility and price efficiency, we use several different measures. We follow Lee, Ready, and Seguin (1994) to first compute the absolute return (dollar and percentage) over a 10-minute interval. Absolute return (dollar and percentage) is computed as the difference between the last trade price before a 10-minute interval and the last trade price in a 10-minute interval. We then follow O'Hara and Ye (2011), to compute the short-term volatility of these returns by finding the daily standard deviations of the absolute returns. This variable is a crude measure of trading irregularities which we interpret to be, the lower the short-term volatility the more efficient. Additionally, we also compute the daily standard deviation of prices as another crude measure of price efficiency and can be interpreted in the same manner as the short-term return volatility.

Our third measure of price efficiency is the variance ratio test (Lo and MacKinlay, 1988). The variance ratio is defined as the absolute value of the ratio of the variance of the 30-minute log returns divided by 3 times the variance of 10-minute log returns minus one.

$$Variance\ Ratio_{kl} = \left|\frac{\sigma_{kl}^2}{\sigma_l^2} - 1\right| \tag{1}$$

Where σ_l^2 and σ_{kl}^2 are the variances of 30-minute returns divided by 3 times the variance of the 10-minute returns for a given stock-day, respectively. As this number moves closer to zero, we interpret this to be that prices increasingly behave like a random walk and thus correspond to a more efficient market. Our final price efficiency measure is the first order autocorrelation of 10-minute returns. We calculate the first order return autocorrelations for each stock-day at 10-minute intraday frequencies. In Equation 2, r_t is the *i*th 10-minute return for a stock-day. We then take the absolute value of the autocorrelation to provide an efficiency metric that captures both under- and overreaction of returns, with larger values indicating greater inefficiency.

$$Autocorrelation_k = Corr(r_t, r_{t-1}) \tag{2}$$

To construct our algorithmic trading (AT) activity measure, we follow the method of Weller (2018) to compute four measures of AT activity: *odd lot-to-volume*, *trade-to-order volume*, *cancel-to-trade ratio*, and *average trade size*. Odd lot-to-volume ratio is the total volume executed in quantities smaller than 100 shares divided by the total volume traded. Trade-to-order volume ratio is the total volume traded divided by the total volume from all orders placed. Cancel-to-trade ratio is the number of full or partial order cancellations divided by the total number of trades. Average trade size is the trade volume in shares divided by the number of trades. Weller finds that odd lot-to-volume and cancel to trade ratios are positively related to algorithmic trading activity, while a higher trade-to-order ratio and average trade size are negatively related to algorithmic trading. As per the methods used by Weller and prescribed by SEC Market Information Data Analytics System (MIDAS), odd lot-to-volume, trade-to-order, and cancel-to-trade ratios are adjusted to exclude those orders reported by the NYSE and NYSE MKT.[79]

In this study we use two methods to measure the level of spatial fragmentation experienced by each stock in the sample. For our first metric, we use the SEC's MIDAS dataset to construct a daily inverse Herfindahl-Hirschman Index (HHI):

$$HHI = \sum_{i=1}^{13}(MS_i)^2 \tag{3}$$

where MS_i equals the market share of volume for each of the 13 lit exchanges identified by the Midas data set during our sample period. Once HHI is calculated, we calculate one minus the usual HHI to allow for an easier interpretation, this being that larger values of 1-HHI now correspond to a greater degree of market fragmentation (from here on, we refer to this measure as simply the HHI). Our second market fragmentation metric is the number of daily venues that a

[79] During this sample period, 13 of the 16 exchange venues were active, while the remaining 3 were still in the process of completing SEC approval. MIDAS collects around a billion feeds from the proprietary feeds of each of the 13 national equity exchanges and of the 13 exchange feeds the NYSE and NYSE MKT report trade size of the initiating order. The other 11 exchanges, however, separate trades by initiating and contra orders. This results in the NYSE number of trades and trade size to not be comparable with other exchanges. Additional MIDAS details and discussion of exchange exclusions are provided on the MIDAS website at http://www.sec.gov/marketstructure/mar_methodology.html

security trades on each day. Midas captures the number of trades and total trade volume for each of the 13 market venues for each stock daily. For a market venue to be included in the daily count of venues, we require that at least one share execute at said exchange. As the number of daily venues that a stock records trading volume at increases, we consider this to be an increase in fragmentation. Likewise, stocks that record trading at a small number of daily venues, we interpret to be an example of a consolidated market.

Panel A and B of Table 1 report summary statistics for both thinly traded and actively traded securities, respectively. First, the summary statistics show that our sampling procedure stays consistent during the actual sample period, in that our sample ADV for thinly traded securities is 49,723 shares and 3,555,930 shares for actively traded securities. A notable difference with our sample compared to that of the SEC OAR study, is that the ADV for thinly traded securities is similar but the ADV for actively traded securities is considerably larger (3.56 million compared to OAR's .562 million). Additionally, the pre sampling procedure in this study is weighted towards larger stocks for both samples. The average price and market capitalization in this study is $1,697.04 and 47.731 billion, respectively, while the average price and market capitalization for the SEC OAR study is $25.10 and 777 million. This is largely due to the sample size of stocks used but also the sampling procedure including the 25 largest market cap stocks in each quartile for each sample. If we look further into the characteristics of each sample, thinly traded and actively traded, we see that the thinly traded sample contains considerably smaller firms which have an average market capitalization of $1.29 billion and an average price of $171.90. Whereas the actively traded sample has an average market capitalization of $95.36 billion and an average price of $3,260.93.

Despite the difference in samples between this study and the SEC's OAR, the evidence initially shows that the market quality for thinly traded securities is substantially worse compared to actively traded securities. In all aspects of market quality, thinly traded stocks have inferior liquidity, higher volatility, and lesser price efficiency. Most notably, thinly traded stocks have an effective spread and average depth of $0.39 and 1,266 shares, while actively traded stocks have an effective spread and depth of $0.03 and 1,504 shares. As for volatility and price efficiency, we can see that the average for thinly traded stocks is higher across all metrics, which signals greater volatility and lesser price efficiency.

IV. EMPIRICAL ANALYSIS

As an initial analysis preceding the hypothesis regressions, we perform a univariate test of significance on the desired variables outlined in the previous section. In this study we are looking to determine what contributing factors predominantly deteriorate liquidity in thinly traded stocks and how these determinants influence daily trading. As noted in Table 1 and confirmed in Table 2, we can see that there are significant differences in market quality and liquidity between thinly traded and actively traded stocks. A popular explanation during the SEC's roundtable discussion for the poor liquidity of thinly traded stocks states that spatial fragmentation, or the number of physical trading venues, is the leading determinant. However, other explanations include temporal fragmentation, arrival of orders at different points in time, and market making difficulties to possible exacerbate the illiquidity of thinly traded stocks.

UNIVARIATE

Table 2 presents the univariate differences between thinly traded and actively traded securities to determine initially which of the three determinants, if any, play a role in the illiquidity of thinly traded stocks. First, we can see that thinly traded stocks are substantially smaller firms when compared to actively traded ones. Market cap is significant, and the difference is large. This supports the contention proposed by the SEC in its Division of Trading and Markets OAR study that thinly traded securities tend to be positively correlated with smaller market cap stocks. Regarding the first determinant, spatial fragmentation, we use two proxies to determine the amount of fragmentation present in each sample: HHI and number of daily lit venues. Both spatial fragmentation metrics show that thinly traded stocks are significantly more consolidated than actively traded stocks. For instance, the average number of daily venues for thinly traded stocks is around 9 daily venues, whereas actively traded stocks trade on average over 12 daily venues. As expected and consistent with prior studies (Bennett and Wei, 2006; Gresse, 2017; Baldauf and Mollner, 2019) and the SEC's OAR study, although there are 13 lit exchanges actively available for thinly traded stocks to execute orders during the sample period, the benefits provided by consolidated order flow may prompt these stocks to be less fragmented.

Although, it seems that spatial fragmentation may be one factor influencing the illiquidity of thinly traded stocks, we next look to examine the impact that temporal fragmentation and the difficulty of market making has on market quality. Temporal fragmentation, the arrival of buy and sell orders at different points in time, is tested indirectly by analyzing the amount of inverted venue activity, seeing as these types of venues have the ability to reduce temporal fragmentation. Likewise, the opaque, non-transparent characteristics of dark venues offer similar benefits that may reduce the timing dislocation of orders and shorten the time these orders spending waiting to execute. Two metrics of interest presented in Table 2 are the dark venue market share and inverted venue market share. If temporal fragmentation plays a role in the deterioration of liquidity for thinly traded securities, we should then see both of metrics be significantly higher for thinly traded than actively traded securities. The dark and inverted venue market share for thinly traded securities is 34.15% and 15.26%, respectively, whereas actively traded stocks are 31.54% and 15.46%. Although the two samples do not have significantly different inverted venue market shares, the mean difference in dark venue market share between thinly traded and actively traded is positive and significant at the 1% level. These results thus provide evidence that supports our prediction that temporal fragmentation may also be a leading determinant in the illiquidity of thinly traded securities.

Lastly, the difficulty in market making for thinly traded stocks and lack of affirmative market making obligation may result in less algorithmic trading activity to be present. To test this prediction, we use the algorithmic trading activity proxies of Weller (2018) which include: odd lot-to-volume, trade-to-order volume, cancel-to-trade ratio, and average trade size. Weller finds that algorithmic trading activity is positively correlated with odd lot-to-volume and cancel-to-trade ratio, but negatively correlated with trade-to-order volume and average trade size. Thus, if our prediction that thinly traded stocks are less appealing to algorithmic and high frequency traders, we should see less odd lot-to-volume and cancel-to-trade but more trade-to-order volume and higher average trade size in thinly traded stocks. Table 2 reports findings showing the opposite of our expectations and all differences are significant at the 1% level. We are cautious to interpret the findings in Table 2 to be conclusive evidence either supporting or contradicting our hypotheses, as these are merely univariate statistics, and this finding may not hold in multivariate analysis.

SPATIAL FRAGMENTATION

The objective for this initial multivariate analysis is to assess the impact that spatial fragmentation, the number of geographical exchanges, has on the liquidity of thinly traded securities compared to actively traded ones. In Figure 1(a), we record the number of daily venues by the number of daily stock observations.[80] As shown in Figure 1(a), for the entire sample of stocks, we see that although all stocks can trade on all thirteen active venues every day, the

[80] For instance, if AAPL were to trade on all thirteen stock venues one day and then trade on twelve stock venues the next day, there would be one observation for twelve venues and one observation for thirteen venues.

majority of our sample trades on either twelve or thirteen venues each day. However, Figure 1(a) shows that not every observation is observed on twelve and thirteen venues, and that there appears to be variation in the number of venues a stock trades on each day, albeit the frequency of these smaller number of daily venues is substantially smaller. As we look further in to this dynamic, we see that Figure 1(b) shows daily trading between the thinly traded and actively traded stocks to differ substantially. Thinly traded stock observations span across all thirteen daily venues and show a much more level distribution with a slightly larger bulk of observations at ten, eleven, and twelve daily venues. However, actively traded stocks trade on no less than eight daily venues and make up an overwhelming majority of the observations at twelve and thirteen venues as shown in figure 1(a).

To assess the impact spatial fragmentation may have on market liquidity, we start by looking at the relation between the daily venues reported by Midas and liquidity (spreads and depth,). First, as we see in Figure 2(a) there is a positive relation between spatial fragmentation and market liquidity, in that, spreads appear to tighten as the number of daily venues grows from one to thirteen. However, in Figure 2(d) we see that depth is depleted as the number of venues grows, which is consistent with Gresse (2017) who shows that small cap stocks have diminished depth as fragmentation increases but not for large cap stocks.

Consistent with our defined spatial fragmentation proxies, Figure 2(b) graphs the relation between daily venues and HHI, showing a positive relation and supports our conjecture that the number of daily venues recording trading is another proxy for fragmentation. Similar to Figure 1(b), we disseminate Figure 2 further, by graphing the separately the two samples, thinly traded and actively traded. Figures 2(e) through 2(i) graph the samples separately in relation to liquidity, off-exchange trading, and inverted venue trading. As we can see in Figures 2(e) and

2(f), both samples show the same pattern whereby spreads tighten as the number of daily venues increases. Figure 2(g) presents evidence consistent with our analysis of Figure 2(d), and shows that depth is depleted for thinly trades stocks as fragmentation increases but actively traded stocks function in the opposite direction and we see an increase in depth as fragmentation increases.

In Hypothesis 1, we contend that previous studies by the U.S. Department of Treasury and the SEC showing thinly traded stocks to be more consolidated can be explained by the benefits of consolidated order flow, and thus we should see more consolidated thinly traded stocks to have better liquidity. However, Figures 2(e) – 2(g) contradict this prediction slightly by showing a positive relation between spreads and depth with the number of daily venues. To more formally test hypothesis 1 that thinly traded stocks have better liquidity at lower levels of fragmentation, we’ll be using a panel regression of only thinly traded stocks to establish a relation between the amount of fragmentation measured by HHI and market quality where the dependent variables include quoted spread, effective spread, quoted depth, short-term volatility, price volatility, variance ratio, and return autocorrelation.

$$Market\ Quality = \alpha_0 + \beta_1 HHI_{it} + \delta_1 Controls_{it} + \varepsilon_{it} \qquad (4)$$

In Table 3 our indicator variable of interest is the HHI and the control variables used include the quoted spread, log volume, log market cap, and log price. We also control for day and stock fixed effects and the errors are clustered by stock. Column 1 and 2 of Table 3, measure the relation of HHI with quoted and effective spread, respectively. We see that the relation between HHI and quoted spread is statistically and economically significant, where an increase in HHI by one standard deviation is associated with an increase in quoted spread by $0.016. However, we do not see significant relation between HHI and effective spread. Additionally, column 3 of Table 3 does not show a significant association between depth and fragmentation, but we do see an improvement in price efficiency measured by return autocorrelation (column 7) as fragmentation increases. There is a strong contention made by the SEC's round table participants that spatial fragmentation, the number of geographical exchanges, to be the main determinant influencing liquidity in thinly traded securities. Other participants, however, believe that the market is capable of creating liquidity on its own without special advantages given to select exchange, contrary to Nasdaq's proposal to suspend UTP privileges for thinly traded securities. The findings in Figure 1 and 2 and Table 3, support the latter, in that spatial fragmentation does marginally widen quoted spreads but has little effect on transaction costs measured by effective spread. Additionally, we don't see spatial fragmentation have an association with smaller spreads or higher volatility.

Despite the benefits provided by increasing fragmentation, we still observe thinly traded stocks to be more consolidated on average than actively traded stocks. We next look to determine what factors may drive traders to seek out other venues thereby increasing fragmentation and improving order execution. In the SEC's round table discussion, Jason Vedder, director of Trading and Operations for an institutional investment firm, stated that trading in thinly traded securities is a "cat and mouse game," where investors have to "hunt across venues for limited pockets of liquidity."[81] In hypothesis 2, we predict that this hunt for liquidity would result in thinly traded securities to have more daily venue changes than actively traded securities. To test this prediction, we use the number of daily venues reported by Midas to count the number of day-to-day changes that are made in the number of daily venues for all stocks. Figure 3 shows

[81] See Equity Market Structure Roundtables April 23, 2018, Mr. Jason Vedder, Director of Trading and Operations, GTS Capital Management

the total number of daily venue changes over our sample period and as we can see there are substantially more venue changes by thinly traded securities than actively traded securities.

To more formally examine this difference, we present both a univariate and multivariate likelihood analysis where the dependent variable is a dummy variable equal to 1 if the number of daily venues for stock *i*, is different on day *t* compared to day *t-1*, and 0 otherwise. Panel A of Table 4 records the univariate results, which show that thinly traded stocks are significantly more likely to change the number of daily venues than actively traded stocks. Panel B of Table 4 uses both a Probit and Linear Probability model as a multivariate analysis regarding the likelihood of thinly traded stocks to change the number of daily venues:

$$Venue\ Change = \beta_1 Thinly\ traded_{it} + \delta_1 Controls_{it} + \varepsilon_{it} \tag{5}$$

Where *venue change* is equal to 1 if the number of daily venues for stock *i*, is different on day *t* compared to day *t-1*, and 0 otherwise. Control variables include log volume, log market cap, and log price. We also control for day and stock fixed effects and the errors are clustered by stock. Both models are qualitatively the same and show that thinly traded stocks are 25.7% to 42.7% more likely to change the number of daily venues.

Next, we look to determine the difference in magnitude of these daily venue changes and look to see what other factors influence the magnitude of these changes. Fragmentation relies upon markets providing participants a host of options to satisfy their needs and certain stock characteristics my influence the preferencing of orders to certain venues or to a certain number of venues. Therefore, we look to assess what role trading costs (spreads), volatility, price, competition, and informational advantages play in causing traders to seek out other venues and causing fluctuations in the number of daily venues a stock trades on. We follow Chordia, Roll, and Subrahmanyam (2001) to determine if changes in trading costs, volatility, and price efficiency are correlated with changes in fragmentation measured by daily venue changes. We us the following panel regression model:

$$\%\Delta Daily\ Venues = \alpha_0 + \beta_1 Thinly\ traded_{it} + \beta_2 \%\Delta Trading\ cost_{it} + \beta_3 \%\Delta Volatility\ _{it} + \beta_4 \%\Delta Price\ Efficency_{it} + \delta_1 Controls_{it} + \varepsilon_{it} \quad (6)$$

Where our dependent variable is the percentage change in the number of trading venues recording an execution on day *t*, for stock *i*. Our main variable of interest is the *Thinly traded*, which we interpret to present the difference in magnitude of daily venue changes between the two samples. Other factors that may influence market fragmentation include percentage changes in quoted and effective spreads, volatility, price efficiency measures, and return. Our control variables remain constant with previous models, as well as a control for fragmentation measured by HHI.

In Panel C of Table 4, we include all variables of interest from equation 6. Our main variable of interest, thinly traded, is significant at the 1% level and shows that thinly traded securities are associated with venues changes that are 24.5% larger than actively traded stock venue changes. Furthermore, we see that there are several other factors that influence changes in daily fragmentation, which include changes in transaction costs, return, and price volatility. Thus far, we see that thinly traded stocks are more likely to change the number of daily venues and that these changes are significantly larger in magnitude compared to actively traded securities, consistent with hypothesis 2. Other factors influencing changes in fragmentation are in line with not only the SEC's roundtable participants highlighting the need to trade across markets to minimize costs, but also with prior studies showing that thinly traded stocks suffer from larger price pressures than actively traded stocks (Chowdry and Nanda, 1991 and Hendershott and Menkveld, 2014).

TEMPORAL FRAGMENTATION

A competing explanation to the poor liquidity in thinly traded stocks identifies temporal fragmentation rather than spatial fragmentation. Temporal fragmentation is the arrival of buy and sell orders at different points in time, prompting investors to be more reluctant to place limit orders that may suffer from information leakage and adverse selection while waiting to execute.[82] A possible remedy that reduces this wait time and information leakage would be placing orders on inverted venues. Inverted venues change the preferences for making versus taking liquidity and influence the probability of execution (Comerton-Forde, Gregoire, and Zhong, 2019; Battalio, Corwin, and Jennings, 2016; Cox, VanNess, and VanNess, 2017; Foucault, Kadan, and Kandel, 2013; Angel, Harris, and Spatt, 2015; and Harris, 2013). In this section we test if the benefits provided by inverted venues by offering faster execution may reduce the costs of temporal fragmentation and allow us to indirectly test whether temporal fragmentation plays a role in the illiquidity of thinly traded stocks. To test hypothesis 3 and 4 that thinly traded securities experience better liquidity when there is more inverted venue activity, we propose the following panel regressions using only thinly traded stocks in equation 7 and the full sample in equation 8:

$$Market\ Quality\ = \alpha_0 + \beta_1 Inverted\ Venue\ Activity_{it} + \delta_1 Controls_{it} + \varepsilon_{it} \tag{7}$$

$$Inverted\ Venue\ Activity = \alpha_0 + \beta_1 Thinly\ traded_{it} + \delta_1 Controls_{it} + \varepsilon_{it} \tag{8}$$

Where the dependent variable for equation 7 is the market quality metrics used in the previous section and for equation 8, we use three metrics to proxy for inverted trading activity:

[82] See Letter from Don Ross, Chief Executive Officer, PDQ Enterprises, LLC (May 10, 2018), available at https://www.sec.gov/comments/265-31/26531-3619683-162360.pdf ("PDQ Letter").

inverted volume, trades, and market share. Using the SECs Midas data set, we're able to observe the amount of volume and trades at each of the four inverted venues.[83] We control for a set of variables that follows the method of Comerton-Forde et al. (2017) and also past market fragmentation studies (O'Hara and Ye, 2011, and Gresse, 2017), which include log of market capitalization, log price, and log daily volume as control variables. Panel A of Table 5 reports the results for equation 8 and the variables of interest measuring the impact of inverted venue activity are log inverted volume and inverted venue market share. In columns 1 and 2, we see a dynamic relation between market quality measured by the two proxies for inverted venue activity. First, one standard deviation increase in the amount of inverted volume is associated with an improvement in market liquidity as both quoted and effective spread tighten by $0.084 and $0.031, respectively. We also see that not only does short-term return volatility decrease, but thinly traded stocks become more efficient by moving closer to a random walk shown by the inverse relation between variance ratio and inverted venue volume.

Although inverted volume is associated with an improvement in market quality, an increase in market share of inverted venues widens quoted spread spreads by $0.035 but inverted venue market share has no significant association with transactions costs measured by effective spread. Likewise, we see that inverted market share is positively associated with the variance ratio but has no significant relation with short-term return volatility. Therefore, the overall impact of increasing both inverted venue volume and market share is associated with an improvement in market quality for thinly traded stocks. The evidence from Panel A of Table 5 is consistent with hypothesis 3 that temporal fragmentation is detrimental to thinly traded securities

[83] Of the 14 U.S. lit exchanges, there are four venues using an inverted fee schedule. These exchanges include the Bats-Y, Nasdaq Boston, Edge-A, and NYSE National.

and inverted venues are a mechanism through which these stocks can reduce the costs from orders arriving at different points in time.

In light of thinly traded stocks being more adversely affected by temporal fragmentation than actively traded stocks, we expect that thinly traded stocks will have a higher proportion of trades which execute at inverted venues. Panel B of Table 5 reports the findings from equation 8, and the variable of interest is a dummy variable identifying thinly traded stocks. The coefficients across the three dependent variables show that thinly traded securities have significantly less inverted venue volume and trades but have a higher market than actively traded securities. The coefficients show that this relation is not only statistically significant but economically significant as well, with thinly traded stocks executing 854,000 shares fewer and 12,682 trades less than actively traded stocks. However, thinly traded securities have an inverted market share that is on average 4.16% higher than actively traded securities. The evidence presented in Table 5 is consistent with both hypothesis 3 and 4 which suggesting that temporal fragmentation plays a large role in the illiquidity of thinly traded stocks and inverted venues are a mechanism through which thinly traded securities can improve their market quality.

Next, we look to see if the substitutability between inverted venues and dark venues (Comerton-Forde et al., 2019) captures the costs of temporal fragmentation. The SEC's OAR report finds that thinly traded securities execute a larger proportion of overall volume on Trade Reporting Facilities (TRFs). The appeal that off-exchange trading provides to thinly traded securities is the speed of execution and the lack of pre-trade transparency that allows traders to hide orders and thus reduce the amount of information leakage they may experience on lit exchanges. These advantages may resolve the effects of temporal fragmentation in a way similar to sending orders to inverted venues. To test our hypothesis 5 that there exists a positive relation between dark venue activity and market quality in thinly traded securities, we will use a model similar to equation 7, but rather than using inverted volume and market share we will test log dark venue volume and market share, obtained from "D" TAQ to the market quality metrics and follows the method of Gresse (2017).

Table 6 reports our findings of the panel regression of dark venue activity to market quality and in line with our prediction we see that there is a dynamic relation between off-exchange activity and market quality similar to inverted venue activity. Dark Volume tightens quoted spreads by $0.087 but as dark venue market share increases so do spreads by $0.05.[84] However, there exists no significant association between dark venue activity and effective spreads, but as for quoted spreads, we see the magnitude difference from an increase in dark venue market share are overcome by an increase in overall dark venue volume. Additionally, we also see that increasing dark volume is correlated with an improvement in price efficiency but as dark venue market increases, we see a deterioration in price efficiency and an increase in return volatility. The effect of dark venue activity on transaction costs, however, still suggest that dark venue activity is another mechanism through which the negative effects of temporal fragmentation can be reduced, but these benefits are accompanied with a reduction in price efficiency.

Although dark venues provide benefits to thinly traded stocks, the findings in Table 6 regarding dark market share suggest that higher levels of dark trading are detrimental to market quality. Comerton-Forde and Putnins (2015) show that when dark trading exceeds 10% of total volume the benefits of information efficiency deteriorate, and dark trading becomes harmful. However, given the differences in their sample compared to ours we contend that this threshold

[84] Economic effects are calculated from a one standard deviation increase in log dark volume and dark market share.

is higher for thinly traded securities. Figure 4 shows the effects of dark trading on information efficiency measured by variance ratio and return autocorrelation. Figure 4 plots the estimated effects of dark trading for thinly traded stocks (Panel A and B) and actively traded stocks (Panel C and D) on the predicted and mean information efficiency measures. The estimated effects of dark trading are obtained from a panel regression where the dependent variables are the information efficiency measures and the independent variables are dummy variables of various ranges of dark venue market share (0–5%, 5–10%, 10–20%, 20–30%, 30–40%, 40–50%, 50–60%, 60–70%, 70%+) and control variables.

In Figure 4, we see evidence that is substantially different than Comerton-Forde and Putnins (2015). For thinly and actively traded securities, dark trading activity at low levels deteriorates information efficiency but once levels pass 20%, they become beneficial until a threshold is met at around 50% of trading in some cases. The threshold before information efficiency deteriorates appears to be higher for thinly traded securities than for actively traded securities. In terms of variance ratio, the threshold is met once dark volume crosses 50% of total trading for thinly traded stocks but for actively traded stocks the threshold is 40%. Overall, the findings in this section support both our predictions and the sentiment held by a few participants of the SEC roundtable that temporal fragmentation rather than spatial fragmentation may be a more influential determinant of the illiquidity of thinly traded securities.

MARKET MAKING

The last determinant considered during the SEC roundtable discussion is the presence of market making activity or lack thereof in thinly traded securities. Several comments made during the discussion highlight the conflict between market maker obligations and the challenges that institutional investors face trading in thinly traded securities. Furthermore, these notions are empirically supported by studies finding that small cap stocks are unattractive for indexing and the negative relation between the amount of AT/HFT activity and off-exchange trading. Taking these factors into consideration may possibly explain the illiquidity of thinly traded stocks, as these types of securities are not able to experience the benefits provided by ATs and HFTs which include reduced trading costs, lower short-horizon volatility, and net positive effects on liquidity provision (Hendershott, Jones, and Menkveld, 2011; Menkveld, 2013; Brogaard, Hendershott, and Riordan, 2014a and 2014b; Conrad, Wahal, Xiang, 2015; Boehmer, Li, and Saar, 2018; Brogaard, Carrion, Moyaert, Riordan, Shkilko, and Solokov, 2018). To test hypothesis 7, we'll look to determine what relation exists between algorithmic trading measured by the four AT proxies of Weller (2018) and thinly traded stocks in the following panel regression:

$$Weller\ (2018)\ AT\ proxies = \alpha_0 + \beta_1 Thinly\ Traded_{it} + \delta_1 Controls_{it} + \varepsilon_{it} \qquad (9)$$

where the dependent variables are odd lot-to-volume, trade-to-order volume, cancel-to-trade ratio, and average trade size for stock *i*, on day *t*. Our indicator variable of interest is the dummy variable for thinly traded securities, which will capture the difference in algorithmic trading between thinly traded and actively traded stocks. The control variables follow the methods of Weller (2018), and include the short-term return volatility, quoted spread, log market cap, and log price. We again control for day and stock fixed effects and the errors are clustered by stock.

First, if we look back to our univariate results in Table 2, we see that initially our prediction do not hold, thinly traded stocks have significantly more algorithmic trading activity than actively traded stocks as all of the proxies are in the direction associated with an more algorithmic trading activity. However, in Table 7 we see a complete reversal from the results in Table 2 and the direction of the coefficients is in the expected direction. First in columns 2 and 3 of Table 7, odd lot-to-volume and cancels-to-trades ratio are significant and negative for the thinly traded coefficient. Thinly traded securities have a 1.02% and 1.495% smaller odd lot-to-volume ratio and cancels-to-trades ratio than actively traded securities. In columns 1 and 4 of Table 7, thinly traded securities have 2.843% larger trades-to-orders ratio and average trade size is .51% larger than actively traded securities. The evidence in Table 7, strongly supports our prediction that thinly traded securities will have significantly less algorithmic trading activity, and, thus, the lack of algorithmic trading activity may play a large roll in the illiquidity of thinly traded securities.

The results in Table 7 demonstrates that there are significant differences in algorithmic trading activity between thinly traded and actively traded securities. This finding can be possibly be explained by the fact that algorithmic traders are not affirmative market makers and thus the unfavorable trading conditions in thinly traded securities may dissuade algorithmic traders from trading in these securities. The unfavorable trading conditions in thinly traded stocks may surface by exploring the difference in liquidity between designated market makers (DMMs), traders that are affirmative market makers, and algorithmic traders. To test this prediction, we'll look to compare the liquidity of thinly traded NYSE-listed stocks, the sole exchange with market makers specialist classified as DMMs, to other exchange-listed thinly traded stocks using a difference-in-difference panel regression:

$$Market\ Quality = \alpha_0 + \beta_1 NYSE\ Listed_{it} + \beta_2 Thinly\ traded_{it} + \beta_3 NYSE\ Listed * Thinly\ traded_{it} + \delta_1 Controls_{it} + \varepsilon_{it} \qquad (10)$$

$NYSE\ Listed_{it}$ is defined as 1 if the stock is listed on the NYSE and 0 otherwise.[85] The interaction, $NYSE\ Listed * Thinly\ traded_{it}$, gives the difference-in-difference estimate for each variable, and our dependent variables include all market quality metrics used in previous sections. In column 1 and 2 of Table 8 we see that contradictory to our expectations, the interaction term has a significant and positive association for both quoted and effective spread. NYSE thinly traded securities have significantly higher transactions costs by $0.505. It appears that despite the presence of DMMs, NYSE listed, thinly traded securities have worse market quality than non-NYSE listed thinly traded securities. Thinly traded stocks have significantly inferior market quality measured by spreads, volatility, and price efficiency than actively traded stocks.

Collectively, both tables 7 and 8 demonstrate that there appears to be both a lack of market making activity and difficulty in market making for thinly traded stocks evident by the deterioration in liquidity for NYSE listed thinly traded securities. Despite affirmative market makers such as DMMs, who have a formal obligation to provide liquidity, we still see significantly wider spreads for thinly traded stocks. We interpret these findings to demonstrate that the lack of formal liquidity provision obligations, which may explain the lack of endogenous liquidity providers, and the difficulty of making a market in thinly traded stocks are determinants in the poor liquidity of thinly traded securities.

[85] Out of the 200 thinly traded and actively traded securities in the sample, 22 are NYSE listed, whereas, 49 actively traded securities are NYSE listed.

V. ROBUSTNESS

In this section, we provide additional tests using the SEC's Tick Size Pilot Program as a robustness check to verify the preceding regression analysis. The SEC's Tick Size Pilot Program, which spanned from October 2016, through September 2018, increased the ticks for smaller capitalization stocks in hopes of increasing overall market quality. The Tick Size Pilot Program created three groups of stocks that would see the minimum tick increment move from $0.01 to $0.05, as well as a fourth control group. Stocks in the first group quoted in $0.05 but would continue to trade at their current price increment. Stocks in the second group quoted and traded in $0.05 increments.[86] The third group was to adhere to the requirements of the second test group, but these stocks would also be subject to a "trade-at" rule which essentially eliminated all dark venue trading except for block size orders or trading at the midpoint (Comerton-Forde et al., 2019).

The focus of this section is to analyze thinly traded securities in the third Tick Size Pilot group who were subject to the "trade-at" rule and determine what impact ending the Tick Size Pilot Program has on the market quality of these stocks. Additionally, we use the ending of the program to further assess which determinants (spatial fragmentation, temporal fragmentation, and market making activity) play a role in the illiquidity of thinly traded securities. Our results in Tables 6, 7, and 8 provide evidence consistent with our predictions that temporal fragmentation

[86] See https://www.finra.org/rules-guidance/key-topics/tick-size-pilot-program

and lack of market making activity play a role in the illiquidity of thinly traded securities. To reduce the effects of temporal fragmentation, we suggest that inverted fee venues and off-exchange trading allow traders to reduce the wait time and offer faster execution at reduce costs through the sub-penny price improvements.

Comerton-Forde et al. (2019) find that in the third Tick Size Pilot group, the sub-tick pricing grid offered by inverted venues becomes valuable when the tick size is large and the ability to trade in dark venues is turned off. The objective in Table 9 through Table 11 is to determine what effect ending the Tick Size Pilot Program has on order routing preferences and whether thinly traded stocks have better liquidity once the “trade-at” rule is lifted and there are no restrictions on dark venue trading. Furthermore, once restrictions in off-exchange trading are removed, do we see a reduction in algorithmic trading considering increased opportunities for stocks to experience queue-jumping activity.

To test this objective, we conduct an event analysis where the event identified is the end of the Tick Size Pilot Program on September 28, 2018. We use data from TAQ, Midas, and CRSP from September through October 2018 to create three samples of stocks that were included in the Tick Size Pilot’s third group and control group. First, we identify all thinly traded stocks in the third group of the Tick Size Pilot that were subject to the “trade-at” rule and label this group as our treatment sample. We next create two separate matched samples from the treatment group of stocks, following the matching method of Davies and Kim (2009), and look to examine differences in inverted venue trading, off-exchange trading, and algorithmic trading.

In Panel A and B of Table 9, we report the matching procedure from Davies and Kim, who show that the best practice is to match firms one-to-one based on market capitalization and share price. Our treatment sample includes 85 thinly traded stocks in group three of the Tick Size

Pilot (TSP) and the first match sample is with 85 thinly traded stocks in the control group of the TSP. The matching procedure yields two samples of stocks that are not significantly different in the matching variables. Our second match sample matches 51 of the 85 thinly traded stocks in TSP group three with 51 actively traded stocks in TSP group 3. Although, this second match sample doesn't differ in price, we can see that actively traded securities in group 3 are significantly larger than the treatment group of thinly traded securities. Furthermore, we can see in Panel C of Table 9 what effect ending the TSP has on all three samples of stocks. While there is no significant change in daily volume across all three samples, the treatment sample of thinly traded stocks and the control sample of actively traded stocks have significant differences in dark venue market share, inverted venue market share, and fragmentation between the first month post TSP (October 2018) and the last month in the TSP (September 2018). The reduction in inverted market share with the simultaneous increase in dark venue market share supports the findings in Comerton-Forde et al. (2019) that inverted venues act as a substitute for off-exchange venues when the ability to trade on off-exchange venues is limited.

Figures 5(a) through 5(c), visually show what impact ending the TSP has on thinly traded securities. Figure 5(a) shows the dark venue market share by date for each sample of stocks, and as we can see on the first day post TSP (October 1, 2018) there is jump in dark market share for the treatment sample while thinly traded control sample experiences no such jump. We can also see that across all three samples, dark venue market share is considerably higher than past literature (Hatheway et al., 2017 report average dark venue market share to be around 26%) and accounts for nearly 40% of trading in the post TSP period. Figure 5(b) shows our initial expectation regarding the substitutability between inverted venues and off-exchange venues, in that, we see a drop in inverted venue market share for the treatment sample and control samples.

Lastly in Figure 5(c) we can see that the end of the TSP shows a drop in fragmentation for both the treatment sample of thinly traded securities and the control sample of actively traded securities.

Table 10 next looks to analyze the market quality differences between the treatment sample and control samples by conducting a difference-in-difference panel regression where the dependent variables include market quality metrics (spreads, depth, volatility, and price efficiency). *Treatment* is equal to 1 if the stock belongs to the treatment sample of thinly traded securities, and 0 if the stock belongs to either control group. *Post TSP* is equal to 1 if the date corresponds to the post TSP period (October 2018) and 0 if the date is in the last month of the TSP (September 2018). The interaction, $Post\ TSP * Treatment$, gives the difference-in-difference estimates between the effect that the ending of the TSP has on the treatment and control samples. Panel A of Table 10 reports the differences in market quality between the treatment sample and the first control sample that includes thinly traded securities in the control group of the TSP. Despite dark venues becoming available for the treatment sample and allowing for sub penny price improvements as well as the ability to hide orders, we see that there is no significant difference in transaction costs nor price efficiency reported for the interaction coefficients $Post\ TSP * Treatment$.

The findings in Panel A of Table 10 show evidence consistent with our prior findings that temporal fragmentation is a determinant in the illiquidity of thinly traded securities. Both inverted venues and off-exchange venues potentially allow for thinly traded securities to improve market quality and reduce the harmful effects brought about by temporal fragmentation. Seeing as these two venues are substitutes (Comerton-Forde et al., 2019), and the results from Table 9 reporting volume moving from inverted venues to off-exchange venues, we should expect there to be no difference between thinly traded stocks in treatment and control sample. However, in Panel B of table 10, where the control sample is actively traded stocks in group 3 of the TSP, we see that post TSP, thinly traded stocks have wider spreads and higher volatility. Thus, thinly traded securities have a deterioration in spread post TSP and this finding can be explained by the change in algorithmic trading activity reported in Table 11.

In Table 11, we provide a difference-in-difference regression analysis of the change in algorithmic trading activity between thinly traded and actively stocks in the post TSP period. Using the four AT proxies of Weller (2018) as the dependent variables, we can see that the interaction term $Post\ TSP * Treatment$ coefficients are significant in the direction that would suggest thinly traded stocks in the post TSP have significantly less algorithmic trading actively than actively traded stocks. The withdrawal of algorithmic traders from thinly traded stocks during the post TSP period is consistent with Yao and Ye (2014) and O'Hara, Saar, and Zhong (2015), who argue that algorithmic trading activity increases in the presence of larger relative tick size. Given that end of the TSP program reverts back to a tick size of $0.01 and Table 7 providing evidence relating to the difficulty of making markets in thinly traded securities, the results in Table 11 further confirms thinly traded securities suffer from a lack of market making activity. This lack of market making activity appears to be a determinant in the illiquidity of thinly traded stocks, which is shown in Table 10, where thinly traded stocks have wider spreads and higher volatility than actively traded stocks in the post TSP period.

VI. CONCLUSION

Market quality and fragmentation remains a relevant discussion as the U.S. market continues to fragment and new efforts by the SEC questioning the "one size fits all" approach to securities market structure has brought about new considerations regarding the optimal market structure for stocks with an average daily volume (ADV) below 100,000 shares. In light of the SEC's recent focus on addressing liquidity concerns for stocks with an ADV below 100,000 shares, thinly traded securities, the goal of this study is to identify possible determinants of the poor liquidity of these types of securities. In October of 2019, the SEC hosted a roundtable discussion to allow other parties to deliberate about what possible explanations exist for the illiquidity of thinly traded securities. Among the commentary of the roundtable, we identify there to be three prominent factors influencing daily liquidity of thinly traded stocks: spatial fragmentation, temporal fragmentation, and market making activity.

The evidence provided in this study suggests that the while a majority of the SEC roundtable participants attribute the poor liquidity of thinly traded securities to spatial fragmentation, number of trading venues in U.S. markets, we find contradicting evidence. We find, using two different metrics to measure spatial fragmentation, that the market is capable of creating liquidity on its own without special advantages given to select exchanges and that spatial fragmentation doesn't appear to be severely impact transactions costs in thinly traded stocks. Temporal Fragmentation and market making activity appear to be more prominent factors contributing to the poor liquidity of thinly traded stocks. We indirectly test temporal fragmentation, arrival of orders at different times, by studying the amount of inverted venue and off-exchange trading activity present in thinly traded stocks compared to actively traded securities. Not only do thinly traded stocks have significantly more inverted venue and off-exchange trading activity as a proportion to overall trading, but as this activity increases, we see improved market quality. Lastly, we find evidence that shows the lack of endogenous liquidity providers and the difficulty of making a market in thinly traded stocks are prominent determinants in the poor liquidity of thinly traded securities. As a robustness check we use the ending of the SEC's Tick Size Pilot Program to confirm that temporal fragmentation and the lack of market makers to be two driving factors influencing the differences in liquidity between thinly traded stocks and actively traded stocks.

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APPENDIX

APPENDIX 1: SUMMARY STATISTICS

Table 1 Summary Statistics – Table reports summary statistics for the sample of data that includes stocks with an average daily volume (ADV) below 100,000 shares identified as thinly traded and those with an ADV greater than 100,000 as actively traded stocks.

Panel A: Thinly Traded Stocks					
Variable	Mean	Median	Std Dev	Min	Max
HHI (1-HHI)	0.6600	0.7275	0.1971	0.00	0.8916
Daily Venues	9.02	10.00	3.18	1.00	13.00
Inverted Market share, %	15.56	14.61	11.8944	0.00	100.00
Dark Market share, %	33.47	29.77	.1856499	0.00	100.00
Quoted Spread, $	0.6685	0.3346	0.8201	0.0100	3.3136
Effective Spread, $	0.3861	0.1788	0.5407	0.0078	2.5215
NBBO Depth, Shares	1,266.16	400.32	9,826.08	100.00	879,601.80
NBBO Depth, Dollar	77,063.85	13,236.00	560,523.20	663.94	49,935,998.00
Realized Spread, $	0.2251	0.0919	0.3388	-0.0345	1.6280
Price Impact, $	0.1266	0.0472	0.2024	-0.0263	0.9730
Price Volatility, bps	0.7043	0.1842	1.7080	0.0090	10.6387
S-T Return Volatility, bps	0.5455	0.3893	0.4791	0.0448	2.3292
Return, (Amihud et al., 94) bps	-0.00001	0.00	0.2339	-1.8194	1.8409
Variance Ratio	0.5924	0.6337	0.2347	0.0153	0.9979
Abs. Return Autocorrelation	0.1761	0.1429	0.1486	0.0000	1.0000
Trade/Order (Midas)	0.3781	0.3467	0.2350	0.0000	1.0000
Odd lot/Volume (Midas)	0.0416	0.0321	0.0367	0.0001	0.1896
Cancel/Order (Midas)	20.18	14.63	17.99	3.90	102.00
Avg. Trade size (Midas)	59.11	47.94	50.00	5.00	295.64
Daily Volume, shares	49,723.39	22,399.00	146,463.30	1.00	9,339,377
Market Capitalization, $ billions	1.2879	.2850	2.6043	.0103	16.5355
Price, $	171.90	25.14	547.26	0.77	4699.00
# of firms	100				

Panel B: Actively Traded Stocks					
Variable	Mean	Median	Std Dev	Min	Max
HHI (1-HHI)	0.7838	0.7943	0.0530	0.4410	0.8913
Daily Venues	12.42	12.00	0.62	9.00	13.00
Inverted Market share, %	15.34	14.45	5.74	2.13	67.40
Dark Market share, %	31.58	30.72	9.37	3.35	88.99
Quoted Spread, $	0.0714	0.0351	0.1182	0.0100	2.3883
Effective Spread, $	0.0328	0.0173	0.0492	0.0078	0.7301
NBBO Depth, Shares	1,504.05	469.57	5,346.72	214.29	310,076.90
NBBO Depth, Dollar	75,659.16	21,288.00	220,255.50	2,385.52	15,187,231.00
Realized Spread, $	0.0097	0.0044	0.0258	-0.0345	0.7709
Price Impact, $	0.0235	0.0128	0.0400	-0.0263	0.9730
Price Volatility, bps	0.3735	0.1864	0.8396	0.0090	10.6387
S-T Return Volatility, bps	0.1938	0.1566	0.1439	0.0448	2.3292
Return, (Amihud et al., 94) bps	0.0006	0.0011	1.0424	-7.3235	7.3408
Variance Ratio	0.4782	0.4887	0.2613	0.0153	0.9979
Abs. Return Autocorrelation	0.1565	0.1335	0.1165	0.0000	0.7973
Trade/Order (Midas)	0.2309	0.2232	0.1081	0.0076	0.7047
Odd lot/Volume (Midas)	0.0489	0.0447	0.0227	0.0001	0.1896
Cancel/Order (Midas)	14.82	13.35	7.25	3.90	102.00
Avg. Trade size (Midas)	78.16	72.10	34.15	6.98	295.64
Daily Volume, shares	3,555,930	719,383	7,674,717	18,214	143,253,494
Market Capitalization, $ billions	95.3551	3.9888	192.4771	.0949	1,304.7647
Price, $	3,260.93	44.57	31,319.59	1.48	340,380.00
# of firms	100				

APPENDIX 2: UNIVARIATE

Table 2: Univariate statistics –. Both t-stats and p-values are reported in the column 4 and 5 of each panel. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Variables	(1) Thinly Traded	(2) Actively Traded	(3) Diff.	(4) t-stat	(5) p-value
Market Capitalization, ($ billion)	1.2546	92.3777	-91.1232	-4.7604***	0.0000
Price	167.1219	1667.591	-1500.469	-0.9553	0.3406
Daily Volume, shares	48573.09	3520582	-3472009	-4.9794***	0.0000
HHI (1-HHI)	.646688	.7842812	-.1375931	-8.4318***	0.0000
Daily Venues	8.777544	12.40939	-3.631844	-12.2073***	0.0000
Quoted Spread, $	.6556272	.0705573	.5850698	7.7321***	0.0000
Effective Spread, $	.3830465	.0324751	.3505714	6.8164***	0.0000
NBBO Depth, Shares	1522.657	1661.165	-138.5072	-0.2064	0.8367
Daily Price Volatility	.6823484	.3682995	.314049	1.7423*	0.0830
S-T return Volatility, LRS 94 bps	.5507078	.1918565	.3588513	9.8387***	0.0000
Variance Ratio	.5975205	.4787102	.1188102	18.3239***	0.0000
Abs. Return Autocorrelation	.1846876	.1566893	.0279983	5.1868***	0.0000
Dark Market share, %	.3414994	.3153547	.0261447	2.4391**	0.0156
Inverted Market Share, %	15.26814	15.46434	-.1962012	- 0.2664	0.7902
Trade/Order (Midas)	.0420828	.0491991	- .0071164	-3.3288***	0.0010
Odd lot/Volume (Midas)	.374211	.2292523	.1449586	7.6269***	0.0000
Avg. Trade size (Midas)	63.14019	78.77399	- 15.63381	-2.9624***	0.0034
Cancel/Order (Midas)	20.63536	14.75369	5.88167	5.3155***	0.0000

APPENDIX 3: SPATIAL FRAGMENTATION AND MARKET QUALITY

Table 3 – Spatial Fragmentation and Market Quality: Reports the regression analysis establish a relation between the amount of fragmentation measured by the HHI and market quality. Where the dependent variables include quoted spread, effective spread, quoted depth, short-term volatility, price volatility, variance ratio, and return autocorrelation. The coefficients for equation 4 include only securities included in the thinly traded sample, 100 total stocks. HHI_{it} is Herfindahl-Hirschman Index, calculated as 1-HHI_{it}, and can be interpreted to be that larger values equate to more fragmentation. The Control variables include quoted spread, log volume, log market cap, and log price. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

VARIABLES	(1) Quoted Spread - $	(2) Effective Spread - $	(3) NNBO Depth - Shares	(4) Price Volatility - bps	(5) S-T Return Volatility (LRS, 94)- bps	(6) Variance Ratio	(7) Return Autocorrelation
HHI_{it}	0.0800**	0.00304	-1,425	-0.200	-0.0370	-0.00640	-0.0461***
	(2.066)	(0.126)	(-1.313)	(-1.237)	(-0.623)	(-0.551)	(-3.301)
Log Volume, shares	-0.0413***	-0.0442***	458.8**	-0.0663	-0.0454***	-0.0211***	-0.0156***
	(-3.085)	(-5.295)	(2.387)	(-1.278)	(-3.634)	(-7.187)	(-6.024)
Log Market Cap.	0.0370	0.0342	-92.18	0.286	0.0230	0.00153	-0.00367
	(0.836)	(0.919)	(-1.165)	(1.025)	(0.422)	(0.722)	(-1.269)
Log price	0.161**	0.129*	-152.8	0.915	0.117	0.00174	-0.00989**
	(2.131)	(1.833)	(-0.925)	(1.599)	(0.747)	(0.397)	(-2.469)
Quoted Spread, $			211.1				
			(1.626)				
Constant	0.0427	-0.171	-585.7	-7.060	0.503	0.751***	0.451***
	(0.0552)	(-0.246)	(-0.204)	(-1.293)	(0.354)	(10.22)	(7.295)
Observations	22,600	22,582	22,334	22,599	22,380	22,205	21,686
R-squared	0.093	0.150	0.014	0.038	0.075	0.024	0.025
Number of Stocks	100	100	100	100	100	100	100
Day Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes	Yes	Yes	Yes

APPENDIX 4: DAILY VENUE CHANGES

Table 4 – Daily Venue Changes: examine this difference in number of daily venue changes between thinly traded and actively traded securities. We present both a univariate and multivariate likelihood analysis. Panel A records the univariate results where the dependent variable is a dummy variable equal to 1 if the number of daily venues for stock *i*, is different on day *t* compared to day *t-1*, and 0 otherwise. Panel B records results for both a Probit and Linear Probability model as a multivariate analysis regarding the likelihood of thinly traded stocks to change the number of daily venues. Column 1 reports the marginal effects from the Probit regression model. Column 2 reports the coefficients from the OLS regression. Where *venue change* is equal to 1 if the number of daily venues for stock *i*, is different on day *t* compared to day *t-1*, and 0 otherwise. Control variables include log volume, log market cap, and log price. In Panel C the dependent variable is the percentage change in $Daily\ Venues_{it}$, calculated as the number of daily venues executing a trade for stock *i*, on day *t*, denoted by %Δ. Our main variable of interest is *Thinly traded*, which is equal to 1 if the security belongs to the thinly traded sample, and 0 otherwise. Other variables of interest include percentage changes in quoted and effective spreads, volatility, price efficiency measures, and return. The Control variables include HHI_{it} as the Herfindahl-Hirschman Index (calculated as 1-HHI_{it}), log volume, log market cap, and log price. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Variables	(1) Thinly Traded	(2) Actively Traded	(3) Diff. (Thinly – Actively)	(4) t-stat
Venue Change Dummy	.70479	.36279	.3420***	16.6151

Panel B: Multivariate Probit and Linear Probability

VARIABLES	(1) Venue Change - Probit	(2) Venue Change - Linear Probability
Thinly Traded	0.427***	0.257***
	(9.597)	(6.975)
Log Market Capitalization	0.00467	0.00448
	(0.667)	(0.710)
Log price	-0.00717	-0.00465
	(-0.507)	(-0.390)
Log Volume, shares	-0.00805	-0.00679*
	(-1.537)	(-1.669)
Constant		0.0952
		(0.396)
Observations	44,584	44,584
R-squared		0.200
Day Fixed Effects	Yes	Yes
Stock Fixed Effects	Yes	Yes
Stock Clustered Errors	Yes	Yes

Panel C			
	(1)	(2)	(3)
VARIABLES	$\%\Delta Daily\ Venues$	$\%\Delta Daily\ Venues$	$\%\Delta Daily\ Venues$
Thinly Traded	0.245***	0.285***	0.254***
	(6.532)	(7.938)	(5.831)
%Δ Effective Spread, $	-0.0673***	-0.0604***	
	(-6.337)	(-4.277)	
%Δ Quoted Spread, $	0.0216***	0.0170***	
	(4.565)	(2.836)	
%Δ S-T Volatility (LRS, 94)	0.00769*		-0.000836
	(1.770)		(-0.217)
%Δ Return (Amihud et al., 94), bps	0.00230***		0.00309***
	(2.949)		(3.744)
%Δ Variance Ratio	-0.00252***		-0.00245***
	(-3.093)		(-2.967)
%Δ Price Volatility, bps	0.0157***		0.00986***
	(5.146)		(3.221)
HHI_{it}	0.659***	0.748***	0.661***
	(10.43)	(11.11)	(10.41)
Log Market Capitalization	0.00264	-0.000213	0.00437
	(0.545)	(-0.0378)	(0.748)
Log price	0.0121	0.0142	0.0163
	(1.005)	(0.932)	(1.116)
Log Volume, shares	0.0444***	0.0538***	0.0455***
	(8.095)	(11.00)	(7.492)
Constant	-1.402***	-1.566***	-1.490***
	(-7.198)	(-7.354)	(-6.824)
Observations	41,526	44,069	41,587
R-squared	0.134	0.157	0.125
Day Fixed Effects	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes

APPENDIX 5: TEMPORAL FRAGMENTATION

Table 5 – Temporal Fragmentation: Reports the regression analysis to indirectly test whether fragmentation plays a role in the illiquidity of thinly traded stocks. We us the following panel regression model: In Panel A the dependent variables are the various market quality metrics (quoted and effective spread and the independent variables include the log of inverted volume and market share of inverted venues. In Panel B, the dependent variables are the amount of volume, number of trades, and market share of inverted venues for stock *i*, on day *t*. Using the SECs Midas data set, we're able to observe the amount of volume and trades at each of the four inverted venues (These exchanges include the Bats-Y, Nasdaq Boston, Edge-A, and NYSE National). The main explanatory variable in Panel B is $Thinly\ traded_{it}$, which is defined equal to 1 if the security belongs to the thinly traded sample, and 0 other wise. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A							
VARIABLES	(1) Quoted Spread - $	(2) Effective Spread - $	(3) NNBO Depth - Shares	(4) Price Volatility	(5) S-T Return Volatility (LRS, 94)- bps	(6) Variance Ratio	(7) Return Autocorrelation
Log Inverted volume	-0.0454***	-0.0166***	-64.39	0.0961**	-0.0331***	-0.00896**	-0.00161
	(-3.456)	(-2.835)	(-1.146)	(2.552)	(-3.048)	(-2.545)	(-0.823)
Inverted Market share	0.00292***	0.000613	6.684	-0.0122***	0.000940	0.000444*	6.73e-05
	(2.908)	(1.372)	(1.040)	(-3.051)	(1.527)	(1.674)	(0.390)
Log Volume, shares	-0.00737	-0.0427***	303.5**	-0.214**	-0.0333**	-0.0187***	-0.00823***
	(-0.301)	(-3.646)	(2.522)	(-2.007)	(-2.035)	(-3.739)	(-3.084)
Log Market Capitalization	0.0383	0.0331	-101.1	0.283	0.0261	0.00204	-0.00225
	(0.892)	(0.951)	(-1.181)	(1.027)	(0.482)	(0.879)	(-1.153)
Log price	0.156**	0.121*	-211.6	0.903	0.118	0.00218	-0.00657**
	(2.203)	(1.872)	(-1.263)	(1.576)	(0.772)	(0.478)	(-2.181)
Quoted Spread, $			30.28				
			(0.505)				
Constant	0.0256	-0.0314	686.4	-6.128	0.450	0.776***	0.321***
	(0.0343)	(-0.0502)	(0.261)	(-1.181)	(0.328)	(9.392)	(6.963)
Observations	20,467	20,449	20,296	20,477	20,439	20,416	20,224
R-squared	0.107	0.180	0.015	0.041	0.089	0.025	0.017
Number of Stocks	100	100	100	100	100	100	100
Day Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Panel B

VARIABLES	(1) Inverted Volume	(2) Inverted Trades	(3) Inverted Market Share
Thinly Traded	-854,737***	-12,682***	4.156*
	(-9.303)	(-24.62)	(1.773)
Log Market Capitalization	-1,443	-23.07	-0.259
	(-0.132)	(-0.383)	(-0.748)
Log price	-39,234*	-211.1*	-1.223*
	(-1.902)	(-1.667)	(-1.877)
Log Volume, shares	31,922***	276.3***	-1.483***
	(4.026)	(4.971)	(-6.587)
Constant	829,304***	12,093***	47.28***
	(2.881)	(6.818)	(4.838)
Observations	44,689	44,689	44,525
R-squared	0.891	0.884	0.314
Day Fixed Effects	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes

APPENDIX 6: OFF-EXCHANGE TRADING AND TEMPORAL FRAGMENTATION

Table 6 – Off-Exchange Trading and Temporal Fragmentation: Reports the regression analysis investigating the relation between off-exchange trading (dark venue trading) for only thinly traded securities and the market quality in these securities. The dependent variables are market quality measures which include quoted and effective spread, depth, volatility, and price efficiency. The independent variables are log of dark volume and dark venue market share, obtained from "D" TAQ. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

VARIABLES	(1) Quoted Spread - $	(2) Effective Spread - $	(3) NNBO Depth - Shares	(4) Price Volatility	(5) S-T Return Volatility (LRS, 94)- bps	(6) Variance Ratio	(7) Return Autocorrelation
Log Dark Volume	-0.0470*	0.00879	-493.8	0.423***	-0.00401	-0.0118**	-0.0159***
	(-1.707)	(0.589)	(-0.698)	(3.034)	(-0.203)	(-2.311)	(-4.887)
Dark Market share	0.269***	0.0463	1,393	-1.301**	0.174*	0.0509**	0.0608***
	(2.999)	(0.824)	(0.586)	(-2.434)	(1.883)	(2.404)	(4.092)
Log Volume, shares	0.00624	-0.0529**	611.0	-0.472***	-0.0510*	-0.0114**	-0.00138
	(0.181)	(-2.625)	(1.069)	(-2.743)	(-1.816)	(-2.090)	(-0.417)
Log Market Capitalization	0.0403	0.0349	-85.22	0.272	0.0245	0.00202	-0.00310
	(0.868)	(0.930)	(-1.032)	(1.023)	(0.452)	(0.854)	(-1.247)
Log price	0.170**	0.130*	-161.7	0.878	0.120	0.00292	-0.00818**
	(2.163)	(1.858)	(-0.908)	(1.612)	(0.772)	(0.632)	(-2.460)
Quoted Spread, $			51.82				
			(0.628)				
Constant	-0.145	-0.189	705.8	-5.998	0.483	0.723***	0.380***
	(-0.180)	(-0.274)	(0.255)	(-1.183)	(0.347)	(9.468)	(7.296)
Observations	22,506	22,489	22,235	22,473	22,317	22,169	21,642
R-squared	0.098	0.156	0.011	0.047	0.081	0.024	0.024
Number of Stocks	100	100	100	100	100	100	100
Day Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes	Yes	Yes	Yes

APPENDIX 7: ALGORITHMIC TRADING AND THINLY TRADED SECURITIES

Table 7 – Regression analysis –Algorithmic Trading Activity and Thinly Traded Securities: Reports the regression analysis investigating the four algorithmic trading (AT) proxies of Weller (2018) thinly traded securities. The Weller measures of AT activity include *odd lot-to-volume*, *trade-to-order volume*, *cancel-to-trade ratio,* and *average trade size*. For each AT measure we use the log as the dependent variable.

$$Weller\ (2018)\ AT\ proxies = \alpha_0 + \beta_1 Thinly\ Traded_{it} + \delta_1 Controls_{it} + \varepsilon_{it}$$

Thinly traded, which is equal to 1 if the security belongs to the thinly traded sample, and 0 otherwise. The control variables include short-term return volatility, quoted spread, log volume, log market cap, and log price. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

VARIABLES	(1) Trades to Orders	(2) Odd lot to Volume	(3) Cancels to Trades	(4) Avg. Trade Size - Midas
Thinly Traded	2.843***	-1.020***	-1.495***	0.510***
	(9.849)	(-3.686)	(-9.569)	(2.609)
Log Market Capitalization	0.0296	0.0656	-0.0194	-0.0418
	(0.723)	(1.419)	(-0.839)	(-1.344)
Log price	0.144*	0.153***	-0.0724	-0.105***
	(1.690)	(2.671)	(-1.393)	(-2.824)
Log Volume, shares	0.516***	-0.342***	-0.214***	0.279***
	(17.94)	(-17.05)	(-15.52)	(18.62)
Quoted Spread, $	-0.158***	0.0812***	0.118***	-0.0602***
	(-4.706)	(3.031)	(4.594)	(-3.834)
S-T Return Volatility (LRS, 94)	-0.134***	0.0667***	0.0738***	-0.0454***
	(-3.642)	(3.381)	(2.688)	(-3.738)
Constant	-13.40***	1.188	7.438***	1.568**
	(-10.71)	(1.049)	(10.01)	(2.051)
Observations	44,406	43,937	44,298	44,298
R-squared	0.540	0.730	0.430	0.802
Day Fixed Effects	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes

APPENDIX 8: AFFIRMATIVE MARKET MAKERS

Table 8 – Regression analysis –Affirmative Market Makers: Reports the regression analysis comparing the liquidity of thinly traded NYSE-listed stocks, the sole exchange with market making specialist classified as DMMs, to other exchange-listed thinly traded stocks using a difference-in-difference panel regression, where our treatment sample is all thinly traded, NYSE-listed stocks. Where the dependent variables are market quality measures which include quoted and effective spread, depth, volatility, and price efficiency. $NYSE\ Listed_{it}$ is equal to 1 if the stock is listed by the NYSE, and 0 otherwise. *Thinly traded*, which is equal to 1 if the security belongs to the thinly traded sample, and 0 otherwise. The interaction, $NYSE\ Listed * Thinly\ traded_{it}$, gives the difference-in-difference estimate for each variable. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

VARIABLES	(1) Quoted Spread - $	(2) Effective Spread - $	(3) NNBO Depth - Shares	(4) Price Volatility	(5) S-T Return Volatility Lee 94	(6) Variance Ratio	(7) Return Autocorrelation
NYSE Listed	0.223	0.119	500.6	1.262	0.128	0.0198	-0.0526***
	(1.433)	(0.903)	(0.926)	(1.248)	(0.475)	(1.215)	(-4.378)
Thinly Traded	0.418	0.217	2,424**	2.143	0.203	-0.00474	-0.116***
	(1.635)	(0.993)	(2.532)	(1.279)	(0.441)	(-0.163)	(-5.295)
$NYSE\ Listed * Thinly\ Traded_{it}$	1.355***	0.505*	-780.4	-1.844	-0.507	-0.0744**	0.0691***
	(4.411)	(1.675)	(-1.324)	(-0.745)	(-0.660)	(-2.373)	(3.722)
Log Volume, shares	-0.0323***	-0.0355***	102.1	-0.0209	-0.0247**	-0.0168***	-0.0127***
	(-3.328)	(-5.108)	(0.744)	(-0.440)	(-2.031)	(-6.660)	(-5.770)
Log Market Capitalization	0.0258	0.0227	999.4***	0.197	0.00440	0.00244	-0.00260
	(0.688)	(0.784)	(2.677)	(0.941)	(0.112)	(0.826)	(-1.297)
Log price	0.138**	0.101*	-1,745***	0.704	0.0760	0.00816*	-0.00590**
	(2.134)	(1.727)	(-2.835)	(1.495)	(0.575)	(1.684)	(-2.167)
Quoted Spread, $			978.1*				
			(1.702)				
Constant	-0.690	-0.452	-17,026**	-7.766	0.235	0.634***	0.471***
	(-0.758)	(-0.590)	(-2.445)	(-1.330)	(0.153)	(6.391)	(6.723)
Observations	44,750	44,731	44,290	44,698	44,423	44,220	43,567
R-squared	0.855	0.890	0.042	0.708	0.542	0.098	0.060
Day and Stock Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes	Yes	Yes	Yes

APPENDIX 9: UNIVARIATE POST TICK SIZE PILOT PROGRAM (ROBUSTNESS)

Table 9: Univariate statistics Post Tick Size Pilot Program (Robustness): –Table reports results for matching procedure and univariate test surrounding the end date of the Tick Size Pilot Program (TSP). There are two match samples that are matched on market capitalization and price (Davies and Kim, 2009). Matched Sample #1 (Panel A) includes the treatment sample, thinly traded stocks in Group 3 of the TSP (85 stocks), and control sample 1, thinly traded stocks in the Control Group of the TSP (85 stocks). Matched Sample #2 (Panel B) includes thinly traded stocks in Group 3 of the TSP (51 stocks), and control sample 2, actively traded stocks in Group 3 of the TSP (51 stocks). Both t-stats and p-values are reported in the column 4 and 5 of each panel. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Matching Test – Thinly traded stocks in Group 3 of Tick Size Pilot and Thinly traded stocks in Control Group of Tick Size Pilot

Variable	(1) Treatment (Thinly Traded in Group 3)	(2) Control 1 (Thinly Traded in Control)	(3) Diff.	(4) t-stat	(5) p-value
Market Capitalization	372,231,005	365,468,274	6,762,731	0.0980	0.9221
Price	35.11	29.73	5.39	0.5995	0.5496
Min error mean	2.8679				

Panel B: Matching Test – Thinly traded stocks and Actively traded stocks in Group 3 of Tick Size Pilot

Variable	(1) Treatment (Thinly Traded in Group 3)	(2) Control 2 (Actively Traded in Group 3)	(3) Diff.	(4) t-stat	(5) p-value
Market Capitalization	520,721,464	70,488,7892	-184,166,428	- 1.6693*	0.0982
Price	30.74	28.51	2.23	0.3556	0.7229
Min error mean	2.4212				

Panel C: Univariate – During and Post Tick Size Pilot Program (Tick Size Pilot end date: September 28, 2018)

Variables	Sample	(1) October 2018 (First month post TSP)	(2) September 2018 (Last month in TSP)	(3) Diff.	(4) t-stat	(5) p-value
Daily Volume, shares	Treatment	27,595.88	29,691.92	-2,096.04	-0.4760	0.6347
	Control 1	30,828.05	28,939.46	1,888.59	0.4997	0.6180
	Control 2	286,794.10	287,213.10	-419.07	-0.0076	0.9939
Dark Market share, %	Treatment	36.7575	24.9332	11.8243	7.4872***	0.0000
	Control 1	36.1733	35.8885	0.2848	0.1608	0.8724
	Control 2	32.5375	27.2644	5.2731	3.1913***	0.0019
Inverted Market Share, %	Treatment	16.8135	31.8126	-14.9991	- 9.8332***	0.0000
	Control 1	15.3896	15.9795	-0.5899	-0.5135	0.6082
	Control 2	18.6502	39.3187	-20.6686	-15.6305***	0.0000
HHI (1-HHI)	Treatment	0.5973	0.6694	-0.0721	- 2.8961***	0.0043
	Control 1	0.6027	0.6229	-0.0202	-0.8183	0.4144
	Control 2	0.7600	0.8177	-0.0577	-6.2244***	0.0000

APPENDIX 10: MARKET QUALITY POST TICK SIZE PILOT PROGRAM (ROBUSTNESS)

Table 10 – Market Quality Post Tick Size Pilot Program (Robustness): Reports the regression analysis to indirectly test whether fragmentation plays a role in the illiquidity of thinly traded stocks. Using a Difference-in-Difference fixed effect model we examine various market quality metrics around the end date of the SEC Tick Size Pilot Program (TSP) on September 28, 2018. We create two different matched sample comparisons, where the samples are matched on market capitalization and price (Davies and Kim, 2009). Matched Sample #1 (Panel A) includes the treatment sample, thinly traded stocks in Group 3 of the TSP (85 stocks), and control sample 1, thinly traded stocks in the Control Group of the TSP (85 stocks). Matched Sample #2 (Panel B) includes thinly traded stocks in Group 3 of the TSP (51 stocks), and control sample 2, actively traded stocks in Group 3 of the TSP (51 stocks). *Treatment* is equal to 1 if the stock is a thinly traded stock in Group 3 of the TSP, and 0 if the stock belongs in controls samples identified previously. *Post TSP* is a dummy variable equal to one for all dates after September 28, 2018, and 0 for all dates before the end of the program. The interaction, *Post TSP*Treatment* gives the difference-in-difference estimate between the effect that the elimination of the TSP had on Thinly Traded securities in the treatment sample and those securities in the control samples. Control Variables include Log Market Capitalization, Log Price, Log Volume, and Quoted Spread. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively

Panel A: Matched Sample #1

VARIABLES	(1) Quoted Spread - $	(2) Effective Spread - $	(3) NNBO Depth - Shares	(4) Realized Spread - $	(5) Price Impact - $	(6) Price Volatility	(7) S-T Return Volatility (LRS, 94)-bps	(8) Variance Ratio	(9) Return Autocorrelation
Post TSP	0.133***	0.169***	-273.9	0.357**	0.000924	0.155***	0.510***	0.0154	-0.0397**
	(2.645)	(2.721)	(-0.425)	(2.085)	(0.854)	(4.111)	(3.158)	(0.560)	(-2.239)
Treatment	-0.645	-1.030	-1,178	0.0644	-0.00328	-0.693	-12.63***	0.153	0.0964
	(-1.239)	(-1.234)	(-0.259)	(0.0960)	(-0.415)	(-1.230)	(-5.311)	(0.758)	(0.822)
*Post TSP*Treatment*	0.0386	0.0283	-894.2***	0.0407	-0.00144**	-0.00888	0.0831	0.0220	-0.00774
	(1.202)	(1.378)	(-2.672)	(1.190)	(-1.992)	(-0.349)	(0.892)	(1.528)	(-0.870)
Log Volume, shares	-0.0380***	-0.0294**	122.2	-0.116	0.00123***	0.000865	-0.0735***	-0.0200***	-0.00901***
	(-4.430)	(-2.170)	(1.476)	(-1.564)	(3.564)	(0.195)	(-3.582)	(-5.846)	(-3.234)
Log Market Capitalization	0.369	0.552	-92.62	0.824	-0.00332	-0.305**	-0.775	-0.0865	-0.00289
	(0.689)	(0.984)	(-0.123)	(0.966)	(-1.068)	(-2.075)	(-1.353)	(-0.571)	(-0.0433)
Log price	-0.228	-0.671	-1,090	-0.461	0.00104	0.129	-4.270***	0.103	0.0278
	(-0.409)	(-0.932)	(-0.514)	(-0.555)	(0.241)	(0.508)	(-4.015)	(0.596)	(0.339)
Quoted Spread, $			55.42						
			(0.760)						
Constant	-5.284	-7.846	5,042	-13.34	0.0551	5.909**	31.95***	2.062	0.230
	(-0.610)	(-0.920)	(0.417)	(-0.955)	(1.090)	(2.315)	(3.214)	(0.849)	(0.214)
Observations	7,011	6,977	7,011	7,014	6,928	7,020	6,978	6,924	6,828
R-squared	0.702	0.460	0.395	0.112	0.094	0.806	0.492	0.080	0.107
Day Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Panel B: Matched Sample #2									
VARIABLES	(1) Quoted Spread - $	(2) Effective Spread - $	(3) NNBO Depth - Shares	(4) Realized Spread - $	(5) Price Impact - $	(6) Price Volatility	(7) S-T Return Volatility (LRS, 94)- bps	(8) Variance Ratio	(9) Return Autocorrelation
Post TSP	-0.00138	0.00573	-211.8	-0.0305	-0.000381	0.0205	0.116	0.0549	-0.0158
	(-0.0849)	(0.476)	(-0.122)	(-0.777)	(-0.621)	(0.799)	(0.926)	(1.341)	(-0.799)
Treatment	-0.0825***	-0.0417***	2,886	-0.0166	-0.000951	-0.0717***	-0.587***	0.0682	-0.0392*
	(-3.005)	(-2.948)	(0.737)	(-1.062)	(-1.218)	(-2.682)	(-3.265)	(1.303)	(-1.865)
*Post TSP*Treatment*	0.132***	0.0636***	419.9	0.0332***	0.000633	0.0397**	0.392***	0.0217	-0.0167**
	(5.541)	(4.940)	(0.683)	(3.067)	(1.255)	(2.034)	(3.682)	(1.034)	(-2.160)
Log Volume, shares	-0.0167***	-0.00697**	1,413*	-0.0190***	0.000438***	0.0415***	-0.0391	-0.0236***	-0.000436
	(-4.737)	(-2.036)	(1.755)	(-2.702)	(2.676)	(4.238)	(-0.959)	(-3.378)	(-0.111)
Log Market Capitalization	0.0138	-0.00967	-3,823	0.0719**	-0.00330**	-0.115*	-0.364	-0.0433	-0.112**
	(0.279)	(-0.453)	(-0.504)	(2.338)	(-2.306)	(-1.833)	(-1.014)	(-0.400)	(-2.585)
Log price	0.0233	0.0333	18,950	-0.0548	0.00126	0.0207	0.00369	0.0802	0.0998**
	(0.372)	(1.222)	(1.152)	(-1.643)	(0.513)	(0.285)	(0.00376)	(0.736)	(2.100)
Quoted Spread, $			683.8*						
			(1.705)						
Constant	-0.0639	0.232	14,063	-1.024**	0.0593**	1.883*	8.225	1.398	2.178***
	(-0.0745)	(0.630)	(0.114)	(-2.073)	(2.417)	(1.755)	(1.401)	(0.730)	(2.859)
Observations	4,277	4,274	4,277	4,267	4,265	4,264	4,255	4,245	4,255
R-squared	0.738	0.500	0.296	0.213	0.154	0.755	0.315	0.101	0.053
Day Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

APPENIX 11: ALGORITHMIC TRADING POST TICK SIZE PILOT PROGRAM (ROBUSTNESS)

Table 11 – Regression analysis –Algorithmic Trading Activity and Thinly Traded Securities Post Tick Size Pilot Program (Robustness): Using a Difference-in-Difference fixed effect model we examine the log of four algorithmic trading (AT) proxies of Weller (2018) in thinly traded and actively traded securities around the end date of the SEC Tick Size Pilot Program (TSP) on September 28, 2018. The Weller measures of AT activity include *odd lot-to-volume*, *trade-to-order volume*, *cancel-to-trade ratio*, and *average trade size*. We create a matched sample comparison, where the samples are matched on market capitalization and price (Davies and Kim, 2009). Matched Sample #2 includes thinly traded stocks in Group 3 of the TSP (51 stocks), and control sample 2, actively traded stocks in Group 3 of the TSP (51 stocks). *Treatment* is equal to 1 if the stock is a thinly traded stock in Group 3 of the TSP, and 0 if the stock belongs in the control sample identified previously. *Post TSP* is a dummy variable equal to one for all dates after September 28, 2018, and 0 for all dates before the end of the program. The interaction, *Post TSP*Treatment* gives the difference-in-difference estimate between the effect that the elimination of the TSP had on Thinly Traded securities in the treatment sample and those securities in the control sample. Control Variables include short-term return volatility (Lee et al., 1994), Log Market Capitalization, Log Price, Log Volume, and Quoted Spread. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

	Matched Sample #2			
VARIABLES	(1) Trades to Orders	(2) Odd lot to Volume	(3) Cancels to Trades	(4) Avg. Trade Size
Post TSP	-0.340***	0.0748**	0.512***	-0.0203
	(-6.107)	(2.140)	(9.529)	(-0.532)
Treatment	-0.0941	0.543**	0.0855	-0.208
	(-0.675)	(2.542)	(0.488)	(-1.650)
*Post TSP*Treatment*	0.0726*	0.0490	-0.179**	-0.0393
	(1.681)	(1.379)	(-2.382)	(-1.447)
Log Market Capitalization	0.418	-0.388	-0.303	0.206
	(1.437)	(-0.907)	(-0.830)	(0.805)
Log price	-0.124	0.571	0.151	-0.0788
	(-0.356)	(1.328)	(0.389)	(-0.240)
Log Volume, shares	0.0613***	-0.0319***	-0.0405**	0.0373***
	(5.903)	(-3.585)	(-2.489)	(4.003)
Quoted Spread, $	-0.0518	-0.0572	0.166**	0.0357
	(-0.772)	(-1.204)	(2.083)	(0.929)
S-T Return Volatility (LRS, 94)	0.0115	0.00213	-0.0230	-0.000698
	(1.348)	(0.296)	(-1.580)	(-0.0916)
Constant	-11.75**	4.502	8.698	0.281
	(-2.292)	(0.592)	(1.355)	(0.0632)
Observations	4,254	4,254	4,254	4,254
R-squared	0.809	0.946	0.742	0.941
Day Fixed Effects	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes

APPENDIX 12: TEMPORAL FRAGMENTATION POST TICK SIZE PILOT (ROBUSTNESS)

Table A (1) – Temporal Fragmentation Post Tick Size Pilot Program (Robustness): Reports the regression analysis to indirectly test whether fragmentation plays a role in the illiquidity of thinly traded stocks. Using a Difference-in-Difference fixed effect model we examine dark market share, inverted venue market share, and maker-taker venue market share around the end date of the SEC Tick Size Pilot Program (TSP) on September 28, 2018. We create two different matched sample comparisons, where the samples are matched on market capitalization and price (Davies and Kim, 2009). Matched Sample #1 (Columns 1-3) includes the treatment sample, thinly traded stocks in Group 3 of the TSP (85 stocks), and control sample 1, thinly traded stocks in the Control Group of the TSP (85 stocks). Matched Sample #2 (Columns 4-6) includes thinly traded stocks in Group 3 of the TSP (51 stocks), and control sample 2, actively traded stocks in Group 3 of the TSP (51 stocks). *Treatment* is equal to 1 if the stock is a thinly traded stock in Group 3 of the TSP, and 0 if the stock belongs in controls samples identified previously. *Post TSP* is a dummy variable equal to one for all dates after September 28, 2018, and 0 for all dates before the end of the program. The interaction, *Post TSP*Treatment* gives the difference-in-difference estimate between the effect that the elimination of the TSP had on Thinly Traded securities in the treatment sample and those securities in the control samples. Control Variables include Log Market Capitalization, Log Price, Log Volume, and Quoted Spread. All regression standard errors are clustered by stock and include both stock and day fixed effects (unless otherwise specified). T-statistics are recorded in the parentheses and asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively

	Matched Sample #1			Matched Sample #2		
	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	Dark Market Share	Inverted Market Share	Maker-Taker Market Share	Dark Market Share	Inverted Market Share	Maker-Taker Market Share
Post TSP	-0.0163	7.353***	-9.966***	0.0196	-14.86***	12.34***
	(-0.788)	(4.396)	(-5.571)	(1.197)	(-6.568)	(5.540)
Treatment	0.415***	28.93*	-29.37*	-0.00703	9.697***	-11.11***
	(2.832)	(1.889)	(-1.760)	(-0.159)	(3.487)	(-3.824)
*Post TSP*Treatment*	0.118***	-14.87***	14.41***	0.0575***	2.262	-2.673
	(10.97)	(-13.72)	(12.08)	(5.004)	(1.362)	(-1.512)
Log Market Capitalization	0.0347	12.38**	-8.065	0.0676	9.710*	-8.693
	(0.574)	(2.276)	(-0.994)	(0.865)	(1.682)	(-1.486)
Log price	0.155*	-0.541	-4.908	-0.154	-7.555	6.713
	(1.781)	(-0.0642)	(-0.467)	(-1.506)	(-0.812)	(0.761)
Log Volume, shares	0.0239***	-2.262***	1.878***	0.0426***	-3.109***	2.417***
	(5.528)	(-7.822)	(6.670)	(5.145)	(-5.059)	(4.386)
Quoted Spread, $	0.0205**	1.255*	-1.300	0.0781***	7.963***	-7.866***
	(2.442)	(1.666)	(-1.519)	(2.935)	(3.565)	(-3.401)
Constant	-0.947	-206.7**	237.0*	-1.232	-94.41	179.8*
	(-0.982)	(-2.377)	(1.810)	(-0.882)	(-0.961)	(1.774)
Observations	6,900	6,956	6,973	4,257	4,269	4,269
R-squared	0.338	0.451	0.422	0.409	0.631	0.596
Day Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Stock Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Stock Clustered Errors	Yes	Yes	Yes	Yes	Yes	Yes

APPENDIX 13: DAILY TRADING VENUES BY UNIQUE STOCK OBSERVATIONS

Figure 1: Daily Midas Venues and Unique Stock Observations

Figures 1(a) and 1(b) provide the relation between the number of unique stock observations the number of daily Midas venues [1, 13]. For instance, if Apple (APPL) were to trade at all 13 lit venues on day t, I report one observation for 13 daily Midas venues and continue this process for all stocks on every day in the sample (2019).

Figure 1a – Daily Venues by Daily Stock Observations

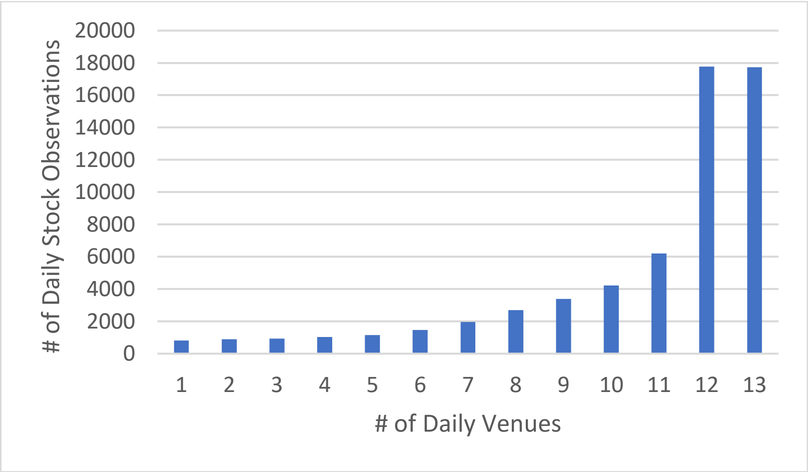
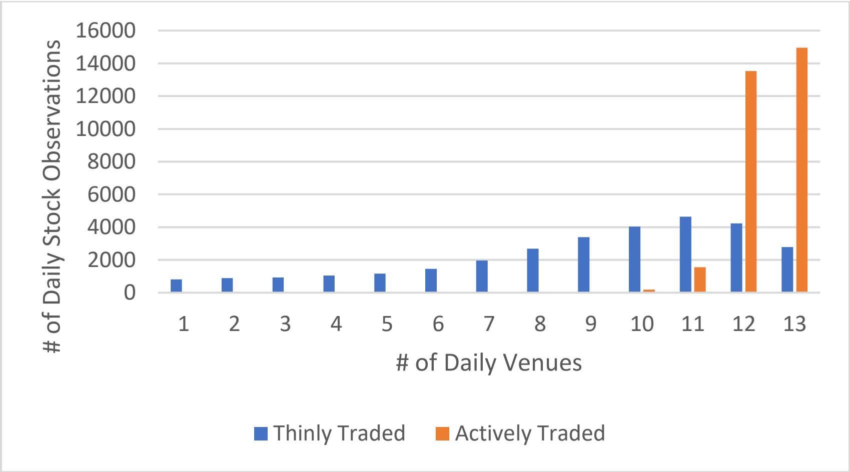


Figure 1b – Daily Venues by fragmentation



APPENDIX 14: FRAGMENTATION, MARKET QUALITY, AND ALGORITHMIC TRADING

Figure 2: Fragmentation and Market Quality

Figure 2 shows the relation between the daily venues reported by Midas and liquidity (spreads and depth. Figures 2a through 2d, include the entire sample of stocks, whereas Figures 2e through 2i are partitioned by thinly and actively traded stocks.

Figure 2a – Daily Impact on Market Liquidity

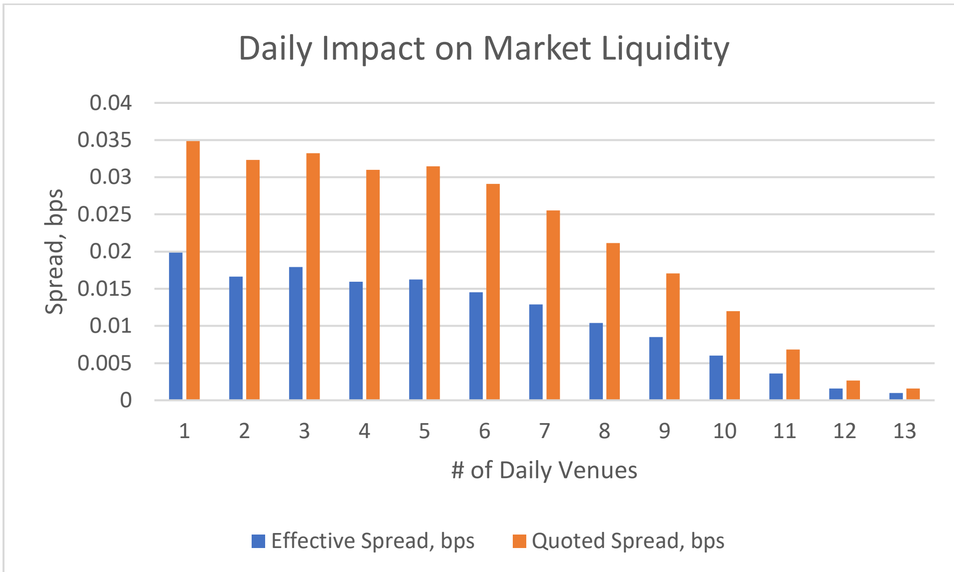


Figure 2b – Daily Venues and HHI

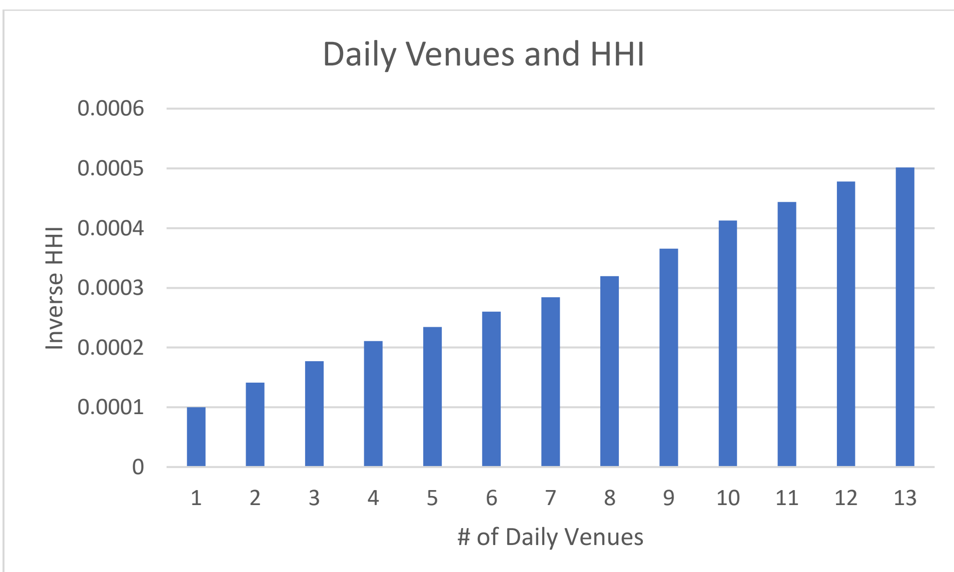


Figure 2c – Daily Venues and Dark Venue Market Share

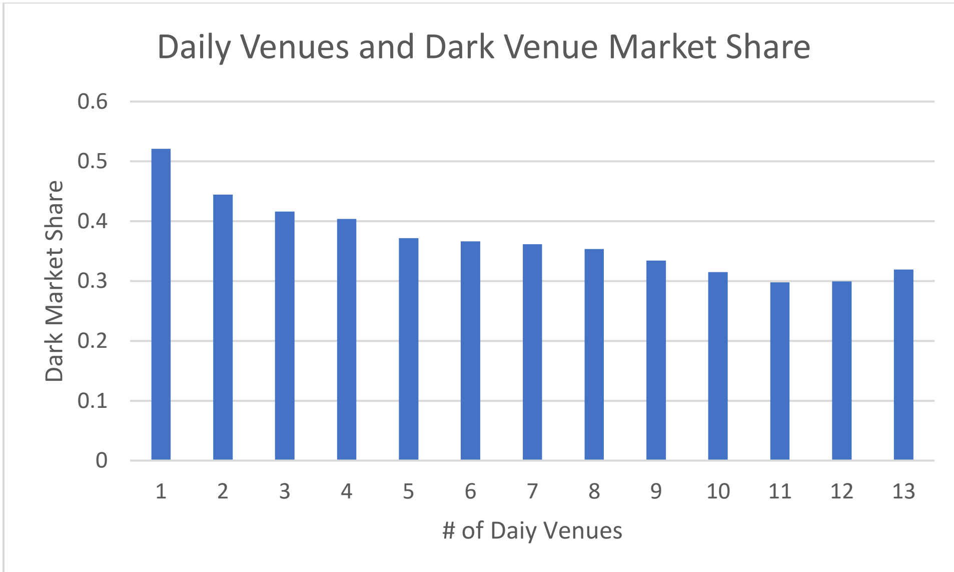


Figure 2d – Daily Venues and NBBO Depth

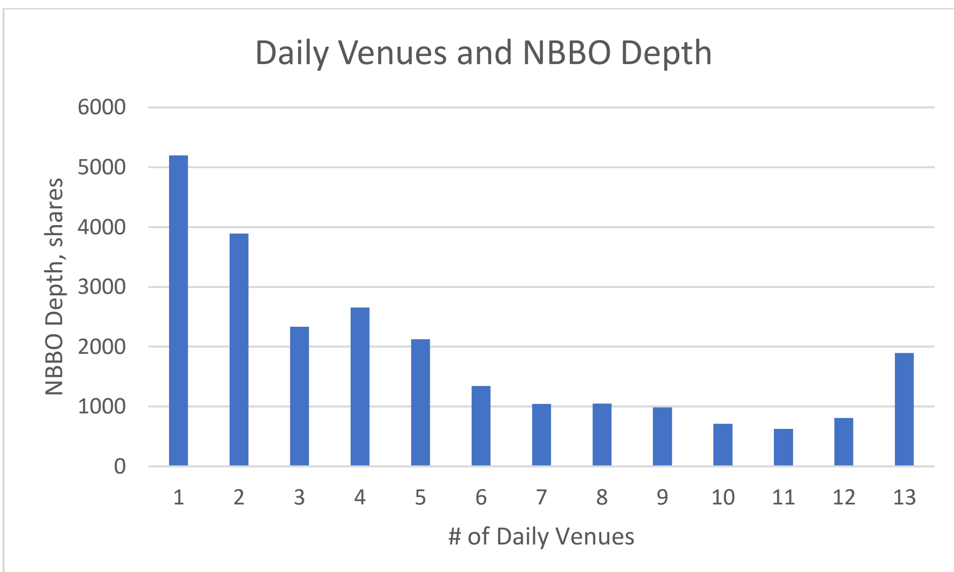


Figure 2e – Daily Venues by Effective Spread, bps

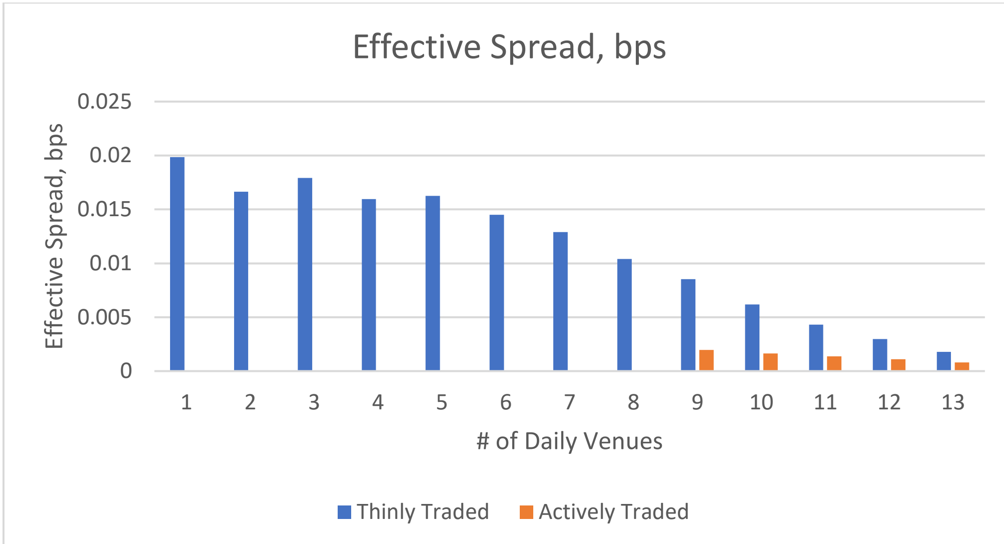


Figure 2f – Daily Venues by Quoted Spread

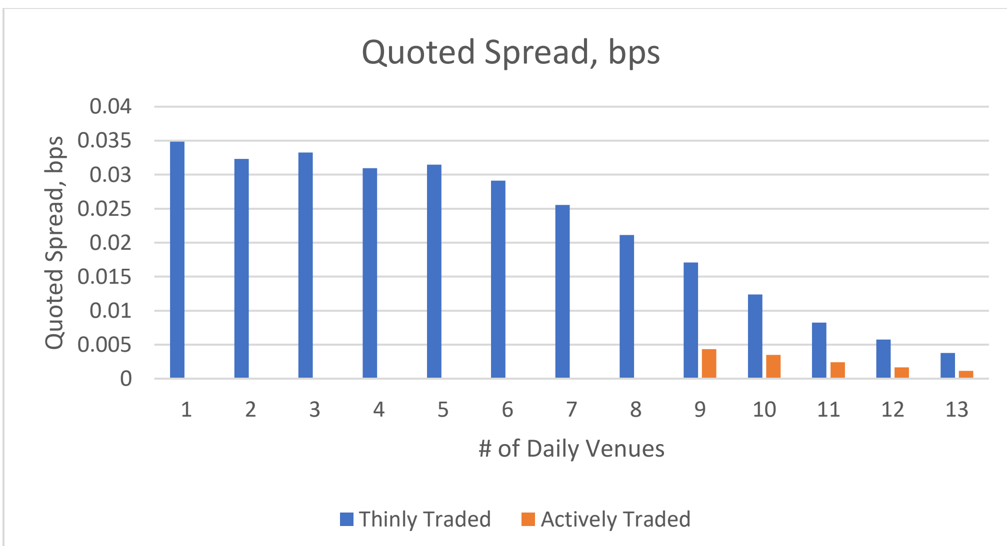


Figure 2g – Daily Venues by NBBO Depth, shares

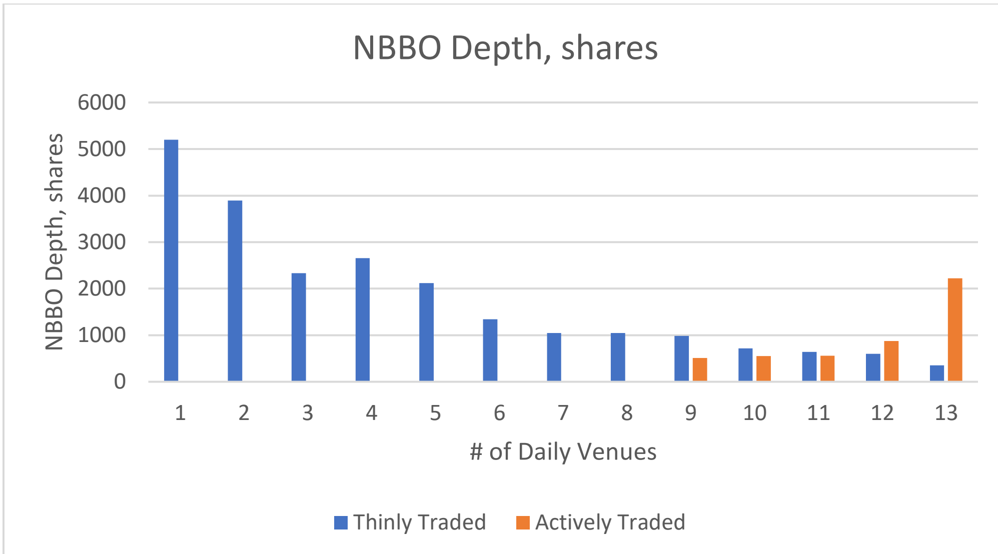


Figure 2h – Daily Venues by Dark Market Share

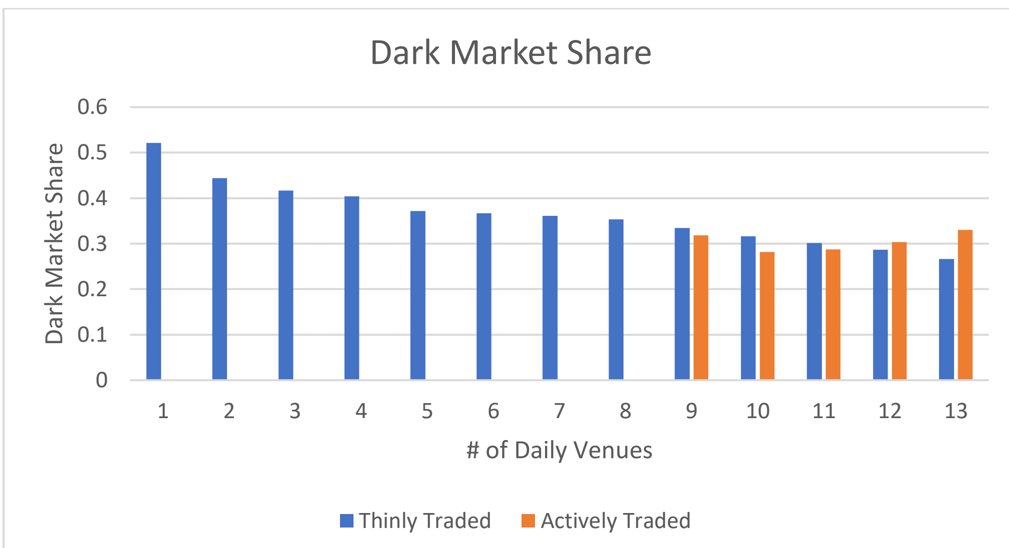
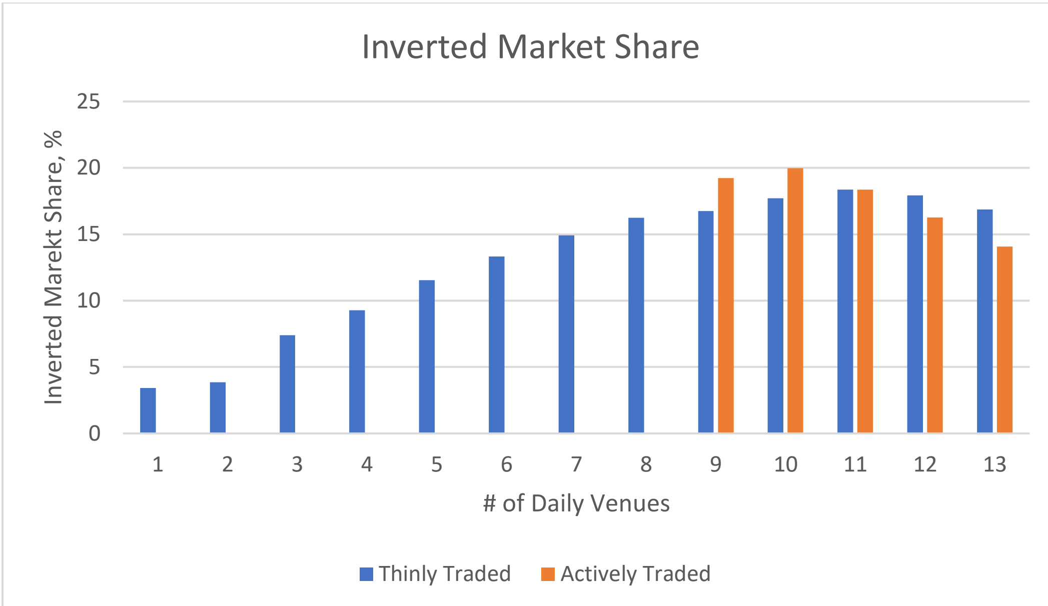


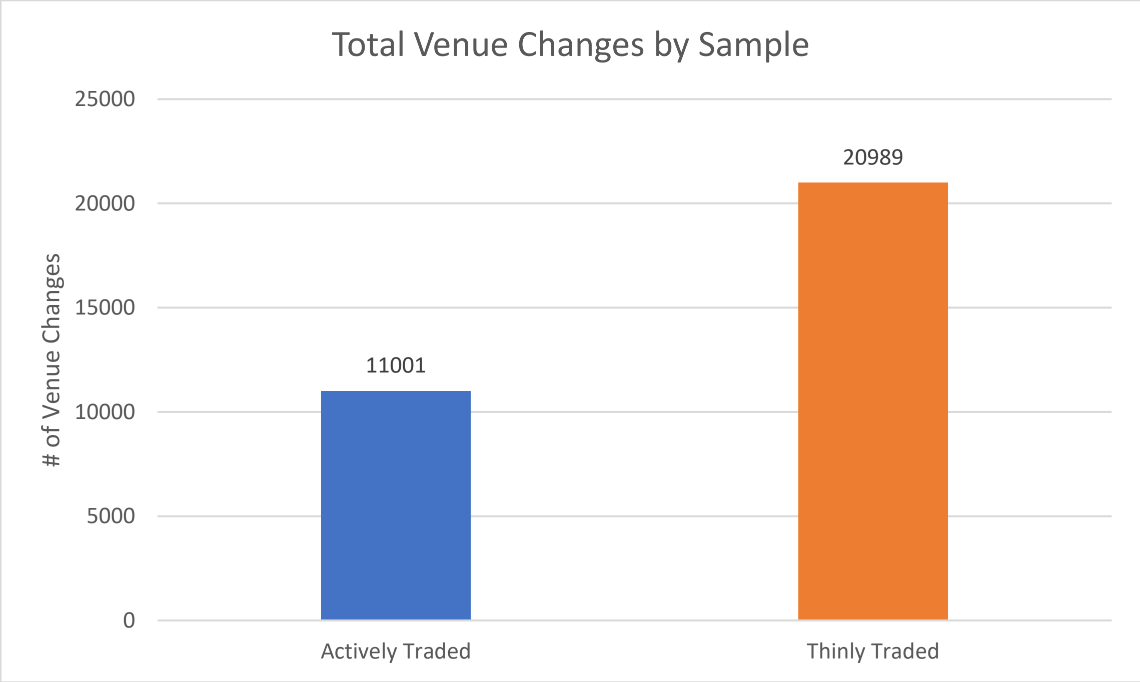
Figure 2i – Daily Venues by Inverted Venue Market Share



APPENDIX 16: CHANGES IN FRAGMENTATION BY THIINLY AND ACTIVELY TRADED SECURITIES

Figure 3: Changes in Fragmentation by Thinly and Actively Traded Securities

Figure 3 provides the total number of daily venue changes over the sample period by thinly traded and actively traded stocks.



APPENDIX 17: PRICE EFFICIENCY AND OFF-EXCHANGE TRADING

Figure 4: Price Efficiency and Off-Exchange Trading

Figure 4 shows the effects of dark trading on information efficiency measured by variance ratio and return autocorrelation. Figure 4 plots the estimated effects of dark trading for thinly traded stocks (Panel A and B) and actively traded stocks (Panel C and D) on the predicted and mean information efficiency measures. The estimated effects of dark trading are obtained from a panel regression where the dependent variables are the information efficiency measures and the independent variables are dummy variables of various ranges of dark venue market share (0–5%, 5–10%, 10–20%, 20–30%, 30–40%, 40–50%, 50–60%, 60–70%, 70%+) and control variables.

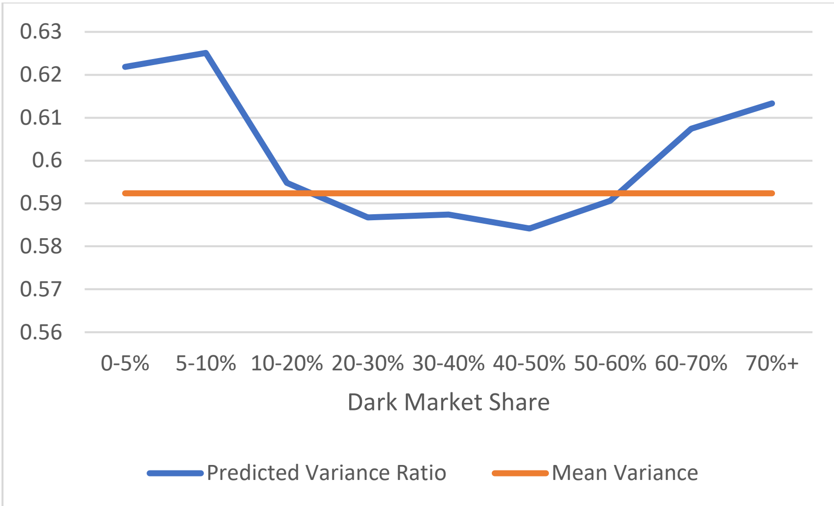


Figure 4a – Variance Ratio (Thinly Traded)

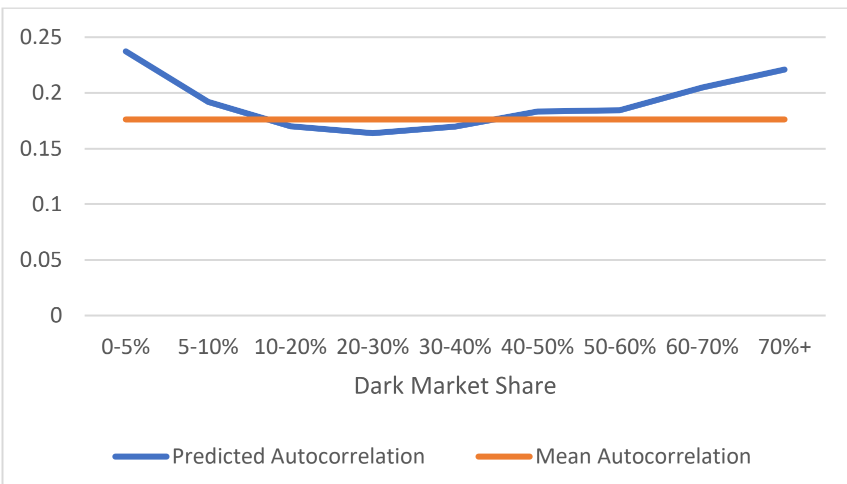


Figure 4b – Return Autocorrelation (Thinly Traded)

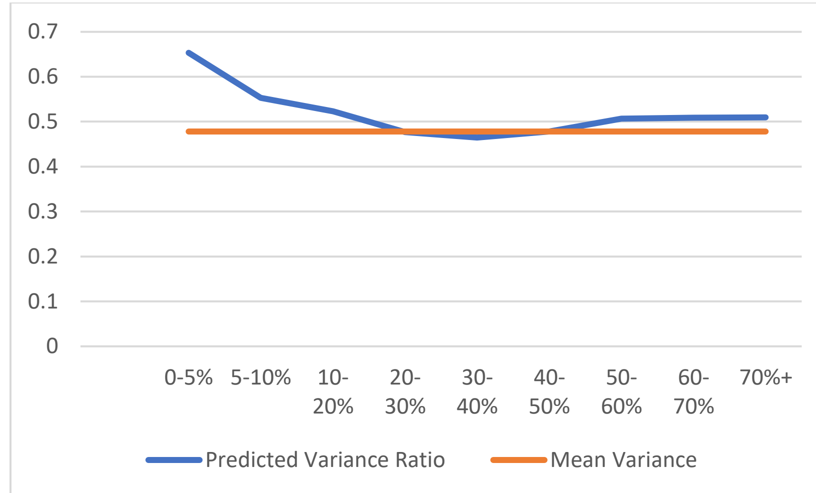


Figure 4c – Variance Ratio (Actively Traded)

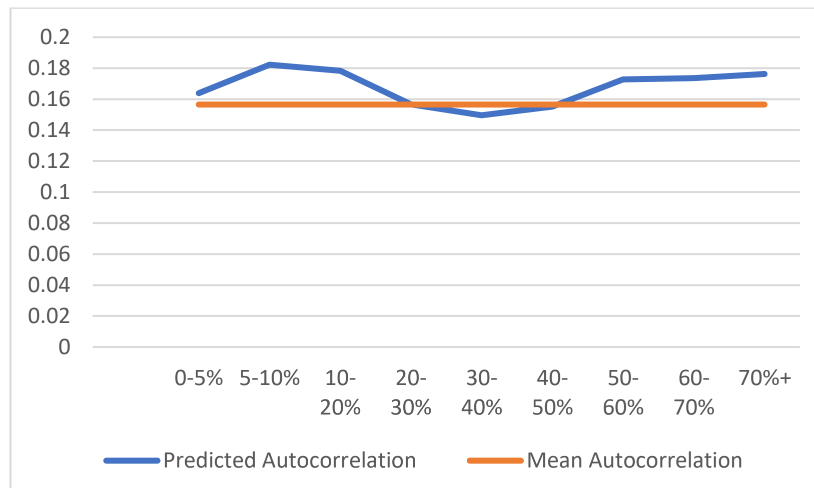


Figure 4d – Return Autocorrelation (Actively Traded

APPENDIX 18: TICK SIZE PILOT PROGRAM EVENT PERIOD

Figure 5: Tick Size Pilot Program Event Period

Figures 5 visually shows what impact ending the TSP has on thinly traded securities, by graphing the mean off-exchange trading, inverted venue activity, and lit fragmentation over the sample period September 1, 2018 through October 31, 2018.

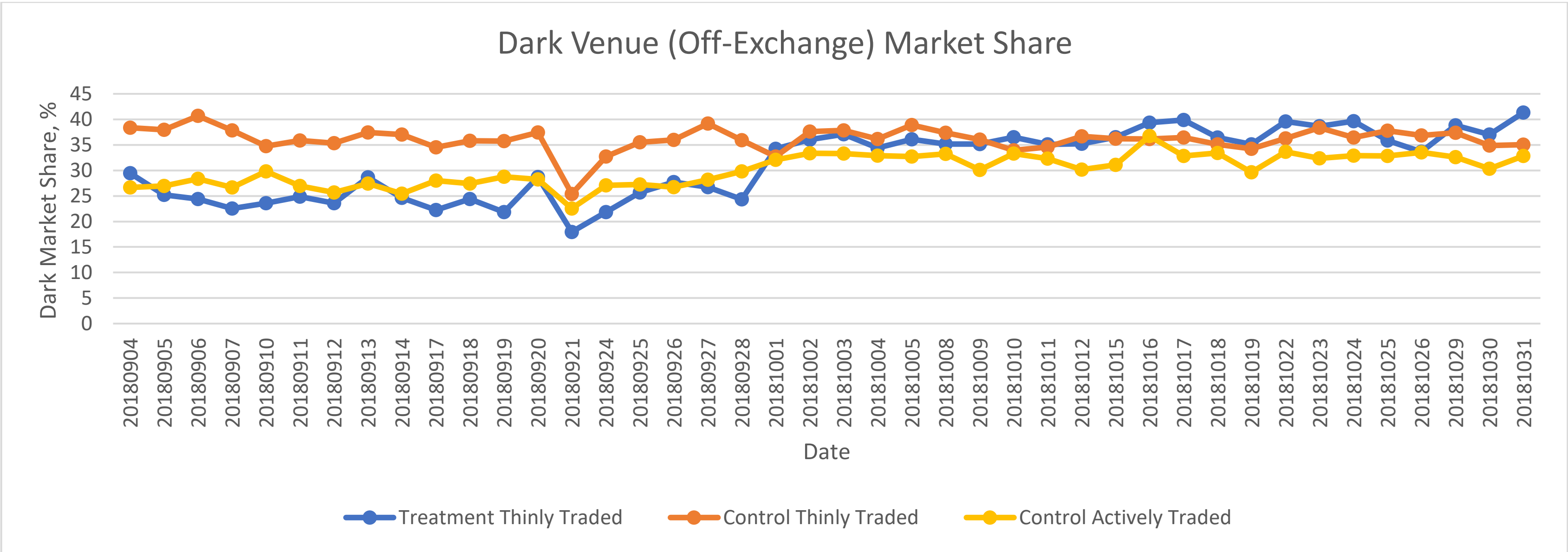


Figure 5a – Tick Size Pilot Program (Robustness): Dark Market Share

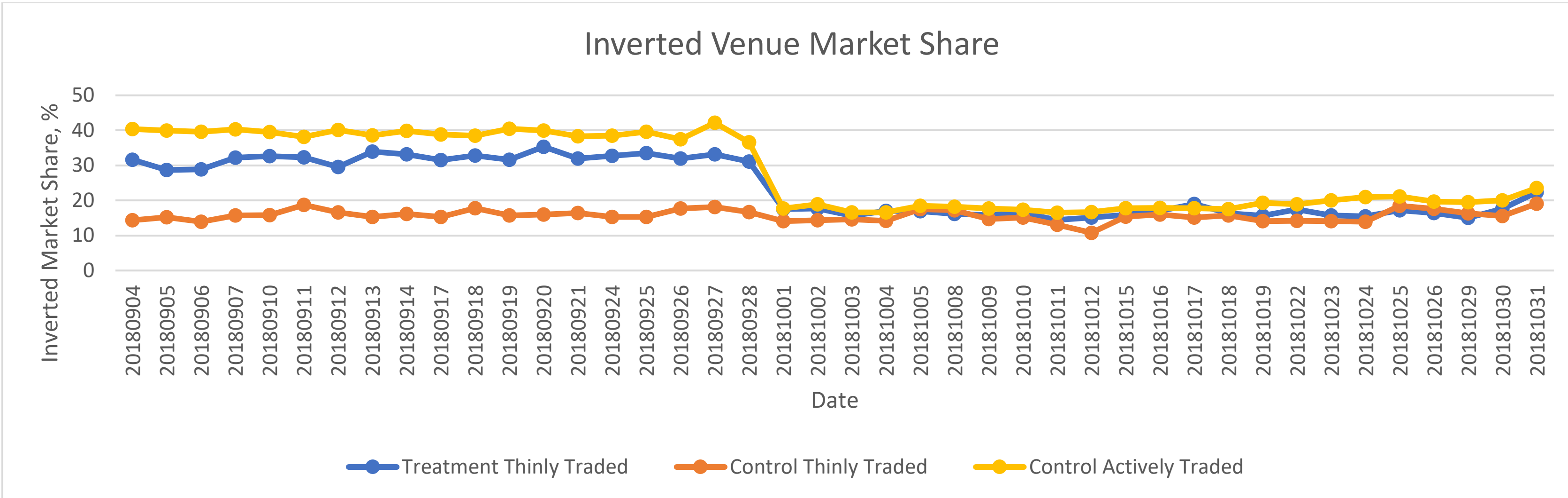


Figure 5b – Tick Size Pilot Program (Robustness): Inverted Venue Market Share

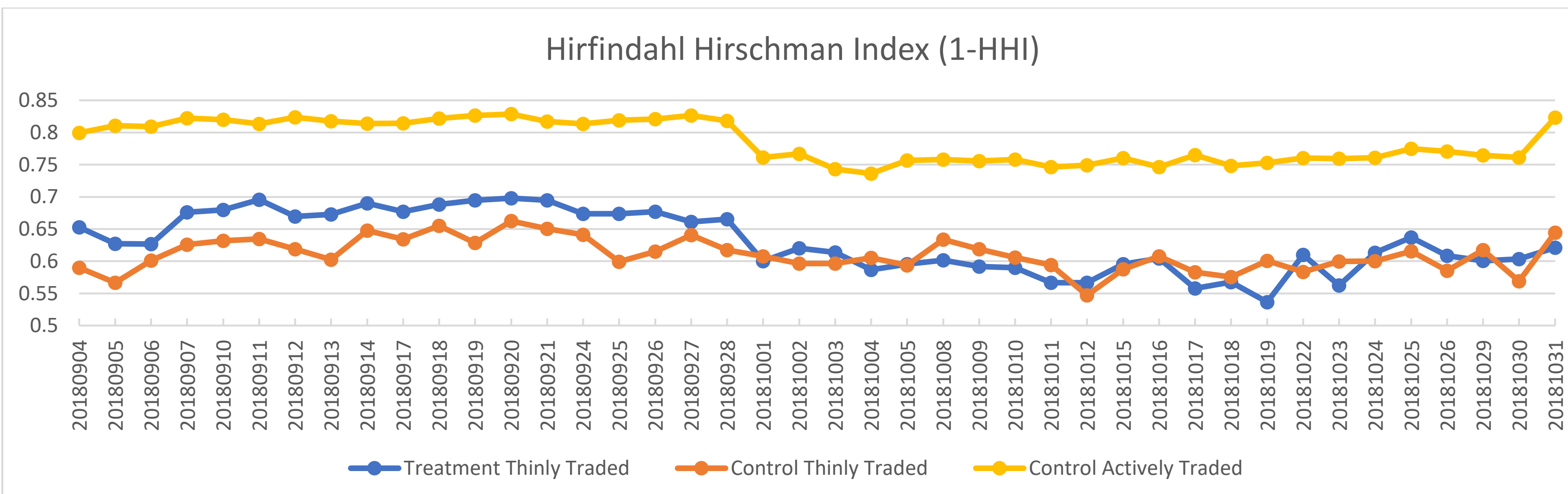


Figure 5c – Tick Size Pilot Program (Robustness): HHI Trade Volume (1-HHI)

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Research Interest

Market Microstructure, Financial Markets, Algorithmic/High frequency trading, Market Making, and Retail trading.

Education

Ph.D. (expected)	Finance	University of Mississippi	2021
M.B.A.	General Business	University of Mississippi	2016-2017
BS	Management	Southwest Minnesota State University	2011-2015
BA	Political Science	Southwest Minnesota State University	2011-2015

WORKING PAPERS AND PUBLICATIONS

1. **Examining the impact Technological Malfunctions can have on Intraday Trading, 2020, Job Market Paper.**

Abstract: In this study we analyze independent technological malfunctions that forced trading halts at various equity exchanges over the past decade. During each halt, all other exchanges remained open. The primary purpose of this study is to examine intraday trading activity before, during, and after a technological malfunction, which are events that are neither driven by an informational event nor an order imbalance. Of the events we record in this study (8 technological malfunctions), a majority document a reduction in liquidity and an increase in short term volatility during and immediately after a technological malfunction. Furthermore, these affects appear to be relatively short-term but, in a few events, we see abnormal trading as far out as 10 days. Additionally, we investigate what impact these events have on algorithmic trading activity but document mixed results that do not provide a clear indication of this impact.

2. **An Examination of the NYSE Retail Liquidity Program, 2020, with Robert Van Ness and Bonnie Van Ness, Working Paper.**

Abstract: We examine the NYSE retail Liquidity program. We find that the number of shares placed by
retail member organizations (RMOs) is positively corrected with NYSE stock volume and that retail share executions are positively related to the percentage of time that the bid or ask has a retail price improvement flag. We also find that retail member organizations share executions, shares placed, number of cancellations, number of odd-lot trades are all positively correlated. RMOs execute more trades in larger and higher trading activity firms.

3. **Retail Investor Trading Activity Around Extreme Price Movements, 2020, Working Paper.**

Abstract: This paper investigates activity of retail traders around extreme price movements (EPMs). Using newly created measures that allow for researches to use publicly available TAQ data to identify retail trades, the evidence provided in this paper suggests that retail alter their trading activity around brief price dislocations. So much so, that these traders are no longer contrarian and trade in the direction of the price change, contradicting past literature on retail traders. The ability to identify the brief changes, also suggests retail traders are informed and have the ability to interpret new information in a short period of time. It is yet to be confirmed whether these price changes are caused by abnormal retail activity or if retail traders adjusting their strategies based on price changes

4. **Exchange-Traded Funds (ETFs) Failure-to-Delivers: ETF Classifications and Fee Model Exchanges, 2018, Working Paper.**

Abstract: Recent literature has highlighted differences between traditional short selling directional selling motivations and operational shorting/liquidity providing motivations in the ETF market. The later motivator has been strongly linked to the short selling activity in ETFs and ultimately elevated failure-to-delivers (FTDs) as a result. In this paper I extrapolate FTDs differences across ETF classifications such as regional, asset class, and country specific differences to determine whether there exists commonality or whether there are underlying differences driving differences in FTDs. I also asses what influence the exchange fee models have on ETF trade executions. I find that there is substantial variation across different ETFs and that these differences seemed to be largely driven by Active participants capturing differences in liquidity by delaying creation of ETF creation baskets to ultimately result in more FTDs as scaled by shares outstanding. These finding are consistent with past literature regarding FTDs to be beneficial to providing liquidity and correcting pricing errors.

<u>Honors and Awards</u>

2020 Graduate Achievement Award for Ph.D. Degree
2019 AFA Student Travel Grant
2015 Academic All-American DII

<u>Conference and Seminar Presentations</u>

2020 University of Mississippi

2019 Financial Management Association (FMA) Annual Conference, discussant

Research and Teaching Experience

- Finance Department, University of Mississippi
    - Instructor, Intermediate Financial Management, Fall 2020
    - Instructor, Intermediate Financial Management, Summer 2020 (Teaching effectiveness: 4.5/5)
    - Instructor, Intermediate Financial Management, Spring 2020 (Teaching effectiveness: 4.3/5)
    - Instructor, Business Finance I, Spring 2020 (Teaching effectiveness: 4.2/5)
    - Instructor, Business Finance I, Fall 2019 (Teaching effectiveness: 3.7/5)
    - Research Assistant, August 2017 – present

Other Professional Experience

- Referee, *Journal of Banking and Finance* 2019 & 2020

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